

Lecture 11

Distorting Taxes and the Welfare Theorems

Hui-Jun Chen

The Ohio State University

June 14, 2022

Overview

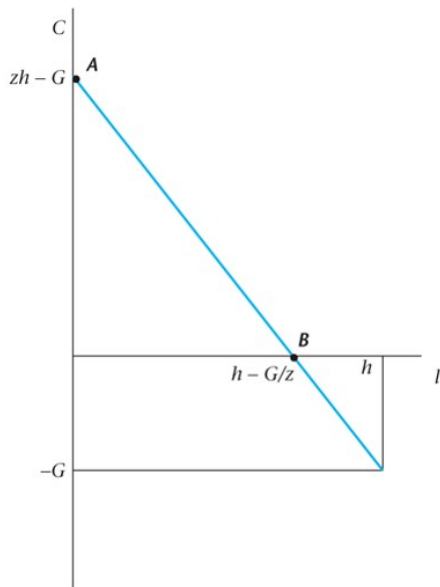
In previous lectures, all the taxes we are discussing is **lump-sum tax**.

- pure **income effect**, no change to consumption-leisure allocation
- satisfy both welfare theorems

In this lecture, the **distorting taxes** will include **substitution effect**, and thus

- creating “wedges” to distort consumption-leisure choice
- violate the welfare theorems ($CE \neq SPP$)

SPP in Simplified Model



Assume production is labor-only technology:

$$Y = zN^d$$

So PPF is

$$C = z(h - l) - G$$

Thus, SPP is

$$\max_l U(z(h - l) - G, l)$$

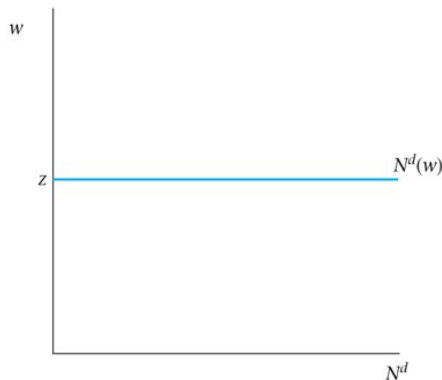
$$\text{FOC: } \frac{D_l U(C, l)}{D_C U(C, l)} = MRS_{l, C}$$

$$= MRT_{l, C} = z = MPN$$

Labor Demand in Simplified Model

$$\max_{N^d} zN^d - wN^d$$

Figure 5.15 The Labor Demand Curve in the Simplified Model



FOC would be $z = w$ (horizontal line)

- if $z < w$: negative profit for every worker hired, choose $N^d = 0$
- if $z > w$: positive profit for every worker hired, choose $N^d = \infty$
- only $z = w$ possible, \therefore linear PPF in previous slide
 - “infinitely elastic” N^d

Competitive Equilibrium w/ Distorting Tax

A competitive equilibrium, with $\{z, G, K\}$ exogenous, is a list of endogenous prices and quantities $\{C, l, N^s, N^d, Y, \pi, w, t\}$ such that:

- ① taking $\{w, \pi\}$ as given, the consumer solves

$$\max_{C, l, N^s} U(C, l) \quad \text{subject to} \quad C = w(1 - t)N^s + \pi \quad \text{and} \quad N^s + l = h$$

- ② taking w as given, the firm solves:

$$\max_{N^d, Y, \pi} \pi \quad \text{subject to} \quad \pi = Y - wN^d \quad \text{and} \quad Y = zN^d$$

- ③ the government spends $G = wtN^s$

- ④ the labor market clears at the equilibrium wage, i.e. $N^s = N^d$

Effect of Distorting Tax

Since the tax is imposed on consumers/workers, it distorted the consumption-leisure decision:

$$MRS_{l,C} = w(1 - t)$$

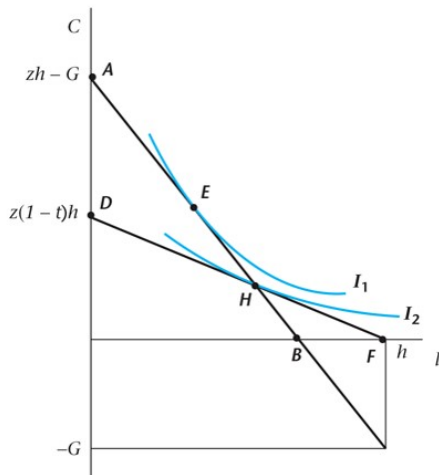
So in the equilibrium, it deviates from SPP:

$$MRS_{l,C} = w(1 - t) < w = z = MPN = MRT_{l,C}$$

Result: CE and SPP lead to different allocation!

