QUANTUM RESERVOIR PROCESSING

NOISY INTERMEDIATE-SCALE QUANTUM ARTIFICIAL INTELLIGENCE

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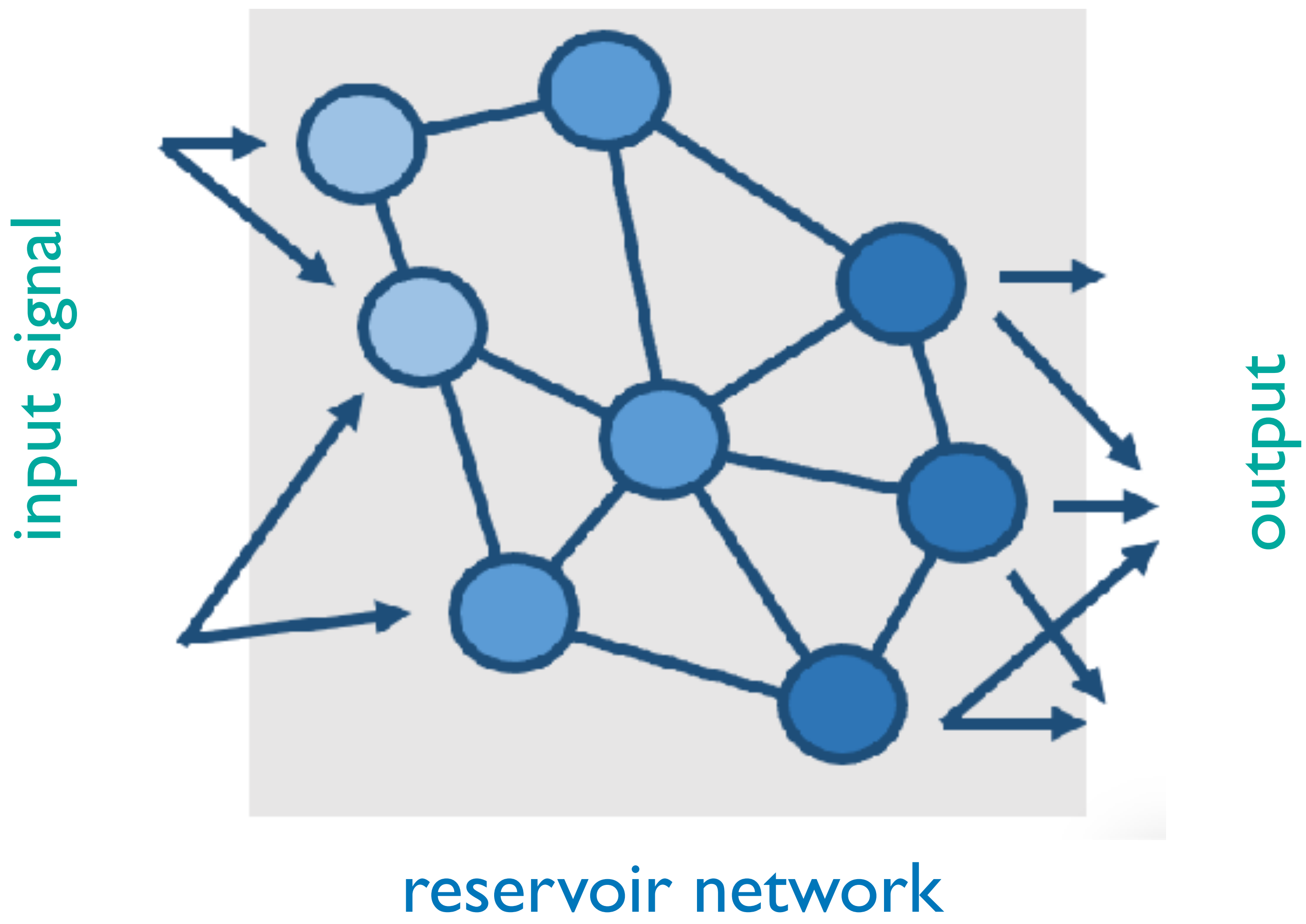
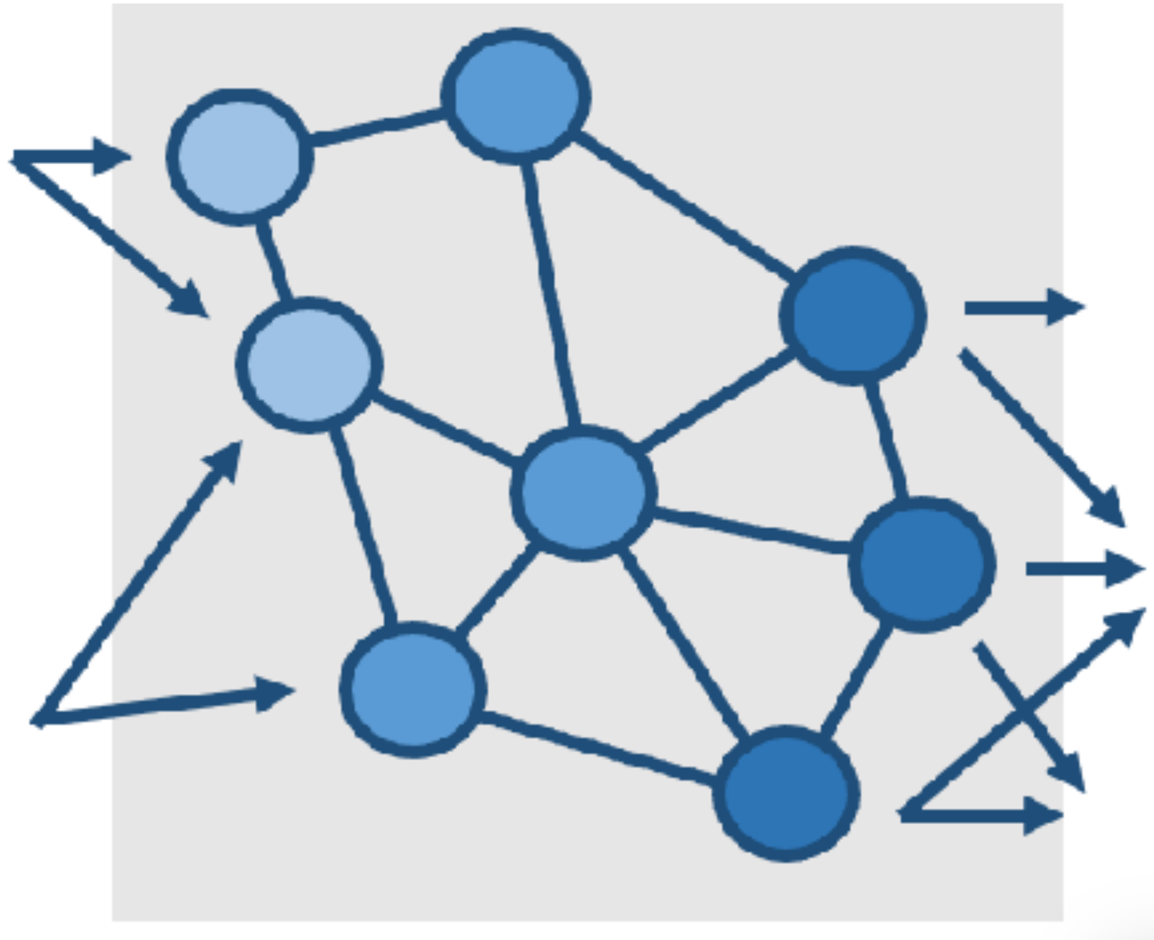


- Random network
- Only output weights are adapted

Classical or quantum input



Classical or quantum output

Network of randomly connected quantum nodes

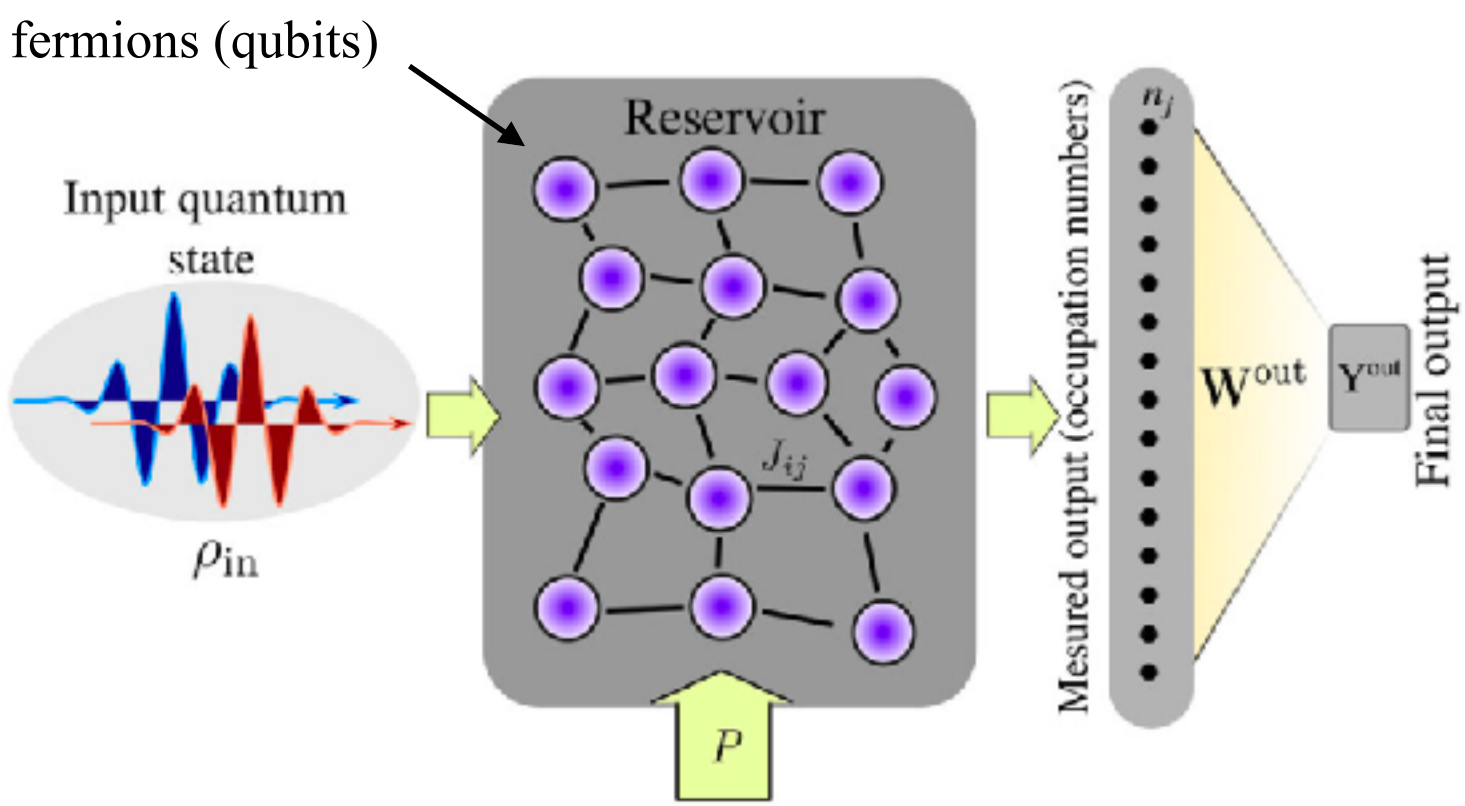
S. Ghosh et al. npj QI **5**, 35 (2019)

WHY IS THIS INTERESTING?

