

ECON 4002.01 Final Exam

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Question 1

Consider the two-period dynamic general equilibrium model with a representative consumer, representative firm and government. The consumer values consumption and leisure in each period, C and l , and provides labour, N_S , in return for a real wage, w . The consumer pays lump-sum taxes T each period and receives all profits from the firm, π .

The representative firm uses labour and capital, N_D and K , to produce output. In the first period, it also chooses investment, I . This determines its capital stock for production in the second period, K' , through the capital accumulation equation $K' = (1 - \delta) K + I$.

The consumer's preferences are

$$U(C, C', N, N') = u(C) - v(N_S) + u(C') - v(N'_S),$$

and the firm's technology is

$$Y = zF(K, N) = zK^\alpha N^{1-\alpha}, \text{ where } \alpha \in (0, 1).$$

Lastly, recall that the government must balance its budget across the two periods, $G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$, where G is government spending and T are taxes.

- ① Assume assumption N1 holds, i.e., substitution effect dominates income effect from a change in real wage, then given interest rate r , consumer will choose the quantity of labor supply N_S by _____

(A) $MRS_{l,C} = w$

(B) $MRS_{C,C'} = r$

(C) $MRS_{l,C} = r$

(D) $MRS_{C,C'} = w$

- ② where the MRS in question 1 is _____

$$(A) \quad MRS_{l,C} = \frac{u'(C)}{u'(C')}$$

$$(B) \quad MRS_{C,C'} = \frac{u'(C)}{u'(C')}$$

$$(C) \quad MRS_{l,C} = \frac{v'(N_S)}{u'(C)}$$

$$(D) \quad MRS_{C,C'} = \frac{v'(N_S)}{u'(C)}$$

③ Following assumption N2, the slope of the labor supply is _____ in wage and thus the labor supply curve has _____ slope

(A) decreasing; positive

(B) increasing; negative

(C) decreasing; negative

(D) increasing; positive

④ Following N1, how does the labor supply curve response to a rise in real interest rate r ? _____

(A) shift to the left

(B) shift to the right

(C) not affected

(D) ambiguous

⑤ In the firm's labor demand, what is the equation that can determine the labor demand curve? _____

$$(A) \quad MPN = w$$

$$(B) \quad MPK = r$$

$$(C) \quad MPN = r$$

$$(D) \quad MPK = w$$

⑥ How does a fall in total factor productivity z affect the equilibrium in labor market? _____

(A) labor supply will shift to the left; labor demand will shift to the right

(B) labor supply will shift to the right; labor demand will shift to the left

(C) labor supply will shift to the right; labor demand not shift

(D) labor supply will not shift; labor demand will shift to the left

⑦ For the goods demand, what is the optimal investment schedule? _____

(A) $MPN' - \delta = w$

(B) $MPN' - \delta = r$

(C) $MPK' - w = r$

(D) $MPK' - \delta = r$

⑧ how does a rise in the real interest rate r changes the investment? _____

(A) $I^d \uparrow$

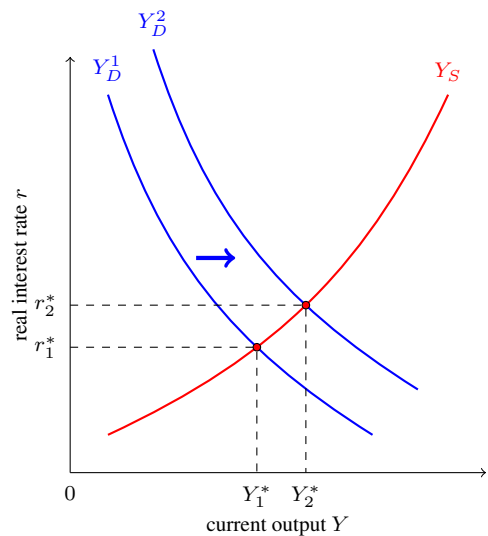
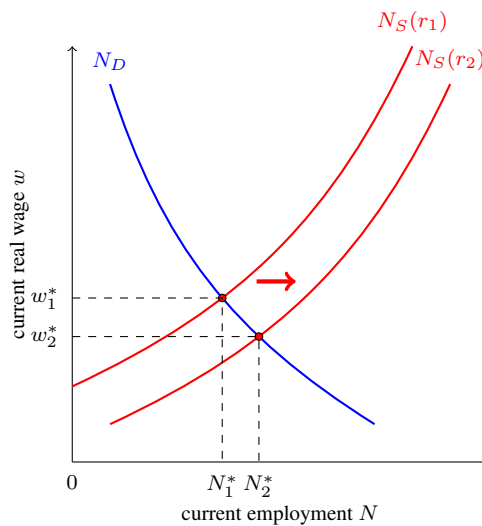
(B) I^d unchanged