Graphical Representation

Figure 5.16 Competitive Equilibrium in the Simplified Model with a Proportional Tax on Labor Income



SPP solution lies at point E:

- \overline{AB} : PPF, slope $-z$
- can reach indifference curve I_1

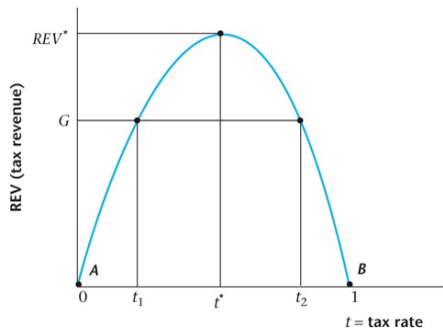
CE solution lies at point H:

- \overline{DF} : consumer's budget line
- can only reach I_2
- proportional tax $\Rightarrow N^s \downarrow$
- $N^s \downarrow \Rightarrow Y \downarrow$, but still need to meet G , so $C \downarrow$: gov't budget critical!

How Much Tax Revenue can be Generated?

equilibrium wage: $w = z$, implies **total tax revenue** by solve consumer problem:

Figure 5.17 A Laffer Curve



$$R(t) = tz(h - l^*(t)),$$

What t maximizes? Solve

$$\max_t R(t) = \max_t tz(h - l^*(t)),$$

- not just $t = 1$! tax **rate** vs tax **base**
- $t = 0$: no revenue because no tax
- $t = 1$: no revenue because no incentive to work

Full Model Elaboration

Let $U(C, l) = \ln C + \ln l$, and $h = z = 1$, by firm's problem we know $w = z = 1$. Consumer has some non-labor income denoted as $x > 0$. FOC leads to

$$\begin{aligned}MRS_{l,C} &= \frac{C}{l} \\&= \frac{(1-t)(1-l) + x}{l} = 1 - t < 1 = MRT_{l,C} \\&\Rightarrow (1-t)(1-l) + x = (1-t)l \\&\Rightarrow 1 - l + \frac{x}{1-t} = l \Rightarrow 2l = 1 + \frac{x}{(1-t)} \\&\Rightarrow l = \frac{1}{2} + \frac{x}{2(1-t)} \\&\Rightarrow N^s(t) = 1 - l = \frac{1}{2} - \frac{x}{2(1-t)}\end{aligned}$$

Maximize Tax Revenue

Total tax revenue is

$$R(t) = tN^s(t),$$

and thus government's problem is

$$\max_t \frac{1}{2}t - \frac{tx}{2(1-t)}.$$

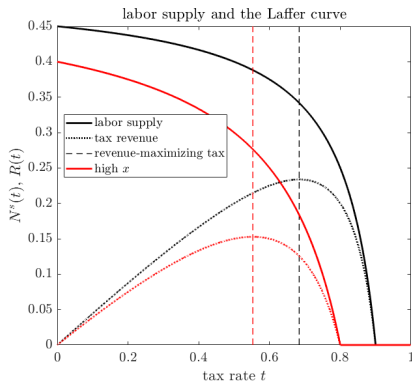
FOC leads to

$$\frac{1}{2} - \frac{x(1-t) + tx}{2(1-t)^2} = 0 \Rightarrow \frac{1}{2} - \frac{x}{2(1-t)^2} = 0$$

$$\frac{1}{2} = \frac{x}{2(1-t)^2} \Rightarrow 1 = \frac{x}{(1-t)^2}$$

$$t = 1 - \sqrt{x}$$

Visualization



Consider two cases:

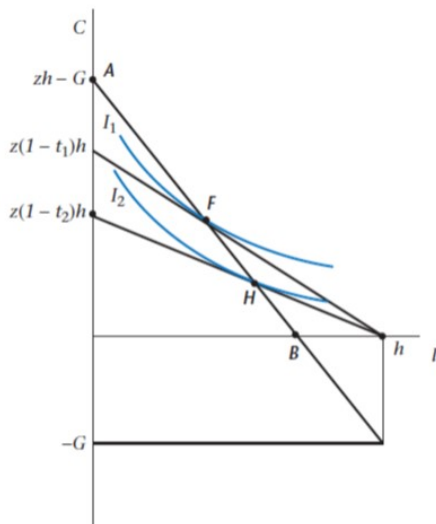
- ① consumer is poor (low x)
- ② consumer is rich (high x)

For a given after tax-wage, rich consumer supplies less labor

- tax revenue shifts down
- Laffer peak shifts left
- many other conditions also impact this analysis!

Multiple Competitive Equilibria Possible

Figure 5.18 Two Competitive Equilibria



Previous slide logic implies the government can choose 2 tax rates for a given required level of G

- both t_1 and t_2 yield the same revenue
- consumer strictly better off under lower tax rate t_1

Tax Revenue

Conclusion

We've focused on the simple case to keep analysis straightforward, but logic applies more broadly.

- SPP: $MRS_{l,C} = MRT_{l,C} = MPN$, since PPF is $C = zF(K, N) - G$
- CE: same distortion as our simple case:
 - consumer problem implies $MRS_{l,C} = w(1 - t)$
 - firm problem implies $MRT_{l,C} = w$
 - same result as simplified model: $MRS_{l,C} \neq MRT_{l,C}$, unlike SPP
 - only difference from simplified model: $MPN = D_N F(K, N) \neq z$