Midterm Exam II

Macroeconomics I
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Each question in Problem 1 worth 2.5 points, each question in Problem 2 worth 3.5 points, and writing down your name and ID number is worth 1 point. The total is 100.

1 Problem 1: Distorting Taxes with Cobb–Douglas Production

Recent U.S. legislation—including the Tax Cuts and Jobs Act (2017) and the One Big Beautiful Bill (2025)—has changed marginal household income tax rates and deduction caps. In our model, these appear as a proportional labor-income tax t that alters the after-tax wage $w(1-t)$ in the production function $Y = zN^{\alpha}$. Utility function is $U = \log C + \log l$. The distortion causes

$$MRS_{l,C} = w(1-t) \neq MRT_{l,C} = MPN = z\alpha N^{\alpha-1}.$$

1. In an undistorted economy, efficiency requires equality between __B__

   (A) $MRS_{l,C} = w(1-t)$

   (B) $MRS_{l,C} = MRT_{l,C} = MPN$

   (C) $MPN > w$

   (D) $MRT_{l,C} = w(1+t)$

2. A proportional labor-income tax primarily __B__

   (A) Shifts the PPF outward

   (B) Reduces the after-tax wage and labor supply

   (C) Raises both consumption and leisure

   (D) Increases productivity directly

3 A lump-sum tax is efficient because __B__

(A) It alters the slope of indifference curves

(B) It does not change the relative price of leisure

(C) It raises marginal utility of income

(D) It lowers government revenue

4 The First Welfare Theorem fails when __C__

(A) Markets are competitive

(B) Preferences are identical

(C) Distorting taxes create wedges

(D) Government spending = 0

5 The tax wedge measures __B__

(A) Difference between disposable and gross income

(B) Gap between $MRS_{l,C}$ and $MRT_{l,C}$

(C) Labor elasticity of substitution

(D) Capital income share

6 When $w(1-t)$ falls, equilibrium labor supply __B__

(A) Rises

(B) Falls due to substitution and income effects

(C) Unchanged

(D) Indeterminate

7 Holding z fixed, a higher t implies __B__

(A) Higher output
(B) Lower output and consumption
(C) Constant GDP
(D) Larger profit

8. Which policy minimizes efficiency loss for a given revenue? B

(A) Raise proportional tax
(B) Lump-sum taxation
(C) Raise sales tax
(D) Subsidize leisure

9. Raising the standard deduction while lowering credits likely B

(A) Raises effective t
(B) Lowers effective t for middle earners
(C) Leaves t unchanged
(D) Has no effect

10. The Laffer curve shows B

(A) Tradeoff between inflation and unemployment
(B) Relationship between t and total tax revenue
(C) Government multipliers
(D) Wage rigidity

11. The firm's wage equals A

(A) $w = z\alpha N^{\alpha-1}$
(B) $w = z(1-\alpha)N^{\alpha}$
(C) $w = zN^{\alpha}$
(D) $w = zN^{1-\alpha}$

(12) Profits satisfy __A__

(A) $\pi = z(1-\alpha)N^\alpha$

(B) $\pi = zN^{\alpha-1}$

(C) $\pi = wN - zN^\alpha$

(D) $\pi = zN - wN$

(13) Equilibrium labor is given by __A__

(A) $N = \dfrac{A}{1+A}, A = \dfrac{\alpha(1-t)}{\alpha(1-t)+1-\alpha}$

(B) $N = (1+A)/A$

(C) $N = A(1+A)$

(D) $N = (1-A)/(1+A)$

(14) Government revenue equals __A__

(A) $R = twN$

(B) $R = (1-t)wN$

(C) $R = tzN^{\alpha-1}$

(D) $R = z(1-t)N^\alpha$

(15) The peak of the Laffer curve occurs where __A__

(A) $\partial R/\partial t = 0$

(B) $R = Y$

(C) $R = w$

(D) $MRS_{l,C} = MRT_{l,C}$

(16) Suppose $Y = zN^\alpha$ with $z = 1$ and $\alpha = 0.5$. If the government imposes a proportional labor-income tax $t = 0.2$, the wage is $w = \alpha z N^{\alpha-1}$, and labor supply satisfies $MRS_{l,C} = w(1-t)$. Compute the equilibrium labor N to be closest to __A__

(A) $N = 0.31$

(B) $N = 0.50$

(C) $N = 0.64$

(D) $N = 0.80$

Solution

Given $z = 1, \alpha = 0.5, t = 0.2$, compute $\alpha(1-t) = 0.5 \times 0.8 = 0.4, 1-\alpha = 0.5$. Thus $A = \frac{0.4}{0.4+0.5} = \frac{0.4}{0.9} = \frac{4}{9} \approx 0.4444$. Then $N = \frac{A}{1+A} = \frac{4/9}{1+4/9} = \frac{4/9}{13/9} = \frac{4}{13} \approx 0.31$

