Problem Set 4

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Question 1

Consider a model that is **similar to** (not exactly the same!) Lecture 17 RBC model but with several differences:

1. Now consumer values leisure in date 1. The lifetime utility function is given by

$$U(C, N, C', N') = \ln C + \ln(1 - N) + \ln C' + \ln(1 - N').$$

First, we start by defining the competitive equilibrium:

1 Given the exogenous quantities _____

(A) $\{G, G', z, z', K\}$ (B) $\{G, G', z, z'\}$
(C) $\{G, G'\}$ (D) $\{z, z', K\}$

a competitive equilibrium is a set of

2 consumer choices _____

(A) $\{C, C', N_S, S\}$ (B) $\{N_S, N_S', l, l', S\}$
(C) $\{C, C', N_S, N_S', l, l', S\}$ (D) $\{C, C', S\}$

3 firm choices _____

(A) $\{Y, Y', N_D, N_D', I, K'\}$ (B) $\{Y, Y', \pi, \pi', N_D, N_D', I, K'\}$
(C) $\{Y, Y', \pi, \pi', I, K'\}$ (D) $\{\pi, \pi', N_D, N_D', I, K'\}$

4 government choices _____

(A) $\{G, G', T, T', B\}$ (B) $\{G, G', B\}$
(C) $\{G, G', T, T'\}$ (D) $\{T, T', B\}$

5 and prices _____

(A) $\{w, w', q\}$ (B) $\{w, w', r\}$
(C) $\{q, q', r\}$ (D) $\{r, r', q\}$

such that

1.

6 Taken _____

(A) $\{w, w', r, \pi, \pi'\}$ (B) $\{w, w', r\}$
(C) $\{w, w', \pi, \pi'\}$ (D) $\{r, \pi, \pi'\}$

as given,

7 consumer chooses _____

(A) $\{r', N_S, N_S'\}$ (B) $\{C', K, K'\}$
(C) $\{r', K, K'\}$ (D) $\{C', N_S, N_S'\}$

to solve

$$\max_{C', N_S, N_S'} \ln\left(wN_S + \pi - T + \frac{w'N_S' + \pi' - T' - C'}{1+r}\right) + \ln C' + \ln(1 - N_S) + \ln(1 - N_S').$$

where we can back out $\{C, S, l, l'\}$.

2.

8 Taken _____ as given,

(A) $\{w, w', q\}$ (B) $\{w, w', r\}$
(C) $\{q, q', r\}$ (D) $\{r, r', q\}$

9 firm chooses _____

(A) $\{H_D, H_D', K'\}$ (B) $\{N_D, N_D', C'\}$
(C) $\{N_D, N_D', K'\}$ (D) $\{\pi, \pi', K'\}$

to solve

$$\max_{N_D, N'_D, K'} zK^\alpha N_D^{1-\alpha} - wN_D - [K' - (1-\delta)K] + \frac{z'(K')^\alpha (N'_D)^{1-\alpha} - w'N'_D + (1-\delta)K'}{1+r}.$$

where we can back out $\{Y, Y', \pi, \pi', I\}$.

3.

(10) Taxes and deficit satisfy _____

(A) $T + \frac{T'}{1+q} = G + \frac{G'}{1+q}$ (B) $T + \frac{T'}{1+r} = G + \frac{G'}{1+r}$

(C) $T + \frac{T'}{1+w} = G + \frac{G'}{1+w}$ (D) $\pi + \frac{\pi'}{1+r} = G + \frac{G'}{1+r}$

and $G - T = B$.

4. All markets clear: (i) labor, $N_S = N_D$ & $N'_S = N'_D$; (ii) goods, $Y = C + G$ & $Y' = C' + G'$; (iii) bonds at date 0, $S = B$.

After defining the competitive equilibrium, now we are going to solve this model.

Step 1: Labor market

(11) From the lecture, we know that the current marginal product of labor (MPN) will equal to current wage. $MPN =$ _____

(A) $z'(1-\alpha)\left(\frac{K}{N_D}\right)^\alpha$ (B) $z(1-\alpha)\left(\frac{K'}{N_D}\right)^\alpha$

(C) $z'(1-\alpha)\left(\frac{K'}{N'_D}\right)^\alpha$ (D) $z(1-\alpha)\left(\frac{K}{N_D}\right)^\alpha$

(12) and thus the current labor demand N_D given the wage w is _____

(A) $N_D = \left(\frac{z'(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$ (B) $N_D = \left(\frac{z(1-\alpha)}{w'}\right)^{\frac{1}{\alpha}} K$

(C) $N_D = \left(\frac{z(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$ (D) $N_D = \left(\frac{z'(1-\alpha)}{w'}\right)^{\frac{1}{\alpha}} K'$

(13) From the lecture, we know that the future marginal product of labor (MPN') will equal to future wage. $MPN' =$ _____

