Final Exam

Macroeconomics I
Hui-Jun Chen

Instruction

Each question worth 2.5 points. The total is 100.

Question 1

Consider the two-period dynamic general equilibrium model with a representative consumer, representative firm and government. The consumer values consumption and leisure in each period, C and l, and provides labour, N_S, in return for a real wage, w. The consumer pays lump-sum taxes T each period and receives all profits from the firm, π.

The representative firm uses labour and capital, N_D and K, to produce output. In the first period, it also chooses investment, I. This determines its capital stock for production in the second period, K', through the capital accumulation equation $K' = (1-\delta)K + I$.

The consumer's preferences are

$$U(C, C', N, N') = u(C) - v(N_S) + u(C') - v(N_S'),$$

and the firm's technology is

$$Y = zF(K, N) = zK^{\alpha}N^{1-\alpha}, \text{ where } \alpha \in (0, 1).$$

Lastly, recall that the government must balance its budget across the two periods, $G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$, where G is government spending and T are taxes.

1. Assume assumption N1 holds, i.e., substitution effect dominates income effect from a change in real wage, then given interest rate r, consumer will choose the quantity of labor supply N_S by __A__

(A) $MRS_{l,C} = w$ (B) $MRS_{C,C'} = r$

(C) $MRS_{l,C} = r$ (D) $MRS_{C,C'} = w$

2 where the MRS in question 1 is <u>C</u>

(A) $MRS_{l,C} = \frac{u'(C)}{u'(C')}$ (B) $MRS_{C,C'} = \frac{u'(C)}{u'(C')}$

(C) $MRS_{l,C} = \frac{v'(N_S)}{u'(C)}$ (D) $MRS_{C,C'} = \frac{v'(N_S)}{u'(C)}$

3 Following assumption N1, the slope of the labor supply is <u>D</u> in wage and thus the labor supply curve has <u>D</u> slope

(A) decreasing; positive (B) increasing; negative

(C) decreasing; negative (D) increasing; positive

4 Following assumption N2, how does the labor supply curve response to a rise in real interest rate r? <u>B</u>

(A) shift to the left (B) shift to the right

(C) not affected (D) ambiguous

5 In the firm's labor demand, what is the equation that can determine the labor demand curve? <u>A</u>

(A) $MPN = w$ (B) $MPK = r$

(C) $MPN = r$ (D) $MPK = w$

6 How does a fall in total factor productivity z affect the equilibrium in labor market? <u>B</u>

(A) labor supply will shift to the left; labor demand will shift to the right

(B) labor supply will shift to the right; labor demand will shift to the left

(C) labor supply will shift to the right; labor demand not shift

(D) labor supply will not shift; labor demand will shift to the left

7 For the goods demand, what is the optimal investment schedule? <u>D</u>

(A) $MPN' - \delta = w$ (B) $MPN' - \delta = r$

(C) $MPK' - w = r$ (D) $MPK' - \delta = r$

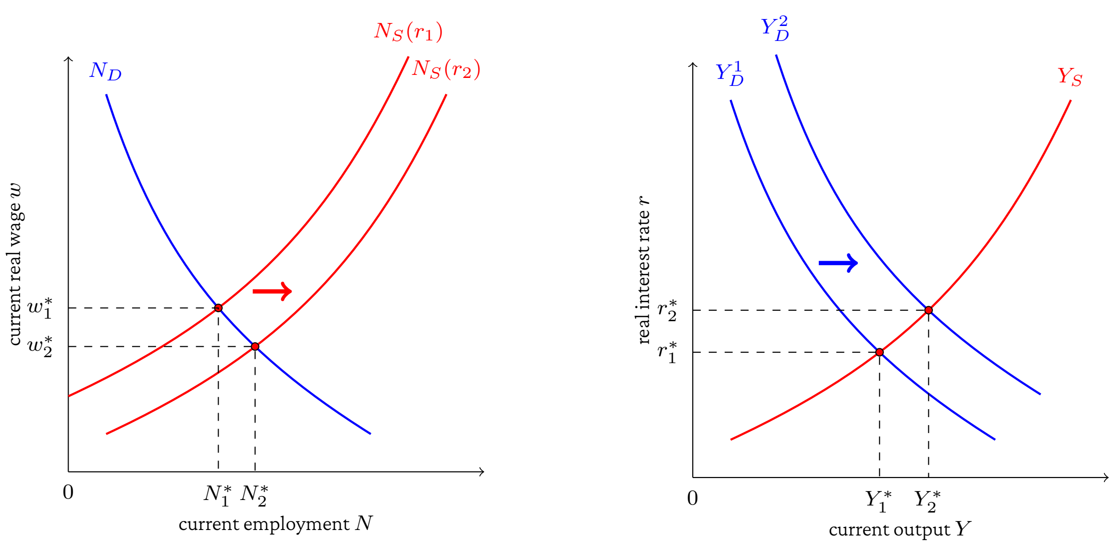
8 how does a rise in the real interest rate r changes the investment? C

