ECON 4002.01 Midterm Exam

Hui-Jun Chen

Instruction

Please submit your answer on Carmen Quiz "Midterm Exam".

All numerical answers are supposed to **round to the second decimal point**[1] unless otherwise noted.

For all algebraic answers,

- There should be no space in your answer in the blank
- the power has to form with bracket, i.e., if want to write K^a, then you should type `K^{a}`
- if you want to express the equilibrium quantity (the star $*$) in the power while there's already something there, let's say K^{d*}, then you should type `K^{d*}`

You **may** consult any note and textbook, but you **cannot** discuss with your classmate or any other person about the exam.

For numerical answer, you are **recommended** to use software to calculate answers.

There will be one T/F choice question that is worth 2 points in the Carmen Quiz "Midterm Exam". The T/F choice question is to confirm: "**I affirm that I have not received or given any unauthorized help on this exam, and that all work is my own.**"

Grades

Question 1 to 40 are worth **2** points, and Question 41 to 44 are worth **5** points. The total grade is **102** points.

[1] round to the second decimal points means that if the third decimal point is a number between 0 to 4, then just get rid of the third decimal point. On the other hand, if the third decimal point is a number between 5 to 9, then round the second decimal point up by adding 1 to the second decimal point number. For example, if the answer you get is 0.534, then round it to 0.53. Yet, if the answer is 0.535, then round it up to 0.54.

Question 1

Considering an one-period general equilibrium model similar to Example in Lecture 08, slide 11 and 12. Also the Experiment 2 from Lecture 07, slide 13 is also a good reference. However, in this model economy, there are two differences:

1. firm rent capital from consumer, and consumers are **endowed** with 2 units of capital ($K^s = 2$).
2. consumer's utility function is $U(C,l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$

1 The competitive equilibrium given $\{G, z, \underline{\ K^s\ }\}$

2 is a set of allocations $\{Y^*, C^*, l^*, N^s, N^d, \pi^*, T^*, \underline{\ K^{d*}\ }\}$

3 and prices $\{w^*, \underline{\ r^*\ }\}$ such that

1. Taken prices and π, T as given, the representative consumer solves

4 $$\max_{\underline{C,l}} U(C,l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$$

5 subject to $C \leq w(h-l) + \underline{\ rK^s\ } + \pi - T$

2. Taken prices as given, the representative firm solves

6 $$\max_{\underline{K^d, N^d}}$$

7 $$zK^aN^{1-a} - wN^d - \underline{\ rK^d\ }$$

3. Government collect taxes to balance budget:

8 $\underline{\ T^* = G\ }$

4. Labor market clear means that the equilibrium wage is w^* such that labor supply equals to labor demand:

9 $\underline{\quad N^s = N^d \quad}$

5. Capital market clear means that the equilibrium rental rate is r^* such that capital supply equals to capital demand:

10 $\underline{\quad K^s = K^d \quad}$

To solve this model economy, we reformulate the competitive equilibrium into the social planner's problem.

First of all, in social planner's problem, all markets must clear, and thus $N^s = N^d = N$, and $K_s = K^d = K = 2$.

Through firm's FOC with respect to N and K, we know w and r are

11 $w = zK^a \underline{\quad (1-a)N^{-a} \quad}$

12 $r = zN^{1-a} \underline{\quad aK^{a-1} \quad}$

which we can use to retrieve wage and rental rate after solving the social planner's problem.

The social planner problem is given by:

13 Objective function is the consumer's utility:

$$\max_{C,l,N,Y,\underline{\quad K \quad}} U(C,l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$$

subject to

a. Aggregate resource constraint

14 $C + G = \underline{\quad Y \quad}$

b. production constraint

15 $Y = \underline{\ zK^aN^{1-a}\ }$

c. labor constraint

16 $N = \underline{\ 1 - l\ }$

d. capital constraint

17 $K = \underline{\ 2\ }$

To solve the social planner's problem, we start with substituting the constraints into utility function:

a. Substituting 23 and 17 into 15, we get

18 $Y = \underline{\ z2^a(1-l)^{1-a}\ }$

b. Substituting 18 into 14, we get

19 $C = \underline{\ z2^a(1-l)^{1-a} - G\ }$

c. Finally, substituting 19 into 13, we get

20 $\max_{\underline{l}}$

21 $U(C(l), l) = \underline{\ \frac{(z2^a(1-l)^{1-a}-G)^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}\ }$

