### Arithmetic Geometry in Shenzhen

### Handbook Last updated: December 3, 2024

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### 1 General info

### Para 1.1 (Important announcement.).

- 1. International guest (who has never been in China before): please definitely read §4 about SIM card and money payment in China.
- 2. We will UPDATE (before conf date) about how to enter campus on TAXI when you first arrive.

  As of today (Dec. 1, 2024), the university temporarily does NOT allow taxi to enter campus (for certain temporary reason).

### Para 1.2 (General info of conference).

- Conference Time: Dec 16-20, 2024.
- Place: M1001, located on 1st Floor of College of Science Building.
- Organizers: Hui Gao (SUSTech), Shizhang Li (MCM), Ruochuan Liu (PKU), Tong Liu (Purdue), Daxin Xu (MCM)
- Website: https://huigaomath.github.io/arith-geom-2024-winter-shenzhen.html

### Para 1.3. Conference secretary/assistants

- Shuyu Lin: current conference secretary.

  (in charge of practical matters: hotel, invitation letter, entry to univ, reimbursement etc)
  email: shuyulin1015@outlook.com cellphone: 13923763431
- Leyuan Shen: former conference secretary.
- Yuetong Li: student, conf assistant. email: ytlii@foxmail.com
- Yimeng Tang: student, conf assistant. email: yimeng.tang.math@gmail.com

# 2 Schedule

Time	Dec 16 (Mon.)	Dec 17	Dec 18	Dec 19	Dec 20
09:45- 10:45	Xinwen Zhu	Stefano Morra	Teruhisa Koshikawa	Naoki Imai	Ziquan Yang
10:45– 11:05			<u></u>	<b>≅</b>	<u>∷</u>
11:05- 12:05	Daqing Wan	Florian Herzig	Dmitry Kubrak	Linus Hamann	Zijian Yao
14:00- 15:00	Ahmed Abbes	Daniel Le	FREE DISCUSSION	Stavros Garoufalidis	END
15:00- 15:30	#II	£9 <u>.</u>	no coffee	;:: <b>□</b>	
15:30- 16:30	Takeshi Tsuji	Jakub Witaszek		Emanuel Reinecke	

<sup>\*:</sup> there is conference dinner on Dec 19 (Thursday). We leave at around 17:00 (by an organized shuttle bus). Due to capacity, this is available for speakers and participating faculties only.

### 3 Title and Abstract

### Dec 16, Monday

Title: TBA

Speaker: Xinwen Zhu

Abstract: TBA

Title: Betti Number Bounds For Varieties and Exponential Sums

Speaker: Daqing Wan

**Abstract:** Using basic properties of perverse sheaves, we give new upper bounds for compactly supported Betti numbers of an affine variety defined by the vanishing of a system of polynomials in n variables of degree at most d. In the complete intersection case, our bound is asymptotically optimal, improving the classical bounds of Katz. As arithmetic applications, we give new total degree bounds for zeta functions and L-functions of varieties over finite fields, improving well-known results of Bombieri, Katz, and Adolphson-Sperber. This introductory talk is based on joint work with Dingxin Zhang.

Title: Twisting Higgs Modules and Applications to the p-adic Simpson Correspondence (Part I)

**Speaker:** Ahmed Abbes

**Abstract:** (This is Part I of a two-parts talk given by AA and TT).

In 2005, Faltings initiated a p-adic analogue of the complex Simpson correspondence, a theory that has since been explored by various authors through different approaches. In this two-lecture series (by AA and TT), we present joint work in progress with Michel Gros, motivated by the goal of comparing the parallel approaches we have developed and establishing a robust framework to achieve broader functoriality results for the p-adic Simpson correspondence.

AA and MG's approach relies on the choice of a first-order deformation and involves a torsor of deformations along with its associated Higgs-Tate algebra, ultimately leading to Higgs bundles. In contrast, TT's approach is intrinsic, relying on Higgs envelopes and producing Higgs crystals. The evaluations of a Higgs crystal on different deformations differ by a twist involving a line bundle on the spectral variety. A similar and essentially equivalent twisting phenomenon occurs in the first approach when considering the functoriality of the p-adic Simpson correspondence by pullback by a morphism that may not lift to the chosen deformations.

