

# Lecture 3

## Business Cycle Measurement

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# Review: Trend and Cycle in Per Capital Real GDP

Figure 1.3: Natural log of Per Capita Real GDP and trend, 1900–2014

$$y = \ln(Y), \text{trend} = \text{HPFilter}(y)$$

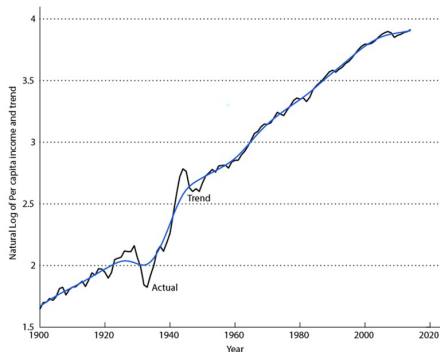
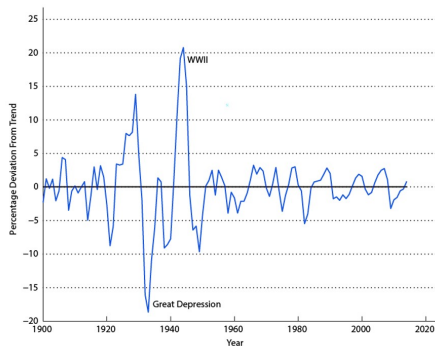


Figure 1.4 Percentage Deviation from Trend in Per Capita Real GDP actual - trend



How do we measure the fluctuation?

# Statistical Concept to measure fluctuation

- **mean/average**  $\bar{X}$ : average level of variable  $X$

$$\bar{X} = E(X) \approx \frac{1}{N} \sum_{i=1}^N X_i, \quad (1)$$

e.g.  $X = \{1, 2, 3, 4, 5\}$ ,  $\bar{X} = \frac{1+2+3+4+5}{5} = 3$

- **variance**  $\sigma_X^2$ : dispersion of variable  $X$  relative to mean

$$\sigma_X^2 = V(X) = E[(X - \bar{X})^2] \approx \frac{1}{N} \sum_{i=1}^N [(X_i - \bar{X})^2], \quad (2)$$

e.g.  $\sigma_X^2 = \frac{(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2}{5} = \frac{4+1+0+1+4}{5} = 2$

**standard deviation** is  $\sigma_X = \sqrt{\sigma_X^2}$ .

# Statistical Concept to measure fluctuation (Cont.)

- **covariance**  $\sigma_{XY}$ : dispersion of variable  $X$  “associates” with  $Y$ .

$$\sigma_{XY} = E[(X - \bar{X})(Y - \bar{Y})] \approx \frac{1}{N} \sum_{i=1}^N [(X_i - \bar{X})(Y_i - \bar{Y})], \quad (3)$$

e.g.  $Y = \{2, 3, 4, 5, 6\}$ ,  $\bar{Y} = 4$ ,

$$\sigma_{XY} = \frac{(1-3)(2-4) + (2-3)(3-4) + (3-3)(4-4) + (4-3)(5-4) + (5-3)(6-4)}{5} = 2$$

- **correlation**  $\rho_{XY}$ : level of association between variables  $X$  and  $Y$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}, \quad (4)$$

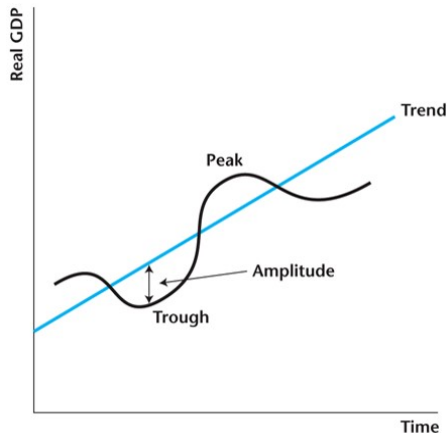
e.g.  $\rho_{XY} = \frac{2}{\sqrt{2} \times \sqrt{2}} = 1$

$\rho_{XY} > 0$ :  $X$  and  $Y$  move **together**;  $< 0$ : move **opposite** directions