(C) $I^d \downarrow$

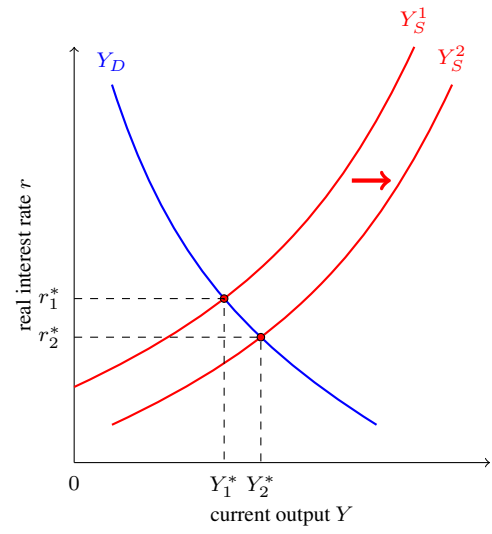
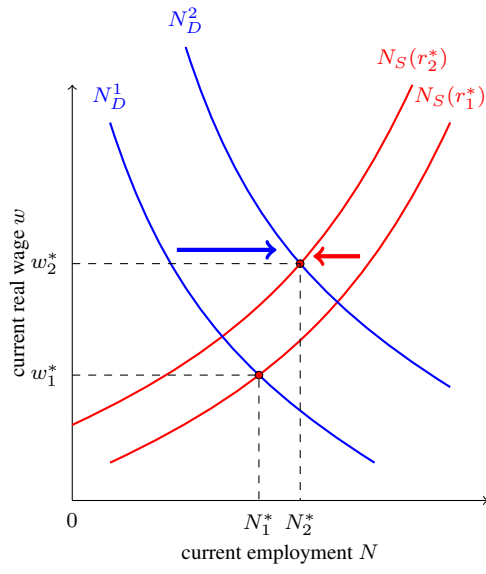
(D) I^d movement is ambiguous

⑨ Consider a rise in future total factor productivity z' . Which of the following figure correctly represents the movement of labor and goods market? _____