17. For $z = 1, \alpha = 0.33, t = 0.3$, equilibrium N is approximately __A__

(A) 0.20

(B) 0.33

(C) 0.40

(D) 0.50

Solution

Given $z = 1, \alpha = 0.33, t = 0.3$, compute $\alpha(1-t) = 0.33 \times 0.7 = 0.231, 1-\alpha = 0.67$. Thus $A = \frac{0.231}{0.231+0.67} = \frac{0.231}{0.901} \approx 0.2564$. Then $N = \frac{A}{1+A} = \frac{0.2564}{1.2564} \approx 0.20$.

18. A rise in productivity z causes the revenue-maximizing tax rate t^* to __A__

(A) Increase

(B) Decrease

(C) Stay constant

(D) Drop to zero

(19) Graphically, a higher t moves equilibrium __A__

(A) From planner optimum to distorted CE below same IC

(B) To higher IC

(C) Vertical shift

(D) Rightward along PPF

(20) In the U.S. tax code, the state and local tax (SALT) *deduction cap* limits how much state/local taxes households can deduct from federal taxable income. When the SALT cap binds in high-tax states, households face a higher *effective* labor-income tax rate t on the margin. In the Cobb – Douglas model $Y = zN^{\alpha}$ with $MRS_{l,C} = w(1-t)$ and $w = \alpha z N^{\alpha-1}$, equilibrium employment in those states will

__B__

(A) rise, because the deduction cap increases work incentives.

(B) fall, due to a higher effective marginal tax rate on labor income.

(C) remain unchanged, since SALT is unrelated to labor supply.

(D) be indeterminate without capital taxation.

2 Problem 2: Lucas Human Capital Accumulation

Reference: Lucas human capital accumulation model (1988 JME)

Credit: Julia K. Thomas

Consider a two-period model, where human capital are accumulated by spending time in education rather than purchasing using output goods.

- the utility function is given by $U(C, C') = u(C) + u(C')$, i.e., consumer doesn't value leisure.
- households are endowed with H of current human capital at date 0, and they accumulate future human capital H' by spending $1 - \phi$ fraction of their time endowment to education. The law of motion for human capital is given by

$$H' = H + (1 - \phi)H, \tag{1}$$

  where $1 - \phi$ is the fraction of the time endowment that goes to education so that households can accumulate human capital.
- households are endowed with K amount of capital, and determine the investment at date 0 to determine their future capital K' at date 1. The usage of these capital for consumer is to rent to the firm and earn the per-unit rent r. The law of motion for physical capital is given by

$$K' = (1 - \delta)K + I. \tag{2}$$

- Firm's production function is given by

$$Y = K^{\alpha}(\phi H)^{1-\alpha}; Y' = K'^{\alpha}(\phi' H')^{1-\alpha}; \tag{3}$$

  Firm pays the per-unit wage w for the labor times the level of human capital supplied by the households and pays per-unit capital renting fee r to consumers.
- Consumer owns the whole firm, and claims the whole profit π.
- There's no government in this model, i.e., $G = G' = T = T' = B = 0$.

First, let's construct the budget constraint for consumers.

(21) Consider the current budget constraint, what is the labor income for consumer? B

(A) rK (B) $w\phi H$ (C) wK (D) $r\phi H$

22. Consider the current budget constraint, what is the capital income for consumer? __A__

(A) rK (B) $w\phi H$ (C) wK (D) $r\phi H$

23. What is the current budget constraint for consumer? __D__

(A) $C \leq wH + r\phi K - I + \pi$ (B) $C \leq w\phi H + r\phi K - I + \pi$
(C) $C \leq wH + rK - I + \pi$ (D) $C \leq w\phi H + rK - I + \pi$

24. What is the profit for the firm? __C__

(A) $\pi = Y - wH - rK$ (B) $\pi = Y - w\phi H - r\phi K$
(C) $\pi = Y - w\phi H - rK$ (D) $\pi = Y - w\phi H - r\phi K$

25. In this economy, does the competitive equilibrium and social planner's problem generate the same result? Why? __B__

(A) No, because the first welfare theorem doesn't holds.
(B) Yes, because the first welfare theorem holds.
(C) Yes, because the first welfare theorem don't holds.
(D) No, because the first welfare theorem holds.

Let's solve this model using the social planner's problem.