# GDP Fluctuations: Definition and Measurement

- **Business cycle:** fluctuations about trend in real GDP
- **Booms/Expansions:** persistent positive deviation
- **Recessions:** persistent negative deviation
- **Peak/Trough:** turning points on deviations
- **Amplitude:** size of deviations
- **Frequency:** # of peaks per year

Figure 3.1 Business Cycle

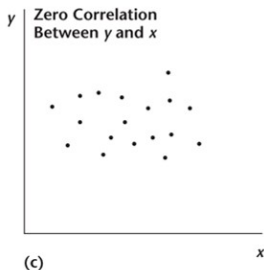
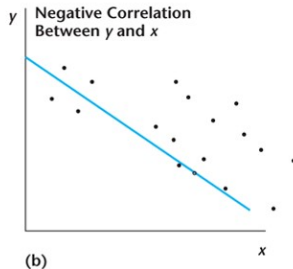
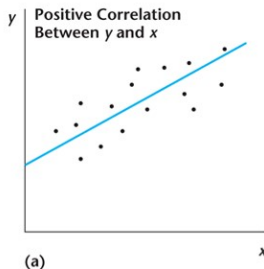


# Measurement on Comovement

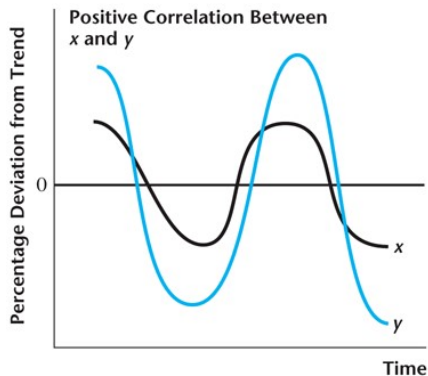
Three key concepts to measure comovement of variables  $X$  with GDP  $Y$ :

- ① **relative volatility**  $\sigma_X/\sigma_Y$ : how noisy is  $X$  relative to GDP
  - $\sigma_X/\sigma_Y$  high  $\Rightarrow X$  is noisy
- ② **cyclical**  $\rho_{XY}$ : how does  $X$  comove with GDP?
  - ① **procyclical**:  $\rho_{XY} > 0$ ,  $X$  and GDP comove **together**
  - ② **acyclical**:  $\rho_{XY} \approx 0$ ,  $X$  and GDP not comoving
  - ③ **countercyclical**:  $\rho_{XY} < 0$ ,  $X$  and GDP move in **opposite** direction
- ③ **lead/coincident/lag**: does  $X$  predict GDP (lead), the opposite (lag) or nor (coincident)?

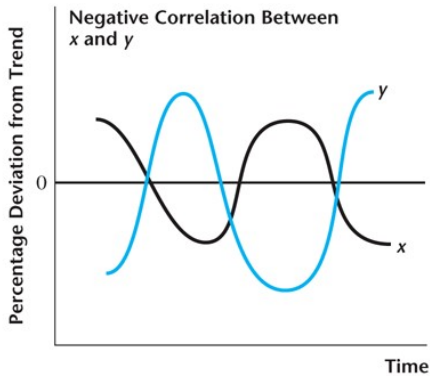
# Visualization of Correlation: Scatter Plots



# Visualization of Correlation: Time Series



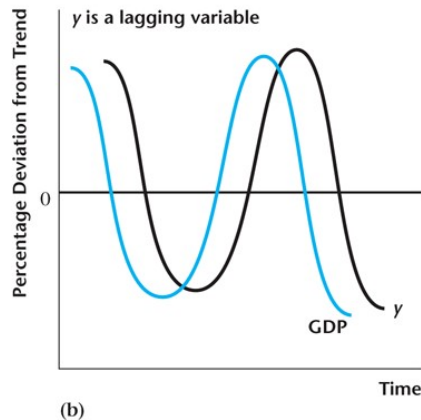
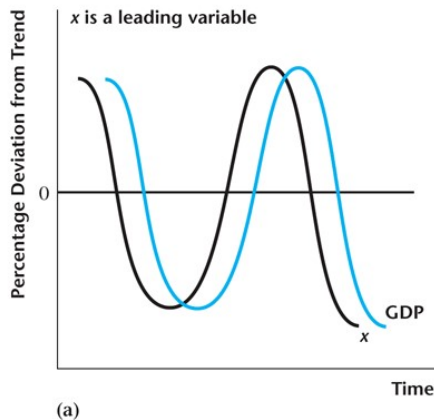
(a)



