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Intermediate Macro Theory

FInal exam review

Outline

Environment

- Spend time in education to accumulate human capital
- Utility: $U(C, C') = u(C) + u(C')$
- ϕ fraction of one unit of time endowment goes to work
- $1 - \phi$ fraction of one unit of time endowment goes to education
- Human capital law of motion: $H' = H + (1 - \varphi)H = (2 - \varphi)H$
- Physical capital law of motion: $K' = (1 - \delta)K + I$
- Production: $Y = K^{\alpha}(\varphi H)^{1-\alpha}; Y' = K'^{\alpha}(\varphi' H')^{1-\alpha}$

Constructing Optimization Problem

- Labor income: $w\varphi H$
- Capital income: rK
- Budget constraints: $C + I \leq w\varphi H + rK + \pi$
- Profit: $\pi = Y - w\varphi H - rK$
- Plug profit into budget constraints to get aggregate resource constraints (Income-Expenditure Identity)

$$\max_{\varphi, K'} \quad u(K^\alpha(\varphi H)^{1-\alpha} + (1-\delta)K - K') + u((K')^\alpha(\varphi'(2-\varphi)H)^{1-\alpha})$$

- $\varphi' = 1$ as there's no third period
- First order conditions yield

$$\begin{aligned}
[\varphi]: \quad & u'(C)(1-\alpha)K^\alpha\varphi^{1-\alpha}H^{-\alpha} = u'(C')(1-\alpha)(K')^\alpha((2-\varphi))^{-\alpha}\varphi H^{-\alpha} \\
& MRS_{C,C'} = \frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^\alpha \left(\frac{2-\varphi}{\varphi}\right)^{-\alpha} \\
[K']: \quad & u'(C) = u'(C')\alpha(K')^{\alpha-1}((2-\varphi)H)^{1-\alpha} \\
& MRS_{C,C'} = \frac{u'(C)}{u'(C')} = \alpha(K')^{\alpha-1}((2-\varphi)H)^{1-\alpha}
\end{aligned}$$

Relationship between two intertemporal assets

- Two equation equates,

$$\left(\frac{K'}{K}\right)^{\alpha}\left(\frac{2-\varphi}{\varphi}\right)^{-\alpha} = \alpha(K')^{\alpha-1}((2-\varphi)H)^{1-\alpha}$$

- Simplify,

$$\frac{\varphi^{\alpha}}{2-\varphi}K' = \alpha K^{\alpha}H^{1-\alpha}$$

  Notice the RHS is constant.

- This is another expressions for optimal investment schedule. The return on human capital, which denotes by decrease in ψ, and the return on physical capital, K' should be equally favarable in equilibrium.

Overview

- Consider a Solow model with economic growth
- Labor productivity grows at rate γ: $X_{t+1} = (1+\gamma)X_t$
- Population grows at rate n: $L_{t+1} = (1+n)n_t$
- Effective labor force: $N_t = X_t L_t$
- Production: $Y_t = AK_t^{\alpha} N_t^{1-\alpha}$
- Consumption is residual of saving, $C_t = (1-s)Y_t$
- Full depreciation on capital, $K_{t+1} = I_t$
- Aggregate resource constraint, $C_t + I_t = Y_t$

Constructing efficiency unit of capital

- Investment is given by $K_{t+1} = I_t = sY_t$
- Growth of effective labor, $\dfrac{N_{t+1}}{N_t} = \dfrac{X_{t+1}L_{t+1}}{X_t L_t} = (1+\gamma)(1+n)$
- Future capital to current effective labor ratio, $\dfrac{K_{t+1}}{N_t} = \dfrac{sY_t}{N_t}$
- Let $k_t = \frac{K_{t+1}}{N_{t+1}}$ denotes the efficiency unit of capital, or capital-labor ratio. The law of motion of k_t is

$$\frac{sY_t}{N_t} = \frac{K_{t+1}}{N_t} = \frac{K_{t+1}}{N_{t+1}}\frac{N_{t+1}}{N_t}$$

$$\frac{sAK_t^{\alpha}N_t^{1-\alpha}}{N_t} = k_{t+1}(1+\gamma)(1+n)$$

$$k_{t+1} = \frac{sA}{(1+\gamma)(1+n)}k_t^{\alpha}$$

Steady state

- In steady state, $k_{t+1} = k_t = k^*$,

$$k^* = \frac{sA}{(1+\gamma)(1+n)} k^{*\alpha}$$
$$k^* = \left(\frac{sA}{(1+\gamma)(1+n)}\right)^{1-\alpha}$$

- Comparing two economy one with large population growth (larger n) and larger saving rate (larger s) than the other, but have constant ratio between the two will exhibit the same efficiency in production, as their efficiency unit of capital should be the same. Just one is larger in scale/size.