- Randomly connected networks model physical devices with manufacturing errors
- It turns out this device operates successfully for a plethora of internal dynamics and couplings — suitable for hardware implementation
- Training is conceptually simple and computationally inexpensive — essentially a linear map
- Simple measurements on individual nodes, i.e. no need for correlated measurements

QUANTUM INPUT — CLASSICAL OUTPUT

THE MODEL



1. Initial condition: steady state of the pumped and decaying reservoir
2. Input state coupled to the reservoir
3. Transient evolution
4. Measurement of the mean occupation of each node

WHAT CAN THIS DO?

- Witness entanglement of the input state
- Estimate entanglement of the input state
- Estimate entropy of the input
- Estimate any function of the input
- Estimate the input (quantum tomography)

Problem: is input state entangled?

Setting for simulations:
Four reservoir nodes modelled as fermions (qubits).
Random nearest-neighbour coupling (Fermi-Hubbard model).

$$\hat{H}_R = \sum_{ij} J_{ij} \left(\hat{b}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{b}_i \right)$$

Random: Jij are uniformly distributed in [-1,1] and normalised to fix available energy.

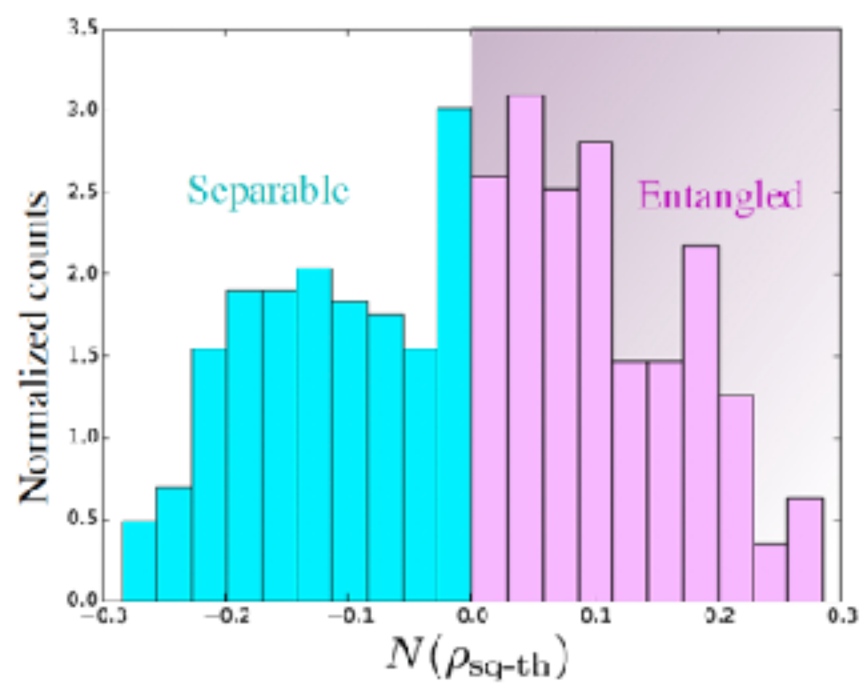
Presented data will be averaged over these random realisations.
This shows that the method is insensitive to the details of the couplings.

Input states encoded in cv of optical two-mode fields.
For training we choose squeezed thermal states:

$$\rho_{\text{in}} = \widehat{S}(\alpha)\rho_{\text{th}}\widehat{S}^{\dagger}(\alpha)$$

$$\hat{S}(\alpha) = \exp(\alpha \hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - \alpha^{*}\hat{a}_1\hat{a}_2)$$

We randomise alpha such that 50% of input states are entangled.
Here we sampled 200 input states.



It turns out this is not very important.
So it could be very simple, say just hoping:

$$\hat{H}_I = \sum_{kj} f_k(t) W_j^{\mathrm{in}} \left(\hat{a}_k^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{a}_k \right)$$

We included possible decay and pumping via master equation:

$$i\hbar\dot{\rho} = [\hat{H}_{\mathbf{tot}}, \rho] + \frac{i\gamma}{2} \sum_j \mathcal{L}(\hat{b}_j) + \frac{iP}{2} \sum_j \mathcal{L}(\hat{b}_j^\dagger)$$

Mean occupation numbers:

$$n_j = \langle \hat{b}_j^\dagger \hat{b}_j \rangle$$

We process them linearly as follows:

$$Y_i^{\text{out}} = \sum_j W_{ij}^{\text{out}} n_j,$$

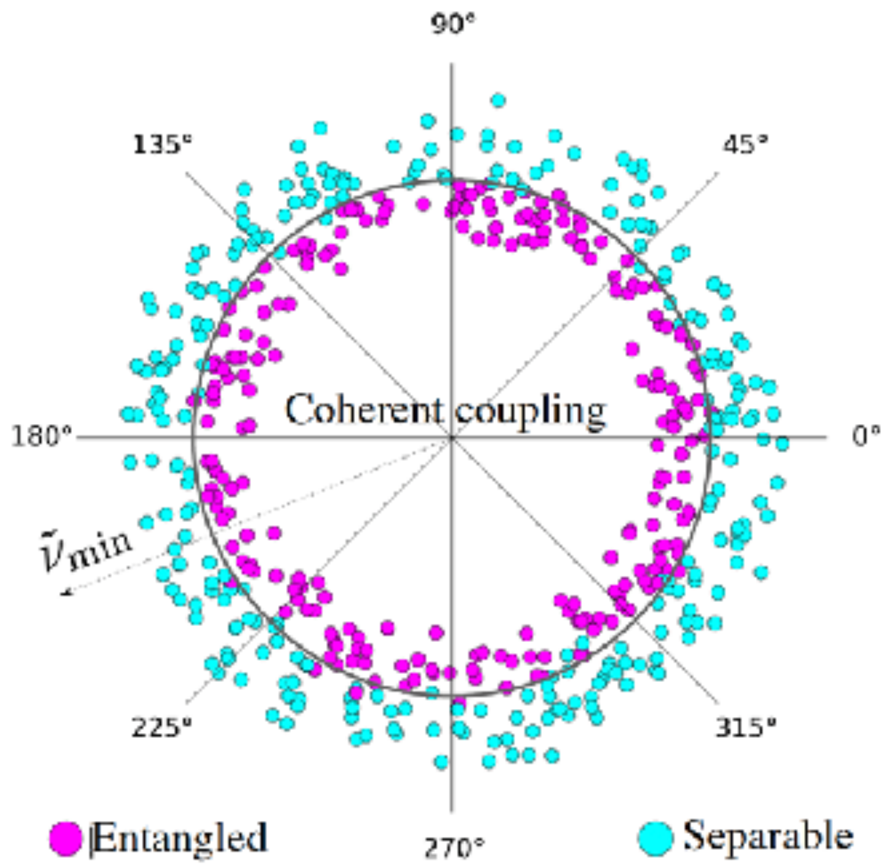
where $\mathbf{Y}^{\text{out}} = (1,\ 0)$ if input is entangled

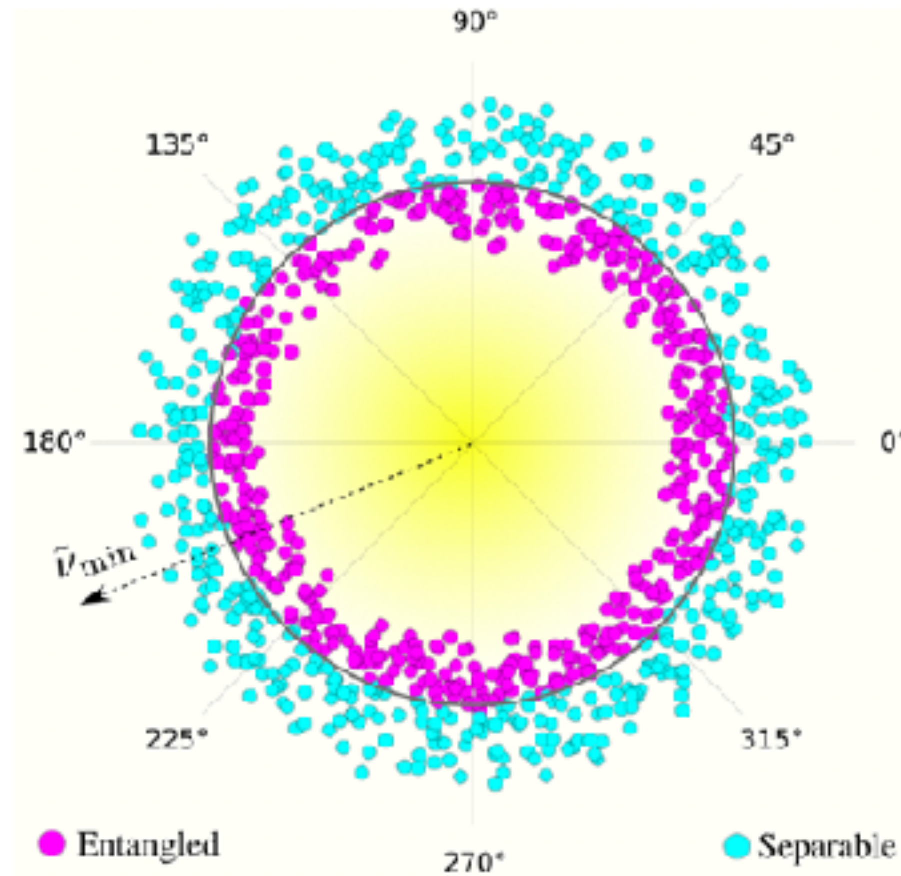
$\mathbf{Y}^{\text{out}} = (0,\ 1)$ otherwise