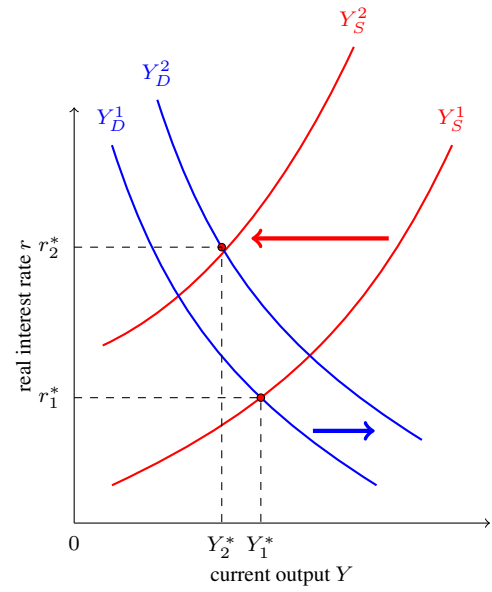
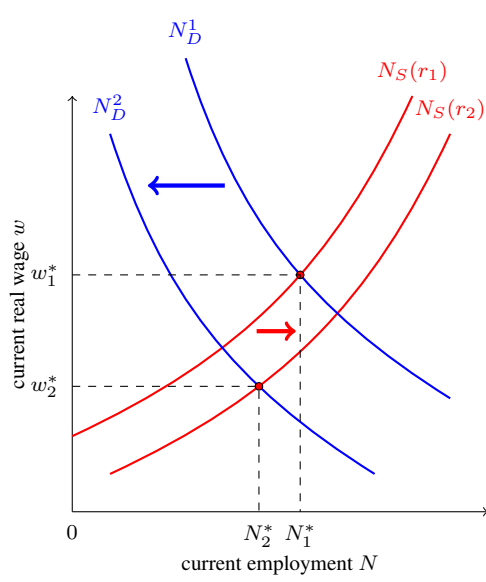
(A)



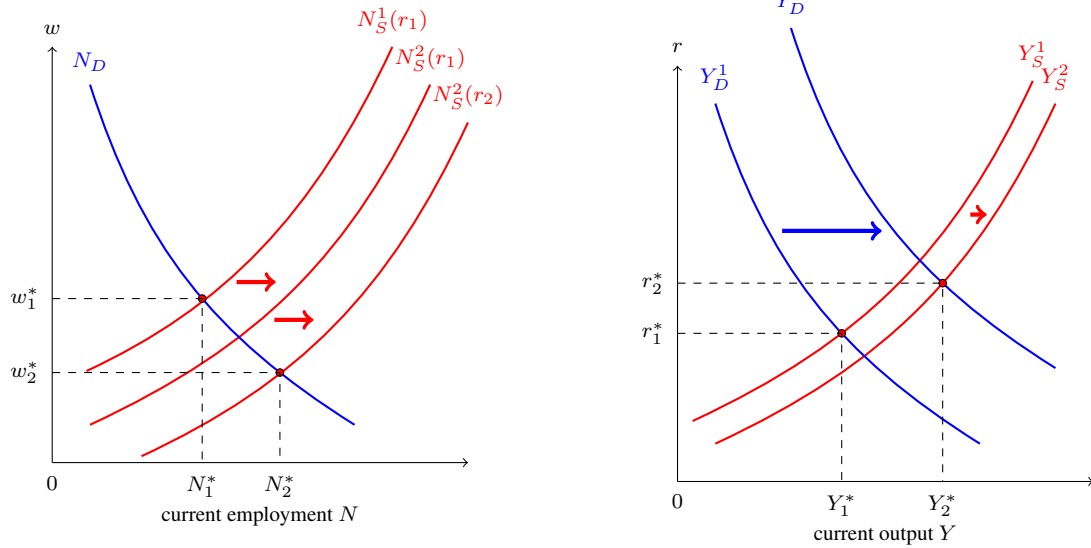
(B)



(C)



(D)



- ⑩ Using the same figures as in the choice of 9, consider a rise in current total factor productivity z . Which of the following figure correctly represents the movement of labor and goods market? _____
- ⑪ Using the same figures as in the choice of 9, consider a rise in government spending G . Which of the following figure correctly represents the movement of labor and goods market? _____
- ⑫ Using the same figures as in the choice of 9, consider a decrease in capital endowment K . Which of the following figure correctly represents the movement of labor and goods market? _____

Question 2

Reference: Lucas human capital accumulation model (1988 JME)

Credit: Julia K. Thomas

Consider a two-period general equilibrium model, where human capital are accumulated by **spending time in education** rather than **purchasing using output goods**.

