Intermediate Macroeconomics Theory

Midterm Review

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Outline

Review of the CE and SPP

1. Competitive Equilibrium (CE) allows all agents to solve their own problems, given exogenous variables and variables determined by other agents
2. Social Planner's Problem (SPP) imposes a benevolent social planner that can directly dictate the allocation of variables, no trade occurs.
    - The most efficient outcome by design
    - Social planner only cares about aggregates: individuals add up to be aggregate
3. When certain condition holds, CE will be as efficient as SPP

Environment

1. Utility function is CRRA: $\frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$
2. Firm rent capital from consumer, and consumer is endowed with 2 unit of capital
3. Production function is Cobb-Douglas: $F(K, N) = K^a N^{1-a}$
4. TFP $z = 1$; consumer is endowed with (1) time $h = 1$ and (2) capital $K^s = 2$

Definition of Competitive Equilibrium

Underlined part is new

A competitive equilibrium given $\{G, z, \underline{K^s}\}$ is a set of allocations $\{Y^*, C^*, l^*, N^{s*}, N^{d*}, \pi^*, T^*, \underline{K^{d*}}\}$ and prices $\{w^*, \underline{r^*}\}$ such that

1. Taken prices w, r and π, T as given, representative consumer solves

$$\max_{C, l \in [0,h]} U(C, l) \quad \text{subject to} \quad C \leq w(h - l) + \underline{rK^s} + \pi - T \tag{1}$$

2. Taken prices w, r as given, the representative firm solves

$$\max_{N^d, K^d \geq 0} zF(K, N^d) - wN^d - \underline{rK^d} \tag{2}$$

3. Government set taxes to balance budget: $T^* = G$
4. Labor market clears: w^* such that $N^{s*} = N^{d*}$
5. <u>Capital market clears: r^* such that $K^s = K^{d*}$</u>

Optimality Conditions

Solve for firms' problem, you get the optimality conditions for N^d and K^d,

$$w = MPN = zK^a(1-a)N^{-a} \tag{3}$$

$$r = MPK = zN^{1-a}aK^{a-1} \tag{4}$$

Rewrite CE into SPP

- Welfare theorem holds, so CE allocation is as efficient as SPP
- Social planner cares about aggregate resources and technological constraint

$$\max_{C,l,N,Y,\underline{K}} \quad U(C(l),l) = \frac{C(l)^{1-b}}{1-b} + \frac{l^{1-d}}{1-d} \qquad \text{(utility function)}$$

$$\text{s.t.} \quad C = Y - G \qquad \text{(aggregate resource constraint)}$$

$$Y = zK^a N^{1-a} \qquad \text{(production constraint)}$$

$$N = 1 - l \qquad \text{(time constraint)}$$

$$K = 2 \qquad \text{(capital constraint)}$$

$$\Rightarrow \quad \max_{l} \quad \frac{(zK^a(1-l)^{1-a} - G)^{1-b}}{1-b} + \frac{l^{1-d}}{1-d} \tag{5}$$

$$\text{s.t.} \quad K = 2 \tag{6}$$

Solving SPP

$$\max_{l} \quad \frac{(zK^a(1-l)^{1-a}-G)^{1-b}}{1-b}+\frac{l^{1-d}}{1-d} \tag{7}$$

$$\text{FOC:} \quad \underbrace{(zK^a(1-l)^{1-a}-G)^{-b}}_{\frac{(\cdot)^{1-b}}{1-b}} \times \underbrace{(1-a)zK^a(1-l)^{-a}}_{zK^a(1-l)^{1-a}} \times \underbrace{(-1)}_{-l} + l^{-d} = 0 \tag{8}$$

$$G=0: \quad z^{-b}K^{-ab}(1-l)^{-b(1-a)} \times (1-a)zK^a(1-l)^{-a} = l^{-d} \tag{9}$$

$$(1-a)z^{1-b}K^{a-ab}(1-l)^{-a-b+ab} = l^{-d} \tag{10}$$

$$a=1/2; \quad b=2; \quad d=3/2 \tag{11}$$

$$\text{Apply:} \quad \frac{1}{2}z^{-1}K^{-\frac{1}{2}}(1-l)^{-\frac{3}{2}} = l^{-\frac{3}{2}} \Rightarrow (\frac{1-l}{l})^{\frac{3}{2}} = \frac{1}{2z\sqrt{K}} \equiv A(z,K) \tag{12}$$

$$\frac{1-l}{l} = (A(z,K))^{\frac{2}{3}} \Rightarrow (1+(A(z,K))^{\frac{2}{3}})l = 1 \Rightarrow l = \frac{1}{1+A(z,k)^{\frac{2}{3}}} \tag{13}$$