We introduce a novel approach to twisting Higgs modules using Higgs-Tate algebras, similar to the first approach of the p-adic Simpson correspondence. In fact, the latter can itself be reformulated as a twist. Our theory provides new twisted higher direct images of Higgs modules, which we apply to study the functoriality of the p-adic Simpson correspondence by higher direct images with respect to a proper morphism that may not lift to the chosen deformations. Along the way, we clarify the relation between our twisting and another twisting construction using line bundles on the spectral variety that appeared recently in other works.

**Title:** Twisting Higgs Modules and Applications to the *p*-adic Simpson Correspondence (Part II)

Speaker: Takeshi Tsuji

**Abstract:** This is Part II, continuing the talk by AA.

### Dec 17, Tuesday

**Title:** Finite Length for Mod p Representations of p-adic  $GL_2$ , Part 1

Speaker: Stefano Morra

**Abstract:** Let p be a prime number and K a finite unramified extension of  $\mathbb{Q}_p$ .

The smooth  $GL_2(K)$  representations appearing in the mod p local Langlands program are expected to satisfy desirable properties; in particular, their "structure" should be predicted by the corresponding 2-dimensional mod p representations of  $Gal(\overline{K}/K)$  (e.g., they are irreducible if and only if the local Galois representation is).

In joint work with C. Breuil, Y. Hu, F. Herzig, and B. Schraen, we show that the smooth mod p representations  $\pi$  of  $GL_2(K)$  appearing in Hecke eigenspaces of the cohomology of Shimura curves are of finite length and satisfy several further constraints coming from the structure of the local Galois representation.

In this talk, we discuss the proof of the special case when K is a quadratic extension of  $\mathbb{Q}_p$ , and introduce important tools which will appear in the proof of the general case, such as  $(\varphi, \Gamma)$ -modules and the Iwahori socle filtration of  $\pi$ .

This is joint work with C. Breuil, F. Herzig, Y. Hu, and B. Schraen.

**Title:** Finite Length for Mod p Representations of p-adic  $GL_2$ , Part 2

**Speaker:** Florian Herzig

**Abstract:** This talk is a continuation of Stefano Morra's talk. We discuss the proof that the  $GL_2(K)$ -representations appearing as Hecke eigenspaces in the mod-p cohomology of Shimura curves are of finite length. Here  $K/\mathbb{Q}_p$  is an arbitrary finite unramified extension. This is joint work with C. Breuil, S. Morra, Y. Hu, and B. Schraen.

Title: Around the Breuil-Mezard and Serre weight conjectures for GSp<sub>4</sub>

Speaker: Daniel Le

Abstract: A generalization of Serre's conjecture predicts that every odd and irreducible mod p Galois representation arises as the reduction of a Galois representation associated to an automorphic representation. The weight part of the conjecture (not yet formulated in general) should predict the set of automorphic weights of such automorphic representations in terms of the restriction of the Galois representation to the decomposition group at p. There are two conjectures due to Gee–Herzig–Savitt: one of a combinatorial nature (for representations which are generic and tamely ramified at p) and one based on the Breuil–Mezard conjecture. The latter approach is inspired by the Kisin–Taylor–Wiles method, which shows that modularity lifting is equivalent to the Breuil–Mezard conjecture and a strong form of the weight part of Serre's conjecture. We show that both GHS conjectures hold for  $GSp_4$  assuming the Galois representation is generic and tamely ramified at p and satisfies additional mild hypotheses. Moreover, we show in this context a strong form of the weight part of Serre's conjecture. The main innovation is a new method for studying components of potentially crystalline deformation rings via the normalization of a local model. This is joint work with Bao Le Hung and Heejong Lee.

Title: TBA

Speaker: Jakub Witaszek

Abstract: TBA

#### Dec 18, Wednesday

**Title:** Cohomology of log prismatic F-crystals

Speaker: Teruhisa Koshikawa

Abstract: TBA

**Title:** Cohomology of K(G, n)**Speaker:** Dmitry Kubrak

Abstract: Given a finite locally free commutative group scheme G over some base scheme S, one can consider the corresponding higher classifying stacks  $B^nG = K(G,n)$ ; these are algebro-geometric versions of the corresponding Eilenberg-Maclane spaces. I will talk about how, given a reasonable cohomology theory  $R_G^2$  (e.g., "?" could be structure sheaf, singular, étale, de Rham, or prismatic cohomology), one can compute the cohomology of K(G,n) in a uniform fashion. More precisely, one can construct a canonical filtration on  $R_G^2(K(G,n))$ , whose associated graded is the free divided power algebra on  $D_2(G)[-n]$ , where  $D_2(G)$  is a certain 2-term complex that can be viewed as the "Dieudonné module" of G relative to the cohomology theory  $R_G^2$ . Moreover, if multiplication by 2 on  $R_G^2$  is invertible, then this

filtration typically splits, giving an explicit formula for  $R_G^?(K(G,n))$  as an  $E_{n-1}$ -algebra. This is joint work with Shizhang Li and Shubhodip Mondal.