$\mathbf{W}^{\text{out}}$ from ridge regression

Testing: another set of squeezed thermal states

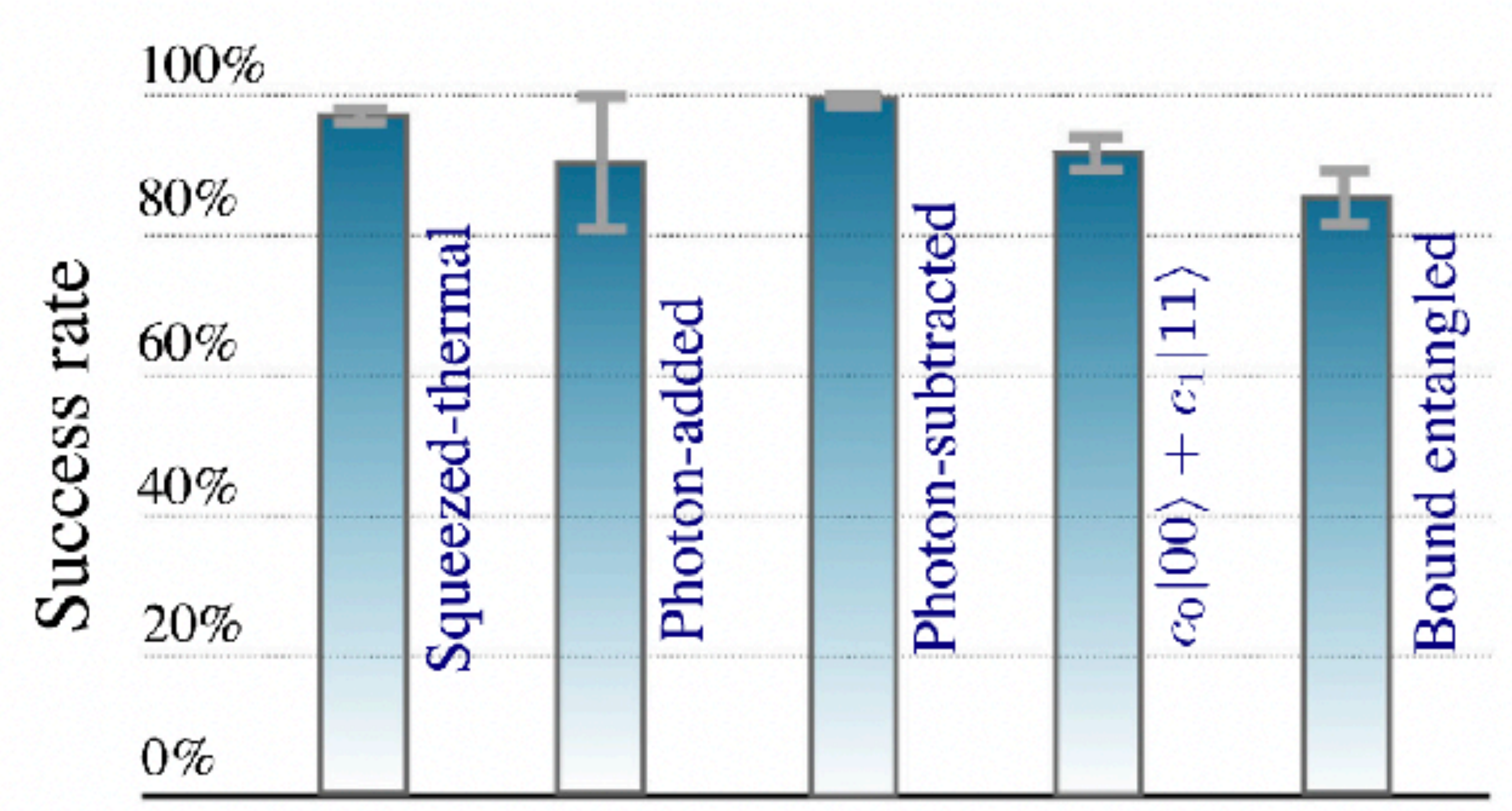
We use the trained Wout and say that input is entangled if upper entry of the output is larger than the lower entry.





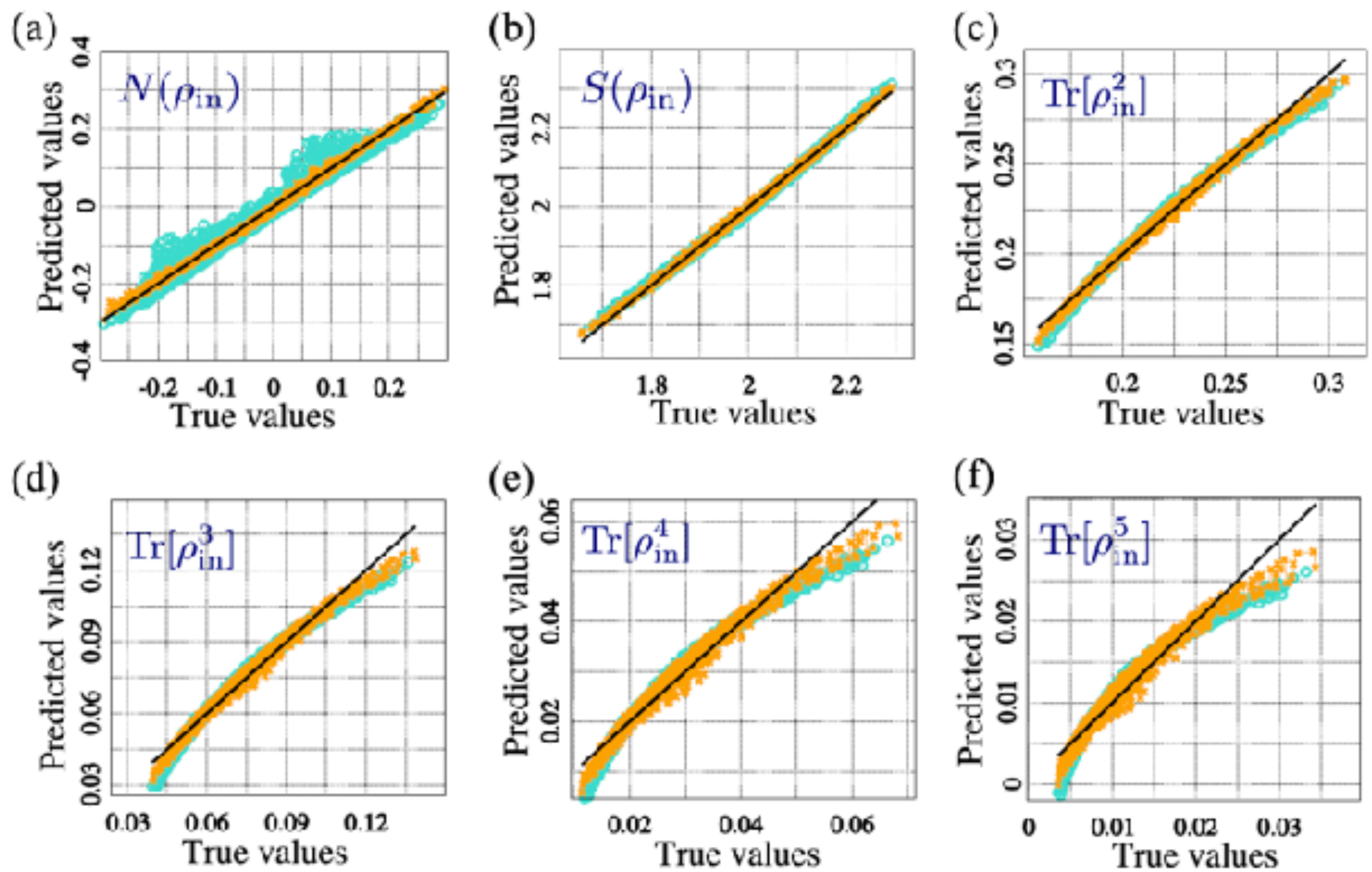
Tested with 100 states, mistake in ~4 states.

Testing with other classes of states



Estimating non-linear functions of input state

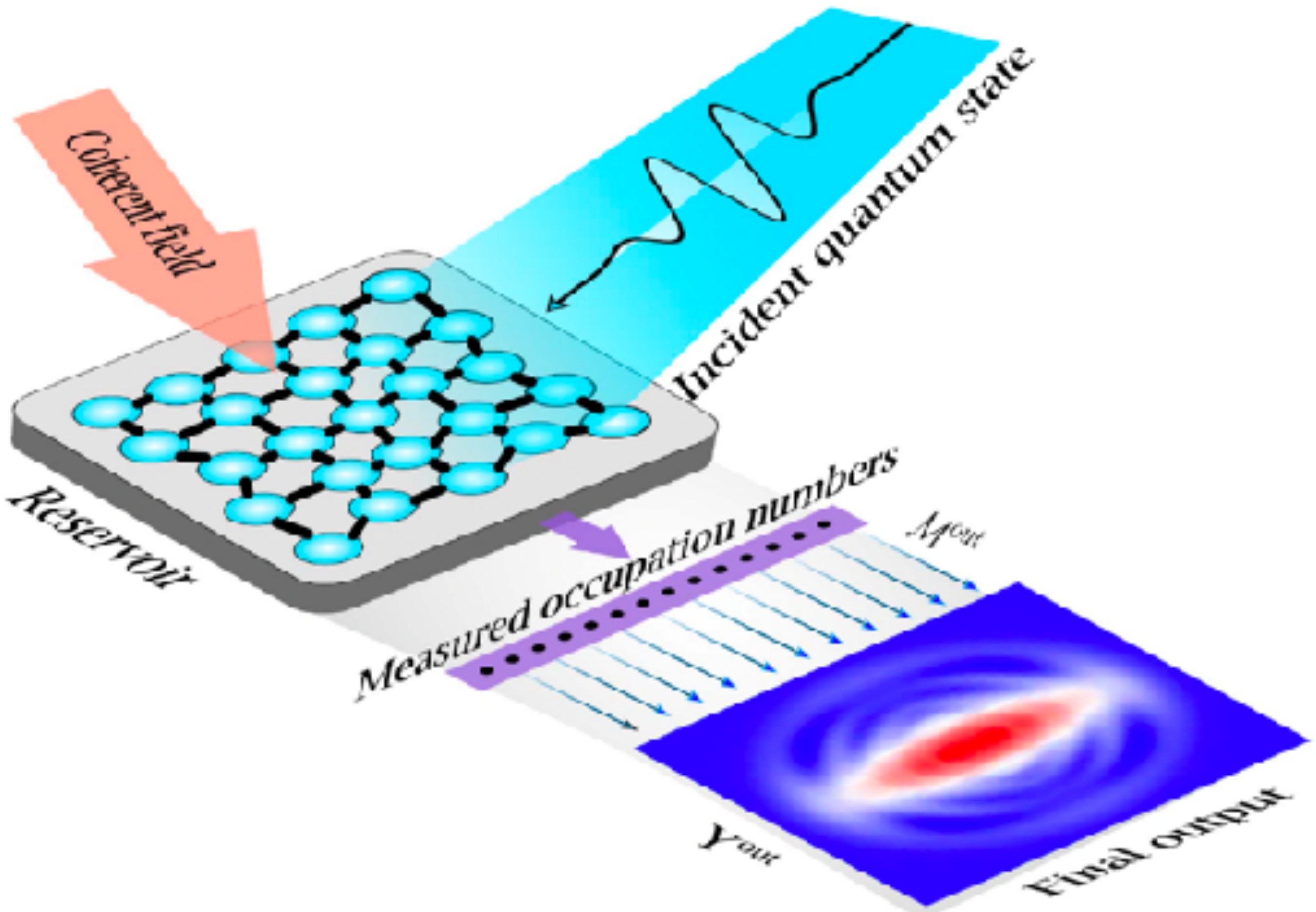
By having higher-dimensional output we may simultaneously estimate a number of parameters. Of course we need to train first, but only once!



