2. Events and Probabilities - Algebra and measurable

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Let us consider that Omega is construct two different elements like Head or Tail:

$$\Omega = \{H, T\}$$

If you consider a collection of subsets, say $\mathcal{F}_0$.

1. What is $\mathcal{F}_0$?

Ans:

The collection $\mathcal{F}_0$ refers to the set of all possible subsets of Ω. This collection is also called the **power set** of Ω, often denoted as $\mathcal{P}(\Omega)$. For your set $\Omega = \{H, T\}$, the power set $\mathcal{F}_0$ will consist of all possible subsets of Ω, including the empty set $\emptyset$ and the set Ω itself. So,

$$\mathcal{F}_0 = \{\emptyset, \{H\}, \{T\}, \{H, T\}\}$$

2. What is the number of elements in $\mathcal{F}_0$?

Ans:

The number of elements in $\mathcal{F}_0$ is the number of all possible subsets of Ω. For any set with n elements, the number of subsets (or elements of the power set) is 2^n. In this case, $\Omega = \{H, T\}$ has 2 elements, so the number of subsets in $\mathcal{F}_0$ is:

$$2^2 = 4$$

Thus, there are 4 elements in $\mathcal{F}_0$: $\emptyset, \{H\}, \{T\}, \{H, T\}$.

3. If Ω is a finite n-element set, what is the number of elements in a collection of all subsets, and why?

Ans:

If Ω is a finite set with n elements, the collection of all subsets of Ω (i.e., the power set of Ω) will contain 2^n elements. This is because for each element in the set, you have two choices when forming a subset: either include the element or exclude it. Therefore, for n elements, there are 2^n possible subsets.

For example:

- If $n = 1$ (e.g., $\Omega = \{H\}$), the power set contains $2^1 = 2$ subsets: $\emptyset$ and $\{H\}$.
- If $n = 2$ (as in the original case $\Omega = \{H, T\}$), the power set contains $2^2 = 4$ subsets.

In general, for any finite set Ω with n elements, the number of elements in the power set is 2^n.