# ECON 4002.01 Problem Set 3
## Hui-Jun Chen

## Question 1

Consider a model that is **similar to** (not exactly!) the Lecture 14 Consumer Problem, but there are three differences:

1. Consumers' utility function is given by $U(C, C', N_S, N'_S) = \log C - bN_S + \log C' - bN'_S$

2. Consumers do **not** own the whole firm, i.e., $\pi = 0$. Instead, they buy shares of the firm $s$ in date $0$ to achieve intertemporal saving at per-unit price $q$. At date $1$, consumers redeem their share to the firm and get $s$ of reward.

3. Consumers are **not** subject to the lump-sum tax, i.e., $T = 0$.

### Budget Constraint

Firstly, let's follow the slide and think about the consumer's budget constraint, you can refer to Lecture 14, slide 4.

(1) there are ___$A$___ choice variables,

    (A) 5         (B) 3         (C) 2         (D) 4

(2) and they are $\{C, C', N_S, N'_S, \underline{\quad C \quad}\}$

    (A) $S$         (B) $S'$         (C) $s$         (D) $s'$

(3) consumers own **part** of the firm and get ___$B$___ of reward

    (A) $\pi$         (B) $s$         (C) $\pi'$         (D) $S$

(4) and they are taken the equilibrium price $\{w, w', \underline{\quad D \quad}\}$ as given.

(A)  $r$         (B)  $r'$         (C)  $q'$         (D)  $q$

After defining all of the variables, consumer's budget constraints in each period are

**5** date 0 budget constraints is ___A___

(A)  $C + qs = wN_S$                (B)  $C + S = wN_S + \pi - T$

(C)  $C = wN_S + qs$                (D)  $C = wN_S + \frac{s}{q} + \pi - T$

**6** date 1 budget constraints is ___C___

(A)  $C' = wN_S + \pi' - T' + (1+r)S$    (B)  $C' = w'N'_S + qs$

(C)  $C' = w'N'_S + s$                    (D)  $C' = w'N'_S + \frac{s'}{q'} + \pi' - T'$

**7** The lifetime budget constraint by combining date 0 and date 1 budget constraints is ___D___

(A)  $C + \frac{C'}{1+r} = wN_S + \frac{w'N'_S}{1+r}$

(B)  $C + \frac{C'}{1+r} = wN_S + \pi - T + \frac{w'N'_S + \pi' - T'}{1+r}$

(C)  $C - qC' = wN_S - qw'N'_S$

(D)  $C + qC' = wN_S + qw'N'_S$

Some calculation details:

$$s = C' - w'N'_S \Rightarrow C + q(C' - w'N'_S) = wN_S$$

$$\Rightarrow C + qC' = wN_S + qw'N'_S$$

## Preference

After finishing consumer's budget constraint, let's turn to the analysis preference:

(8) Accroding to the consumer's utility mentioned before, the derivative of consumer's utility function $U(C, C', N_S, N'_S)$ with respect to current consumption $C$ is __A__

(A) $\frac{1}{C}$      (B) $\frac{1}{C'}$      (C) $\frac{C'}{C}$      (D) $\frac{C}{C'}$

(9) Similarly, the derivative of consumer's utility function $U(C, C', N_S, N'_S)$ with respect to future consumption $C'$ is __B__

(A) $\frac{1}{C}$      (B) $\frac{1}{C'}$      (C) $\frac{C'}{C}$      (D) $\frac{C}{C'}$

(10) Similarly, the derivative of consumer's utility function $U(C, C', N_S, N'_S)$ with respect to current labor supply $N_S$ is __D__

(A) $\frac{1}{N'_S}$      (B) $\frac{1}{N_S}$      (C) $-bN_S$      (D) $-b$

(11) Similarly, the derivative of consumer's utility function $U(C, C', N_S, N'_S)$ with respect to future labor supply $N'_S$ is __C__

(A) $\frac{1}{N'_S}$      (B) $\frac{1}{N_S}$      (C) $-b$      (D) $-bN_S$

(12) After deriving four derivatives of the utility function, consumer's marginal rate of substitution between $C$ and $C'$, $MRS_{C,C'}$ is __C__

(A) $\frac{1}{C}$      (B) $\frac{1}{C'}$      (C) $\frac{C'}{C}$      (D) $\frac{C}{C'}$

Some calculation details:

$$MRS_{C,C'} = \frac{u'(C)}{u'(C')} = \frac{1/C}{1/C'} = \frac{C'}{C}$$

(13) Similarly, $MRS_{l,C} =$ __A__

(A) $bC$      (B) $bC'$      (C) $-bC$      (D) $-bC'$

Some calculation details:

$$MRS_{l,C} = -MRS_{N_S,C} = \frac{v'(N_S)}{u'(C)} = \frac{b}{1/C} = bC$$

(14) Similarly, $MRS_{l',C} = \underline{\quad A \quad}$

(A) $bC$        (B) $bC'$        (C) $-bC$        (D) $-bC'$

Some calculation details:

$$MRS_{l',C} = -MRS_{N'_S,C} = \frac{v'(N'_S)}{u'(C)} = \frac{b}{1/C} = bC$$

## Representative Consumer's Problem

Since in this model the share purchasing $s$ is indeed the saving for the consumer, and consumer's share purchasing decision is implied by the combination of its consumption and labor supply decision, and thus in equilibrium, consumers are not choosing shares.