Trained and tested on squeezed thermal states

Reservoir size: 2 (blue) and 4 (orange)

QUANTUM STATE TOMOGRAPHY



S. Ghosh et al. IEEE TNNLS **32**, 3148 (2021)

It is solving a system of linear equations.

Any density matrix can be written as sum of D^2 linearly independent states:

$$\rho_{\text{in}} = \sum_i \alpha_i \rho_i, \quad \text{with} \quad \sum_i \alpha_i = 1$$

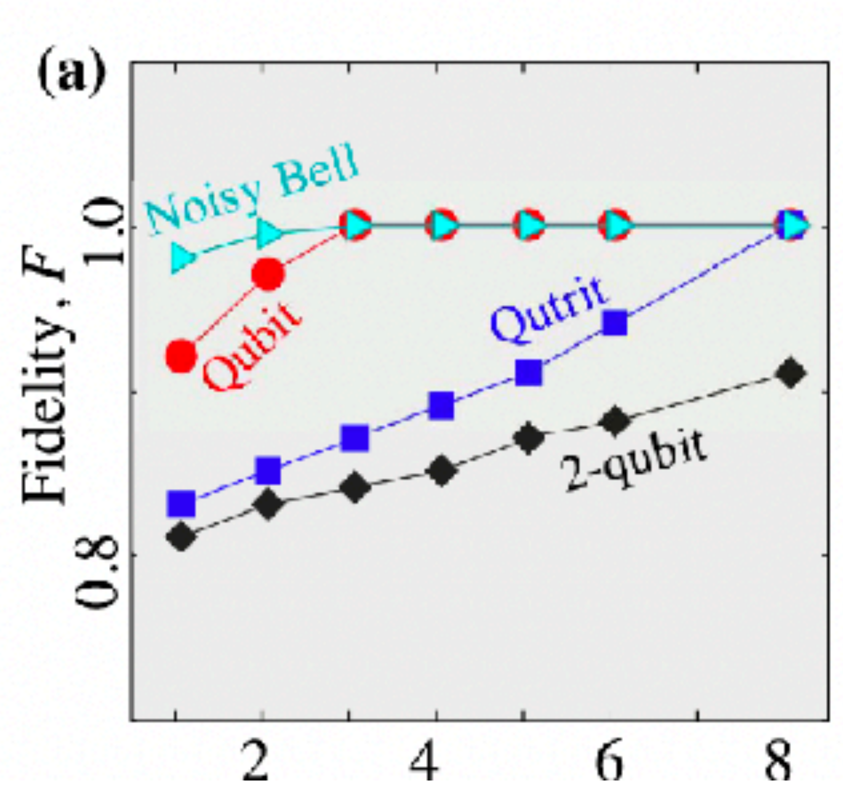
Training solves the following set of D^2 equations for D^2 random input states:

$$\mathcal{M}^{\text{out}} \vec{n}_i + \vec{m} = \vec{\rho}_i$$

Since the occupation measurement is effectively a POVM on the input state, it is linear in this state and we have:

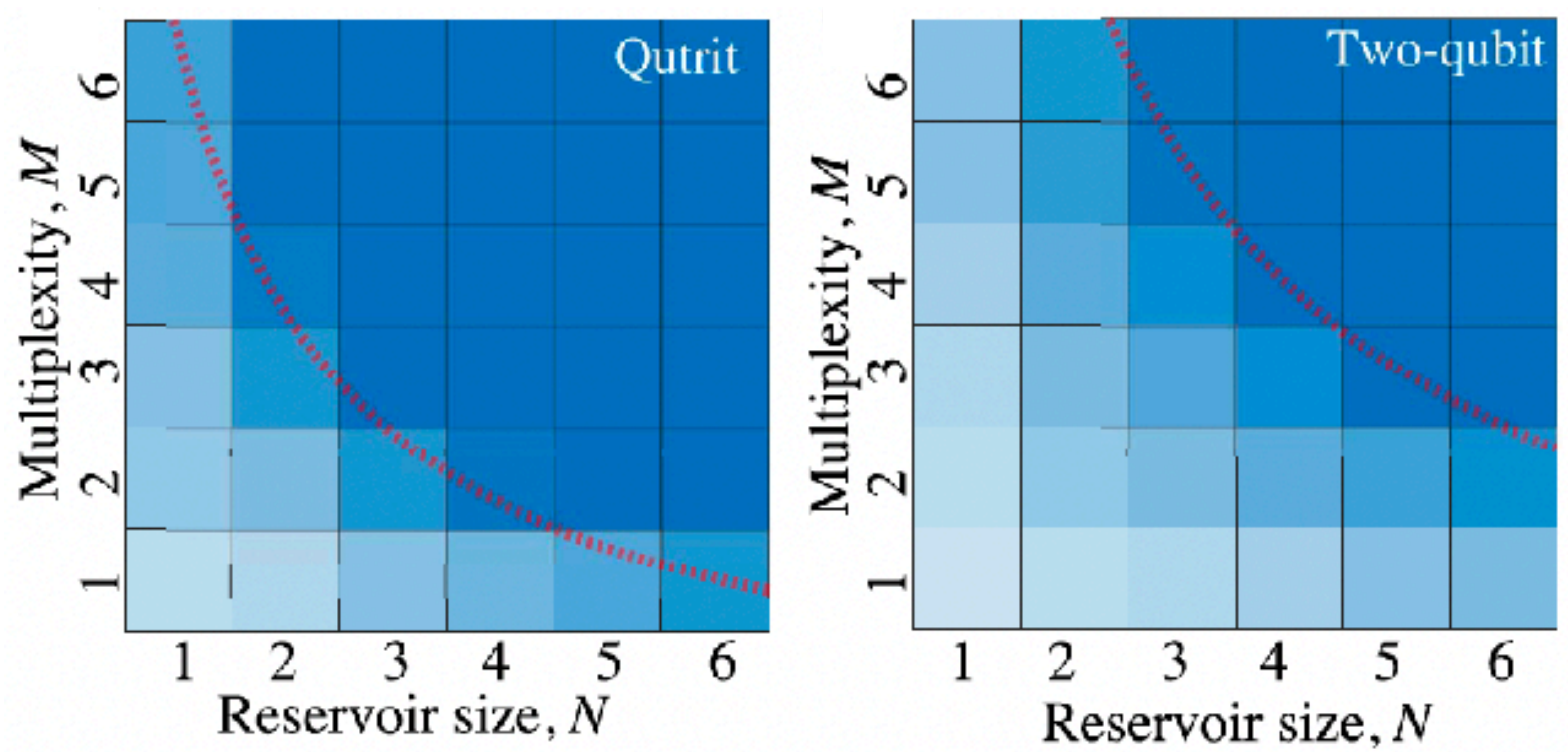
$$\vec{n} = \sum_i \alpha_i \vec{n}_i \xrightarrow[\text{QRP is using a linear map}]{} \mathcal{M}^{\text{out}} \vec{n} + \vec{m} = \vec{\rho}_{\text{in}}$$

This is the number of independent real parameters



D is exponential in the number of input systems.

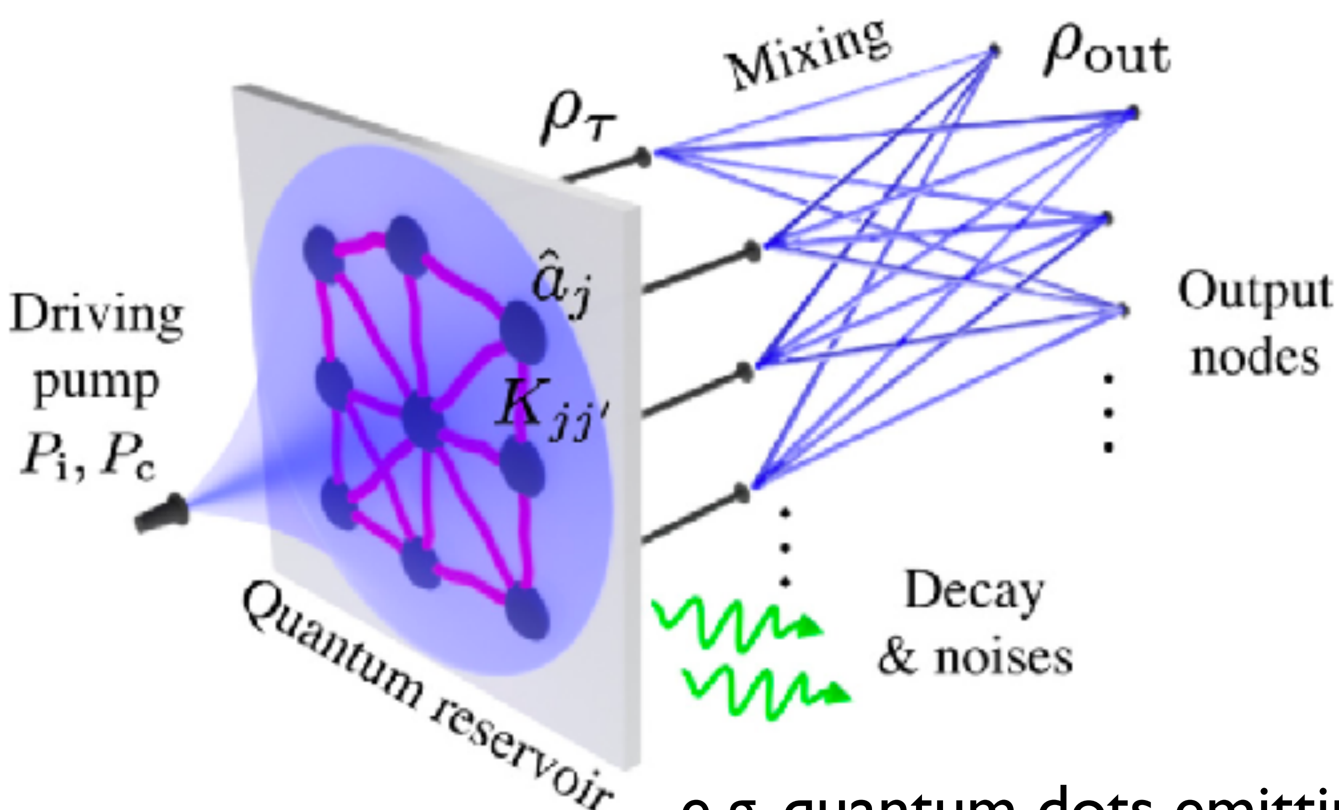
Yet, instead of exponentiating the number of reservoir nodes, we can measure them many times…



