ECON 4002.01 Midterm Exam

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Instruction

Please submit your answer on Carmen Quiz "Midterm Exam".

You **may** consult any note and textbook, but you **cannot** discuss with your classmate or any other person about the exam.

There will be one T/F choice question that is worth 2 points on the answer sheet. The T/F choice question is to confirm: "**I affirm that I have not received or given any unauthorized help on this exam, and that all work is my own.**"

Grades

Question 1 to 40 are worth **2** points, and Question 41 to 44 are worth **5** points. The total grade is **102** points.

Question 1

Considering an one-period general equilibrium model **similar to** (but not exactly the same as) Example in Lecture 08, slide 11 and 12. Also the Experiment 2 from Lecture 07, slide 13 is also a good reference. However, in this model economy, there are two differences:

1. firm rent capital from consumer, and consumers are **endowed** with 2 units of capital ($K^s = 2$).
2. consumer's utility function is $U(C, l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$

(1) The competitive equilibrium given $\{G, z, \_\_\_\_\_\}$

(A) K^{d*} (B) K^s (C) w^* (D) r^*

(2) is a set of allocations $\{Y^*, C^*, l^*, N^s, N^d, \pi^*, T^*, \_\_\_\_\_\}$

(A) K^{d*} (B) K^s (C) w^* (D) r^*

(3) and prices $\{w^*, \_\_\_\_\_\}$ such that

(A) K^{d*} (B) K^s (C) w^* (D) r^*

1. Taken prices and π, T as given, the representative consumer solves

(4) $\max_{\_\_\_\_} U(C, l) = \dfrac{C^{1-b}}{1-b} + \dfrac{l^{1-d}}{1-d}$

(A) C, d (B) b, d (C) C, l (D) b, l

(5) subject to $C \leq w(h - l) + \_\_\_\_\_ + \pi - T$

(A) rN^d (B) rK^s (C) rK^{d*} (D) rN^s

2. Taken prices as given, the representative firm solves

$$\underbrace{\max}_{\text{Q6}} \; z(K^d)^a(N^d)^{1-a} - wN^d - \underbrace{\_\_\_\_}_{\text{Q7}}$$

6 (A) K^s, N^s (B) K^s, N^d (C) K^d, N^d (D) K^d, N^s

7 (A) wN^s (B) wN^d (C) rK^s (D) rK^d

3. Government collect taxes to balance budget:

8 _____

(A) $N^s = N^d$ (B) $T^* = G$ (C) $K^s = K^d$ (D) $w = r$

4. Labor market clear means that the equilibrium wage is w^* such that labor supply equals to labor demand:

9 _____

(A) $N^s = N^d$ (B) $T^* = G$ (C) $K^s = K^d$ (D) $w = r$

5. Capital market clear means that the equilibrium rental rate is r^* such that capital supply equals to capital demand:

10 _____

(A) $N^s = N^d$ (B) $T^* = G$ (C) $K^s = K^d$ (D) $w = r$

To solve this model economy, we reformulate the competitive equilibrium into the social planner's problem.

First of all, in social planner's problem, all markets must clear, and thus $N^s = N^d = N$, and $K^s = K^d = K$ (also $K = 2$, but for question 11 to 16, still remain the symbol K).

Through firm's FOC with respect to N and K, we know w and r are

11 $w = zK^a$ _____

(A) $(1-a)N^{-a}$ (B) $(a)N^{-a}$ (C) $(1-a)N^{a-1}$ (D) $(a)N^{a-1}$

12 $r = zN^{1-a}$ _____

(A) $(1-a)K^a$ (B) aK^{1-a} (C) $(1-a)K^{a-1}$ (D) aK^{a-1}

which we can use to retrieve wage and rental rate after solving the social planner's problem.

The social planner problem is given by:

13 Objective function is the consumer's utility:

$$\max_{C,l,N,Y,\_\_\_\_\_} U(C,l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$$

