

# Lecture 18

## The Real Business Cycle Model

### Part 5: Application and Matching Data

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- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

Analysis on  $z \uparrow$

$$\uparrow z F(\underline{p}, \underline{n})$$

$z \uparrow$

$z' \uparrow$

$K \downarrow$

$G \uparrow$

Suppose current TFP increases from  $z_1$  to  $z_2$ ,  $z_2 > z_1$

- labor demand (firm):  $z \uparrow \Rightarrow MPN \uparrow$ , and thus  $N_{D,2} > N_{D,1}, \forall w$
- labor supply (consumer): no direct effect, but  $r^* \downarrow$  leads to  $N_S(r)$  shifts in
- labor market clearing: demand  $\uparrow$  ( $w \uparrow$ ) and  $N^*(r) \uparrow$ , hold  $r$  fixed
- output supply: shifts out  $\because$  labor market,  $Y_{S,2}(r) > Y_{S,1}(r), \forall r$
- output demand: no effect, only move along the curve, because
  - firm: current TFP is not changing optimal investment schedule

- consumer: no direct effect

- government: no direct effect

$G \uparrow \Rightarrow \frac{G}{K} \uparrow = \tau + \frac{I}{K} \uparrow$

$K^* = \alpha Y$   
 $N^* = (1-\alpha)Y$

$\downarrow MPN' - \delta = (Y) \downarrow$   
 $K^* \uparrow$   $N_D^* \uparrow$   $Y_D^* \uparrow$

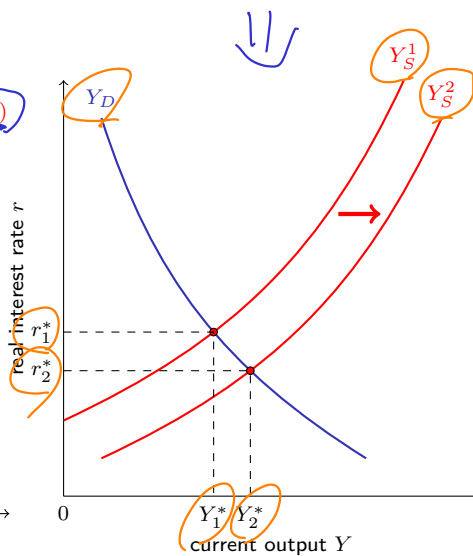
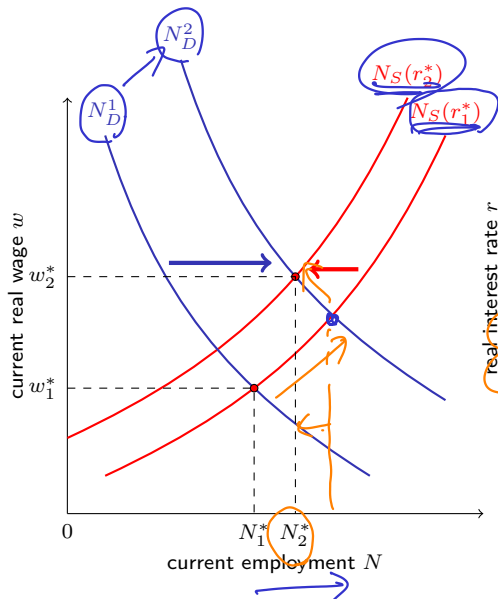
# Equilibrium Effects of $z \uparrow$

$z \uparrow$

$z' \uparrow$

$K \downarrow$

$G \uparrow$



## Taking Stock: $z \uparrow$

Output supply curve shifts out, while output demand remain the same

- output  $\uparrow$ ,  $Y_2 > Y_1$
- real interest rate  $\downarrow$ ,  $r_2 < r_1$
- decreases in  $r$  make labor supply shifts in
  - saving  $S$  become less desirable, so no need to work that much
- wage increase because of the shifts in demand and supply
- effect on  $N^*$  is theoretically ambiguous, yet data shows that the effect of intertemporal substitution of leisure ( $N_S \downarrow$ ) is small

Recall **business cycle facts**: procyclical labor, real wage, and average labor productivity.

$$z \uparrow \Rightarrow N^* \uparrow, w^* \uparrow$$

All consistent with model prediction!