Dashed line shows relation $NM = D^2 - 1$

CLASSICAL INPUT — QUANTUM OUTPUT

From simple input to complex output



e.g. quantum dots emitting photons
rho_tau is the steady state

S. Ghosh et al. Phys. Rev. Lett. **123**, 260404 (2019)

T. Krisnanda et al. Neu. Net. **136**, 141 (2021)

THE MODEL

The reservoir and coherent pump: $$\hat{H} = \sum_j E_j \hat{a}_j^\dagger \hat{a}_j + \sum_{\langle jj' \rangle} K_{jj'}(\hat{a}_j^\dagger \hat{a}_{j'} + \hat{a}_{j'}^\dagger \hat{a}_j) + \sum_j (P_{\mathrm{C},j} \hat{a}_j^\dagger + P_{\mathrm{C},j}^* \hat{a}_j)$$

Decay and incoherent pump: $$\dot{\rho} \equiv \mathcal{L}[\rho] = -\frac{i}{\hbar}(\hat{H}\rho - \rho\hat{H}) + \sum_j \frac{\gamma_j}{\hbar} L[\rho, \hat{a}_j] + \frac{P_{\mathrm{I},j}}{\hbar} L[\rho, \hat{a}_j^\dagger]$$

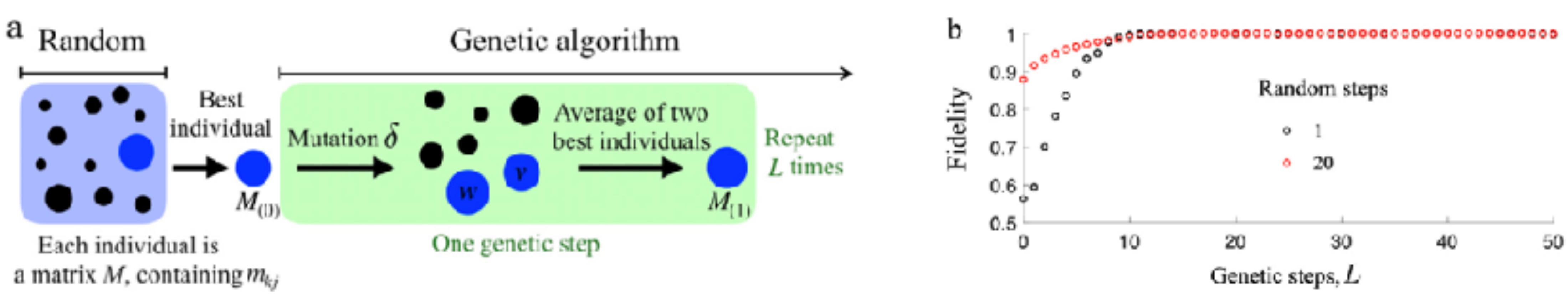
Noise: $$\rho(t + \Delta t) = M_{\mathrm{dp}} M_{\mathrm{ds}}[\rho(t) + \Delta t \mathcal{L}[\rho(t)]]$$

Linear mixing of the emissions: $$\hat{C}_k = \sum_j m_{kj} \hat{B}_j$$

final output modes

photonic modes

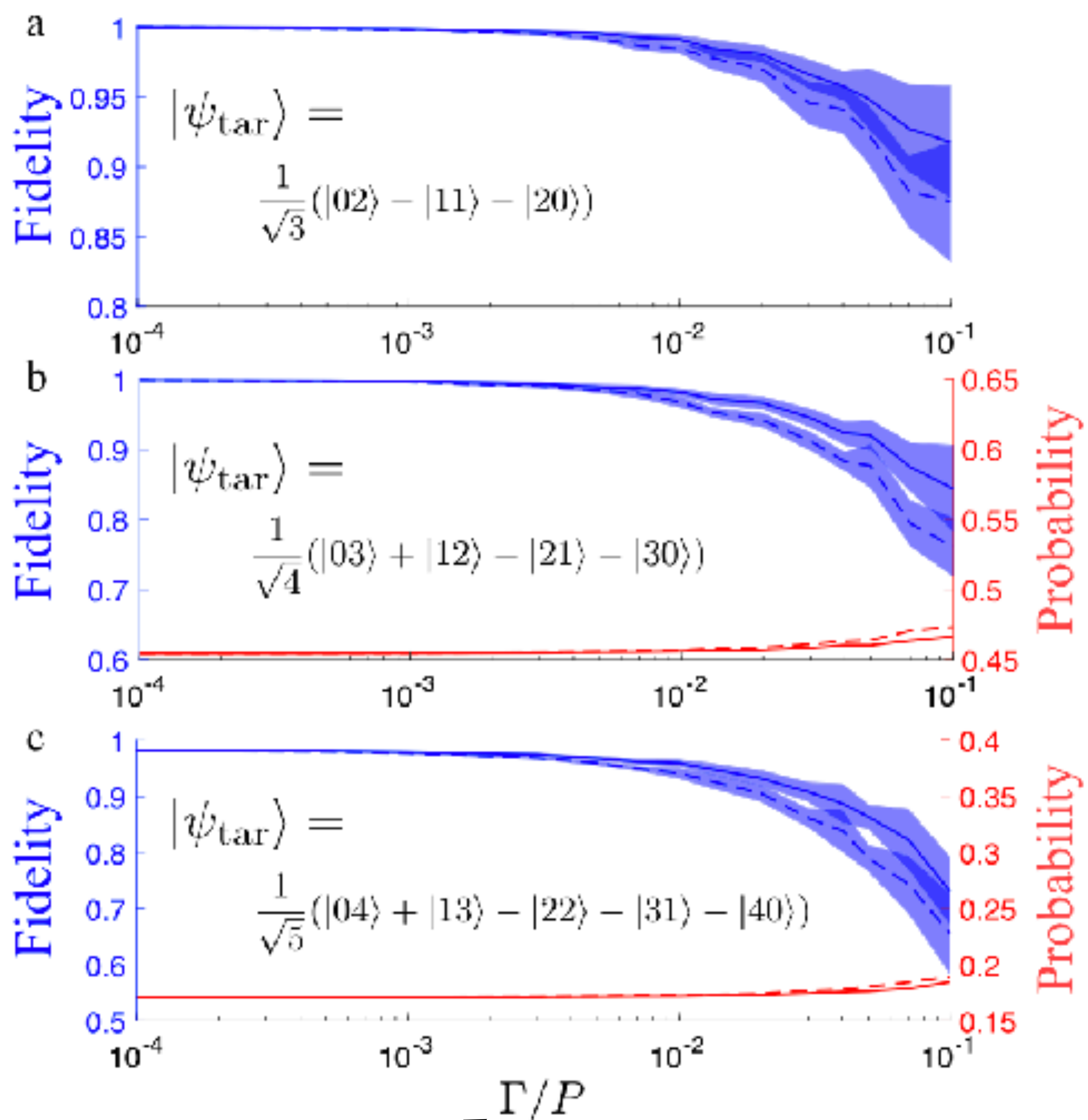
weights to be trained (by maximizing fidelity)



PREPARATION OF MAXIMALLY ENTANGLED STATES IN HIGHER D

without noise

with noise



reservoir with 2 nodes

reservoir with 4 nodes: 2 post-selected

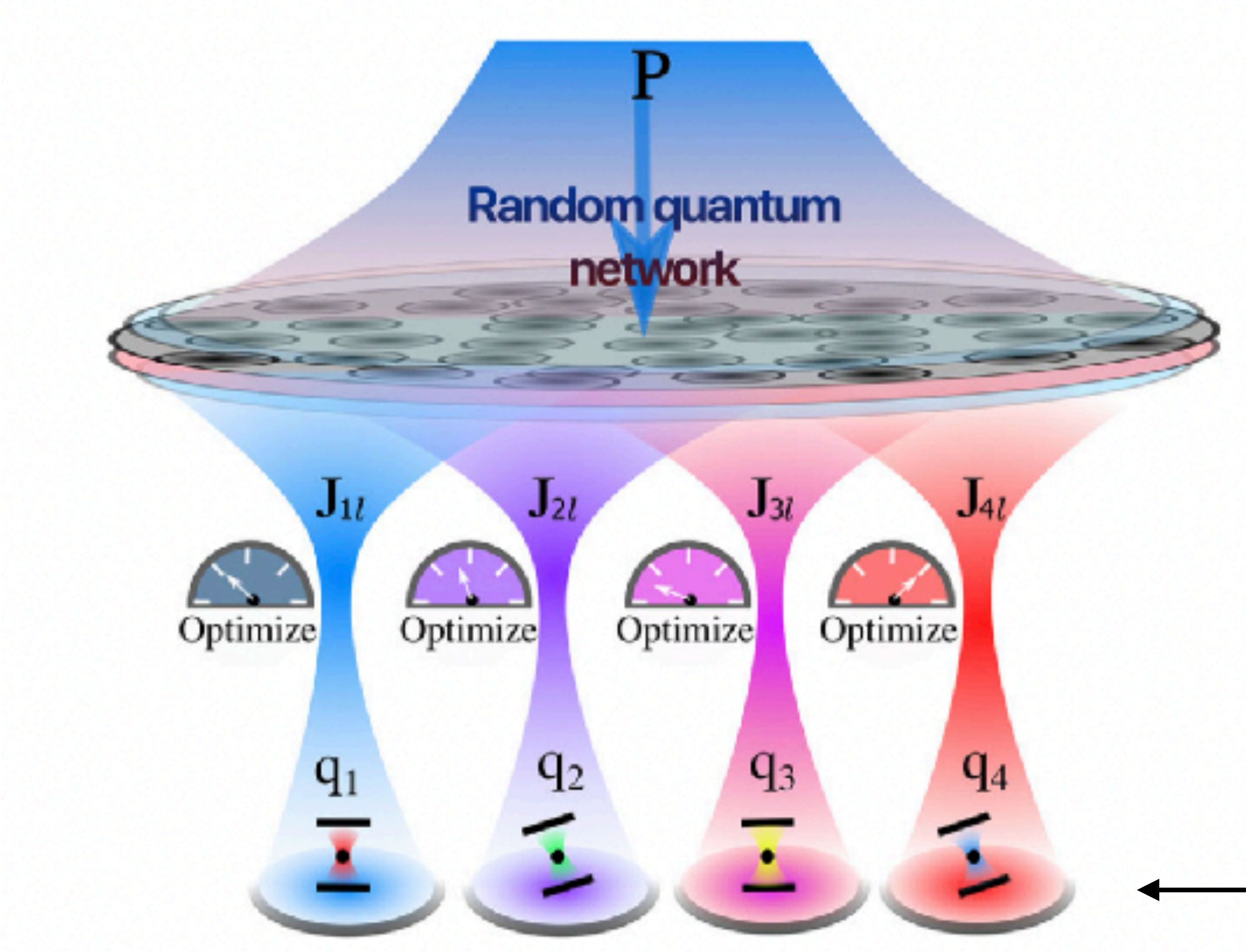
reservoir with 4 nodes: 2 post-selected

highest decay rate

each node pumped with the same strength

- NOON states — quantum metrology
- W states — multipartite entanglement
- Cluster states — universal quantum computing
- Single-photon states — quantum technologies