(A) $I^d \uparrow$ (B) I^d unchanged

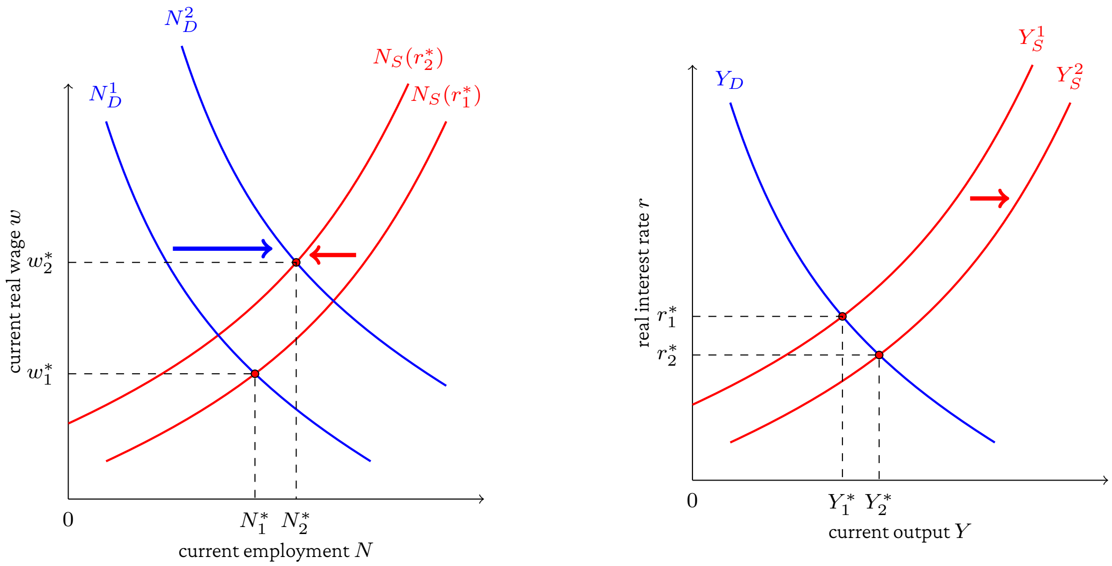
(C) $I^d \downarrow$ (D) I^d movement is ambiguous

9 Consider a rise in future total factor productivity z'. Which of the following figure correctly represents the movement of labor and goods market? A

(A)