- the utility function is given by $U(C, C') = u(C) + u(C')$, i.e., consumer doesn't value leisure.
- households are endowed with H of current human capital at date 0, and they accumulate future human capital H' by **spending $1 - \phi$ fraction of their time endowment to education**. The law of motion for human capital is given by

$$H' = H + (1 - \phi)H, \quad (\text{A})$$

where $1 - \phi$ is the fraction of the time endowment that goes to education so that households can accumulate human capital.

- households are endowed with K amount of capital, and determine the investment at date 0 to determine their future capital K' at date 1. The usage of these capital for consumer is to rent to the firm and earn the per-unit rent r . The law of motion for physical capital is given by

$$K' = (1 - \delta)K + I. \quad (\text{B})$$

- Firm's production function is given by

$$Y = K^\alpha (\phi H)^{1-\alpha}; Y' = K'^\alpha (\phi' H')^{1-\alpha}; \quad (\text{C})$$

Firm pays the per-unit wage w for the labor supplied by the households and pays per-unit capital renting fee r to consumers.

- Consumer owns the whole firm, and claims the whole profit π .
- There's no government in this model, i.e., $G = G' = T = T' = B = 0$.

First, let's construct the budget constraint for consumers.

- ⑬ Consider the current budget constraint, what is the labor income for consumer?

(A) rK (B) $w\phi H$ (C) wK (D) $r\phi H$

- ⑭ Consider the current budget constraint, what is the capital income for consumer?

(A) rK (B) $w\phi H$ (C) wK (D) $r\phi H$

- ⑮ What is the current budget constraint for consumer? _____
- (A) $C \leq wH + r\phi K - I + \pi$ (B) $C \leq w\phi H + r\phi K - I + \pi$
 (C) $C \leq wH + rK - I + \pi$ (D) $C \leq w\phi H + rK - I + \pi$
- ⑯ What is the profit for the firm? _____
- (A) $\pi = Y - wH - rK$ (B) $\pi = Y - w\phi H - r\phi K$
 (C) $\pi = Y - w\phi H - rK$ (D) $\pi = Y - w\phi H - r\phi K$
- ⑰ In this economy, does the competitive equilibrium and social planner's problem generate the same result? Why? _____
- (A) No, because the first welfare theorem doesn't holds.
 (B) Yes, because the first welfare theorem holds.
 (C) Yes, because the first welfare theorem don't holds.
 (D) No, because the first welfare theorem holds.

Let's solve this model using the social planner's problem.

- ⑱ Combine your answers in 15 and 16, in the perspective of social planner, we can rewrite household's current budget constraint as _____
- (A) $C \leq Y - I$ (B) $C \leq Y - rK - w\phi H - I$
 (C) $C \leq Y - r\phi K - wH - I$ (D) $C \leq Y - r\phi K - w\phi H - I$

Since the budget constraint is binding, we can replace consumption as your answer in 18.

Social planner's problem is then given by

$$\max_{C, C', \phi, K', H'} u(C) + u(C') \quad (\text{D})$$

$$\text{s.t. } C = \text{your answer in 18} \quad (\text{E})$$

$$C' = Y' \quad (\text{F})$$

$$H' = H + (1 - \phi)H \quad (\text{G})$$

$$K' = (1 - \delta)K + I \quad (\text{H})$$

Replace consumption with your answer in 18 and investment with equation H, we can rewrite social planner's problem as

$$\begin{aligned} \max_{\underbrace{\quad}_{19}} u(\underbrace{\quad}_{20}) + u(K'^\alpha (\phi' H')^{1-\alpha}) \\ \text{s.t. } H' = H + (1 - \phi)H \end{aligned}$$

- 19 (A) ϕ, K', H', C' (B) ϕ, K', C'
(C) ϕ, K', H' (D) K', H', C'

- 20 (A) $K^\alpha (\phi H)^{1-\alpha} + (1 - \delta)K - K'$
(B) $K^\alpha (\phi H)^{1-\alpha}$
(C) $K^\alpha (\phi H)^{1-\alpha} + (1 - \delta)K - K' + rK$
(D) $K^\alpha (\phi H)^{1-\alpha} + (1 - \delta)K - K' + w\phi H$