### Dec 19, Thursday

**Title:** Local Langlands Correspondence for *p*-adic Covering Groups

Speaker: Naoki Imai

**Abstract:** Recently, Fargues–Scholze constructed the local Langlands correspondence for *p*-adic reductive groups and formulated the categorical conjecture. In this talk, we discuss its generalization to covering groups of *p*-adic reductive groups. This talk is based on discussions in progress with Tony Feng, Teruhisa Koshikawa, and Yifei Zhao.

Title: Geometric constant terms of Whittaker sheaves

Speaker: Linus Hamann

Abstract: We let G be a connected reductive group over the p-adic numbers with a Borel B and maximal torus T. We study the geometric analogue of the following simple question in smooth representation theory: "What are the Jacquet modules of the Whittaker model?". In particular, we will replace the Whittaker model with a Whittaker sheaf supported on the neutral component of the moduli stack  $\operatorname{Bun}_G$  of G-bundles on the Fargues-Fontaine curve, and Jacquet functors with geometric constant terms defined in terms of the moduli stack of B-bundles  $\operatorname{Bun}_B$ . The calculation of these constant terms will involve considering certain relative compactifications  $\overline{\operatorname{Bun}}_B$  of the moduli stack of B-bundles. In the case that  $G = \operatorname{GL}_2$ , this compactification has the special property that it is smooth and that its dualizing complex can be explicitly computed in terms of the modulus character of B. The answer will be a sheaf described in terms of a Galois cohomology group attached to the adjoint representation of the dual torus on the unipotent radical of the dual Borel. This Chevalley complex will also compute the spectral incarnation of these constant terms on the Galois side of the conjectured categorical equivalence between sheaves on  $\operatorname{Bun}_{\operatorname{GL}_2}$  and  $\operatorname{Ind}$ -coherent sheaves on the moduli stack of 2-dimensional Weil group representations. In particular, the calculation will verify certain instances of constant term compatibility under the categorical equivalence. This compatibility is being leveraged in work of Hansen-Mann to prove the full categorical conjecture for  $\operatorname{GL}_2$ . This is joint work in progress with Naoki Imai.

Title: The Habiro ring of a number field

Speaker: Stavros Garoufalidis

Abstract: TBA.

Title: Relative Poincare duality in nonarchimedean geometry

Speaker: Emanuel Reinecke

**Abstract:** In my talk, I will explain a new, essentially diagrammatic proof of mod-p Poincaré duality for smooth and proper morphisms of rigid-analytic varieties over a p-adic field. In the course of the argument, we will see a novel construction of trace maps for any smooth morphism of rigid-analytic varieties. In the end, I will state suitable versions of Poincaré duality and trace maps for any proper morphism. Joint work with Shizhang Li and Bogdan Zavyalov.

#### Dec 20, Friday

Title: Some Questions about Crystalline Riemann-Hilbert Functors

Speaker: Ziquan Yang

**Abstract:** Liu–Zhu proved a remarkable rigidity property for de Rham local systems, namely, over a connected base, if the local system is de Rham at a single classical point, then it is de Rham everywhere. This is achieved by studying the base change properties of the relative  $D_{\rm dR}$  functor.

Recently, Haoyang and I showed that, similarly, crystallinity and semi-stability of a local system can be checked at sufficiently many classical points. Meanwhile, we defined and studied the crystalline and semi-stable analogues of  $D_{\rm dR}$ . Since we have already given talks about this result at different incidences, I plan to focus more on some of the open questions left pertaining to these functors.

Title: p-adic local systems: monodromy and rigidity

Speaker: Zijian Yao

**Abstract:** I will discuss some recent work on crystalline local systems on algebraic varieties (mostly we focus on the situation of curves) over p-adic fields. The notion of such a local system can be viewed as a family of crystalline Galois representations parametrized by a variety, and is intimately related to relative p-adic Hodge theory. I will explain some recent results on rigidity of certain properties of such local systems, and its relation with monodromy. This is mostly based on joint work with Hansheng Diao.