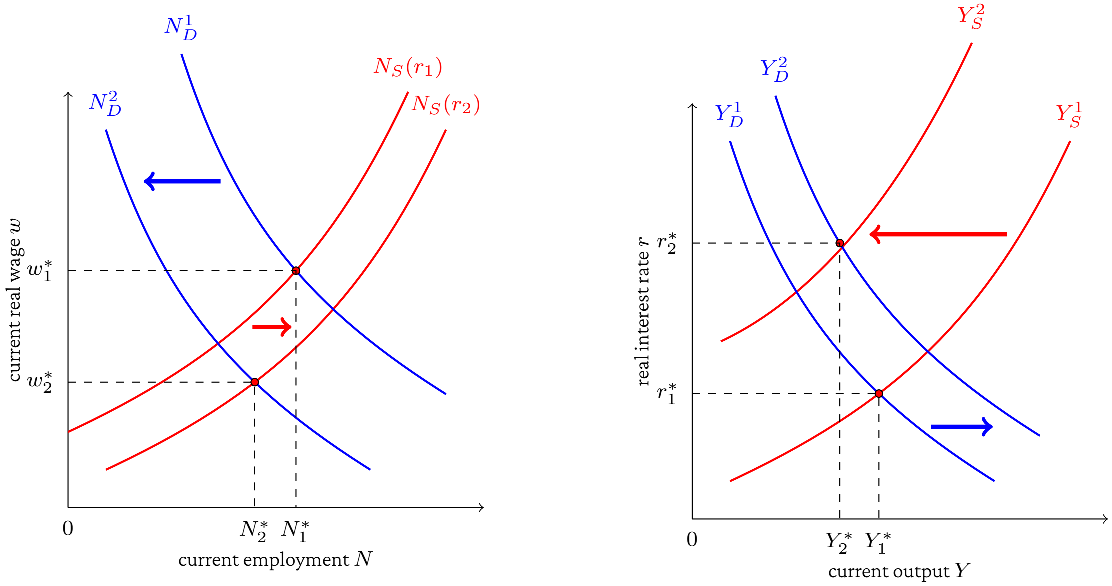


(B)



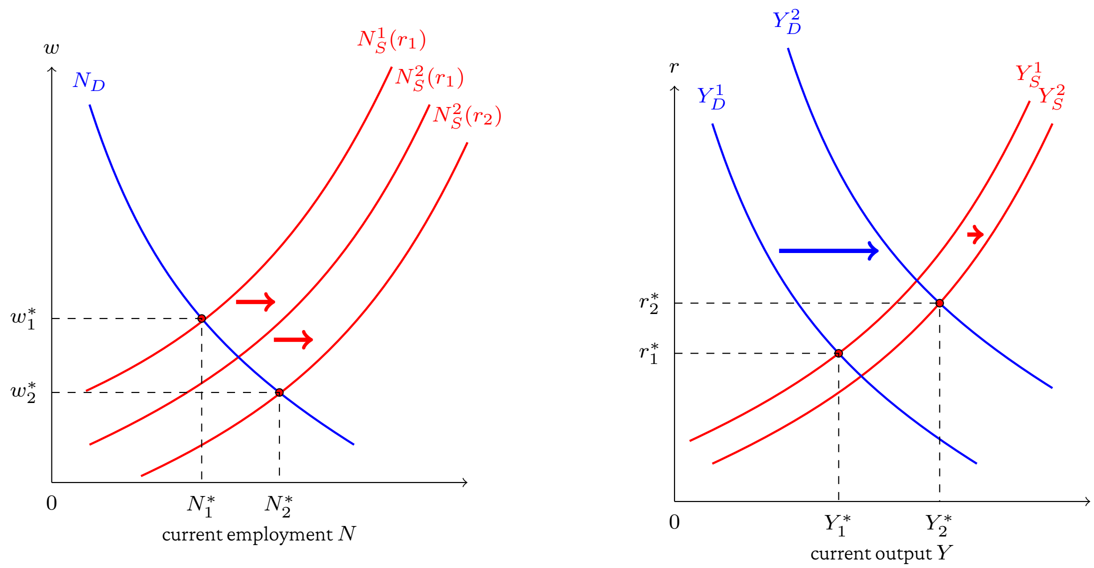
N_D^2
N_D^1
$N_S(r_2^*)$
$N_S(r_1^*)$
current real wage w
w_2^*
w_1^*
0
N_1^* N_2^*
current employment N
Y_D
Y_S^1
Y_S^2
real interest rate r
r_1^*
r_2^*
0
Y_1^* Y_2^*
current output Y

(C)



N_D^1
N_D^2
$N_S(r_1)$
$N_S(r_2)$
current real wage w
w_1^*
w_2^*
0
N_2^* N_1^*
current employment N
Y_S^2
Y_D^2
Y_D^1
Y_S^1
real interest rate r
r_2^*
r_1^*
0
Y_2^* Y_1^*
current output Y