# Analysis on $z' \uparrow$

$z \uparrow$

$z' \uparrow$

$K \downarrow$

$G \uparrow$

- **labor demand**: no direct effect
- **labor supply**: no direct effect, yet  $r \uparrow$  cause supply to shift to the right
- **output supply**: no direct effect
- **output demand**: higher  $z' \Rightarrow$  higher  $MPK' \Rightarrow$  firm's investment demand is higher  $\Rightarrow$  demand shifts to the right
  - no direct effect from consumer and government

$$\uparrow MPK' - \delta = \uparrow$$

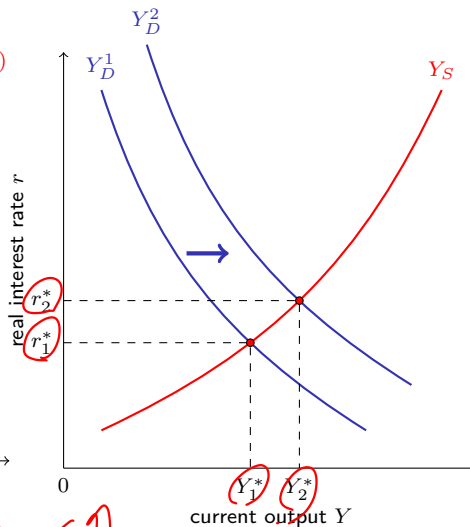
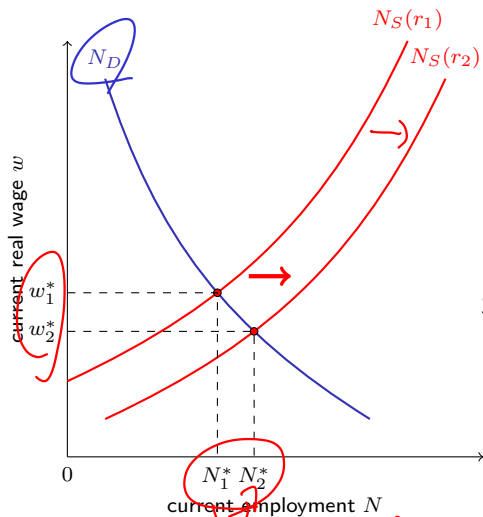
# Equilibrium Effect of $z' \uparrow$

$z \uparrow$

$z' \uparrow$

$K \downarrow$

$G \uparrow$



$N \uparrow \Rightarrow Y \uparrow$

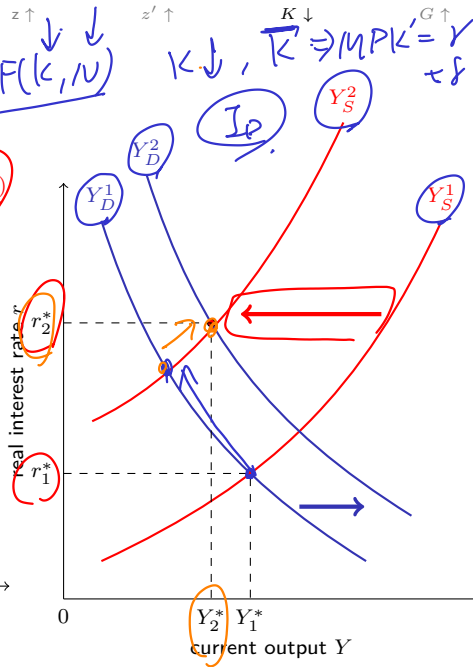
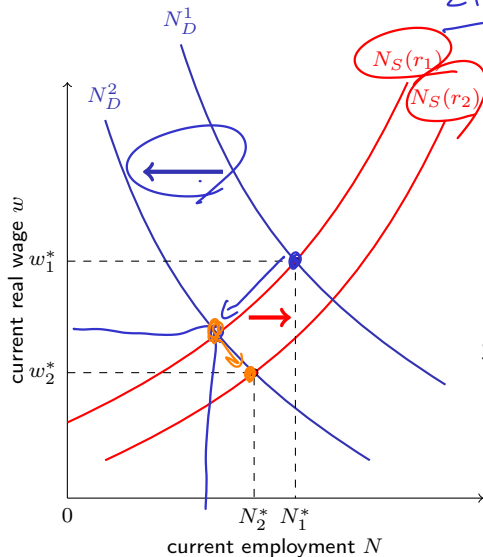
# Analysis on Destruction of Initial Capital $K \downarrow$

Suppose a natural disaster destroys some initial capital:  $K_1 \rightarrow K_2$ , where  $K_2 < K_1$ .