- 21 There's one result directly from our model assumption. Since this is a two-period model, and agents don't live to the third period, we know that $\phi' = \underline{\hspace{1cm}}$
(A) 0.3 (B) 0.5 (C) 0 (D) 1

Using your answer in 21 as well as substitute $H' = (2 - \phi)H$ into the utility function, we can write the social planner's problem as

$$\max_{\phi, K'} u(\underbrace{\quad}_{20}) + u(\underbrace{\quad}_{22}).$$

- (A) $K'^{\alpha}((2 - \phi)H)^{-\alpha}$ (B) $K'^{\alpha}((2 - \phi)H)$
 (C) $K'^{\alpha}((2 - \phi)H)^{1-\alpha}$ (D) $K'^{\alpha}H^{1-\alpha}$

②③ The first order condition with respect to K' would leads to _____

- (A) $\frac{u'(C)}{u'(C')} = K'^{\alpha}((2 - \phi)H)^{1-\alpha}$
 (B) $\frac{u'(C)}{u'(C')} = \alpha K'^{\alpha-1}H^{1-\alpha}$
 (C) $\frac{u'(C)}{u'(C')} = K'^{\alpha}((2 - \phi)H)^{1-\alpha}$
 (D) $\frac{u'(C)}{u'(C')} = \alpha K'^{\alpha-1}((2 - \phi)H)^{1-\alpha}$

②④ The first order condition with respect to ϕ would leads to _____

- (A) $\frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{1-\alpha} \left(\frac{2-\phi}{\phi}\right)^{-\alpha}$
 (B) $\frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{\alpha} \left(\frac{2-\phi}{\phi}\right)^{-\alpha}$
 (C) $\frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{\alpha}$
 (D) $\frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{\alpha-1} \left(\frac{2-\phi}{\phi}\right)^{-\alpha}$

Remember that $MRS_{C,C'} = \frac{u'(C)}{u'(C')}$, and thus your answer in 23 and 24 should equal to each other.

②⑤ Simplify the above equation and we can get _____

- (A) $\frac{\phi^{\alpha}}{2-\phi}K' = \alpha K^{\alpha}H^{1-\alpha}$ (B) $\frac{\phi^{\alpha}}{2-\phi}K'^{\alpha-1} = \alpha K^{\alpha}H^{1-\alpha}$
 (C) $\frac{\phi}{2-\phi}K' = \alpha K^{\alpha}H^{1-\alpha}$ (D) $\frac{\phi^{1-\alpha}}{2-\phi}K' = \alpha K^{\alpha}H^{1-\alpha}$

From the above equation, we can see that the choice variables, ϕ and K' , are equal to $\alpha K^{\alpha}H^{1-\alpha}$. Remember that both K and H are the endowments and α is the parameter of production function.

②⑥ What is the economics intuition of the this equation? _____

- (A) The investment on human capital is a more favorable option than the investment on physical capital in equilibrium
- (B) The investment on human capital is a less favorable option than the investment on physical capital in equilibrium
- (C) The investment on human capital and the investment on physical capital are equally favorable options in equilibrium
- (D) We cannot determine which investment is more favorable in equilibrium

Question 3

Credit: Aubhik Khan

Note: In the lecture I only teach two-period model. This question is meant to be a taste for what graduate level of macroeconomics looks like. The infinite period model is what most contemporary macroeconomics model looks like, and this question would guide you to solve infinite period model.