26. Combine your answers in 23 and 24, in the perspective of social planner, we can rewrite household's current budget constraint as __A__

(A) $C \leq Y - I$ (B) $C \leq Y - rK - w\phi H - I$
(C) $C \leq Y - r\phi K - wH - I$ (D) $C \leq Y - r\phi K - w\phi H - I$

Since the budget constraint is binding, we can replace consumption as your answer in 26.

Social planner's problem is then given by

$$\max_{C,C',\phi,K',H'} u(C) + u(C') \tag{4}$$
$$\text{s.t. } C = \text{ your answer in 26} \tag{5}$$
$$C' = Y' \tag{6}$$
$$H' = H + (1-\phi)H \tag{7}$$
$$K' = (1-\delta)K + I \tag{8}$$

Replace consumption with your answer in 26 and investment with equation 8, we can rewrite social planner's problem as

$$\max_{\underbrace{\underline{\text{C}}}_{27}} u(\underbrace{\underline{\text{A}}}_{28}) + u(K'^{\alpha}(\phi' H')^{1-\alpha})$$
$$\text{s.t. } H' = H + (1-\phi)H$$

27 (A) ϕ, K', H', C' (B) ϕ, K', C'
(C) ϕ, K', H' (D) K', H', C'

28 (A) $K^{\alpha}(\phi H)^{1-\alpha} + (1-\delta)K - K'$
(B) $K^{\alpha}(\phi H)^{1-\alpha}$
(C) $K^{\alpha}(\phi H)^{1-\alpha} + (1-\delta)K - K' + rK$
(D) $K^{\alpha}(\phi H)^{1-\alpha} + (1-\delta)K - K' + w\phi H$

29 There's one result directly from our model assumption. Since this is a two-period model, and agents don't live to the third period, we know that $\phi' =$ __D__

(A) 0.3 (B) 0.5 (C) 0 (D) 1

Using your answer in 29 as well as substitute $H' = (2-\phi)H$ into the utility function, we can write the social planner's problem as

$$\max_{\phi, K'} u(\underbrace{\underline{\quad A \quad}}_{28}) + u(\underbrace{\underline{\quad C \quad}}_{30}).$$

30 (A) $K'^{\alpha}((2-\phi)H)^{-\alpha}$ (B) $K'^{\alpha}((2-\phi)H)$

(C) $K'^{\alpha}((2-\phi)H)^{1-\alpha}$ (D) $K'^{\alpha}H^{1-\alpha}$

31 The first order condition with respect to K' would leads to __D__

(A) $\frac{u'(C)}{u'(C')} = K'^{\alpha}((2-\phi)H)^{1-\alpha}$

(B) $\frac{u'(C)}{u'(C')} = \alpha K'^{\alpha-1}H^{1-\alpha}$

(C) $\frac{u'(C)}{u'(C')} = K'^{\alpha}((2-\phi)H)^{1-\alpha}$

(D) $\frac{u'(C)}{u'(C')} = \alpha K'^{\alpha-1}((2-\phi)H)^{1-\alpha}$

32 The first order condition with respect to ϕ would leads to __B__

(A) $\frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{1-\alpha}\left(\frac{2-\phi}{\phi}\right)^{-\alpha}$

(B) $\frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{\alpha}\left(\frac{2-\phi}{\phi}\right)^{-\alpha}$

(C) $\frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{\alpha}$

(D) $\frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{\alpha-1}\left(\frac{2-\phi}{\phi}\right)^{-\alpha}$

Remember that $MRS_{C,C'} = \frac{u'(C)}{u'(C')}$, and thus your answer in 31 and 32 should equal to each other.

33 Simplify the above equation and we can get __A__

(A) $\frac{\phi^{\alpha}}{2-\phi}K' = \alpha K^{\alpha}H^{1-\alpha}$ (B) $\frac{\phi^{\alpha}}{2-\phi}K'^{\alpha-1} = \alpha K^{\alpha}H^{1-\alpha}$

(C) $\frac{\phi}{2-\phi}K' = \alpha K^{\alpha}H^{1-\alpha}$ (D) $\frac{\phi^{1-\alpha}}{2-\phi}K' = \alpha K^{\alpha}H^{1-\alpha}$

From the above equation, we can see that the choice variables, ϕ and K', are equal to $\alpha K^{\alpha} H^{1-\alpha}$. Remember that both K and H are the endowments and α is the parameter of production function.

(34) What is the economics intuition of the this equation? __C__

(A) The investment on human capital is a more favorable option than the investment on physical capital in equilibrium

(B) The investment on human capital is a less favorable option than the investment on physical capital in equilibrium

(C) The investment on human capital and the investment on physical capital are equally favorable options in equilibrium

(D) We cannot determine which investment is more favorable in equilibrium