- **labor demand**:  $K \downarrow \Rightarrow MPN \downarrow \Rightarrow N_D^2(w) < N_D^1(w), \forall w$
- **labor supply**: no direct effect, but  $r^* \uparrow \Rightarrow N_S(r) \downarrow$
- **labor market clearing**: lower wage and quantity of labor, hold  $r$  fixed
- **output supply**: shifts in,  $\therefore$  labor market effects,  $Y_S^2(r) < Y_S^1(r), \forall r$
- **output demand**: shifts out, because
  - firm:  $K \downarrow$ , so must  $I_D \uparrow$  to meet same amount of  $K'$ 
    - remember capital accumulation process  $K' = I_D + (1 - \delta)K$
  - consumer and government have no direct effects



# Equilibrium Effect of $K \downarrow$



# Analysis on Government Spending Increase $G \uparrow$

$z \uparrow$     $z' \uparrow$     $K \downarrow$     $G \uparrow$

Suppose  $G \uparrow$ , holding  $G'$  fixed. This is more complicated...

- **example:** wartime spending (WWII), Stimulus in recession (COVID check)

Need to trace individual decisions and market clearing conditions to find **overall equilibrium effect**.

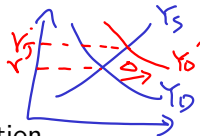
- **simplification:** assume  $MPC$  is constant
- **interpretation:** slope  $< 1$  in output demand curve
- **example:**  $U(C, C') = \ln C + \beta \ln C' \Rightarrow C' = \underbrace{\beta(1+r)}_{C = \frac{1}{\beta(1+r)} C'}$   $C$ , which implies

$$\underline{C = \frac{1}{1+\beta} \left( \underline{Y - T} + \frac{Y' - T'}{1+r} \right) \Rightarrow \frac{dC}{dY} = \frac{1}{1+\beta}}$$

# Impact on Output Demand

$G \uparrow$  causes a  $\Delta$  amount of shift in the output demand curve. How big is  $\Delta$ , and where do the change comes from?

① direct effect:  $G_2 - G_1 > 0$



② indirect effect: increase in taxes decreases the consumption

- $\because G_2 > G_1$ ,  $T_2 + \frac{T'_2}{1+r} > T_1 + \frac{T'_1}{1+r}$ , and thus consumer's income  $\downarrow$  by the amount of  $G_2 - G_1$ .

C  $\downarrow$

- effect on consumption:  $MPC \times (G_2 - G_1)$

③ indirect effect: consumer perceives as  $Y_D$  changes  $\Delta$  amount, and thus consumption changes.

- translate to consumption:  $MPC \times \Delta$

$$(1-MPC)\Delta = (1-MPC)(G_2-G_1)$$

$$\Delta = G_2 - G_1 + MPC \times (G_2 - G_1) + MPC \times \Delta \Rightarrow \Delta = G_2 - G_1$$

note: more complicated if MPC is not constant, or varies across people!