## 4 SIM card and Money (For international guests)

If you are a foreigner who has never been in China before, money payment could be a problem. Most Chinese people have NOT used cash payment in China for MANY years. Credit card is NOT widely accepted (e.g., not in small store, not in taxis).

Para 4.1 (Money payment solution). The easiest solution.

- 1. Buy a SIM card (so you have data connection). It might be easier to do so before your flight; or maybe at Chinese airport. (Your own SIM card might also work in China, using data roaming.)
- 2. Bundle your (international) credit card with Alipay or Wechat on your cellphone. (there are many guides on the internet: it is very easy). Please do this already before your arrival.
- 3. With Alipay/Wechat (and your mobile data), you can pay anywhere in China.
- 4. You can also try to bring some cash (say,  $\geq$  200 RMB) just in case. Although it is not a good solution... (e.g., Taxi drivers could have little/none change...)
- 5. Your first (and hence urgent) use of money would be from Airport to SUSTech, please be prepared early on!

## 5 Entering Shenzhen (from Hongkong)

Para 5.1 (From HKG airport to Shenzhen border). If you fly to HKG airport, you have to cross Hongkong/Shenzhen border—involving Passport Control for foreign citizens— before arrival in Shenzhen city.

- 1. Upon arrival in HKG airpot, We recommend using shuttle/coaches. see: <a href="https://www.hongkongairport.com/en/transport/mainland-connection/mainland-coaches/">https://www.hongkongairport.com/en/transport/mainland-connection/mainland-coaches/</a> (taxi also works, but won't save you much time, and it costs more).
- 2. There are several border control locations on Hongkong/Shenzhen border. A most convenient one is Shenzhen Bay Port (tell your shuttle service counter). (Other "Port" (border control) works equally well; and different shuttle service companies work equally well).
- 3. Typical time from HKG to Shenzhen Bay Port (border) via shuttle: 30min.
- 4. Passport Control time: varies depending on crowd. Normally,  $\leq 30$  min.
- 5. Taxi from Shenzhen/HK border to SUSTech area (for your hotels): 45min-1 hour.
- 6. For Hotel info, Taxi-to-hotel notes: see §6. Caution: there are 2 possible hotels!

## 6 Going to Hotel (and food) / 酒店与餐饮

Contents Please be VERY careful about your HOTEL name!

- Hotel 1: Genpla Hotel/君璞酒店. §6.1.
- Hotel 2: SUSTech Guest House/ 南科大专家公寓 §6.2
- Food for both hotels. §6.3

## 6.1 Hotel 1: Genpla Hotel (outside SUSTech campus)/君璞酒店

Para 6.1 (Hotel info : Genpla Hotel). . (Pronounced as JunPu Hotel in Chinese.)

- 1. Address: Shenzhen City LiuXian Road No. 3333. Tang-Lang-Cheng Block C, Floor 7 南山区留仙大道 3333 号塘朗城西区 C 座 7 楼。
- 2. Taxi notes in English. (around 100–150 RMB, 40-60min, cf. §4 for money payment method!.)

Notes to Taxi driver: Please bring me to Junpu (Genpla) Hotel.

Chinese: 请带我去深铁塘朗城君璞酒店。靠近塘朗地铁站。

准确地址:南山区留仙大道 3333 号塘朗城西区 C座 7楼。

Chinese version on taxi. 强烈建议打车直接到酒店。

从塘朗地铁站找到酒店会花费一些时间;因为酒店位于一个复杂的 mall 内部。

- 3. Check-in method: Tell them that you have a room reserved by the math department 人住方式: 报自己姓名, 说南科大数学系预订的即可
- 4. Hotel phone number: +86-0755-27776988 (+86 is China code, 0755 is Shenzhen code)

#### Remark 6.2. Chinese only (if you take subway, which we do NOT recommend for newcomer).

如果是坐地铁到达,不要按照高德地图指示,而是从塘朗地铁站 A 出口直接进入塘朗城,往右前方走,进入乐购超市和万宁之间的走道,然后左转,可以看到君璞酒店的广告牌,广告牌左边即为酒店的电梯,乘坐电梯到 7 楼,7 楼为前台大厅及餐厅。如果乘坐计程车,直接到酒店 1 楼门口(背向留仙大道一侧),乘坐电梯上 7 楼。