Consider the Solow Growth Model. Labour productivity grows at the rate $\gamma > 0$, $X_{t+1} = (1 + \gamma) X_t$, for $t = 0, 1, \dots$, and population grows at the rate $n > 0$, $L_{t+1} = (1 + n) L_t$. The effective labour force at date t is $N_t = X_t L_t$. Let aggregate production be given by

$$Y_t = A K_t^\alpha N_t^{1-\alpha} \text{ where } 0 < \alpha < 1, \quad (\text{I})$$

A is total factor productivity and K_t is the present capital stock. Total consumption is a constant fraction of output,

$$C_t = (1 - s) Y_t, \quad (\text{J})$$

where $0 < s < 1$ is the savings rate. There is full depreciation of the capital stock each period, $\delta = 1$. Thus, with I_t representing aggregate investment, the capital stock next period is

$$K_{t+1} = I_t. \quad (\text{K})$$

The aggregate resource constraint is

$$C_t + I_t = Y_t. \quad (\text{L})$$

- (27) Use (J) to eliminate C_t in (L) and solve for I_t in terms of Y_t as _____
- (A) sC_t (B) $(1-s)Y_t$ (C) $(1-s)C_t$ (D) sY_t
- (28) Substitute your result in 27 into (K) accumulation process and get K_{t+1} as _____
- (A) sC_t (B) $(1-s)Y_t$ (C) $(1-s)C_t$ (D) sY_t

Define capital per efficiency unit of labour as $k_t = \frac{K_t}{N_t}$ (so that $k_{t+1} = \frac{K_{t+1}}{N_{t+1}}$).

- (29) Express $\frac{N_{t+1}}{N_t}$ using only labor productivity growth rate γ and population growth rate n as _____
- (A) γn (B) $(1 + \gamma)(1 + n)$
(C) $(1 + \gamma)n$ (D) $\gamma(1 + n)$
- (30) Using your answer in 28 and express $\frac{K_{t+1}}{N_t}$ using Y_t as _____
- (A) $\frac{sC_t}{N_t}$ (B) $\frac{(1-s)Y_t}{N_t}$ (C) $\frac{(1-s)C_t}{N_t}$ (D) $\frac{sY_t}{N_t}$
- (31) Find the law of motion of the efficiency unit of capital, i.e., the g function in $k_{t+1} = g(k_t)$ as _____ (Hint: $k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{N_{t+1}}{N_t} \frac{K_{t+1}}{N_{t+1}}$.)
- (A) $k_{t+1} = \frac{sA}{(1+r)n} k_t^\alpha$ (B) $k_{t+1} = \frac{sA}{rn} k_t^\alpha$
(C) $k_{t+1} = \frac{sA}{(1+r)(1+n)} k_t^\alpha$ (D) $k_{t+1} = \left(\frac{sA}{(1+r)(1+n)} k_t \right)^\alpha$

In the infinite period model, what we want to find is “**steady state**”, which means that “the variables (called state variables) which define the behavior of the system or the process are unchanging in time.” (from wikipedia)

- ③ Find the steady state efficiency unit of capital of this economy, k^* , is ____ (Hint: not changing means $k_{t+1} = k_t = k^*$)

(A) $\left(\frac{sA}{rn}\right)^{\frac{1}{1-\alpha}}$

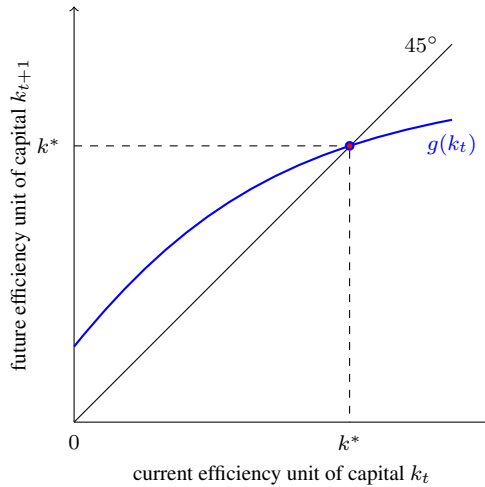
(B) $\left(\frac{sA}{(1+r)(1+n)}\right)^{\frac{1}{1-\alpha}}$