(D)



(10) Using the same figures as in the choice of 9, consider a rise in current total factor productivity z. Which of the following figure correctly represents the movement of labor and goods market? __B__

(11) Using the same figures as in the choice of 9, consider a rise in government spending G. Which of the following figure correctly represents the movement of labor and goods market? __D__

(12) Using the same figures as in the choice of 9, consider a decrease in capital endowment K. Which of the following figure correctly represents the movement of labor and goods market? __C__

Question 2

Consider a model that is **similar to** (not exactly!) the Lecture 14 Consumer Problem, but there are three differences:

1. Consumers' utility function is given by $U(C, C', N_S, N_S') = \log C - bN_S + \log C' - bN_S'$
2. Consumers do **not** own the whole firm, i.e., $\pi = 0$. Instead, they buy shares of the firm s in date 0 to achieve intertemporal saving at per-unit price q. At date 1, consumers redeem their share to the firm and get s of reward.

3. Consumers are **not** subject to the lump-sum tax, i.e., $T = 0$.

Budget Constraint

Firstly, let's follow the slide and think about the consumer's budget constraint, you can refer to Lecture 14, slide 4.

13 there are $\underline{\quad A \quad}$ choice variables,

(A) 5 (B) 3 (C) 2 (D) 4

14 and they are $\{C, C', N_S, N'_S, \underline{\quad C \quad}\}$

(A) S (B) S' (C) s (D) s'

15 consumers own **part** of the firm and get $\underline{\quad B \quad}$ of reward

(A) π (B) s (C) π' (D) S

16 and they are taken the equilibrium price $\{w, w', \underline{\quad D \quad}\}$ as given.

(A) r (B) r' (C) q' (D) q

After defining all of the variables, consumer's budget constraints in each period are

17 date 0 budget constraints is $\underline{\quad A \quad}$

(A) $C + qs = wN_S$ (B) $C + S = wN_S + \pi - T$

(C) $C = wN_S + qs$ (D) $C = wN_S + \frac{s}{q} + \pi - T$

18 date 1 budget constraints is $\underline{\quad C \quad}$

(A) $C' = wN_S + \pi' - T' + (1+r)S$ (B) $C' = w'N'_S + qs$

(C) $C' = w'N'_S + s$ (D) $C' = w'N'_S + \frac{s'}{q'} + \pi' - T'$

(19) The lifetime budget constraint by combining date 0 and date 1 budget constraints is ___D___

(A) $C + \frac{C'}{1+r} = wN_S + \frac{w'N_S'}{1+r}$

(B) $C + \frac{C'}{1+r} = wN_S + \pi - T + \frac{w'N_S' + \pi' - T'}{1+r}$

(C) $C - qC' = wN_S - qw'N_S'$

(D) $C + qC' = wN_S + qw'N_S'$

Some calculation details:

$s = C' - w'N_S' \Rightarrow C + q(C' - w'N_S') = wN_S$

$\Rightarrow C + qC' = wN_S + qw'N_S'$

Preference

After finishing consumer's budget constraint, let's turn to the analysis preference:

(20) Accroding to the consumer's utility mentioned before, the derivative of consumer's utility function $U(C, C', N_S, N_S')$ with respect to current consumption C is ___A___

(A) $\frac{1}{C}$ (B) $\frac{1}{C'}$ (C) $\frac{C'}{C}$ (D) $\frac{C}{C'}$

(21) Similarly, the derivative of consumer's utility function $U(C, C', N_S, N_S')$ with respect to future consumption C' is ___B___

(A) $\frac{1}{C}$ (B) $\frac{1}{C'}$ (C) $\frac{C'}{C}$ (D) $\frac{C}{C'}$

(22) Similarly, the derivative of consumer's utility function $U(C, C', N_S, N_S')$ with respect to current labor supply N_S is ___D___

