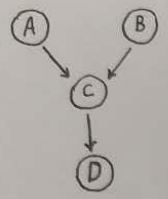
1. (a)



(b) $P(A, B, C, D) = P(A)\,P(B)\,P(C|A,B)\,P(D|C)$

parameters : $1+1+4+2=8$

2. (a) $B \perp C \mid A$

$D \perp A \mid B, C$

(b) $P(B, C, D|\neg A) = \frac{P(\neg A, B, C, D)}{P(\neg A)}$

$= \frac{P(D|B,C)\,P(B|\neg A)\,P(C|\neg A)\,\cancel{P(\neg A)}}{\cancel{P(\neg A)}}$

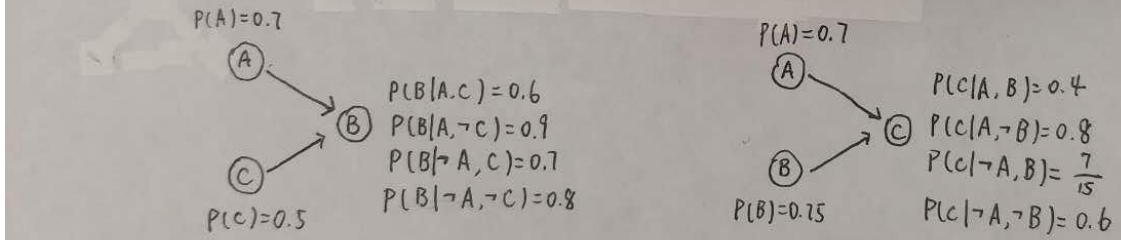
B	C	D	$P(B, C, D \mid \neg A)$
0	0	0	0.24375
0	0	1	0.13125
0	1	0	0.1125
0	1	1	0.0125
1	0	0	0.28125
1	0	1	0.09375
1	1	0	0.0875
1	1	1	0.0375

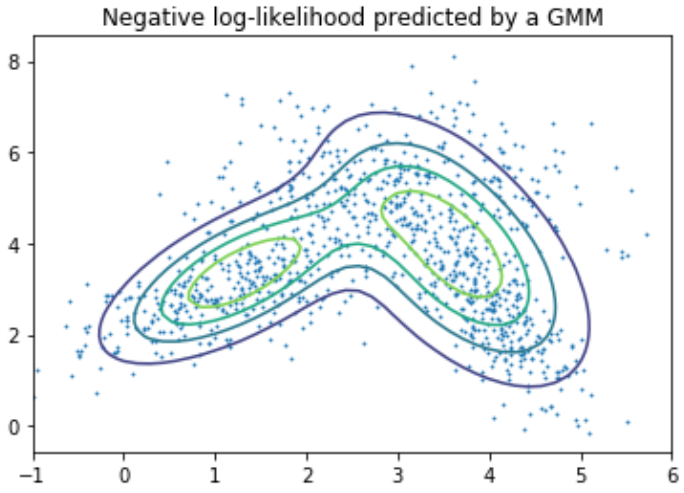
4. (a) K=2

$P(\neg A)$

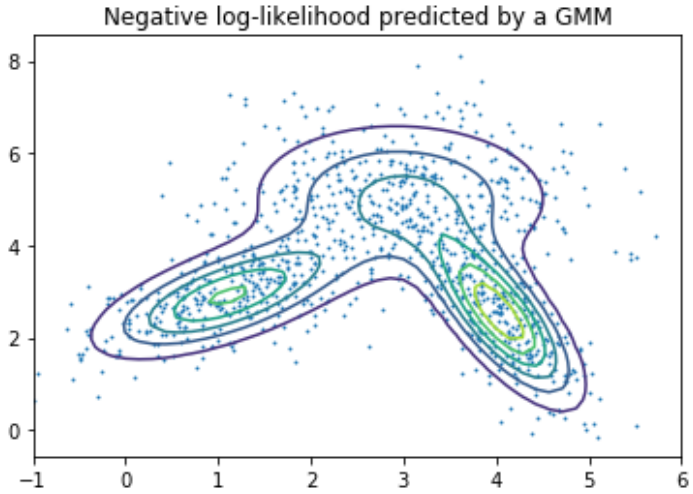
B	C	D	$P(B, C, D \mid \neg A)$
0	0	0	0.24375
0	0	1	0.13125
0	1	0	0.1125
0	1	1	0.0125
1	0	0	0.28125
1	0	1	0.09375
1	1	0	0.0875
1	1	1	0.0375

From the table, by calculation, we get that $A \perp B$, $A \perp C$, $B \not\perp C$, $A \not\perp B \mid C$, $A \not\perp C \mid B$, $B \not\perp C \mid A$