(A) C (B) N (C) K (D) Y

subject to

a. Aggregate resource constraint

14 $C + G =$ _____

(A) C (B) N (C) K (D) Y

b. production constraint

15 $Y = z$ _____

(A) $K^{1-a}N^{1-a}$ (B) K^aN^{1-a} (C) K^aN^a (D) $K^{1+a}N^{1+a}$

c. labor constraint

16 $N =$ _____

(A) $1 - Y$ (B) $1 - C$ (C) $1 - N$ (D) $1 - l$

d. capital constraint

(17) $K =$ _____

(A) 2 (B) 3 (C) 4 (D) 5

To solve the social planner's problem, we start with substituting the constraints into utility function:

a. Substituting 16 and 17 into 15, we get

(18) $Y = z$ _____

(A) $2^{a-1}(1-l)^{1-a}$ (B) $2^{a}(1-l)^{a}$
(C) $2^{a-1}(1-l)^{a}$ (D) $2^{a}(1-l)^{1-a}$

b. Substituting 18 into 14, we get

(19) $C = z$ _____

(A) $2^{a-1}(1-l)^{1-a} - G$ (B) $2^{a}(1-l)^{a} - G$
(C) $2^{a-1}(1-l)^{a} - G$ (D) $2^{a}(1-l)^{1-a} - G$

c. Finally, substituting 19 into 13, we get

$$\underbrace{\max}_{\text{Q20}} \; U(C(l), l) = \frac{(z \overbrace{\qquad\quad}^{\text{Q21}})^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$$

(20) (A) N (B) l (C) C (D) K

(21) (A) $2^{a-1}(1-l)^{1-a} - G$ (B) $2^{a}(1-l)^{a} - G$
(C) $2^{a-1}(1-l)^{a} - G$ (D) $2^{a}(1-l)^{1-a} - G$

Let $z = 1, G = 0, a = \frac{1}{2}, b = 2, d = \frac{3}{2}$ and solve for all unknowns,

22 $l =$ _____

(A) 1.23 (B) 0.33 (C) 0.67 (D) 0.20

23 $N =$ _____

(A) 1.23 (B) 0.33 (C) 0.67 (D) 0.20

24 $w^* =$ _____

(A) 1.23 (B) 0.33 (C) 0.67 (D) 0.20

25 $r^* =$ _____

(A) 1.23 (B) 0.33 (C) 0.67 (D) 0.20

The following is the calculation for the answer from 22 to 25:

Question 2

Consider a model economy with distorting labor taxes similar to Lecture 11, but having two difference:

1. the production function is Cobb-Douglas requiring only labor input, i.e., $Y = zN^a$, and
2. consumer get "disutility" from working, and the utility function is given by $U(C, N) = \ln C - bN$, where b is a parameter.

Other than the two changes mentioned above, the definition of the general equilibrium is exactly the same as slide 5 in Lecture 11. Therefore, this question focus mainly on algebraic calculation.

26 The derivative of utility function with respect to consumption C is ______

(A) aC^{a-1} (B) C^{-a} (C) $\frac{1}{C}$ (D) C

27 The derivative of utility function with respect to labor N is ______

(A) $(1-a)b^{-a}$ (B) b^{a-1} (C) b (D) $-b$

28 The marginal rate of substitution between labor and consumption ($MRS_{N,C} \equiv \frac{D_N U}{D_C U}$) is ______

(A) $(1-a)b^{-a}aC^{a-1};$ (B) $b^{a-1}C^{-a}$

(C) $\frac{b}{C}$ (D) $-bC$

29 According to the slide 6 and 9, Lecture 11, in equilibrium MRS is going to be equal to the after-tax wage, i.e., _____

(A) w (B) $w(1-t)$ (C) wt (D) $(1-t)$

30 According to the slide 5, Lecture 11, consumer's budget constraint is saying that $C =$ _____ $N + \pi$

(A) w (B) $w(1-t)$ (C) wt (D) $(1-t)$

31 Different from slide 5, Lecture 11, now the production function is $Y = zN^a$, and thus by solving the firm's problem, the equilibrium wage as a function of labor demand N must be $w = z$ ______

(A) $(1-a)N^{-a}$ (B) aN^{-a} (C) $(1-a)N^a$ (D) aN^{a-1}

32 Following 31, firm's profit $\pi = z$ ______

(A) $(1-a)N^{-a}$ (B) aN^{-a} (C) $(1-a)N^a$ (D) aN^{a-1}

The $MRS_{N,C}$ we calculated above is MRS between labor and consumption. The equilibrium condition that in the lecture 11 is according to the $MRS_{l,C}$, the MRS between leisure and consumption. Recall that $l = 1 - N$, and thus $MRS_{l,C} = -MRS_{N,C}$.

