

# Lecture 16

## The Real Business Cycle Model

### Part 3: Competitive Equilibrium

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- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

# Review: Consumer's Problem

Taken  $\{w, w', r, T, T', \pi, \pi'\}$  as given, a representative consumer chooses  $\{C', N_S, N'_S\}$  to solve

$$\begin{aligned} \max_{C', N_S, N'_S} \quad & u \left( wN_S + \pi - T + \frac{w'N'_S + \pi' - T' - C'}{1 + r} \right), \\ & - v(N_S) + u(C') - v(N'_S) \end{aligned} \quad (1)$$

which we can back out  $C, S, l, l'$ .

# Review: Firm's Problem

Taken  $\{w, w', r\}$  as given, a representative firm chooses  $\{N_D, N'_D, K'\}$  to solve

$$\begin{aligned} \max_{N_D, N'_D, K'} \quad & zF(K, N_D) - wN_D - [K' - (1 - \delta)K] \\ & + \frac{z'F(K', N'_D) - w'N'_D + (1 - \delta)K'}{1 + r}, \end{aligned} \quad (2)$$

which we can back out  $Y, Y', \pi, \pi', I$

# Government Budget Constraint

Government behaves exactly the same in two-period model:

- current budget constraint:  $G = T + B$
- future budget constraint:  $G' + (1 + r)B = T'$
- lifetime budget constraint:  $G + \frac{G'}{1 + r} = T + \frac{T'}{1 + r}$

Taken  $\{r\}$  as given, government satisfy lifetime budget constraint by choosing  $\{T, T', B\}$ .

# Market Clear

There are three markets to clear:

① labor markets clear at each date determines wage:

- find  $w$  such that  $N_S = N_D$
- find  $w'$  such that  $N'_S = N'_D$

② goods markets clear at each date determines consumption and investment:

- date 0 (today):  $Y = C + I + G$
- date 1 (tomorrow):  $Y' = C' + I' + G'$

③ bonds market clears at date 0 determines real interest rate:

- find  $r$  such that  $S = B$

# Competitive Equilibrium: RBC Model

Given exogenous quantities  $\{G, G', z, z', K\}$ , a **competitive equilibrium** is a set of

- ① consumer choices

$$\{C, C', N_S, N'_S, l, l', S\},$$

- ② firm choices

$$\{Y, Y', \pi, \pi', N_D, N'_D, I, K'\},$$

- ③ government choices  $\{T, T', B\}$ ,  
and

- ④ prices  $\{w, w', r\}$

such that

- ① consumer solves problems in (1),

- ② firm solves problems in (2),

- ③ government balances its budget,  
and

- ④ all three markets clear.

# Plan to analyze the Model

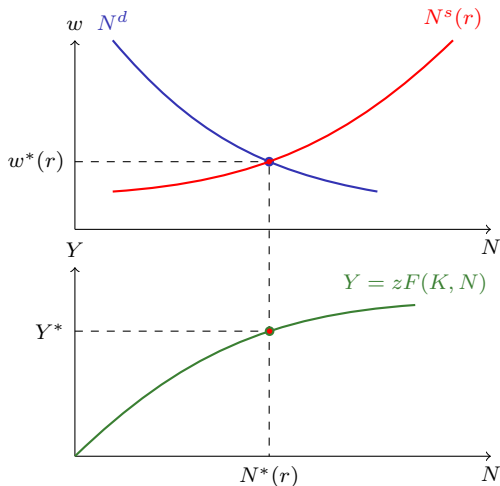
In the following slides, we are going to use graphical analysis on **two markets** in **current period**:

- ① **current labor market**: interaction of firm and consumer today
  - similar to static model: **labor supply** and **labor demand** curves
  - what's new: both curves reflect the dynamic tradeoff through interest rate
- ② **current goods market**: interaction of firm, consumer and government today
  - new: construct and analyze **output supply** and **output demand** curves



# The Current Labor Market

Figure 11.14 Determination of Equilibrium in the Labor Market Given the Real Interest Rate  $r$



■ **consumer optimality:** ceteris paribus,  $N^s \uparrow$  in  $w$

- **N1:**  $\frac{dN^s}{dw} > 0$ , substitution dominates income effect

■ **firm optimality:**  $N^d \downarrow$  in  $w$

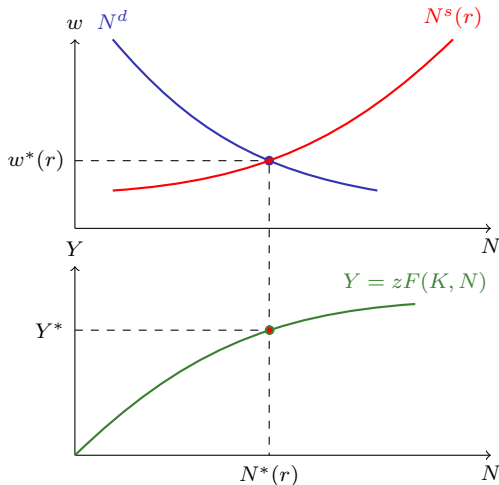
- $MPN = w$ ,  $\therefore$  diminishing MPN