QUANTUM INPUT — QUANTUM OUTPUT

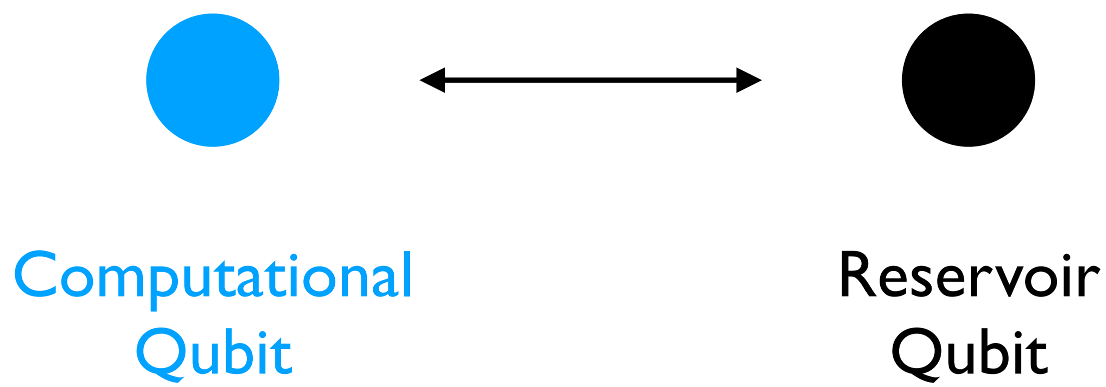


Computational
Qubits

S. Ghosh et al. Comms. Phys. 4, 105 (2021)

THE MODEL: SINGLE QUBIT OPERATIONS

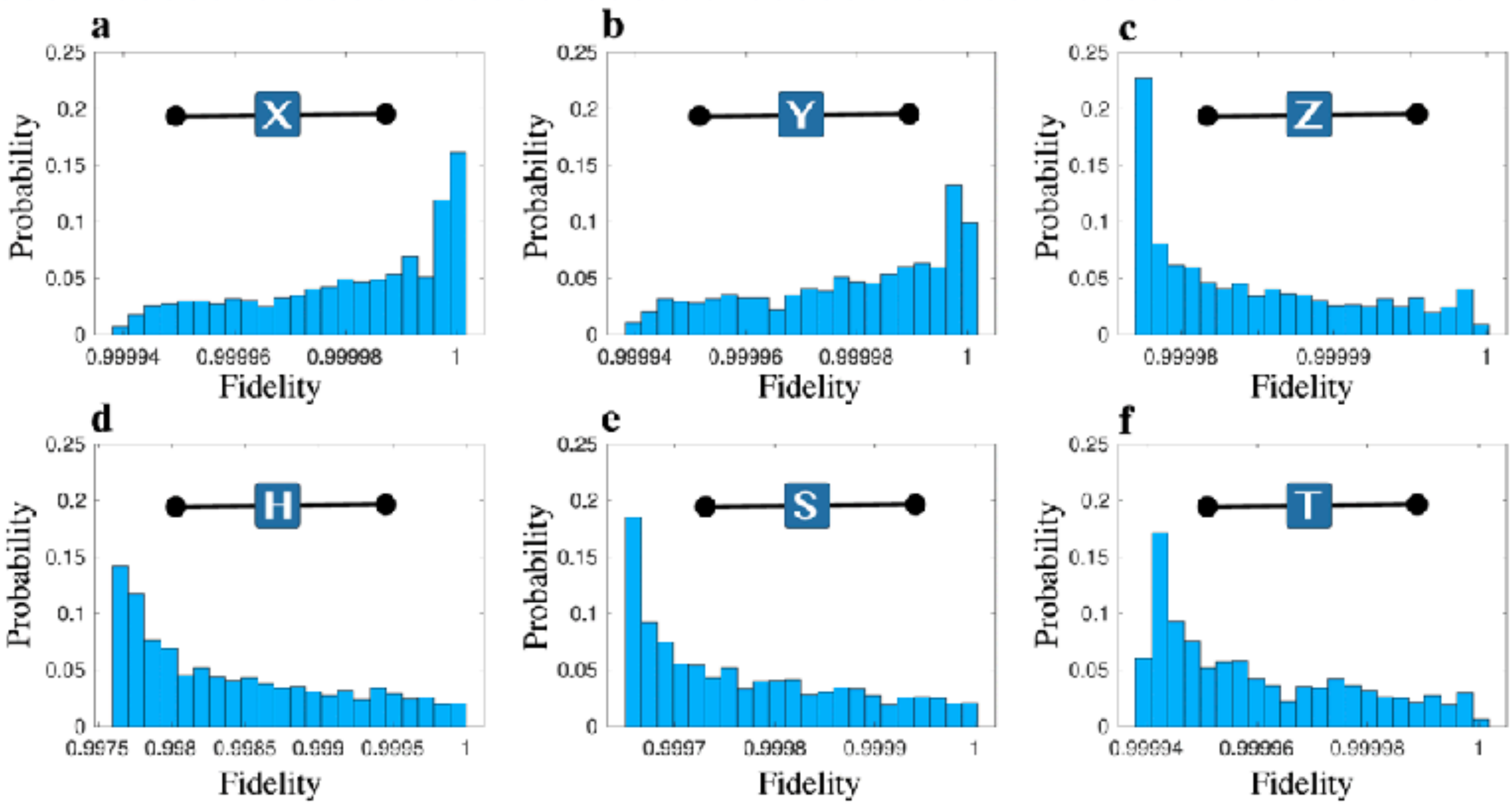
In toy examples learning can be done (almost) analytically



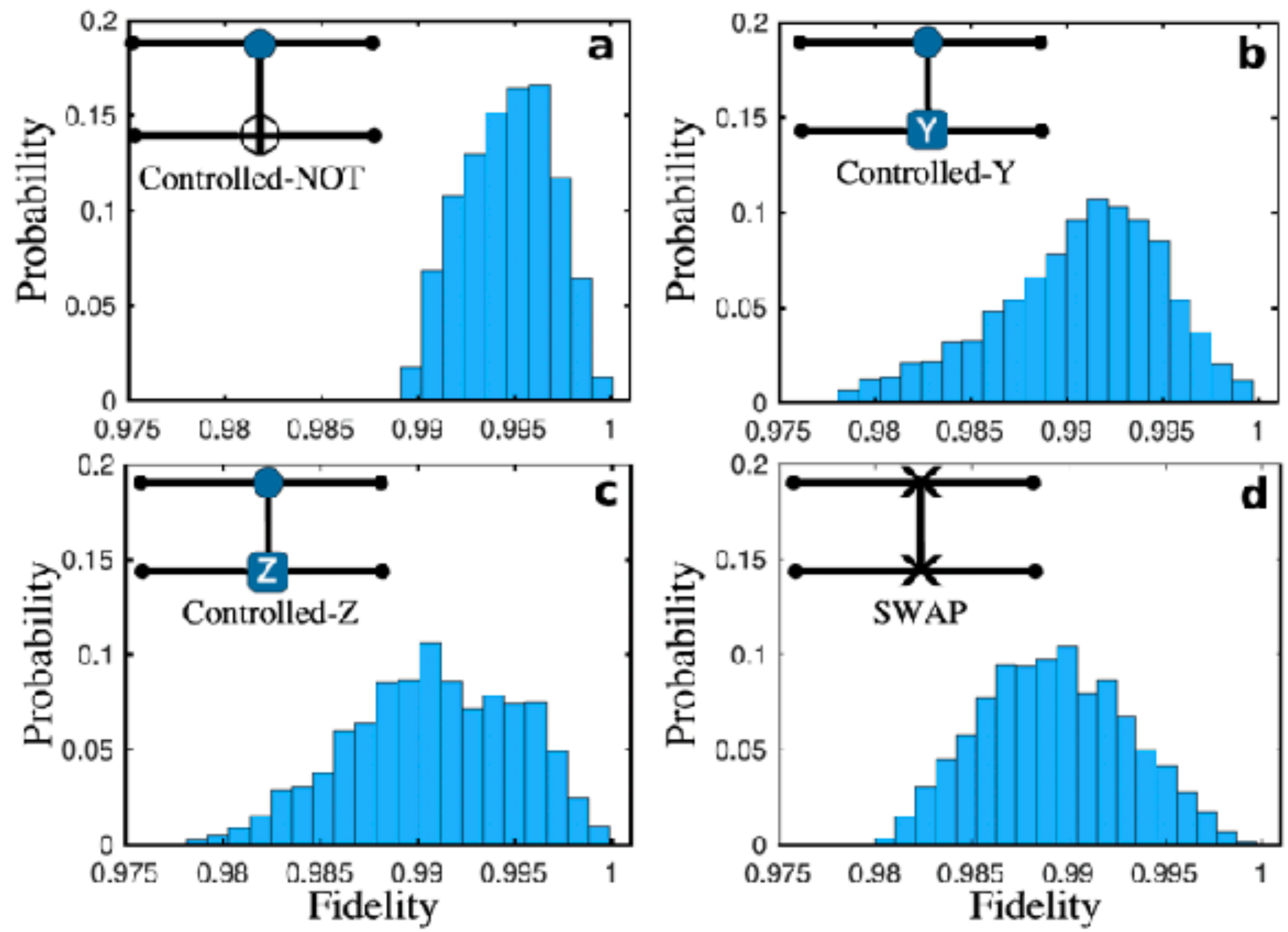
1. Compute Kraus operators on the computational qubit
2. Use closed form expression for gate fidelity
3. Maximise on a computer

$$
\begin{aligned}
F &= \int_{S^{2n-1}} \langle\psi|U_0^\dagger \mathcal{G}(|\psi\rangle\langle\psi|)U_0|\psi\rangle dV \\
&= \sum_k \int_{S^{2n-1}} |\langle\psi|M_k|\psi\rangle|^2 dV \\
&= \frac{1}{n(n+1)}\left\{\mathrm{Tr}\Big(\sum_k M_k^\dagger M_k\Big) + \sum_k |\mathrm{Tr}(M_k)|^2\right\}
\end{aligned}
$$