(A) $\frac{1}{N_S'}$ (B) $\frac{1}{N_S}$ (C) $-bN_S$ (D) $-b$

(23) Similarly, the derivative of consumer's utility function $U(C, C', N_S, N_S')$ with respect to future labor supply N_S' is ___C___

(A) $\frac{1}{N'_S}$ (B) $\frac{1}{N_S}$ (C) $-b$ (D) $-bN_S$

24. After deriving four derivatives of the utility function, consumer's marginal rate of substitution between C and C', $MRS_{C,C'}$ is __C__

(A) $\frac{1}{C}$ (B) $\frac{1}{C'}$ (C) $\frac{C'}{C}$ (D) $\frac{C}{C'}$

Some calculation details:

$$MRS_{C,C'} = \frac{u'(C)}{u'(C')} = \frac{1/C}{1/C'} = \frac{C'}{C}$$

25. Similarly, $MRS_{l,C} =$ __A__

(A) bC (B) bC' (C) $-bC$ (D) $-bC'$

Some calculation details:

$$MRS_{l,C} = -MRS_{N_S,C} = \frac{v'(N_S)}{u'(C)} = \frac{b}{1/C} = bC$$

26. Similarly, $MRS_{l',C} =$ __A__

(A) bC (B) bC' (C) $-bC$ (D) $-bC'$

Some calculation details:

$$MRS_{l',C} = -MRS_{N'_S,C} = \frac{v'(N'_S)}{u'(C)} = \frac{b}{1/C} = bC$$

Representative Consumer's Problem

Since in this model the share purchasing s is indeed the saving for the consumer, and consumer's share purchasing decision is implied by the combination of its consumption and labor supply decision, and thus in equilibrium, consumers are not choosing shares.

Consumer's Problem is to maximize utility function by choosing C, C', N_S, N'_S, subject to the lifetime budget constraint 19.

27. First step, we substitute C with all the other terms in the lifetime budget constraint and get __A__

(A) $\max_{C', N_S, N'_S} \log(wN_S + qw'N'_S - qC') - bN_S + \log C' - bN'_S$

(B) $\max_{C, N_S, N'_S} \log C - bN_S + \log(wN_S + qw'N'_S - qC) - bN'_S$

(C) $\max_{C, N_S, N'_S} \log C - bN_S + \log\left(\frac{wN_S + qw'N'_S - C}{q}\right) - bN'_S$

(D) $\max_{C', N_S, N'_S} \log(wN_S + qw'N'_S - qC) - bN_S + \log C' - bN'_S$

Note: read what should be substitute into other terms!

28. The FOC w.r.t. C' is $\underline{\quad C \quad}$

(A) $b = \frac{qw'}{wN_S + qw'N'_s - qC'}$ (B) $b = \frac{w}{wN_S + qw'N'_s - qC'}$

(C) $\frac{1}{C'} = \frac{q}{wN_S + qw'N'_S - qC'}$ (D) $\frac{1}{C} = \frac{1}{wN_S + qw'N'_S - qC'}$

29. The FOC w.r.t. N_S is $\underline{\quad B \quad}$

(A) $b = \frac{qw'}{wN_S + qw'N'_s - qC'}$ (B) $b = \frac{w}{wN_S + qw'N'_s - qC'}$

(C) $\frac{1}{C'} = \frac{q}{wN_S + qw'N'_S - qC'}$ (D) $\frac{1}{C} = \frac{1}{wN_S + qw'N'_S - qC'}$

30. The FOC w.r.t. N'_S is $\underline{\quad A \quad}$

(A) $b = \frac{qw'}{wN_S + qw'N'_s - qC'}$ (B) $b = \frac{w}{wN_S + qw'N'_s - qC'}$

(C) $\frac{1}{C'} = \frac{q}{wN_S + qw'N'_S - qC'}$ (D) $\frac{1}{C} = \frac{1}{wN_S + qw'N'_S - qC'}$

Question 3

Credit: Aubhik Khan

Note: In the lecture I only teach two-period model. This question is meant to be a taste for what graduate level of macroeconomics looks like. The infinite period model is what most contemporary macroeconomics model looks like, and this question would guide you to solve infinite period model.