(15) Consumer's Problem is to maximize utility function by choosing $\underline{\quad D \quad}$,

(A) $C, C', S, S'$                   (B) $S, S', N_S, N'_S$

(C) $C, C', s, s'$                    (D) $C, C', N_S, N'_S$

subject to the lifetime budget constraint 7.

Consumer's Problem formulation:

$$\max_{C,C',N_S,N'_S} \log C - bN_S + \log C' - bN'_S \text{ subject to } C + qC' = wN_S + qw'N'_S$$

(16) First step, we substitute $C$ with all the other terms in the lifetime budget constraint and get $\underline{\quad A \quad}$

(A) $\displaystyle\max_{C',N_S,N'_S} \log\left(wN_S + qw'N'_S - qC'\right) - bN_S + \log C' - bN'_S$

(B) $\displaystyle\max_{C,N_S,N'_S} \log C - bN_S + \log\left(wN_S + qw'N'_S - qC\right) - bN'_S$

(C) $\max\limits_{C,N_S,N'_S} \log C - bN_S + \log\left(\dfrac{wN_S + qw'N'_S - C}{q}\right) - bN'_S$

(D) $\max\limits_{C',N_S,N'_S} \log\left(wN_S + qw'N'_S - qC\right) - bN_S + \log C' - bN'_S$

Note: read what should be substitute into other terms!

(17) The FOC w.r.t. $C'$ is ___$C$___

(A) $b = \dfrac{qw'}{wN_S + qw'N'_s - qC'}$    (B) $b = \dfrac{w}{wN_S + qw'N'_s - qC'}$

(C) $\dfrac{1}{C'} = \dfrac{q}{wN_S + qw'N'_S - qC'}$    (D) $\dfrac{1}{C} = \dfrac{1}{wN_S + qw'N'_S - qC'}$

(18) The FOC w.r.t. $N_S$ is ___$B$___

(A) $b = \dfrac{qw'}{wN_S + qw'N'_s - qC'}$    (B) $b = \dfrac{w}{wN_S + qw'N'_s - qC'}$

(C) $\dfrac{1}{C'} = \dfrac{q}{wN_S + qw'N'_S - qC'}$    (D) $\dfrac{1}{C} = \dfrac{1}{wN_S + qw'N'_S - qC'}$

(19) The FOC w.r.t. $N'_S$ is ___$A$___

(A) $b = \dfrac{qw'}{wN_S + qw'N'_s - qC'}$    (B) $b = \dfrac{w}{wN_S + qw'N'_s - qC'}$

(C) $\dfrac{1}{C'} = \dfrac{q}{wN_S + qw'N'_S - qC'}$    (D) $\dfrac{1}{C} = \dfrac{1}{wN_S + qw'N'_S - qC'}$

## Question 2

Consider a model that is **similar to** (not exactly!) the Lecture 15 Firm's Problem, but there are two differences:

1. Consider that when firm is hiring workers, it is also doing the job training, i.e., the firm is also accumulating the **human capital** for itself.

- In date $0$, firms are hiring workers, paying wage $w$ and investing the job training cost $I^h$.

- In date $1$, firms pays wage $w'$ to the workers.

- the human capital accumulation process based on two parts. The first part is that the remaining human capital is depreciated by $\delta_h$. The second part is the human capital investment, $I^h$. Initial human capital at date $0$ is $1$, i.e., $H = 1$.

- Human capital **cannot** be liquidated after date $1$.

2. Production function is $Y = zK^\alpha(HN)^{1-\alpha}$, where $\alpha \in [0, 1]$, and the labor cost is $wN$ in date $0$ and $w'N'$ in date $1$.