L. H. Pedersen, K. Molmer, N. M. Moller, Phys. Lett. A **367**, 47 (2007)

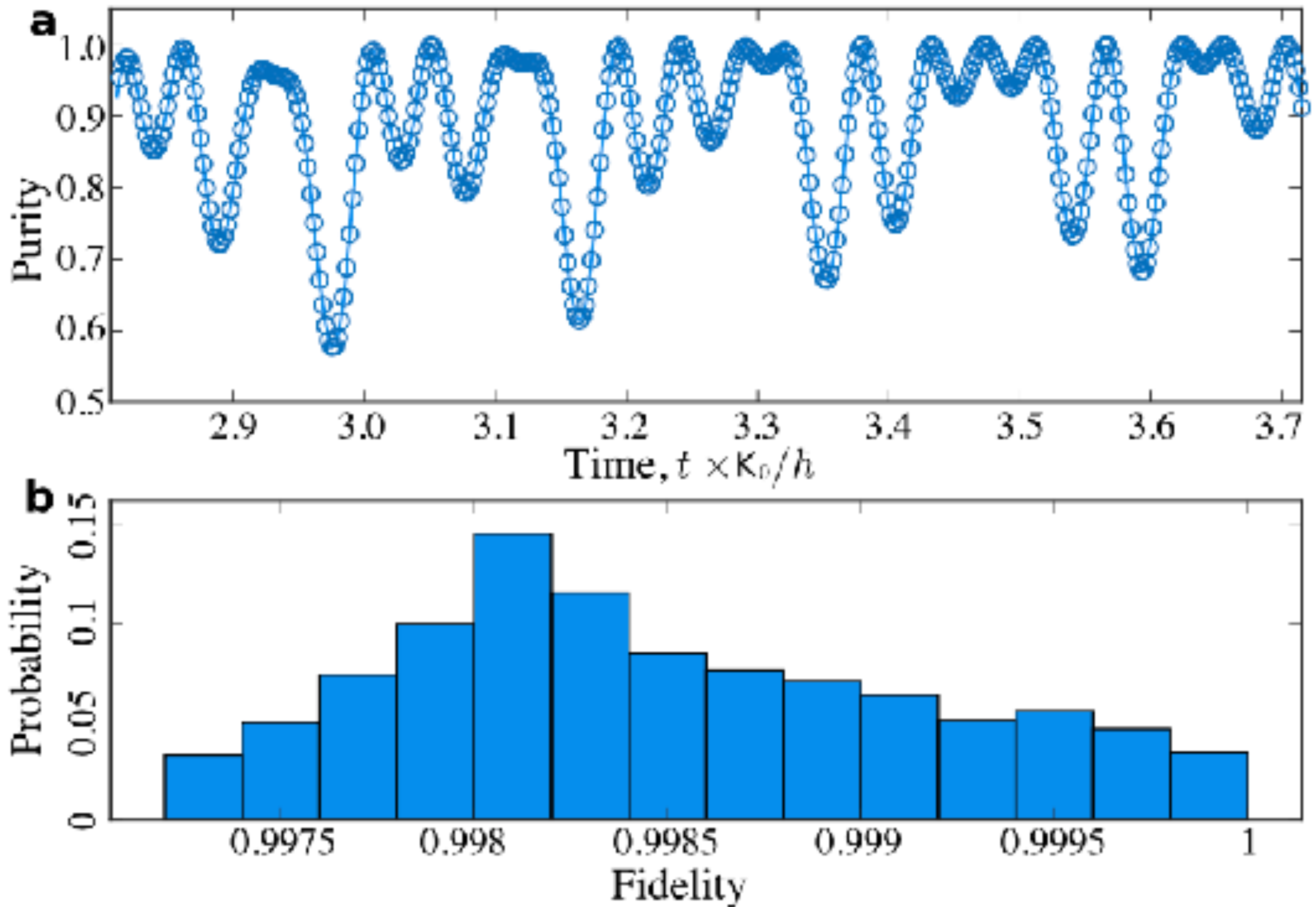


Most of the gates on average above 0.999



Reservoir of 6 nodes. Fidelity of 2000 random states.
(10 random pure state for training — max of average fidelity)

Altogether this provides a universal set of gates.



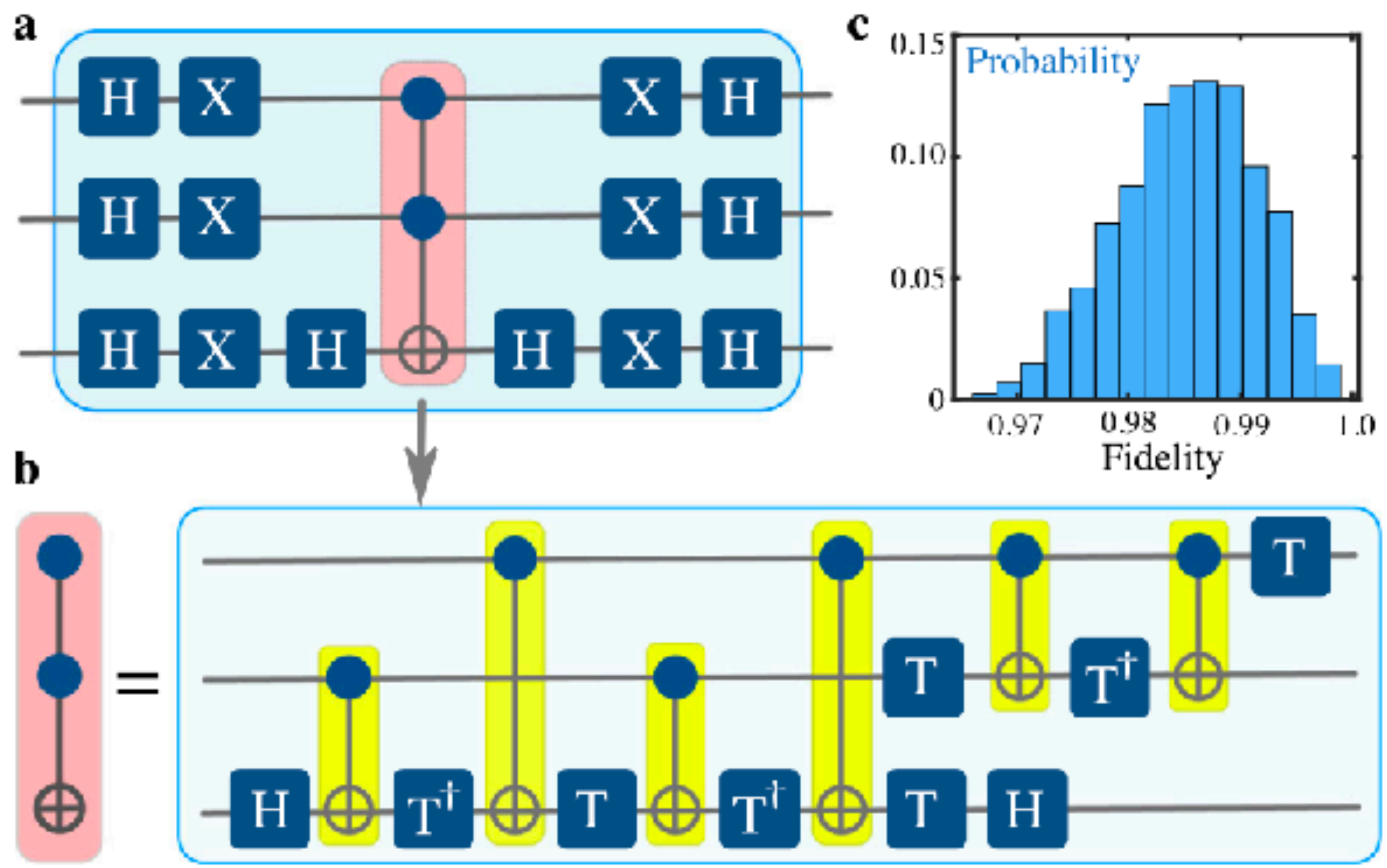
Purity of a single qubit coupled to a single reservoir node

Fidelity between QRP operation and the map induced by the master equation:

$$\hbar\dot{\rho} = (\gamma/2)(2\sigma^{-}\rho\sigma^{+} - \sigma^{+}\sigma^{-}\rho - \rho\sigma^{+}\sigma^{-})$$

Three-qubit Grover's unitary

In conventional circuit model is requires 29 gates



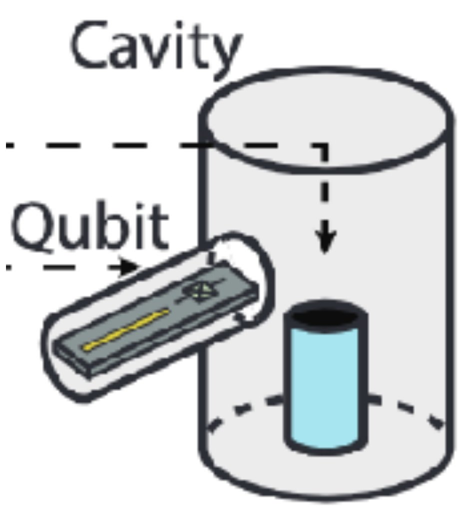
QRP trained with 10 random states gives mean F > 0.98