■ account for **multiple markets:** intersect at  $N^*(r)$

- note: top figure is given  $r, \pi$
- labor mkt clearing  $w$  is  $w^*(r)$

# The Current Labor Market (Cont.)

Figure 11.14 Determination of Equilibrium in the Labor Market Given the Real Interest Rate  $r$



## ■ $r$ increases?

- **N2 (consumer):**  $N^s(r) \uparrow$  in  $r$   
 $\Rightarrow w \downarrow, N^*(r) \uparrow$

- **firm:**  $\because MPN = w$ , same

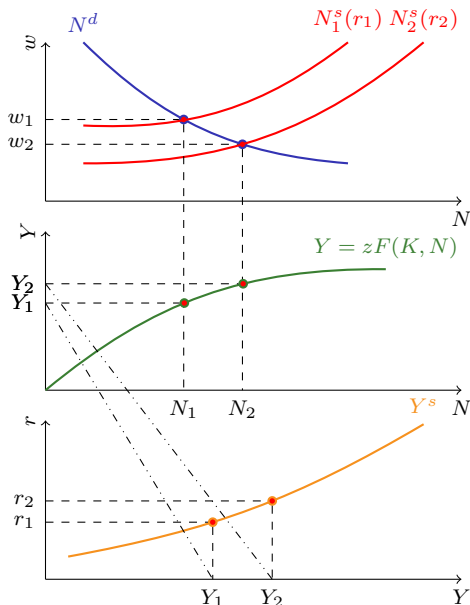
## ■ consumer wealth increases?

- **N3 (consumer):**  
 $N^s(r) \downarrow \Rightarrow w \uparrow, N^*(r) \downarrow$

- **firm:** nothing

## ■ Bottom chart: $N^*(r) \rightarrow Y^*(r)$ , output supply!

# The Output Supply Curve



Using our insight from labor market, we can repeat our analysis for any real interest rate  $r$

- Top: each  $r$  implies a different “labor market equilibrium”, i.e.,  $w$  &  $N$
- Middle: each  $N(r)$  yields production  $Y^S(r)$
- Bottom: combined to show  $\frac{dY^S}{dr} > 0$

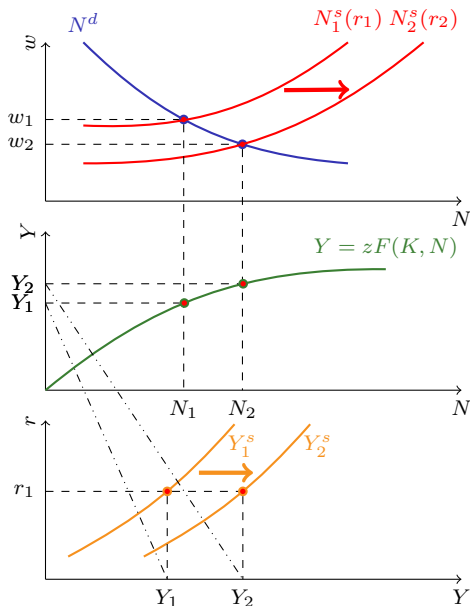
# Shifts in the Output Supply Curve

How do changes in exogenous variables shift  $Y^S(r)$ ? Consider 2 cases:

- ① shift in lifetime wealth (for example, by gov't spending or taxation)
- ② change in total factor productivity (TFP) or capital stock
  - recall static model: with  $K$  fixed, these have the same effect

In each case, we can start our analysis with the current labor market.

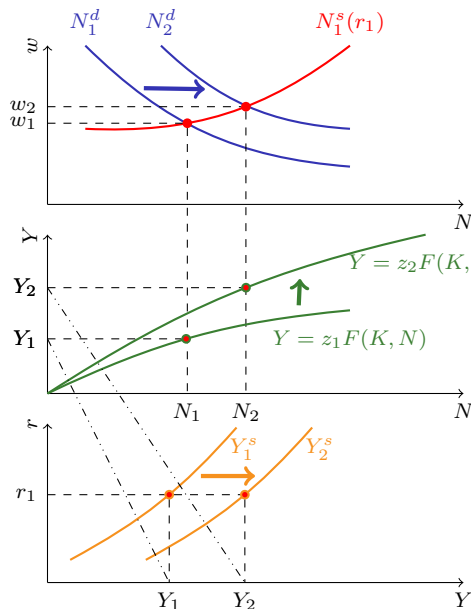
# Wealth and Output Supply



Suppose  $G \uparrow$  or  $G' \uparrow$ .

- gov. budget:  $T \uparrow$  or  $T' \uparrow$
- consumer budget:  $we \downarrow$
- **N3:**  $dN^S/d(we) < 0$ ,  $N^S(r) \uparrow$   
(shift to the right, top panel)
- Middle:  $N \uparrow \Rightarrow Y^S \uparrow$
- bottom: combine, get rightward shift in output supply

## TFP / Capital and Output Supply



Suppose TFP  $z \uparrow$ .