33 Combining your answer in 28 and 29 together, let $MRS_{l,C} = -MRS_{N,C}$, and substitute consumption from your answer in 30, we get the optimal condition is $b($ ______ $N + \pi) = w(1-t)$

(A) w (B) $w(1-t)$ (C) wt (D) $(1-t)$

The following three blanks are corresponds to 34 to 36. Combining your answer in 31 and 32 and substitute w and π into your answer in 33, we get

$$b(\ z \underbrace{\rule{4cm}{0.4pt}}_{w \text{ part}} + z \underbrace{\rule{3.5cm}{0.4pt}}_{\pi \text{ part}}) = z \underbrace{\rule{2.5cm}{0.4pt}}_{w \text{ part } 2} (1-t)$$

34 $z \underbrace{\rule{1.5cm}{0.4pt}}_{w \text{ part}}$

(A) $aN^{a-1}(1-t)$ (B) $aN^a t$

(C) $aN^a(1-t)$ (D) $(1-a)N^a(1-t)$

35. $z\underbrace{\rule{2cm}{0.4pt}}_{\pi \text{ part}}$

(A) $(1-a)N^a$ (B) aN^a

(C) $(1-a)N^{a-1}$ (D) aN^{a-1}

36. $z\underbrace{\rule{2cm}{0.4pt}}_{w \text{ part 2}}$

(A) aN^{a-1} (B) aN^a

(C) aN^a (D) $(1-a)N^a$

37. Solve for N as a function of t, we get $N = \frac{a(1-t)}{b(\rule{1cm}{0.4pt}+(1-a))}$

(A) $a(t-1)$ (B) $(a-1)(1-t)$ (C) $(1-t)$ (D) $a(1-t)$

Some calculation details:

____

____

____

____

For the following questions, choose the closest answer to your calculation.

38. Now let $z = 1$, $a = 0.33$, $b = 2.15$ and $t = 0.5$, the N you calculated in 37 is _____

(A) 0.09 (B) 1.66 (C) 0.30 (D) 0.74

39. after calculate the approximated value for N to the second decimal point, you can also calculate $w =$ _____

(A) 0.09 (B) 1.66 (C) 0.30 (D) 0.74

(40) Same for $\pi =$ _____

(A) 0.09 (B) 1.66 (C) 0.30 (D) 0.74

(41) For the tax revenue generate by $t = 0.5$, how much government spending G can the government pay off? $G = R(t) = wtN =$ _____

(A) 0.009 (B) 1.066 (C) 0.030 (D) 0.074

(42) According to Laffer curve, there's also another tax rate t_2 such that it can also generate same amount of revenue to pay for the government spending you've calculated 41. What is t_2? $t_2 \approx$ _____

(A) 0.9387 (B) 0.7677 (C) 0.9285 (D) 0.2541

If you want to solve 42 numerically, you can open the following link to get a `julia` terminal:

https://www.tutorialspoint.com/execute_julia_online.php

and use `q2.jl` file in the Carmen module `Midterm` or copy from the code at the end of this file to the online julia terminal. Note that if you copy the code from this pdf file, you need to make sure that all indentation/space is correct.

(43) Given that $z = 1, a = 0.33$, and $b = 2.15$, if government wants to maximize the labor tax revenue $R(t) = wtN$, the optimal tax rate $t^* =$ _____,

(A) 0.9387 (B) 0.7677 (C) 0.9285 (D) 0.2541

(44) and the optimal labor tax revenue $R(t^*) =$ _____

(A) 0.9387 (B) 0.7677 (C) 0.9285 (D) 0.2541

You can play with this code and explore more!

Note: Due to julia's restriction and latex compilation, if you copy this to the julia terminal, you will get error (on line 8). Please use the content in the `q2.jl`.

```julia
# parameters
a = 0.33
b = 2.15
z = 1
t = 0.5

# implicit functions
labor(a, b, t) = (a*(1-t)) / (b*(a*(1-t) + (1-a)))
wage(a, z, N) = a*z*N^(a-1)
gov(w, t, N) = w*t*N

## find the G level at tax = 0.5
N = labor(a, b, t)
w = wage(a, z, N)
G = gov(w, t, N)
Gtarget = G

## iterate all possible tax value and calculate corresponding
tnum = 1000
tvec = collect( range(0.0001, 0.999, tnum) )
Gvec = Array{Float64, 1}(undef, tnum)

for indt = 1:1:tnum
    local t, N, w, G
    t = tvec[indt]
    N = labor(a, b, t)
    w = wage(a, z, N)
    G = gov(w, t, N)
    Gvec[indt] = G
    if (abs(G - Gtarget) < 0.0001)
        println("Potential answer for Q42: \
                At tax = $t, G = $G, \
                Target - G = $(G - Gtarget)")
    end
end

Gmax = maximum(Gvec)
Gmaxidx = argmax(Gvec)
Tmax = tvec[Gmaxidx]
```



```julia
println("Q43: maximum G is achieved at tax rate $Tmax")
println("Q44: maximum G is $Gmax")
```