Huibo Zhao hz2480

Part 1

$\neg$ Myth $\rightarrow$ (Mortal $\wedge$ Mammal) 1.

Myth $\rightarrow$ ($\neg$ Mortal). 2.

($\neg$ Mortal $\vee$ Mammal) $\rightarrow$ Horned 3.

Horned $\rightarrow$ magic 4.

a) The unicorn is horned

Mortal $\rightarrow$ $\neg$ Myth 5 from 2 (Contrapositive)

Mortal $\rightarrow$ Mortal $\wedge$ Mammal 6 from 5 and 1 (hypothetical syllogism as suggested on Piazza)

$\neg$ Mortal $\vee$ (Mortal $\wedge$ Mammal) 7 from 6 (implication)

($\neg$ Mortal $\vee$ Mortal) $\wedge$ ($\neg$ Mortal $\vee$ Mammal) 8 from 7 (distributive)

$\neg$ Mortal $\vee$ Mammal 9 from 8 ($\neg$ Mortal $\vee$ Mortal must be T)

horned 10 (from 3 and 9 modus ponens)

b) The unicorn is magical

We just proved horned. So we can get magic from 4 and 10 (modus ponens)

c) We can't show unicorn is mythical

Mortal $\rightarrow$ $\neg$ Myth implication from 2

$\neg$ Mortal $\vee$ $\neg$ Mammal $\rightarrow$ Myth implication from 1

We find if Mortal = T, Myth = F

if Mortal = F, Myth = T

We can only infer horned and magic from knowledge base. We don't know mortal for sure.

Given soundness: we can't infer formulas that are not logically entailed by KB.

Thus, the statement the Unicorn is mythical can't be deducted.

Part 2

1.

p	q	$p \leftrightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	T	T	T

$p \leftrightarrow q \models \neg p \vee q$ is Correct

2.

p	q	r	$p \wedge q$	$(p \wedge q) \to r$	$p \to r$	$q \to r$	$(p \to r) \vee (q \to r)$
F	F	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	T	F	F	T	T	F	T
F	T	T	F	T	T	T	T
T	F	F	F	T	F	T	T
T	F	T	F	T	T	T	T
T	T	F	T	F	F	F	F
T	T	T	T	T	T	T	T

$(p \wedge q) \to r \models (p \to r) \vee (q \to r)$ is Correct

3.

p	q	r	$\neg p \wedge \neg q$	$r \vee (\neg p \wedge \neg q)$	$r \vee (\neg p \wedge \neg q) \to r$	$p \to r$	$q \to r$	$(q \to r) \wedge (q \to r)$
F	F	F	T	T	F	T	T	T
F	F	T	T	T	T	T	T	T
F	T	F	F	F	T	T	F	F
F	T	T	F	T	T	T	T	T
T	F	F	F	F	T	F	T	F
T	F	T	F	T	T	T	T	T
T	T	F	F	F	T	F	F	F
T	T	T	F	T	T	T	T	T

Statement $(r \vee (\neg p \wedge \neg q)) \to r \equiv (p \to r) \wedge (q \to r)$ is not correct.

4.

p	q	$p \leftrightarrow q$	$\neg p$	$\neg p \to q$	$(p \leftrightarrow q) \wedge (\neg p \to q)$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	F	T	T	F
F	F	T	T	F	F

It is satisfiable when both q and p are true.

Part 3

$B \land C \to A$	② B	$D \land E \to C$	④ $E \lor F$	$D \land \neg F$
$\neg(B \land C) \lor A$		$\neg(D \land E) \lor C$		⑤ D
① $\neg B \lor \neg C \lor A$		③ $\neg D \lor \neg E \lor C$		⑥ $\neg F$

b) E (④ and ⑥ unit resolution rule) ⑦

$\neg B \lor A \lor \neg D \lor \neg E$ (① and ③ full resolution rule) ⑧

$A \lor \neg D \lor \neg E$ (⑧ and ② unit resolution rule) ⑨

$A \lor \neg E$ (⑨ and ⑤ unit resolution rule) ⑩

A (⑩ and ⑦ unit resolution rule)

Part 4

- Every dog likes toys = $\forall x.\ dog(x) \to like(x, toys)$ (like(a,b) means a like b)
- There are some toys that no dog likes = $\exists x.\ toys(x) \land \neg like(dogs, x)$
- Every dog who sleeps snores, but dogs that do not sleep do not snore = $(\forall x[dog(x) \land sleep(x)] \to Snore(x)) \land$ $(\forall x[dog(x) \land \neg sleep(x)] \to \neg Snore(x))$
- Everybody who takes W4701 needs to take two exams.
  $\forall x\,[Person(x) \land Take(W4701, x)] \to Take(2exams, x)$
- There is a barber who shaves all men in town who do not shave themselves
  $\exists x.\ barber(x) \land \forall y\ Men(y) \land In(y, town) \land \neg Shave(y, y) \to shave(x, y)$ (Shave(a,b) means a shaves b)

Sentence ex. : Every kid likes a cookie.

Part 5

Method A: $P(\text{have disease} \mid \text{positive test}) = \dfrac{P(\text{positive test} \mid \text{have disease}) \cdot P(\text{have disease})}{P(\text{positive test})}$

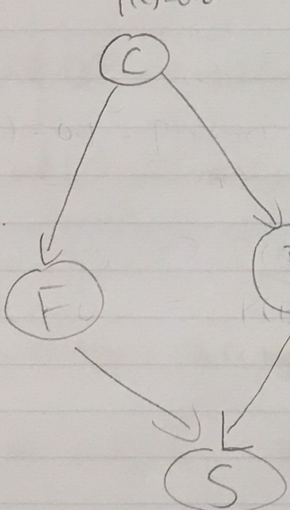
$$= \frac{0.95 \cdot 0.01}{0.01 \cdot 0.95 + 0.99 \cdot 0.1} = 0.088$$

Method B: $P(\text{have disease} \mid \text{positive test}) = \dfrac{0.9 \times 0.01}{0.9 \times 0.01 + 0.05 \times 0.99} = 0.154$

Given a positive result, method B has greater chance of that patient have the disease.

Part 6

$P(C) = 0.6$



C	P(F)
T	0.2
F	0.1

C	P(B)
T	0.5
F	0

B	F	P(S)
T	T	0.9
F	T	0.6
T	F	0.5
F	F	0.1

a. $P(C \mid S) = \dfrac{P(C \wedge S)}{P(S)} = P(S \mid C) \cdot \dfrac{0.6}{1}$

$= 0.6 \cdot \big(P(S \mid B, F)\, P(B \mid C)\, P(F \mid C) +$
$P(S \mid B, \neg F)\, P(B \mid C)\, P(\neg F \mid C) +$
$P(S \mid \neg B, \neg F)\, P(\neg B \mid C)\, P(\neg F \mid C) +$
$P(S \mid \neg B, F)\, P(\neg B \mid C)\, P(F \mid C) \big)$

$= 0.6 \cdot (0.9 \times 0.5 \times 0.2 +$
$0.5 \times 0.5 \times 0.8 +$
$0.1 \times 0.8 \times 0.5 +$
$0.6 \times 0.5 \times 0.2) = 0.234$

b. $P(F \mid S) = P(F \wedge S) = P(F \wedge S \wedge C \wedge B) + P(F \wedge S \wedge \neg C \wedge B) + P(F \wedge S \wedge C \wedge \neg B) + P(F \wedge S \wedge \neg C \wedge \neg B)$

$= 0.6 \times 0.2 \times 0.5 \times 0.9 + 0 + 0.6 \times 0.2 \times 0.5 \times 0.6 + 0$

$= 0.09$