EXPERIMENT

Centre for
Quantum
Technologies

Bosonic Circuit QED setup

Cavity mode — reservoir
Qubit — readout device

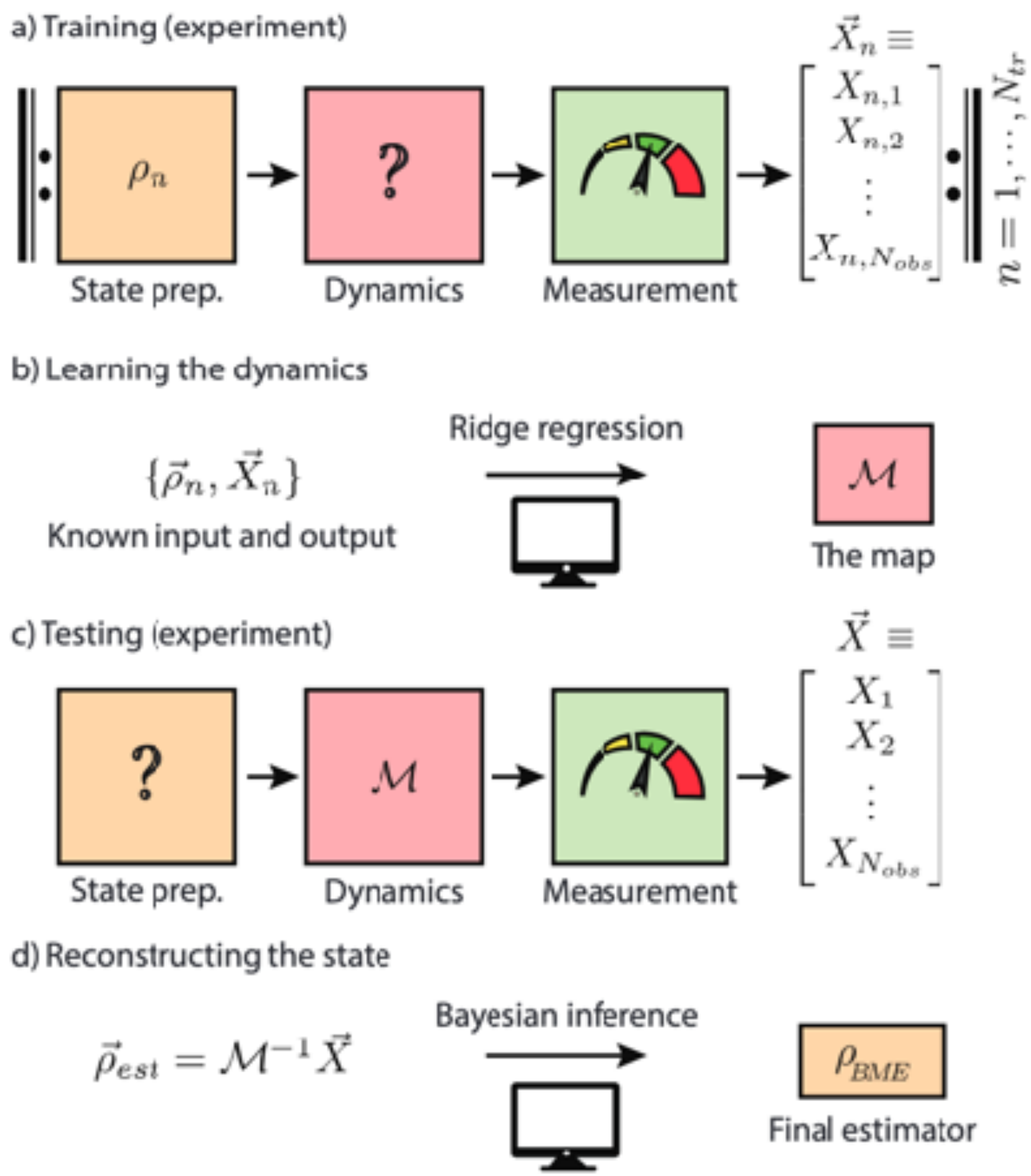


$$
\begin{aligned}
\hat{H}_0/\hbar &= \sum_{k=q,c,r} \omega_k \hat{k}^\dagger \hat{k} - \frac{\chi_{kk}}{2} \hat{k}^\dagger \hat{k}^\dagger \hat{k}\hat{k} \\
&\quad - \chi_{cq} \hat{c}^\dagger \hat{c} \hat{q}^\dagger \hat{q} - \chi_{qr} \hat{q}^\dagger \hat{q} \hat{r}^\dagger \hat{r} - \chi_{cr} \hat{c}^\dagger \hat{c} \hat{r}^\dagger \hat{r} \\
&\quad - \chi'_{cq} \hat{c}^\dagger \hat{c}^\dagger \hat{c}\hat{c} \hat{q}^\dagger \hat{q},
\end{aligned}
$$

$$
\begin{aligned}
\hat{H}_d/\hbar &= \epsilon_q(t)\hat{q}e^{i\omega_{dq}t} + \epsilon_q^*(t)\hat{q}^\dagger e^{-i\omega_{dq}t} \\
&\quad + \epsilon_c(t)\hat{c}e^{i\omega_{dc}t} + \epsilon_c^*(t)\hat{c}^\dagger e^{-i\omega_{dc}t}
\end{aligned}
$$

T. Krisnanda et al. QST **10**, 035041 (2025)

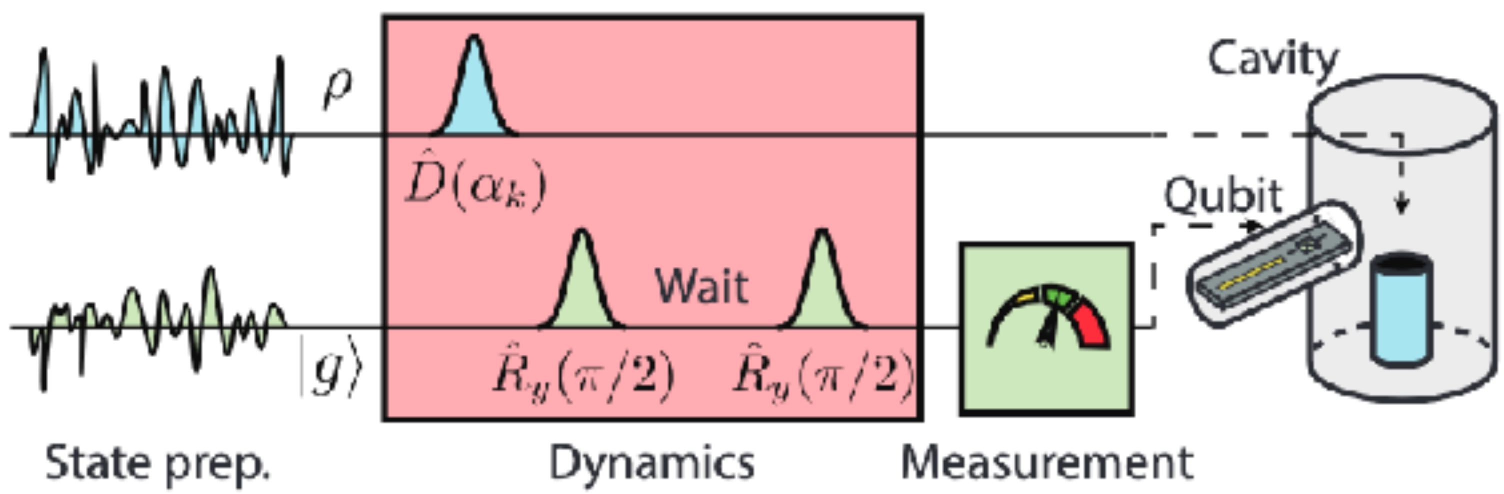
THE PROTOCOL FOR PROCESS AND STATE TOMOGRAPHY



Process tomography

State tomography

For reservoir dimensions 2 to 6.



Parity measurement — noisy translation from the reservoir to the qubit

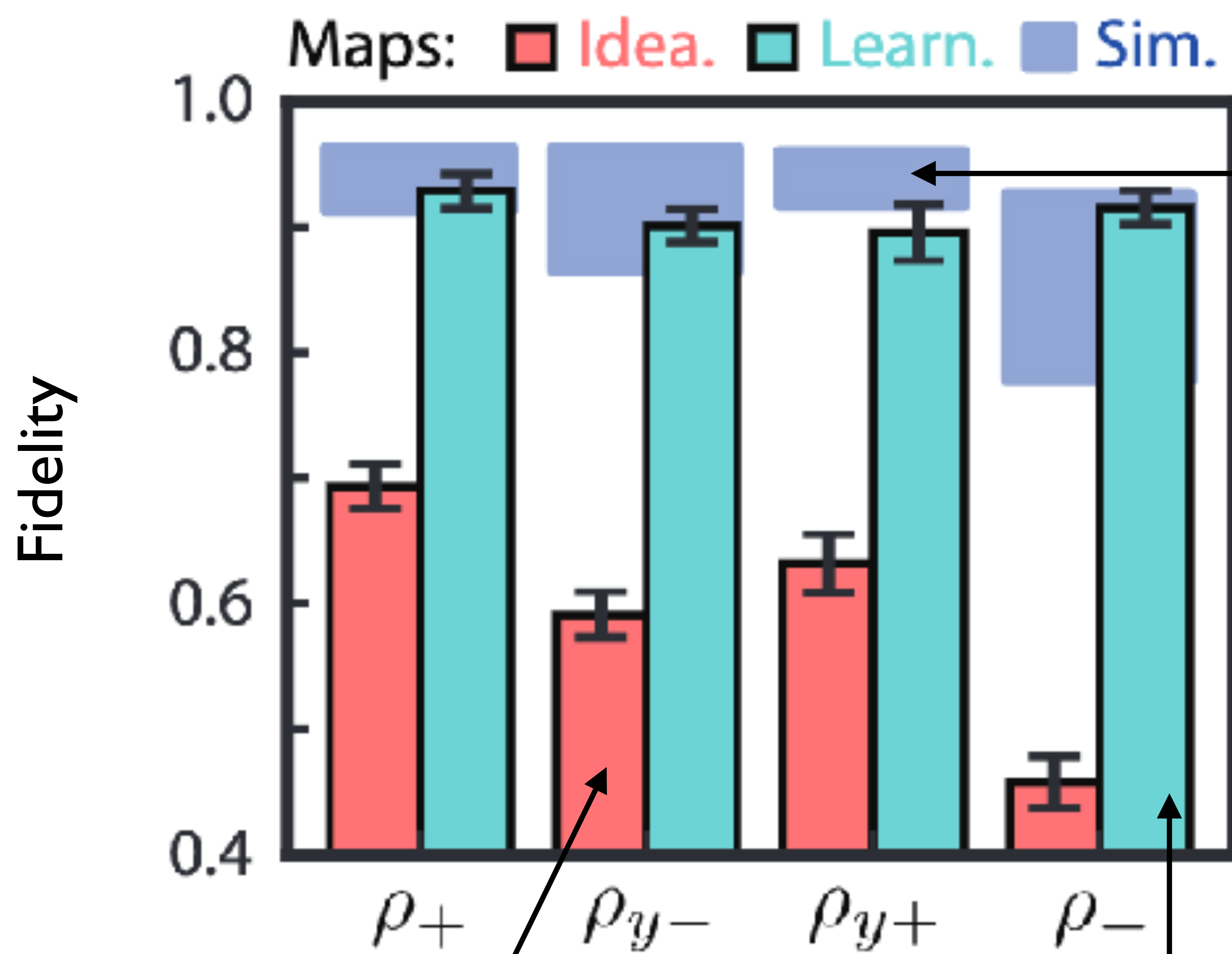
D^2 input states (independently verified to have average fidelity ~0.97):

D Fock states $(|0\rangle, |1\rangle, \cdots, |D-1\rangle)$

$D^2 - D$ of their superpositions $(|l\rangle + e^{i\Phi}|m\rangle)/\sqrt{2}$ $\quad l < m = 0, 1, \cdots, D-1$ and $\Phi = \{0, \pi/2\}$

For every input state the parity is measured after D^2-1 different displacements, 1000 times for each, to estimate the mean. Then ridge regression returns the map.

TESTING RESULTS



Using fully blown simulation, with noisy coupling strength

Kitten states

$|\alpha\rangle \pm |-\alpha\rangle$

$|\alpha\rangle \pm i|-\alpha\rangle$

Standard treatment (idealised map, assuming perfect mapping of parity)

Using the learnt map.

Ellimination of systematic errors

In this setup it is the noisy parity mapping

Economic in terms of resources

For D = 6, we trained with 36 states and used a reservoir of ~2.5 qubits
Classical neural networks have already been used for this problem.
They utilised ~700 neurons and 7000 traning states.

A. M. Palmieri et al. npj Quant. Inf. **6**, 12 (2020)

CONCLUSIONS

QRP: live long and prosper!

INTERSPECIES EDUCATION