Let $z = 1, G = 0, a = \frac{1}{2}, b = 2, d = \frac{3}{2}$ and solve for all unknowns,

22 $l = \underline{\ 0.72\ }$

23 $N = \underline{\ 0.28\ }$

24 $w^* = \underline{\ 2.67\ }$

(25) $r^* = \underline{\ 0.19\ }$

The following is the calculation for the answer from 22 to 25:

FOC results in $l^{-d} = z^{-b}2^{-ab}(1-l)^{-b+ab}(1-a)z(1-l)^{-a}$

$l^{-d} = z^{1-b}2^{-ab}(1-a)(1-l)^{-a-b+ab}$

$l^{-\frac{3}{2}} = \frac{1}{4z}(1-l)^{-\frac{3}{2}}$

$\left(\frac{1-l}{l}\right)^{\frac{3}{2}} = \frac{1}{4z} = \frac{1}{4}$

$\left(\frac{1-l}{l}\right) = \left(\frac{1}{4}\right)^{\frac{2}{3}} \approx 0.3968 \Rightarrow 1-l = 0.3968l \Rightarrow l \approx 0.7158 \approx 0.72$

Question 2

Consider a model economy with distorting labor taxes similar to Lecture 11, but having two difference:

1. the production function is Cobb-Douglas requiring only labor input, i.e., $Y = zN^a$, and
2. consumer get "disutility" from working, and the utility function is given by $U(C, N^s) = \ln C - bN$, where b is a parameter.

Other than the two changes mentioned above, the definition of the general equilibrium is exactly the same as slide 5 in Lecture 11. Therefore, this question focus mainly on algebraic calculation.

26 The derivative of utility function with respect to consumption C is $\underline{\quad \frac{1}{C} \quad}$

27 The derivative of utility function with respect to leisure l is $\underline{\quad b \quad}$

28 The marginal rate of substitution between leisure and consumption ($MRS_{l,C}$) is $\underline{\quad bC \quad}$

29 According to the slide 6 and 9, Lecture 11, in equilibrium MRS is going to be equal to $\underline{\quad w(1-t) \quad}$

30 According to the slide 5, Lecture 11, consumer's budget constraint is saying that $C = \underline{\quad w(1-t) \quad} N^s + \pi$

31 Different from slide 5, Lecture 11, now the production function is $Y = zN^a$, and thus by solving the firm's problem, the equilibrium wage as a function of labor demand N must be $w = \underline{\quad azN^{a-1} \quad}$

32 Following 31, firm's profit $\pi = \underline{\quad (1-a)zN^a \quad}$

33 Combining your answer in 28 and 29 together, substitute consumption from your answer in 30, and imposing labor market clearing condition so that $N^s = N$, we get the optimal condition is $b(\underline{\quad (w(1-t)N) + \pi \quad}) = w(1-t)$

The following three blanks are corresponds to 34 to 36. Combining your answer in 31 and 32 and substitute w and π into your answer in 33, we get

$$b(\underbrace{\underline{\quad azN^a(1-t) \quad}}_{w \text{ part}} + \underbrace{\underline{\quad (1-a)zN^a \quad}}_{\pi \text{ part}}) = \underbrace{\underline{\quad azN^{a-1} \quad}}_{w \text{ part 2}}(1-t)$$

34 $\underbrace{\underline{\quad azN^a(1-t) \quad}}_{w \text{ part}}$

35 $\underbrace{\underline{\quad (1-a)zN^a \quad}}_{\pi \text{ part}}$

36 $\underbrace{\underline{\quad azN^{a-1} \quad}}_{w \text{ part 2}}$

37 Solve for N as a function of t, we get $N = \underline{\quad \frac{a(1-t)}{b(a(1-t)+(1-a))} \quad}$

Some calculation details:

$$b(azN^a(1-t) + (1-a)zN^a) = azN^{a-1}(1-t)$$

$$\Rightarrow bzN^a(a(1-t) + (1-a)) = azN^{a-1}(1-t)$$

$$\Rightarrow bN(a(1-t) + (1-a)) = a(1-t)$$

$$\Rightarrow N = \frac{a(1-t)}{b(a(1-t)+(1-a))}$$

38 Now let $z = 1$, $a = 0.33$, $b = 2.15$ and $t = 0.5$, the N you calculated in 37 is $\underline{\quad 0.0919 \approx 0.09 \quad}$