(20) Assume that firm is discounting in the same way as consumer is, i.e., firms are discounting in the real interest rate $r$. Firm's object function is __$D$__,

(A) $\displaystyle\max_{N,N'} V = \pi + \frac{\pi'}{1+r'}$

(B) $\displaystyle\max_{N,N',K',I^h} V = \pi + \frac{\pi'}{r}$

(C) $\displaystyle\max_{N,N',K',H',I,I^h} V = \pi + q\pi'$

(D) $\displaystyle\max_{N,N',K',H',I,I^h} V = \pi + \frac{\pi'}{1+r}$

(21) where $\pi$ is __$B$__

(A) $zK^\alpha(HN)^{1-\alpha} - wN - I$

(B) $zK^\alpha(N)^{1-\alpha} - wN - I - I^h$

(C) $zK^\alpha(HN)^{1-\alpha} - wN - I - I^h$

(D) $zK^\alpha(HN)^{1-\alpha} - wN - I^h$

(22) and $\pi'$ is __$D$__

(A) $z'K'^\alpha(H'N')^{1-\alpha} - w'N' + (1-\delta)K' + (1-\delta_h)H'$

(B) $z'K'^\alpha(H'N')^{1-\alpha} - w'N' + (1-\delta)K' - (1-\delta_h)H'$

(C) $z'K'^\alpha(H'N')^{1-\alpha} - w'H'N' + (1-\delta)K'$

(D) $z'K'^\alpha(H'N')^{1-\alpha} - w'N' + (1-\delta)K'$

6

(23) subject to the capital accumulation process $K' = (1-\delta)K + I$ and human capital accumulation process __A__

(A) $H' = (1 - \delta_h)H + I^h$      (B) $H' = (1 - \delta)H + I^h$

(C) $H' = (1 - \delta_h)H + I$      (D) $H' = (1 - \delta)H + I$

By substituting $\pi, \pi', Y, Y', I$, and $I^h$ into the firm's problem, we get

$$\max_{N,N',K',H'} zK^\alpha(HN)^{1-\alpha} - wN - \underbrace{C}_{24} + \frac{z'K'^\alpha(H'N')^{1-\alpha} - w'N' + \overbrace{A}^{25}}{1+r}.$$

(24)   (A) $[K' - (1-\delta)K]$

     (B) $[H' - (1-\delta_h)H]$

     (C) $[K' - (1-\delta)K] - [H' - (1-\delta_h)H]$

     (D) $[K' - (1-\delta)K] + [H' - (1-\delta_h)H]$

(25)   (A) $(1-\delta)K'$      (B) $(1-\delta)K' + (1-\delta_h)H'$

     (C) $(1-\delta)K' + (1-\delta_h)H'N'$      (D) $(1-\delta)rK' + (1-\delta_h)H'N'$

Formal formulation:

$$\max_{N,N',K',H'} zK^\alpha(HN)^{1-\alpha} - wN - [K' - (1-\delta)K] - [H' - (1-\delta_h)H]$$

$$\frac{z'K'^\alpha(H'N')^{1-\alpha} - w'N' + (1-\delta)K'}{1+r}$$

(26) The FOC w.r.t. $N$ is __A__

(A) $(1-\alpha)zK^\alpha(HN)^{-\alpha}H = w$

(B) $-1 + \dfrac{(1-\alpha)z'K'^\alpha(H'N')^{-\alpha}N}{1+r} = 0$

7

(C) $(1-\alpha)z'K'^{\alpha}(H'N')^{-\alpha}H' = w'$

(D) $-1 + \dfrac{\alpha z'K'^{\alpha-1}(H'N')^{1-\alpha} + (1-\delta)K}{1+r} = 0$

(27) The FOC w.r.t. $N'$ is __*C*__

(A) $(1-\alpha)zK^{\alpha}(HN)^{-\alpha}H = w$

(B) $-1 + \dfrac{(1-\alpha)z'K'^{\alpha}(H'N')^{-\alpha}N}{1+r} = 0$

(C) $(1-\alpha)z'K'^{\alpha}(H'N')^{-\alpha}H' = w'$

(D) $-1 + \dfrac{\alpha z'K'^{\alpha-1}(H'N')^{1-\alpha} + (1-\delta)K}{1+r} = 0$

(28) The FOC w.r.t. $K'$ is __*D*__

(A) $(1-\alpha)zK^{\alpha}(HN)^{-\alpha}H = w$

(B) $-1 + \dfrac{(1-\alpha)z'K'^{\alpha}(H'N')^{-\alpha}N}{1+r} = 0$

(C) $(1-\alpha)z'K'^{\alpha}(H'N')^{-\alpha}H' = w'$

(D) $-1 + \dfrac{\alpha z'K'^{\alpha-1}(H'N')^{1-\alpha} + (1-\delta)K}{1+r} = 0$

(29) The FOC w.r.t. $H'$ is __*B*__

(A) $(1-\alpha)zK^{\alpha}(HN)^{-\alpha}H = w$

(B) $-1 + \dfrac{(1-\alpha)z'K'^{\alpha}(H'N')^{-\alpha}N}{1+r} = 0$

(C) $(1-\alpha)z'K'^{\alpha}(H'N')^{-\alpha}H' = w'$

(D) $-1 + \dfrac{\alpha z'K'^{\alpha-1}(H'N')^{1-\alpha} + (1-\delta)K}{1+r} = 0$