

# Lecture 9

## Social Planner's Problem

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# Overview

After constructing both **consumers'** and **firms'** problem, we start to bring them together in **one-period model**:

- Lecture 8: **competitive equilibrium** (CE)
  - each agent solve their problems individually
  - aggregate decision determines “prices” (wage, rent, etc.)
- Lecture 9: **social planer's problem** (SPP)
  - imaginary and benevolent social planner determines the allocation
  - should be the most efficient outcome
- Lecture 10: CE and SPP examples

# What is Social Planner?

- **Benevolent dictator** whose goal is to maximize **social welfare** given **technological constraint**
- **Social welfare**: joint “happiness” of every agent in this economy
  - **consumer**: tangency between IC and budget line in  $(C, l)$ -plane
  - **firm**:  $Y = zF(K, N) = zF(K, h - l)$ 
    - labor market clearing:  $N = N^s = N^d$
    - consistent with consumer behavior:  $N = h - l$
  - **government**: income-expenditure identity,  $C = Y - G$ 
    - government is not necessary the social planner! (also one of the agents)
- **Technological constraint**: production possibility frontier

# Production Possibility Frontier (PPF)

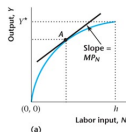
- **Def:** technological possibilities for the whole economy

$$C = zF(K, h - l) - G \quad (1)$$

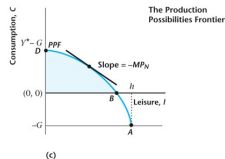
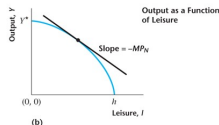
- **Marginal rate of transformation (MRT):** rate to transform leisure to consumption (through work)

$$\begin{aligned} MRT_{l,C} &= -\frac{dC}{dl} \\ &= zD_N F(K, N) \quad (2) \\ &= MPN \end{aligned}$$

Figure 5.2 The Production Function and the Production Possibilities Frontier

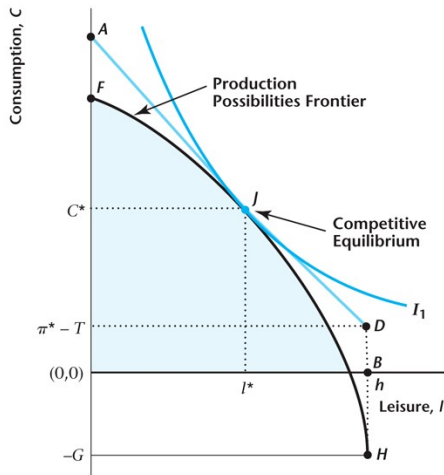


Production Function



# Competitive Equilibrium: Graphical Representation

Figure 5.3 Competitive Equilibrium



Combine PPF with IC:

- $\overline{AD}$ : tangent to consumer's IC  $I_1$  and PPF  $\overline{FH}$
  - negative slope of  $\overline{AD}$ : equilibrium wage  $w$ 
    - $\therefore \overline{AD}$  is budget line
  - Recall Lecture 8 & last slide:
    - consumer:  $MRS_{l,C} = w$
    - firm:  $MPN = w$
    - efficiency:  $MRT_{l,C} = MPN$
- $$MRS_{l,C} = MRT_{l,C} = MPN$$

# Concept: Pareto Improvement / Optimal

A competitive equilibrium is **Pareto optimal** or **Pareto efficient** if there is no way to rearrange production or to reallocate goods so that someone is made better off without making someone else worse off.

- only one consumer, so relatively straightforward
- but, still a powerful concept:
  - free markets can produce socially efficient outcomes
  - often easier to analyze social optimum than competitive equilibrium
- caveats:
  - “efficiency” in economics is a statement about a model
  - very narrow: e.g. having Jeff Bezos pay for a meal for someone in need.

# Social Planner's Problem

objective: consumer's utility  $\max_{C,l,N,Y} U(C,l)$

subject to

agg. resource constraint  $C + G \leq Y$

production constraint  $Y = zF(K, N)$

labor constraint  $N = h - l$

- **What's here:** GDP accounting, physical / technological constraints, required government spending, consumer preferences
- **What's not:** consumer's budget constraint, the wage rate, consumer's / firm's individual problems, profits, taxes

# Solving Social Planner's Problem

We know all constraints bind, so by substituting:

$$\max_l U(zF(K, h - l) - G, l) \quad (3)$$

**FOC:**

$$\begin{aligned} D_l U(zF(K, h - l) - G, l) \\ = D_C U(zF(K, h - l) - G, l) (zD_N F(K, h - l)) \end{aligned} \quad (4)$$

**Rearrange:**

$$\frac{D_l U(zF(K, h - l) - G, l)}{D_C U(zF(K, h - l) - G, l)} = zD_N F(K, h - l) \Rightarrow MRS_{l,C} = MRT_{l,C} \quad (5)$$

Same Result! Why?

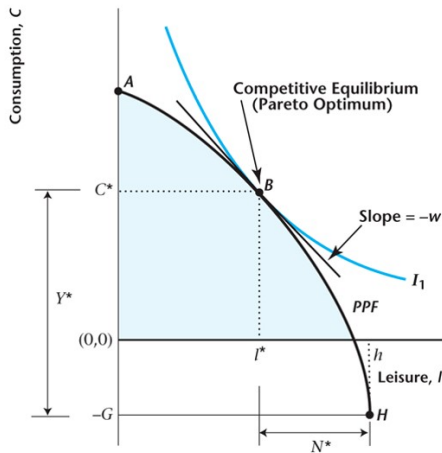


# Welfare Theorem

- **First welfare theorem:** under certain conditions, the allocation under a competitive equilibrium is Pareto optimal
- **Second welfare theorem:** under certain conditions, a Pareto optimal allocation is the allocation for a competitive equilibrium.
- straightforward to show here (we already have!) but not always so.
  - conditions not always met!
- SPP and CE often coincide if not identical, serves as a good benchmark

# Social Planner's Problem: Graphical Representation

Figure 5.4 Pareto Optimality



Apply SPP & 2nd welfare theorem for competitive equilibrium:

- $l^*$  determined by SPP at  $B$
- $C^*, N^*, Y^*$  by plugging into constraints
- $w^* = MPN = MRT_{l,C} = MRS_{l,C}$

# What Can Go Wrong? Cases when $SPP \neq CE$

- ① Externalities: activity for which an individual does not take account of all associated costs and benefits: can be positive or negative
  - example: pollution must be cleaned up, but firm doesn't have to
- ② Distorting taxes: lead to “wedges” between MRS, MP, and MRT
  - example: proportional labor income tax vs lump-sum tax
- ③ Non-competitive / monopolistic behavior: firms or consumers may not be price takers
  - examples: local media markets, negotiations