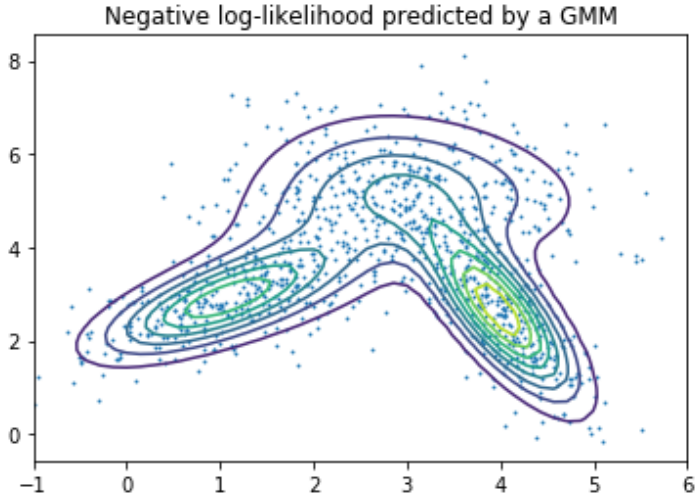



Negative log-likelihood predicted by a GMM
8
6
4
2
0
−1
0
1
2
3
4
5
6

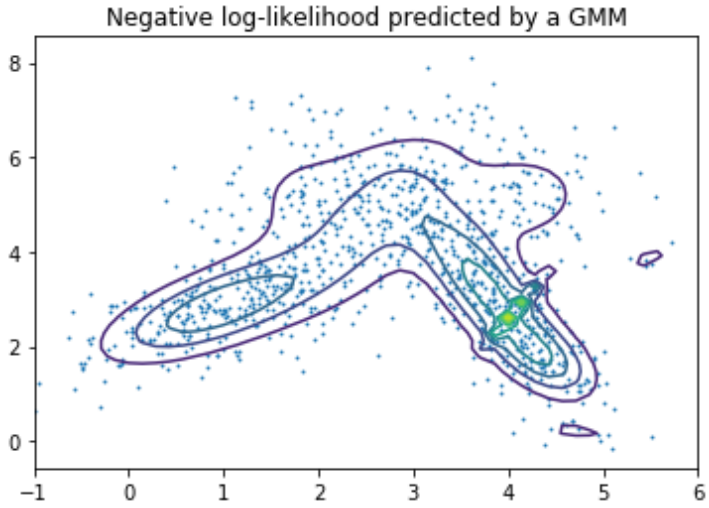
K=3


Negative log-likelihood predicted by a GMM
8
6
4
2
0
−1
0
1
2
3
4
5
6

K=5

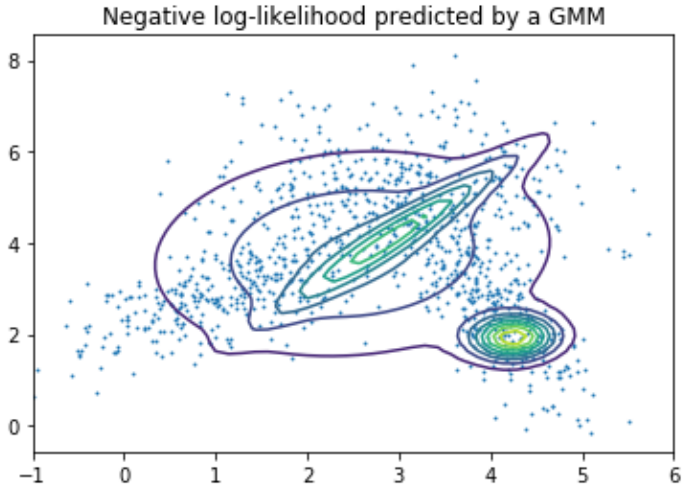

Negative log-likelihood predicted by a GMM
8
6
4
2
0
−1
0
1
2
3
4
5
6

K=10

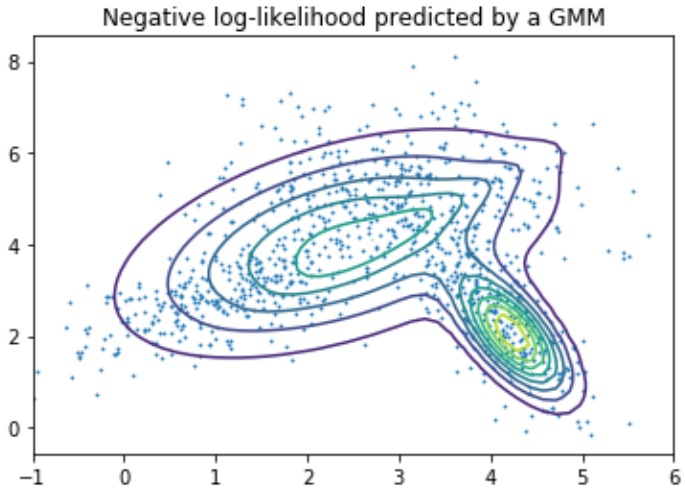


(b)