Consider the Solow Growth Model. Labour productivity grows at the rate $\gamma > 0$, $X_{t+1} = (1+\gamma) X_t$, for $t = 0, 1, \ldots$, and population grows at the rate $n > 0$, $L_{t+1} = (1+n) L_t$. The effective labour force at date t is $N_t = X_t L_t$. Let aggregate production be given by

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha} \text{ where } 0 < \alpha < 1, \tag{1}$$

A is total factor productivity and K_t is the present capital stock. Total consumption is a constant fraction of output,

$$C_t = (1-s) Y_t, \tag{2}$$

where $0 < s < 1$ is the savings rate. There is full depreciation of the capital stock each period, $\delta = 1$. Thus, with I_t representing aggregate investment, the capital stock next period is

$$K_{t+1} = I_t. \tag{3}$$

The aggregate resource constraint is

$$C_t + I_t = Y_t. \tag{4}$$

31 Use (2) to eliminate C_t in (4) and solve for I_t in terms of Y_t as D

(A) sC_t (B) $(1-s)Y_t$ (C) $(1-s)C_t$ (D) sY_t

32 Substitute your result in 31 into (3) accumulation process and get K_{t+1} as D

(A) sC_t (B) $(1-s)Y_t$ (C) $(1-s)C_t$ (D) sY_t

Define capital per efficiency unit of labour as $k_t = \frac{K_t}{N_t}$ (so that $k_{t+1} = \frac{K_{t+1}}{N_{t+1}}$).

(33) Express $\frac{N_{t+1}}{N_t}$ using only labor productivity growth rate γ and population growth rate n as __B__

(A) γn (B) $(1+\gamma)(1+n)$

(C) $(1+\gamma)n$ (D) $\gamma(1+n)$

(34) Using your answer in 32 and express $\frac{K_{t+1}}{N_t}$ using Y_t as __D__

(A) $\frac{sC_t}{N_t}$ (B) $\frac{(1-s)Y_t}{N_t}$ (C) $\frac{(1-s)C_t}{N_t}$ (D) $\frac{sY_t}{N_t}$

(35) Find the law of motion of the efficiency unit of capital, i.e., the g function in $k_{t+1} = g(k_t)$ as __C__ (Hint: $k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{N_{t+1}}{N_t}\frac{K_{t+1}}{N_{t+1}}$.)

(A) $k_{t+1} = \frac{sA}{(1+\gamma)n}k_t^\alpha$ (B) $k_{t+1} = \frac{sA}{\gamma n}k_t^\alpha$

(C) $k_{t+1} = \frac{sA}{(1+\gamma)(1+n)}k_t^\alpha$ (D) $k_{t+1} = \left(\frac{sA}{(1+\gamma)(1+n)}k_t\right)^\alpha$

In the infinite period model, what we want to find is "steady state", which means that "the variables (called state variables) which define the behavior of the system or the process are unchanging in time." (from wikipedia)

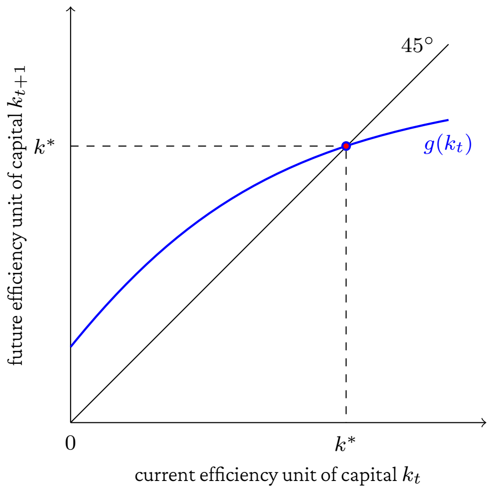
(36) Find the steady state efficiency unit of capital of this economy, k^*, is __B__ (Hint: not changing means $k_{t+1} = k_t = k^*$)

(A) $\left(\frac{sA}{\gamma n}\right)^{\frac{1}{1-\alpha}}$ (B) $\left(\frac{sA}{(1+\gamma)(1+n)}\right)^{\frac{1}{1-\alpha}}$