# Impact on Output Demand (Cont.)

 $z \uparrow$  $z' \uparrow$  $K \downarrow$  $G \uparrow$ 

The **elasticity** of output demand with respect to government spending is defined as the **demand multiplier**:

$$m_D = \frac{\Delta}{G_2 - G_1} = 1$$

- implication: rightward shift of the demand curve is exactly 1-1
- because of 1-1 relationship, we know  $Y_D^2(r) = Y_D^1(r) + \Delta$

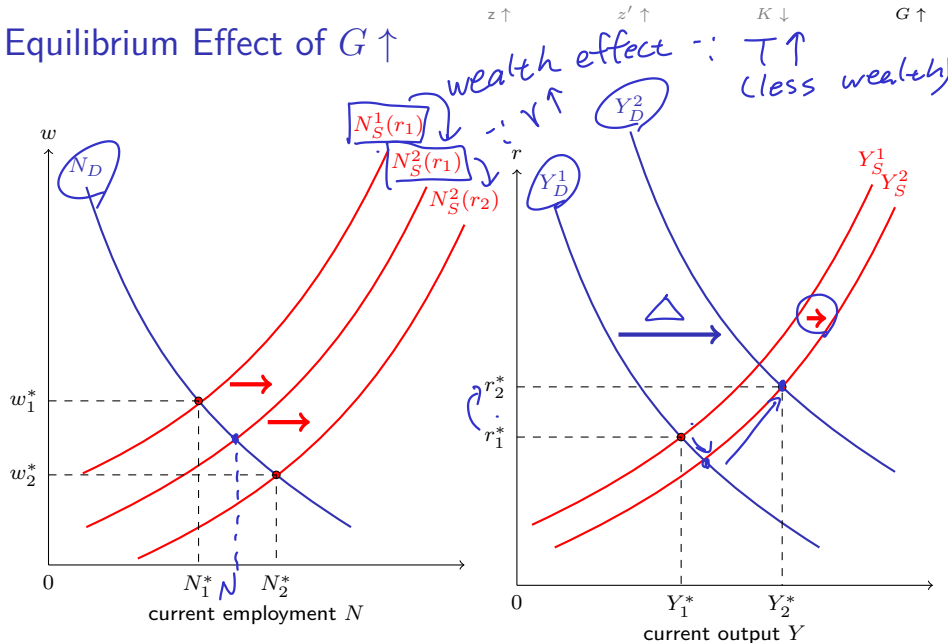
# Impact on Output Supply

 $z \uparrow$  $z' \uparrow$  $K \downarrow$  $G \uparrow$ 

- labor demand: no effect
- labor supply: outward shift,  $\because$  wealth effect of  $T, T' \uparrow$ 
  - holding  $r$  fixed,  $N_S^2(r_1) > N_S^1(r_1)$
  - in equilibrium of next slide,  $r^* \uparrow$ , and thus saving become desirable,  $N_S^2(r_2) > N_S^2(r_1)$
- output supply: shifts out, given labor supply shifts

Combine effects:  $Y^* \uparrow$ ,  $N^* \uparrow$ ,  $w^* \downarrow$ , yet  $r^*$  depends on the amount of movement for both demand and supply.

# Equilibrium Effect of $G \uparrow$



# Taking Stock: Output

What is the **total government expenditure multiplier**?

- **definition**: the **equilibrium** (as opposed to demand or supply only) ratio of **increase in output** to the **increase in government spending**.
- **result**: must be **less than 1** without “large” shifts in supply curve
  - shift in output demand curve is  $G_2 - G_1$  for each  $r$
  - supply curve slopes up: **equilibrium effect**  $< G_2 - G_1$  (before shift)
  - what determines size of supply curve shift?
    - size of wealth effect on labor supply (small)
    - size of intertemporal substitution effect on labor supply (small)
  - “Keynesian” stimulus: multiplier may be **positive in recessions**, but need some **sort of economic inefficiency** for this result.

# Taking Stock: Everything Else

$z \uparrow$

$z' \uparrow$

$K \downarrow$

$G \uparrow$



Imagine supply curve is horizontal:

- equilibrium effect:  $Y_2 - Y_1 = G_2 - G_1$ , no change in  $r$
- would have to come from no change in consumer's lifetime wealth, and so would induce no change in current consumption.

With upward slope sufficient to make  $r_2 > r_1$  (empirically plausible case):

- consumption falls due to intertemporal substitution effect  
 $K' \downarrow \Leftrightarrow \uparrow MPK' - \delta = r \uparrow$
- investment falls due to higher opportunity cost of investing in capital
- “crowding out:” government expenditures here also limit future production
- total: higher output, but at what cost?