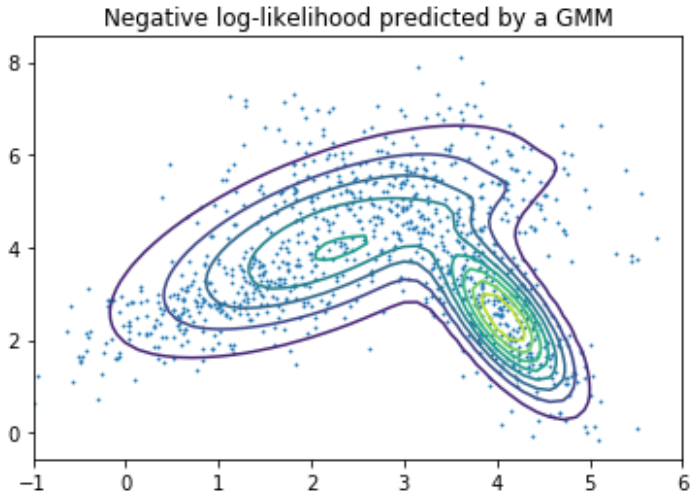
1 step:



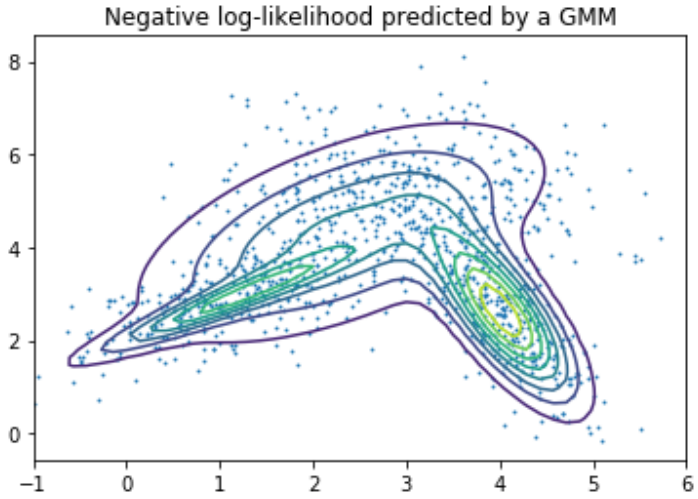
5 steps:



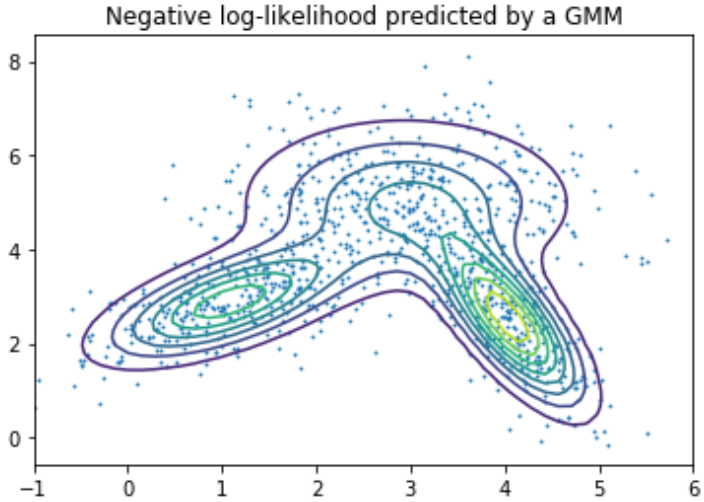
10 steps:



20 steps:



50 steps:



Model parameters:

π:

[0.29085370724411608, 0.41871863656465769, 0.29042765619122624]
μ:
[[1.0552311330913857, 2.8599948528529326],
 [2.9603051263510638, 5.0069983235458944],
 [4.0518992929984305, 2.4815516213917506]]
Σ:
[array([[0.62283072, 0.36501207],
        [0.36501207, 0.53719646]]),
array([[1.01032542, -0.01279441],
        [-0.01279441, 1.05066972]]),
array([[0.21541969, -0.36190934],
        [-0.36190934, 1.12336943]])]