(A) $z'(1-\alpha)\left(\frac{K}{N_D}\right)^{\alpha}$

(B) $z(1-\alpha)\left(\frac{K'}{N_D}\right)^{\alpha}$

(C) $z'(1-\alpha)\left(\frac{K'}{N_D'}\right)^{\alpha}$

(D) $z(1-\alpha)\left(\frac{K}{N_D}\right)^{\alpha}$

(14) and thus the future labor demand N_D' given the future wage w' is _____

(A) $N_D' = \left(\frac{z'(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$

(B) $N_D' = \left(\frac{z(1-\alpha)}{w'}\right)^{\frac{1}{\alpha}} K$

(C) $N_D' = \left(\frac{z(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$

(D) $N_D' = \left(\frac{z'(1-\alpha)}{w'}\right)^{\frac{1}{\alpha}} K'$

In the labor supply part, we know that the marginal rate of substitution between leisure and consumption $MRS_{l,C}$ equals to the wage.

(15) $MRS_{l,C} =$ _____

(A) $\frac{C}{1-N_S}$

(B) $\frac{1-N_S}{C}$

(C) $\frac{N_S}{1-C}$

(D) $\frac{N_S'}{1-N_S}$

In the saving part, we know that the marginal rate of substitution between current and future consumption $MRS_{C,C'}$ equals to the real interest rate $(1+r)$

(16) $MRS_{C,C'} =$ _____

(A) $\frac{N_S'}{N_S}$

(B) $\frac{C}{C'}$

(C) $\frac{C'}{C}$

(D) $\frac{N_S}{N_S'}$

(17) Solve for C', we get _____

(A) $C' = (1+r)N_S$

(B) $C' = (1+r)C$

(C) $C' = (1+r)C'$

(D) $C' = (1+r)N_S'$

Start from now we denote the income that is not directly affected by consumer choice as x and x', similar to Lecture 17.

18. Substitute C' using your answer in 17 into the budget constraint and solve for C, we get _____

(A) $C = \frac{1}{2}\left(wN_S + x + \frac{x'}{1+r}\right)$

(B) $C = \frac{1}{1+\beta}\left(wN_S + x + \frac{x'}{1+r}\right)$

(C) $C = \frac{1}{1+\beta}\left(wN_S + C' + \frac{C'}{1+r}\right)$

(D) $C = \frac{1}{2}\left(wN_S + N_S' + \frac{N_S'}{1+r}\right)$

19. Substitute your answer of 18 into your answer in 15, we can solve the labor supply $N_S =$ _____

(A) $\frac{1}{3} - \frac{2}{3w}\left(x + \frac{x'}{1+r}\right)$

(B) $\frac{2}{3} - \frac{w}{3}\left(x + \frac{x'}{1+r}\right)$

(C) $\frac{2}{5} - \frac{5}{3w}\left(x + \frac{x'}{1+r}\right)$

(D) $\frac{2}{3} - \frac{1}{3w}\left(x + \frac{x'}{1+r}\right)$

20. From 12 we solve for labor demand N_D. From 19 we solve for labor supply N_S. If for this question we let $\alpha = 1$, then we can solve the wage w as a function of real interest rate r as _____

(A) $w^*(r) = x + \frac{x'}{1+r}$

(B) $w^*(r) = \frac{1}{3}\left(x + \frac{x'}{1+r}\right)$

(C) $w^*(r) = \frac{1}{2}\left(x + \frac{x'}{1+r}\right)$

(D) $w^*(r) = zK\left(x + \frac{x'}{1+r}\right)$

For the output demand curve, we know that the optimal investment schedule is given by $MPK' - \delta = r$.

21. We know that the MPK' is _____

(A) $\alpha z K^{\alpha-1} N^{1-\alpha}$

(B) $\alpha z' K'^{\alpha-1} N'^{1-\alpha}$

(C) $(1-\alpha) z' K'^{\alpha} N'^{-\alpha}$

(D) $\alpha z K^{\alpha} N^{-\alpha}$

22. We can solve the optimal investment schedule and get $K' =$ _____

(A) $\left(\frac{z'\alpha}{q+\delta}\right)^{\frac{1}{1-\alpha}} N'$

(B) $\left(\frac{z'\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} N$

(C) $\left(\frac{z'\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} N'$

(D) $\left(\frac{z'\alpha}{q+\delta}\right)^{\frac{1}{1-\alpha}} N$

(23) and the investment I_D is determined by capital accumulation process $K'-(1-\delta)K$ and is _____

(A) $\left(\frac{z'\alpha}{q+\delta}\right)^{\frac{1}{1-\alpha}} N' - (1-\delta)K$

(B) $\left(\frac{z'\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} N - (1-\delta)K$

(C) $\left(\frac{z'\alpha}{q+\delta}\right)^{\frac{1}{1-\alpha}} N - (1-\delta)K$

(D) $\left(\frac{z'\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} N' - (1-\delta)K$

(24) Based on your answer in 23, the investment demand I_D is _____ in future labor N'.

(A) increasing

(B) no related

(C) decreasing