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1 General info

Para 1.1 (General info of conference).

- Conference Time: Dec 16–20, 2024.
- Place: M1001, located on 1st Floor of College of Science Building.
- Organizers: Hui Gao (SUSTech), Shizhang Li (MCM), Ruochuan Liu (PKU), Tong Liu (Purdue), Daxin Xu (MCM)
- Website: <https://huigaomath.github.io/arith-geom-2024-winter-shenzhen.html>

Para 1.2. Conference secretary/assistants

When contacting the secretaries/assistants, please feel free to cc:

Hui Gao: mathnature@gmail.com

- Ms. Min Zhang (English name: Mia)
email: zhangm7@mail.sustech.edu.cn
- Mr. Yuetong Li: student, conf assistant. email: ytlai@foxmail.com
- Mr. Yimeng Tang: student, conf assistant. email: yimeng.tang.math@gmail.com

2 Schedule

Place: M1001, located on 1st Floor of College of Science Building.

(All in person talk; no video recording).

Time	Dec 16 (Mon.)	Dec 17	Dec 18	Dec 19	Dec 20
09:35- 09:45	Qing Xiang (opening)				
09:45- 10:45	Xinwen Zhu	Stefano Morra	Teruhisa Koshikawa	Naoki Imai	Ziquan Yang
10:45- 11:05	☕ (<i>coffee</i>)	☕	☕	☕	☕
11:05- 12:05	Daqing Wan	Florian Herzig	Dmitry Kubrak	Linus Hamann	Zijian Yao
					Efim Zelmanov (closing)
14:00- 15:00	Ahmed Abbes	Daniel Le	FREE DISCUSSION	Stavros Garoufalidis	END
15:00- 15:30	☕	☕	no coffee	☕	
15:30- 16:30	Takeshi Tsuji	Jakub Witaszek		Emanuel Reinecke	

*: there is conference dinner on Dec 19 (Thursday). We leave at around 17:00 (by an organized shuttle bus). Due to capacity, this is available for speakers and participating faculties only.

3 Title and Abstract

Dec 16, Monday

Title: Newton stratification of adjoint quotient of loop groups

Speaker: Xinwen Zhu

Abstract: I will first review the Newton stratification for family of F -isocrystals (such as weak Grothendieck conjecture and purity). Then I will review the moduli space of F -isocrystals, which can be regarded as the quotient of the loop group (of $\mathrm{GL}(n)$) by Frobenius conjugation action. I will then report a joint work with Aron Heleodoro and Xuhua He, in which we define and study the Newton stratifications of the quotient of the loop group by ordinary conjugation action. In fact, our method is applicable to the case of Frobenius conjugation action as well, leading to completely new proofs of the above mentioned results about Newton stratification of F -isocrystals.

Title: Betti Number Bounds For Varieties and Exponential Sums

Speaker: Daqing Wan

Abstract: Using basic properties of perverse sheaves, we give new upper bounds for compactly supported Betti numbers of an affine variety defined by the vanishing of a system of polynomials in n variables of degree at most d . In the complete intersection case, our bound is asymptotically optimal, improving the classical bounds of Katz. As arithmetic applications, we give new total degree bounds for zeta functions and L -functions of varieties over finite fields, improving well-known results of Bombieri, Katz, and Adolphson-Sperber. This introductory talk is based on joint work with Dingxin Zhang.

Title: Twisting Higgs Modules and Applications to the p -adic Simpson Correspondence (Part I)

Speaker: Ahmed Abbes

Abstract: (This is Part I of a two-parts talk given by AA and TT).

In 2005, Faltings initiated a p -adic analogue of the complex Simpson correspondence, a theory that has since been explored by various authors through different approaches. In this two-lecture series (by AA and TT), we present joint work in progress with Michel Gros, motivated by the goal of comparing the parallel approaches we have developed and establishing a robust framework to achieve broader functoriality results for the p -adic Simpson correspondence.

AA and MG's approach relies on the choice of a first-order deformation and involves a torsor of deformations along with its associated Higgs-Tate algebra, ultimately leading to Higgs bundles. In contrast, TT's approach is intrinsic, relying on Higgs envelopes and producing Higgs crystals. The evaluations of a Higgs crystal on different deformations differ by a twist involving a line bundle on the spectral variety. A similar and essentially equivalent twisting phenomenon occurs in the first approach when considering the functoriality of the p -adic Simpson correspondence by pullback by a morphism that may not lift to the chosen deformations.