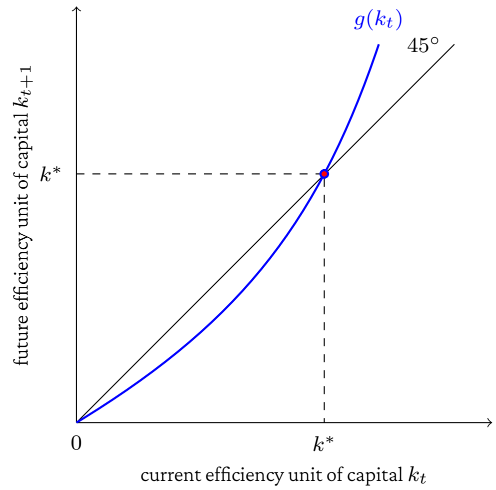
(C) $\left(\frac{sA}{(1+\gamma)n}\right)^{\frac{1}{1-\alpha}}$ (D) $\left(\frac{sA}{(1+\gamma)(1+n)}\right)^{\frac{\alpha}{1-\alpha}}$

(37) On a figure of k_t on the x-axis and k_{t+1} on the y-axis, which of the following figure correctly plots the $g(k_t)$ function? __D__

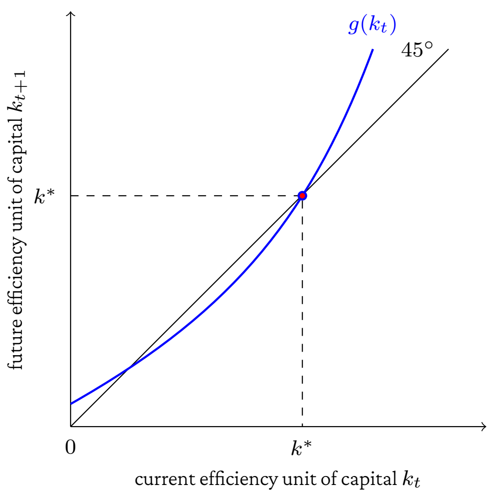
(A)



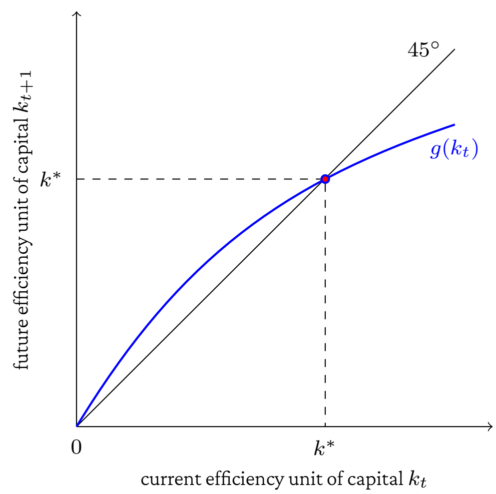
(B)



(C)



(D)



38 What is the 45° line means in question 37? B

(A) $k_{t+1} > k_t$ (B) $k_{t+1} = k_t$ (C) $k_{t+1} < k_t$

39 Among all the figures in the choice of question 37, which graph will the $g(k_t)$ function be if $\alpha > 1$? B

Consider two economies, a and b, where economy b has a higher savings rate, *and a higher rate of technological progress*, compared to economy a. In other words, $\gamma_b > \gamma_a$, and $s_b > s_a$. Moreover, assume that

$$\frac{s_b}{1+\gamma_b} = \frac{s_a}{1+\gamma_a},$$

and the rate of population growth is the same ($n_a = n_b$).

40 Denote the g function for both economy as $g_a(k_t)$ and $g_b(k_t)$, what is the relationship of two g function? __B__

(A) $g_a(k_t)$ is on top of $g_b(k_t)$, for all k_t

(B) $g_a(k_t)$ and $g_b(k_t)$ is the same curve

(C) $g_a(k_t)$ is below $g_b(k_t)$, for all k_t

(D) $g_a(k_t)$ intersects with $g_b(k_t)$