(b)



# Visualization of Leading and Lagging: Time Series



# Data: Comovement of $C$ and $I$

Figure 3.9 Percentage Deviations from Trend in Real Consumption and Real GDP

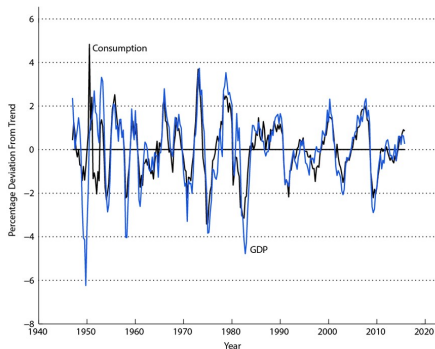
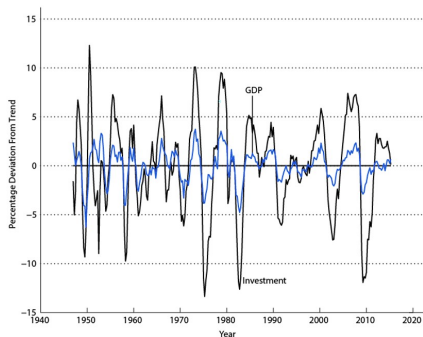


Figure 3.10 Percentage Deviations from Trend in Real Investment and Real GDP



# Data: Correlation of Imports

Figure 3.5 Time Series (Detrended)

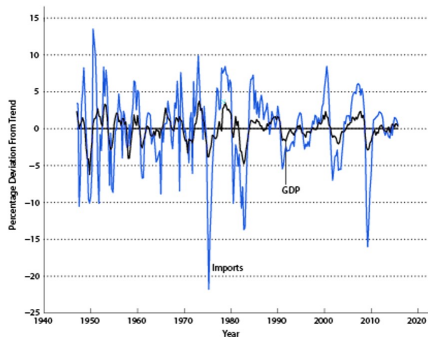
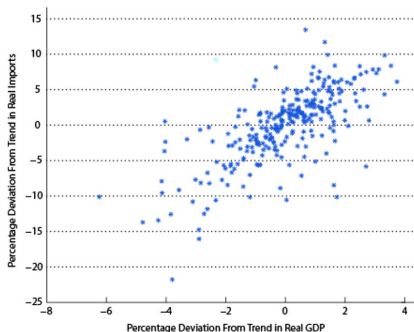
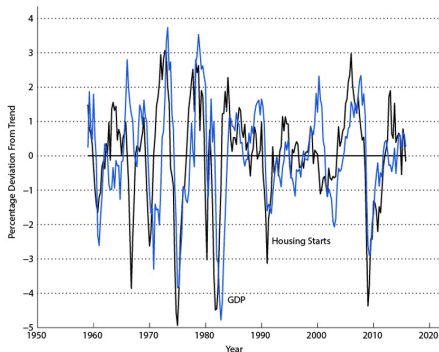


Figure 3.6 Scatter Plot: Time Series



# Data: Leading Indicator, Housing Starts

Figure 3.8 Percentage Deviations in Real GDP and Housing Starts



- **Def:** the construction project is started for a private dwelling
- **Question:** Why housing start is predicting GDP?
- **Ans:** **commitment** to a quantity of residential **investment**

# Summary

Variable	Corr. Coef.	Std Dev (% of S.D. GDP)	Cyclical	Lead/Lag	Variation Relative to GDP
consumption	0.77	77	Procyclical	Coincident	Smaller
investment	0.80	301	Procyclical	Coincident	Larger
employment	0.78	65	Procyclical	Lagging	Smaller
average labor productivity	0.77	63	Procyclical	Coincident	Smaller

What about the properties of government spending? Expectation?