We introduce a novel approach to twisting Higgs modules using Higgs-Tate algebras, similar to the first approach of the p -adic Simpson correspondence. In fact, the latter can itself be reformulated as a twist. Our theory provides new twisted higher direct images of Higgs modules, which we apply to study the functoriality of the p -adic Simpson correspondence by higher direct images with respect to a proper morphism that may not lift to the chosen deformations. Along the way, we clarify the relation between our twisting and another twisting construction using line bundles on the spectral variety that appeared recently in other works.

Title: Twisting Higgs Modules and Applications to the p -adic Simpson Correspondence (Part II)

Speaker: Takeshi Tsuji

Abstract: This is Part II, continuing the talk by AA.

Dec 17, Tuesday

Title: Finite Length for Mod p Representations of p -adic GL_2 , Part 1

Speaker: Stefano Morra

Abstract: Let p be a prime number and K a finite unramified extension of \mathbb{Q}_p .

The smooth $\mathrm{GL}_2(K)$ representations appearing in the mod p local Langlands program are expected to satisfy desirable properties; in particular, their “structure” should be predicted by the corresponding 2-dimensional mod p representations of $\mathrm{Gal}(\overline{K}/K)$ (e.g., they are irreducible if and only if the local Galois representation is).

In joint work with C. Breuil, Y. Hu, F. Herzig, and B. Schraen, we show that the smooth mod p representations π of $\mathrm{GL}_2(K)$ appearing in Hecke eigenspaces of the cohomology of Shimura curves are of finite length and satisfy several further constraints coming from the structure of the local Galois representation.

In this talk, we discuss the proof of the special case when K is a quadratic extension of \mathbb{Q}_p , and introduce important tools which will appear in the proof of the general case, such as (φ, Γ) -modules and the Iwahori socle filtration of π .

This is joint work with C. Breuil, F. Herzig, Y. Hu, and B. Schraen.

Title: Finite Length for Mod p Representations of p -adic GL_2 , Part 2

Speaker: Florian Herzig

Abstract: This talk is a continuation of Stefano Morra’s talk. We discuss the proof that the $\mathrm{GL}_2(K)$ -representations appearing as Hecke eigenspaces in the mod- p cohomology of Shimura curves are of finite length. Here K/\mathbb{Q}_p is an arbitrary finite unramified extension. This is joint work with C. Breuil, S. Morra, Y. Hu, and B. Schraen.

Title: Around the Breuil–Mezard and Serre weight conjectures for GSp_4

Speaker: Daniel Le

Abstract: A generalization of Serre’s conjecture predicts that every odd and irreducible mod p Galois representation arises as the reduction of a Galois representation associated to an automorphic representation. The weight part of the conjecture (not yet formulated in general) should predict the set of automorphic weights of such automorphic representations in terms of the restriction of the Galois representation to the decomposition group at p . There are two conjectures due to Gee–Herzig–Savitt: one of a combinatorial nature (for representations which are generic and tamely ramified at p) and one based on the Breuil–Mezard conjecture. The latter approach is inspired by the Kisin–Taylor–Wiles method, which shows that modularity lifting is equivalent to the Breuil–Mezard conjecture and a strong form of the weight part of Serre’s conjecture. We show that both GHS conjectures hold for GSp_4 assuming the Galois representation is generic and tamely ramified at p and satisfies additional mild hypotheses. Moreover, we show in this context a strong form of the weight part of Serre’s conjecture. The main innovation is a new method for studying components of potentially crystalline deformation rings via the normalization of a local model. This is joint work with Bao Le Hung and Heejong Lee.

Title: Mixed characteristic analogues of F -split singularities

Speaker: Jakub Witaszek

Abstract: In my talk I will discuss a way to generalise the notion of Frobenius splitting to mixed characteristic. This is based on joint work with Bhargav Bhatt, Linquan Ma, Zsolt Patakfalvi, Karl Schwede, Kevin Tucker, and Joe Waldron.

Dec 18, Wednesday

Title: Cohomology of log prismatic F -crystals

Speaker: Teruhisa Koshikawa

Abstract: TBA

Title: Cohomology of $K(G, n)$

Speaker: Dmitry Kubrak

Abstract: Given a finite locally free commutative group scheme G over some base scheme S , one can consider the corresponding higher classifying stacks $B^n G = K(G, n)$; these are algebro-geometric versions of the corresponding Eilenberg-MacLane spaces. I will talk about how, given a reasonable cohomology theory $R_G^?$ (e.g., “?” could be structure sheaf, singular, étale, de Rham, or prismatic cohomology), one can compute the cohomology of $K(G, n)$ in a uniform fashion. More precisely, one can construct a canonical filtration on $R_G^?(K(G, n))$, whose associated graded is the free divided power algebra on $D_?(G)[-n]$, where $D_?(G)$ is a certain 2-term complex that can be viewed as the “Dieudonné module” of G relative to the cohomology theory $R_G^?$. Moreover, if multiplication by 2 on $R_G^?$ is invertible, then this filtration typically splits, giving an explicit formula for $R_G^?(K(G, n))$ as an E_{n-1} -algebra. This is joint work with Shizhang Li and Shubhodip Mondal.