■ **firm optimality:**

$$MPN = z D_N F(\cdot) \uparrow \Rightarrow N^d \uparrow$$

■ **Top:**  $N^d$  shifts out  $\Rightarrow$   
 $w^* \uparrow, N^* \uparrow$

■ **Middle:** production fcn shifts  
 up,  $\because z \uparrow$

■ **Bottom:** combine, outward shift  
 in output supply

# Summary: Current Labor Market

We have constructed most of the model!

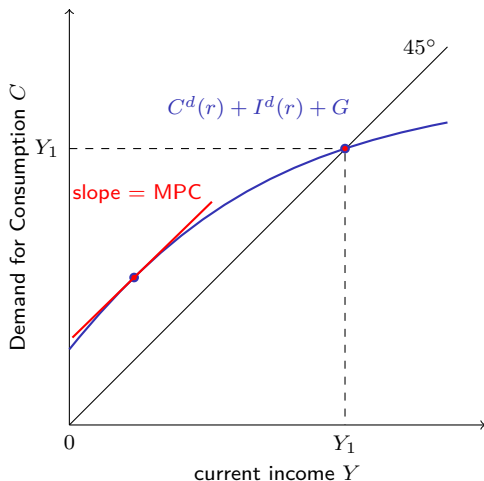
- **labor market clearing**, *conditional on* the interest rate
- trace through production function to get **output supply curve**

Now we need to determine the **equilibrium interest rate**,  $r^*$ .

- pair the **output supply curve** with the **output demand curve**
- who demands goods today, and how much?
  - consumer: consumption  $C^d(r, Y)$
  - firm: investment  $I^d(r)$
  - government: expenditures  $G$
  - use GDP accounting to get aggregate demand for goods

# Current Goods Demand

Figure 11.18 The Demand for Current Goods



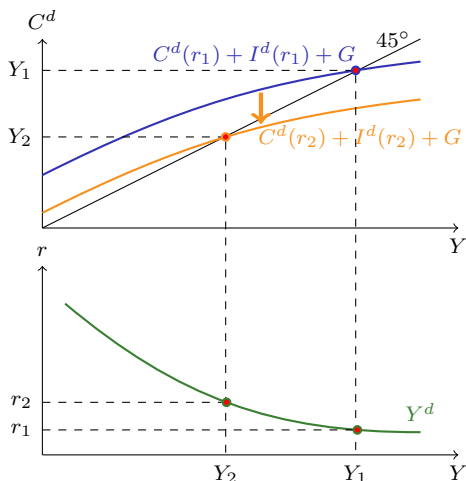
$$D(r, Y) = C^d(r, Y) + I^d(r) + G$$

- plot  $D(r, Y)$  on  $y$ -axis,  $Y$  on  $x$ -axis
- $C^d$  depends on wealth:  $w e = w N + \pi - T + \frac{w' N' + \pi' - T'}{1 + r}$ , which depends on income.
- Not true for  $I$  and  $G$ 
  - $MPC < 1$ : flatter than 45° line
  - $MPC$  diminishing: concave
  - combine: cross 45° line at  $Y^d(r)$



# Constructing Output Demand

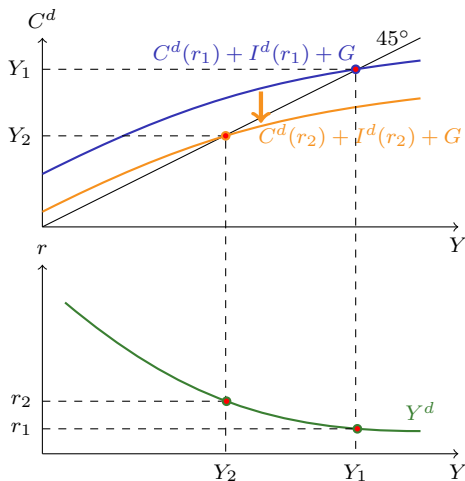
How different  $r$  affect output demand?



- **C2 (consumer):**  $C^d(r) \downarrow$  if  $r \uparrow$  (substitution effect dominates)
- **firm:** optimal investment schedule ( $r = MPK' - \delta$ ),  $r \uparrow \Rightarrow MPK' \uparrow \Rightarrow I^d \downarrow$
- **gov:** no change,  $\because G$  exogenous
- **Combine:**
  - ① intersection with 45° line is lower

# Constructing Output Demand (Cont.)

Combine:



① intersection with  $45^\circ \downarrow$

② **output demand curve  $Y^d$**   
downward sloping

$Y^d(r)$  shift to the right if

① present value of taxes  $\downarrow \Rightarrow C^d \uparrow$

② future income  $\uparrow \Rightarrow C^d \uparrow$

③ future TFP  $\uparrow \Rightarrow I^d \uparrow$

④ current capital  $\downarrow \Rightarrow I^d \uparrow$

Other changes (e.g., current TFP)  
are ambiguous in general!

# Competitive Equilibrium

Figure 11.21 The Complete Real Intertemporal Model