Retrive aggregates

After solve for optimal leisure, l^*,

1. Solve for optimal labor with $N^* = 1 - l^*$
2. Solve for optimal output with $Y^* = zK^a(N^*)^{1-a}$
3. Solve for optimal consumption with $C^* = Y^*$ $(G = 0)$
4. Solve for optimal wage with $w = MPN$
5. Solve for optimal rental rate with $r = MPK$

Outline

Review of Distorting Tax

1. Labor tax violates the welfare theorems by introducing wedges in consumption-leisure decisions
   - $MRS_{l,C} = w \underbrace{(1 - t)}_{\text{wedges}} < w = MRT_{l,C} = MPN$

2. Gov collect labor tax revenue, $R(t) = wtN$, to pay for gov spending $(G = R(t))$

3. Gov's objective may be pay fix amount of exogenous G or maximizing revenue $R(t)$

Environment

- Production function: $Y = zN^a$
- Utility: $U(C, N) = \ln C - bN$
- Labor tax is paid by households

Optimality Conditions

- $D_C U(C,N) = \frac{1}{C}$
- $D_N U(C,N) = -b$
- $MRS_{N,C} = -bC$
- $MRS_{l,C} = -MRS_{N,C} = bC$
- Consumer's optimality condition: $MRS_{l,C} = bC = w(1-t)$
- Firms' optimality conditions:
  - $w = MPN = azN^{a-1} \Rightarrow wN = azN^a = aY$
  - $\pi = Y - wN = (1-a)zN^a = (1-a)Y$

Solve for optimal labor given labor tax rate

$$bC = w(1-t) \tag{14}$$

$$b[w(1-t)N + \pi] = w(1-t) \tag{15}$$

$$b[azN^a(1-t) + (1-a)zN^a] = azN^{a-1}(1-t) \tag{16}$$

$$zN^a b[a(1-t) + (1-a)] = azN^{a-1}(1-t) \tag{17}$$

$$Nb[a(1-t) + (1-a)] = a(1-t) \tag{18}$$

$$N = \frac{a(1-t)}{b[a(1-t) + (1-a)]} \tag{19}$$

Labor tax revenue

- $w(t) = azN^{a-1} = az\left(\frac{a(1-t)}{b[a(1-t)+(1-a)]}\right)^{a-1}$
- $R(t) = w(t)N(t)t = azN^{a}t = az\left(\frac{a(1-t)}{b[a(1-t)+(1-a)]}\right)^{a} \times t$
- Recall: gov's objective may be pay exogenous G or maximizing revenue $R(t)$
  - Given a $G = \bar{G}$, there are two tax rates, t_1 and t_2, that can fulfill the same $\bar{G}$
    - t_1 is less distortionary than t_2
  - Maximizing $R(t)$ requires $\max_t R(t) = taz\left(\frac{a(1-t)}{b[a(1-t)+(1-a)]}\right)^{a}$, which may be hard to solve
- Either way, we can solve it numerically using Julia.

Julia Code i

```julia
# parameters
a = 0.33
b = 2.15
z = 1
t = 0.5

# implicit functions
labor(a, b, t) = (a*(1-t)) / (b*(a*(1-t) + (1-a)))
wage(a, z, N) = a*z*N^(a-1)
gov(w, t, N) = w*t*N

## find the G level at tax = 0.5
N = labor(a, b, t)
w = wage(a, z, N)
G = gov(w, t, N)
Gtarget = G

## iterate all possible tax value and calculate corresponding G value
tnum = 1000
tvec = collect( range(0.0001, 0.999, tnum) )
Gvec = Array{Float64, 1}(undef, tnum)
```

```julia

for indt = 1:1:tnum
    local t, N, w, G
    t = tvec[indt]
    N = labor(a, b, t)
    w = wage(a, z, N)
    G = gov(w, t, N)
    Gvec[indt] = G
    if (abs(G - Gtarget) < 0.0001)
        println("Potential answer for Q42: \
                At tax = $t, G = $G, \
                Target - G = $(G - Gtarget)")
    end
end

Gmax = maximum(Gvec)
Gmaxidx = argmax(Gvec)
Tmax = tvec[Gmaxidx]

println("Q43: maximum G is achieved at tax rate $Tmax")
println("Q44: maximum G is $Gmax")
```