39 after calculate the approximated value for N to the second decimal point, you

can also calculate $w = \underline{\quad 1.66 \quad}$

40 Same for $\pi = \underline{\quad 0.30 \quad}$

41 For the tax revenue generate by $t = 0.5$, how much government spending G can the government pay off? **For this question, please round to the fourth decimal point** $G = R(t) = wtN = \underline{\quad 0.0746999 \approx 0.0747 \quad}$

42 According to Laffer curve, there's also another tax rate t_2 such that it can also generate same amount of revenue to pay for the government spending you've calculated 41. What is t_2? $t_2 \approx \underline{\quad 0.9387 \approx 0.94 \quad}$

Given that $z = 1, a = 0.33$, and $b = 2.15$, if government wants to maximize the labor tax revenue $R(t) = wtN$,

43 the optimal tax rate $t^* = \underline{\quad 0.76768 \approx 0.77 \quad}$,

44 and the optimal labor tax revenue $R(t^*) = \underline{\quad 0.09285 \approx 0.0929 \quad}$ (**For this question, please round to the fourth decimal point**)

Answers

1	K^s
2	K^{d*}
3	r^*
	C, l
5	rK^s
	K^d, N^d
	rK^d
8	$T^* = G$
9	$N^s = N^d$
10	$K^s = K^d$
11	$(1-a)N^{-a}$
12	aK^{a-1}

K

14 Y

15 zK^aN^{1-a}

16 $1-l$

17 2

18 $z2^a(1-l)^{1-a}$

19 $z2^a(1-l)^{1-a}-G$

l

21 $\frac{(z2^a(1-l)^{1-a}-G)^{1-b}}{1-b}+\frac{l^{1-d}}{1-d}$

22 0.72

23 0.28

24 2.67

25 0.19

FOC results in $l^{-d}=z^{-b}2^{-ab}(1-l)^{-b+ab}(1-a)z(1-l)^{-a}$

$l^{-d}=z^{1-b}2^{-ab}(1-a)(1-l)^{-a-b+ab}$

$l^{-\frac{3}{2}}=\frac{1}{4z}(1-l)^{-\frac{3}{2}}$

$\left(\frac{1-l}{l}\right)^{\frac{3}{2}}=\frac{1}{4z}=\frac{1}{4}$

$\left(\frac{1-l}{l}\right)=\left(\frac{1}{4}\right)^{\frac{2}{3}}\approx 0.3968 \Rightarrow 1-l=0.3968l \Rightarrow l\approx 0.7158\approx 0.72$

26 $\frac{1}{C}$

27 b

28 bC

29 $w(1-t)$

30 $w(1-t)$

31 azN^{a-1}

32 $(1-a)zN^a$

33 $(w(1-t)N)+\pi$

$azN^a(1-t)$

$(1-a)zN^a$

azN^{a-1}

34 $azN^a(1-t)$

35 $(1-a)zN^a$

36 azN^{a-1}

37 $\frac{a(1-t)}{b(a(1-t)+(1-a))}$

$b(azN^a(1-t)+(1-a)zN^a)=azN^{a-1}(1-t)$

$\Rightarrow bzN^a(a(1-t)+(1-a))=azN^{a-1}(1-t)$

$\Rightarrow bN(a(1-t)+(1-a))=a(1-t)$

$\Rightarrow N=\frac{a(1-t)}{b(a(1-t)+(1-a))}$

38 $0.0919 \approx 0.09$

39 1.66

40 0.30

41 $0.0746999 \approx 0.0747$

42 $0.9387 \approx 0.94$

43 $0.76768 \approx 0.77$

44 $0.09285 \approx 0.0929$