## 6.2 Hotel 2: SUSTech Guest House/ 南科大专家公寓

#### Para 6.3. Hotel info.

- 1. It is located inside campus.
- 2. Address: Xueyuan Ave. 1088, SUSTech. Guest House Building 1. Address: 南山区学苑大道 1088 号南方科技大学专家公寓 1 号楼 (cf. para 8.4 for pic)
- 3. Taxi notes. (around 100–150 RMB, 40-60min, cf. §4 for money payment method!.)
  Notes to Taxi driver: Please bring me to SUSTech Guesthouse.
  Chinese: 请带我去南山区学苑大道 1088 号南方科技大学专家公寓 1 号楼 (在校园内部,从 1 号门或者 2 号门可以入校)。
  - You need to pass university gate through taxi, please refer to §7 on how to enter campus.
- 4. Check-in method: Tell them that you have a room reserved by the math department 人住方式: 报自己姓名, 说南科大数学系预订的即可
- 5. possible early check-out? The rooms are reserved for check-in Dec 15, and check-out Dec 21.

  If you depart earlier than Dec 21, please inform them when you check-in. (this should cause no fee).
- 6. Hotel phone number: +86-0755-86664284 (+86 is China code, 0755 is Shenzhen code)

## 6.3 Food (餐饮)

### Para 6.4 (About Food).

- 1. (Each) Hotel provides Breakfast.
- 2. During conference talk days (Dec 16-20), we provide meal tickets (餐券) for Lunch and Supper.
- 3. Inside campus, there are KFC/Subway/Starbucks; also some convenience store.
- 4. For Dec 15 and/or Dec 21, you need to arrange food by your own.

## 7 Enter university campus / 人校

The university requires (certain) registration in order to enter campus, which unfortunately is not very friendly for foreign visitors...

### Para 7.1 (How to enter SUSTech campus: International Guests).

- Try showing your passport to entrance guard at the gate and saying you are visiting Dept of Mathematics. Maybe they don't speak English... in the worst case, you can try asking students walking by.
- Or, show following Chinese texts to entrance guard.
   您好,我是访问南科大数学系高辉老师的外国教授,高辉的电话为: 13262959192.
   (Above reads: hello, I am an international professor to Hui Gao in Dept of Math in SUSTech, Hui Gao's cellphone is 13262959192.)
- Another helpful item. Show entrance guard my ID picture.



### Para 7.2 (Chinese version: 入校申请: 微信小程序).

- 1. 微信小程序中搜索: 校外人员进校申请
- 2. 点击"入校申请", 然后填写相关信息, 邀请码: 由秘书在会议前 email 发送, 请等待
- 3. 审核完毕后, 进校时出示小程序中的"通行码"即可进校。
- 4. 进校需要出示通行码。请提前至少1天申请入校。
- 5. 每次预约可预约访问期至多7天,超过预约时间后再进校前需要重新申请

## 8 From Hotel to Math Dept/conference venue / 去会议地点

#### Contents.

- From Genpla hotel to Math dept. §8.1.
- From SUSTech Guesthouse to Math dept. §8.2.

### 8.1 From Genpla Hotel (君璞酒店) outside campus

Para 8.1 (English version: From Genpla hotel to Math Dept ).

- 1. It should be easy to locate TangLang metro station (it is close to JunPu hotel); circled in bottom of picture. (cf pic in para 8.3.)
- 2. go north and follow the obvious road leading to Main Gate 1 of SUSTech.
- 3. How to enter SUSTech campus through Gate 1: see §7.
- 4. Once inside, pass the bridge in front of you, then you are at Science Building. There is an *overpass* between two buildings and the RHS is the entrance to **Math Dept.** Room M1001 is on the 1st floor of Math Department.

Para 8.2 (Chinese version: 从君璞酒店到报告厅).

- 1. 找到南科大 1 号门 (cf pic in para 8.3.)
- 2. 入校门指南: see §7.
- 3. 步行过桥就到了理学院。理学院有个楼之间天桥, 那里右转最近的人口就是数学系 1 楼。
- 4. 实在找不到就找到瑞幸咖啡, 那里紧挨着数学系。

Para 8.3 (Picture: from Genpla hotel to Math Dept).



## 8.2 From SUSTech Guest House (专家公寓) inside campus

Para 8.4 (Guest House (专家公寓) to Math Dept).

1. Very easy (you are ALWAYS inside campus). Locate track field nearby, then follow road along river. 10-15 minutes walk.