(C) $\left(\frac{sA}{(1+r)n}\right)^{\frac{1}{1-\alpha}}$

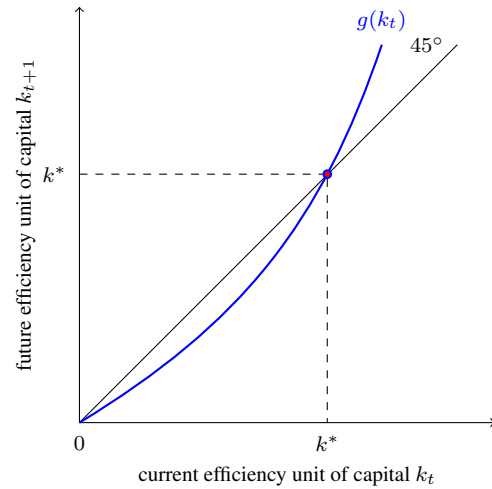
(D) $\left(\frac{sA}{(1+r)(1+n)}\right)^{\frac{\alpha}{1-\alpha}}$

33 On a figure of k_t on the x -axis and k_{t+1} on the y -axis, which of the following figure correctly plots the $g(k_t)$ function? _____

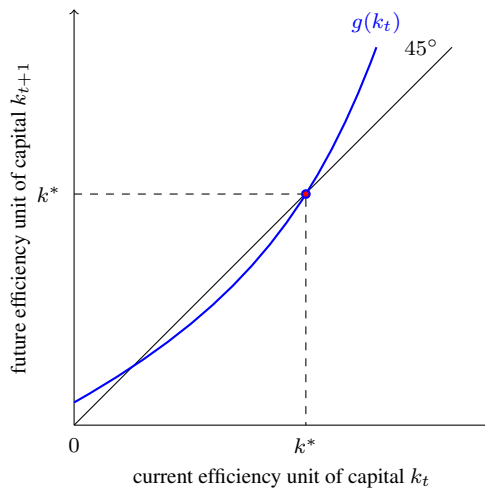
(A)



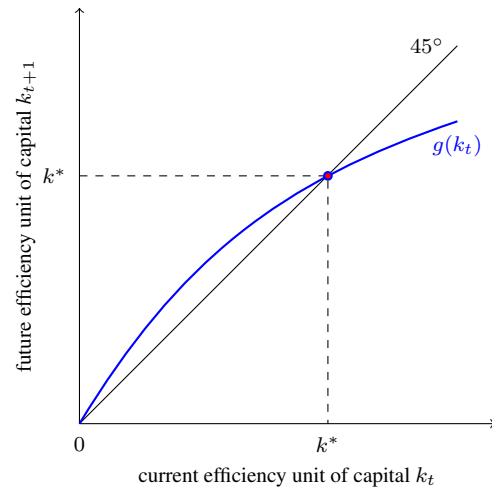
(B)



(C)



(D)



34 What is the 45° line means in question 33? _____

(A) $k_{t+1} > k_t$

(B) $k_{t+1} = k_t$

(C) $k_{t+1} < k_t$

- 35) Among all the figures in the choice of question 33, which graph will the $g(k_t)$ function be if $\alpha > 1$? _____

Consider two economies, a and b , where economy b has a higher savings rate, and a higher rate of technological progress, compared to economy a . In other words, $\gamma_b > \gamma_a$, and $s_b > s_a$. Moreover, assume that

$$\frac{s_b}{1 + \gamma_b} = \frac{s_a}{1 + \gamma_a},$$

and the rate of population growth is the same ($n_a = n_b$).

- 36) Denote the g function for both economy as $g_a(k_t)$ and $g_b(k_t)$, what is the relationship of two g function? _____

- (A) $g_a(k_t)$ is on top of $g_b(k_t)$, for all k_t
- (B) $g_a(k_t)$ and $g_b(k_t)$ is the same curve
- (C) $g_a(k_t)$ is below $g_b(k_t)$, for all k_t
- (D) $g_a(k_t)$ intersects with $g_b(k_t)$