Dec 19, Thursday

Title: Local Langlands Correspondence for p -adic Covering Groups

Speaker: Naoki Imai

Abstract: Recently, Fargues–Scholze constructed the local Langlands correspondence for p -adic reductive groups and formulated the categorical conjecture. In this talk, we discuss its generalization to covering groups of p -adic reductive groups. This talk is based on discussions in progress with Tony Feng, Teruhisa Koshikawa, and Yifei Zhao.

Title: Geometric constant terms of Whittaker sheaves

Speaker: Linus Hamann

Abstract: We let G be a connected reductive group over the p -adic numbers with a Borel B and maximal torus T . We study the geometric analogue of the following simple question in smooth representation theory: “What are the Jacquet modules of the Whittaker model?”. In particular, we will replace the Whittaker model with a Whittaker sheaf supported on the neutral component of the moduli stack Bun_G of G -bundles on the Fargues-Fontaine curve, and Jacquet functors with geometric constant terms defined in terms of the moduli stack of B -bundles Bun_B . The calculation of these constant terms will involve considering certain relative compactifications $\overline{\mathrm{Bun}}_B$ of the moduli stack of B -bundles. In the case that $G = \mathrm{GL}_2$, this compactification has the special property that it is smooth and that its dualizing complex can be explicitly computed in terms of the modulus character of B . The answer will be a sheaf described in terms of a Galois cohomology group attached to the adjoint representation of the dual torus on the unipotent radical of the dual Borel. This Chevalley complex will also compute the spectral incarnation of these constant terms on the Galois side of the conjectured categorical equivalence between sheaves on $\mathrm{Bun}_{\mathrm{GL}_2}$ and Ind-coherent sheaves on the moduli stack of 2-dimensional Weil group representations. In particular, the calculation will verify certain instances of constant term compatibility under the categorical equivalence. This compatibility is being leveraged in work of Hansen-Mann to prove the full categorical conjecture for GL_2 . This is joint work in progress with Naoki Imai.

Title: The Habiro ring of a number field

Speaker: Stavros Garoufalidis

Abstract: TBA. (but cf. the recent preprint by Garoufalidis–Scholze–Wheeler–Zagier with same title, <https://arxiv.org/abs/2412.04241v1>.)

Title: Relative Poincaré duality in nonarchimedean geometry

Speaker: Emanuel Reinecke

Abstract: In my talk, I will explain a new, essentially diagrammatic proof of mod- p Poincaré duality for smooth and proper morphisms of rigid-analytic varieties over a p -adic field. In the course of the argument, we will see a novel construction of trace maps for any smooth morphism of rigid-analytic varieties. In the end, I will state suitable versions of Poincaré duality and trace maps for any proper morphism. Joint work with Shizhang Li and Bogdan Zavyalov.

Dec 20, Friday

Title: Some Questions about Crystalline Riemann–Hilbert Functors

Speaker: Ziquan Yang

Abstract: Liu-Zhu proved a remarkable rigidity property for de Rham local systems, namely, over a connected base, if the local system is de Rham at a single classical point, then it is de Rham everywhere. This is achieved by studying the base change properties of the relative D_{dR} functor.

Recently, Haoyang and I showed that, similarly, crystallinity and semi-stability of a local system can be checked at sufficiently many classical points. Meanwhile, we defined and studied the crystalline and semi-stable analogues of D_{dR} . Since we have already given talks about this result at different incidences, I plan to focus more on some of the open questions left pertaining to these functors.

Title: p -adic local systems: monodromy and rigidity

Speaker: Zijian Yao

Abstract: I will discuss some recent work on crystalline local systems on algebraic varieties (mostly we focus on the situation of curves) over p -adic fields. The notion of such a local system can be viewed as a family of crystalline Galois representations parametrized by a variety, and is intimately related to relative p -adic Hodge theory. I will explain some recent results on rigidity of certain properties of such local systems, and its relation with monodromy. This is mostly based on joint work with Hansheng Diao.