Problem Set 2

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Instruction

Due at 11:59 PM (Eastern Time) on Sunday, June 14, 2022.

Please answer this problem set on Carmen quizzes "Problem Set 2". In the following problems, the part that is in **red and bold** are the order of questions that should be answered on Carmen quizzes.

Problem 1

Remember the Example in Lecture 8.

Consumer: $\max_{C,l} \ln C + \ln l \quad$ subject to $\quad C \leq w(1-l) + \pi$

$$\text{FOC} \quad \frac{C}{l} = w \tag{1}$$

$$\text{Binding budget constraint} \quad C = w(1-l) + \pi \tag{2}$$

$$\text{Time constraint} \quad N^s = 1 - l \tag{3}$$

Firm: $\max_{N^d} (N^d)^{\frac{1}{2}} - wN^d$

$$\text{FOC} \quad \frac{1}{2}(N^d)^{-\frac{1}{2}} = w \tag{4}$$

$$\text{Output definition} \quad Y = (N^d)^{\frac{1}{2}} \tag{5}$$

$$\text{Profit definition} \quad \pi = Y - wN^d \tag{6}$$

Market clear:

$$N^s = N^d \tag{7}$$

Fill the following blanks for the step-by-step guide for algebraic calculation:

1. Step 1: Impose Market clear condition, so shrink all 7 equations to **Q1** ______ equations

Consumer: $\max_{C,l} \ln C + \ln l \quad$ subject to $\quad C \leq w(1-l) + \pi$

$$\text{FOC} \quad \frac{C}{l} = w \tag{8}$$
$$\text{Binding budget constraint} \quad C = w(1-l) + \pi \tag{9}$$
$$\text{Time constraint} \quad N = 1 - l \tag{10}$$

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

$$\text{FOC} \quad \frac{1}{2}(N)^{-\frac{1}{2}} = w \tag{11}$$
$$\text{Output definition} \quad Y = (N)^{\frac{1}{2}} \tag{12}$$
$$\text{Profit definition} \quad \pi = Y - wN \tag{13}$$

2. Step 2: replace l in terms of N using $l = 1 - N$

Consumer: $\max_{C,l} \ln C + \ln l \quad$ subject to $\quad C \leq w(1-l) + \pi$

$$\text{FOC} \quad \frac{C}{(\underline{\quad \textbf{Q2} \quad})} = w \tag{14}$$
$$\text{Binding budget constraint} \quad C = w(\underline{\quad \textbf{Q3} \quad}) + \pi \tag{15}$$

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

$$\text{FOC} \quad \frac{1}{2}(N)^{-\frac{1}{2}} = w \tag{16}$$
$$\text{Output definition} \quad Y = (N)^{\frac{1}{2}} \tag{17}$$
$$\text{Profit definition} \quad \pi = Y - wN \tag{18}$$

3. Step 3: replace π and Y as N

Consumer: $\max_{C,l} \ln C + \ln l \quad$ subject to $\quad C \leq w(1-l) + \pi$

$$\text{FOC} \quad \frac{C}{(\underline{\quad \textbf{Q2} \quad})} = w \tag{19}$$
$$\text{Binding budget constraint} \quad C = w(\underline{\quad \textbf{Q3} \quad}) + \pi \tag{20}$$

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

$$\text{FOC} \quad \frac{1}{2}(N)^{-\frac{1}{2}} = w \tag{21}$$

$$\text{Profit definition} \quad \pi = (\underline{\quad \textbf{Q4} \quad}) - wN \tag{22}$$

4. Step 4: Substitute $\pi(N)$ into Binding budget constraint and get

$$C = (\underline{\quad \textbf{Q5} \quad}) \tag{23}$$

5. Step 5: With consumer's FOC and firm's FOC both equate to w, we can get another expression of C:

$$C = (\underline{\quad \textbf{Q2} \quad}) \times (\underline{\quad \textbf{Q6} \quad}) \tag{24}$$

6. Step 6: Let (23) equate (24) and we get N as

$$N = (\underline{\quad \textbf{Q7} \quad}) \tag{25}$$

7. Step 7: Trace back to all unknowns given the value of N, we get

$$C = (\underline{\quad \textbf{Q8} \quad}) \tag{26}$$

$$l = (\underline{\quad \textbf{Q9} \quad}) \tag{27}$$

$$Y = (\underline{\quad \textbf{Q10} \quad}) \tag{28}$$

$$\pi = (\underline{\quad \textbf{Q11} \quad}) \tag{29}$$

$$w = (\underline{\quad \textbf{Q12} \quad}) \tag{30}$$

Problem 2

Credit: Sungmin Park

Suppose that the only consumption good in the state of Ohio is soybeans, and suppose that Ohio's output Y of number of soybeans is determined by the following production function

$$F(K, L) = zK^{\frac{1}{3}}L^{\frac{2}{3}}, \tag{31}$$

where z is TFP, K is its current stock of capital, and L is the size of its current labor force. Suppose also that the current technological level is $z = 1$, the current stock of capital is 1,000, and the current labor force is 8,000.

(a) At the current technological level, capital stock, and labor force, what is the total output of soybeans? What is the marginal product of capital? What is the marginal product of labor?

$MPK =$ **Q13**

$MPN =$ **Q14**

Suppose that the markets for capital and labor are competitive—that is, there are many firms (soybean farms) in Ohio, so that they are price-takers.

(b) What is the real rental rate of capital? What is the real wage? How many soybeans go to the capitalists? (i.e. what is the capital income?) How many soybeans go to the workers? (i.e. what is the labor income?) What is the total income?

rental rate $R =$ **Q15**

wage $w =$ **Q16**

Capital income $R \times K$ **Q17**

Labor income $w \times L$ **Q18**

Total income $R \times K + w \times L$ **Q19**

Now suppose that the demand for soybeans in Ohio are as follows. First, the demand for the consumption of soybeans is given as

$$C = 0.5(Y - T) + 500, \tag{32}$$

where T is the number of soybeans taxed by the Ohio state government. Second, the demand for investing soybeans for future production is given as

$$I = 1500 - 100r, \tag{33}$$

where r is the real interest rate in percent. Finally, the demand for soybeans from the Ohio state government is as the government collect the same amount as taxes T and spend it on running the Ohio State University.

(c) What are the equilibrium interest rate, investment, and consumption?

$r =$ **Q20**

$I =$ **Q21**

$C =$ **Q22**

(d) Suppose the Ohio state government reduces the taxation to $T = 200$ while maintaining the same spending of $G = 400$, as a part of COVID-19 stimulus package. What is the government surplus/deficit? What is the equilibrium interest rate, investment, and consumption after this government policy?

government surplus/deficit: $T - G =$ **Q23**

$r =$ **Q24**

$I =$ **Q25**

$C =$ **Q26**

Suppose instead that Ohio state residents' consumption demand also depends on the real interest rate:

$$C = 0.5(Y - T) + 600 - 50r. \tag{34}$$

That is, for every percentage point increase in the interest rate, the Ohio state resident would rather not consume 50 soybeans and save them instead.

(e) Analyze again the effects of reduced taxation from $T = 400$ to $T = 200$, using the new consumption demand but using everything else the same as (d)

government surplus/deficit: $T - G =$ **Q27**

$r =$ **Q28**

$I =$ **Q29**

$C =$ **Q30**