

# Lecture 15

## The Real Business Cycle Model

### Part 2: Firm

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- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

# Demand for Consumption Goods

Ultimately, 3 markets will have to clear in the current period (date 0):

- ① labor (like static model)
- ② credit (like dynamic model)
- ③ consumption goods (implied in each case by Walras' Law)

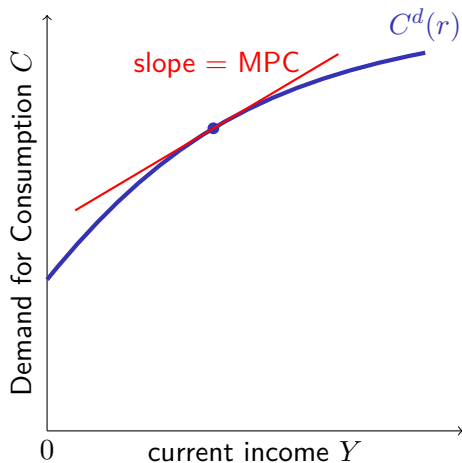
Recall our insights from last classes. Primary determinants of consumption:

- over lifetime: permanent income / lifetime wealth
- across periods: interest rate, current vs future income

Based on this, we'll construct a **demand curve for current consumption** goods that depends on lifetime wealth and the interest rate

# Current Goods Demand and Current Income

Figure 11.4 Consumer's Current Demand for Consumption Goods Increases with Income



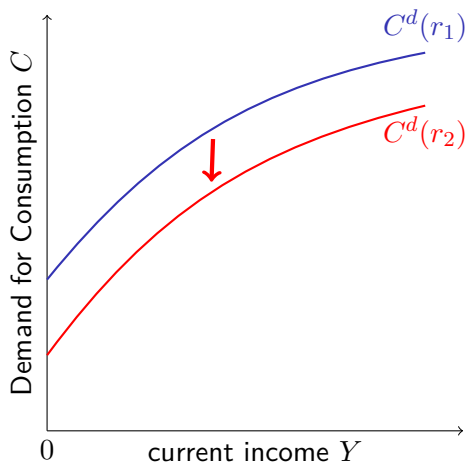
## Assumption C1: demands for goods

↑ in income

- Recall **pure income effect**
- Slope of tangent line is **marginal propensity to consume (MPC)**
  - what fraction of  $Y \uparrow$  goes to  $C$ ?
  - $MPC = dC_D/dY$
- normal goods: both  $C$  and  $C' \uparrow$ , so saving  $S \uparrow$ 
  - usually  $MPC < 1$ , i.e., not all  $Y \uparrow$  goes to  $C$ .

# Current Goods Demand and Real Interest Rate

Figure 11.5 Real Interest Rate  $\uparrow$  Shifts the Demand for Consumption Goods Down

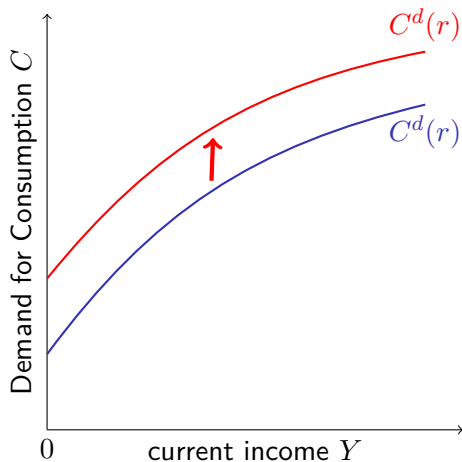


**Assumption C2:** demands for goods  
 $\downarrow$  in real interest rate

- Recall both **income** and **substitution effect** (from dynamic model)
- Income effect: ambiguous (for borrowers and lenders)
- Substitution effect: always negative (for borrowers and lenders)
- **C2** assumes substitution effect dominates

# Current Goods Demand and Lifetime Wealth

Figure 11.6 An Increase in Lifetime Wealth Shifts the Demand for Consumption Goods Up



**Assumption C3:** demands for goods

↑ in lifetime wealth

■ similar to pure income effect

Note: consumer's demand is only one part of the GDP:

$$Y = C + I + G.$$

We'll discuss  $I$  and  $G$  in next lecture

# Overview: Firm Decision

■ **production:** needs both capital  $K$  and labor  $N$ ,  $Y = zF(K, N)$

■ **endowment:** firm is endowed with initial capital  $K$

■ **firm decision:**

- **both dates:** labor ( $N$ ), profit ( $\pi$ ), and output ( $Y$ ) by production

$$Y = zF(K, N) \text{ and } Y' = z'F(K', N')$$

- **date 0 (today): investment** ( $I$ ) determines future capital  $K'$  given initial capital  $K$  and **depreciation rate**  $\delta \in [0, 1]$ ,

$$K' = (1 - \delta)K + I$$

■ **Assumptions:**

- ① investment made in consumption goods
- ② remaining capital  $(1 - \delta)K'$  liquidates tomorrow ( $\because$  model ends)

# Firm's Optimization Problem

Firm maximizes the discounted present value of profits:

$$\max_{N_D, N'_D, K', I} V = \pi + \frac{\pi'}{1+r} \quad \text{subject to} \quad K' = (1-\delta)K + I,$$

where  $\pi = Y - wN - I$ , and  $\pi' = Y' - w'N' + \underbrace{(1-\delta)K'}_{\text{liquidate}}$ .

Notice: since we assume that **consumer owns the firm**, so firm calculates present value using **real interest rate  $r$** , i.e., how consumer discounts.

By substituting  $\pi$ ,  $\pi'$ ,  $Y$ ,  $Y'$  and  $I$  into above problem, we get

$$\begin{aligned} \max_{N_D, N'_D, K'} & zF(K, N_D) - wN_D - [K' - (1-\delta)K] \\ & + \frac{z'F(K', N'_D) - w'N'_D + (1-\delta)K'}{1+r} \end{aligned} \quad (1)$$



# Firm's Optimality Conditions

$$[N_D] : z D_N F(K, N_D) = w$$

$$[N'_D] : z' D_N F(K', N'_D) = w'$$

$$[K'] : -1 + \frac{z' D_{K'} F(K', N'_D) + (1 - \delta)}{1 + r} = 0$$

- FOCs on current and future labor are **the same** as static model!
  - Why? Since labor choice is **static**: choose labor for **current** production
- FOC on future capital equalize the **marginal cost and benefit** of investment
  - cost: loss in current consumption (incurred today)
  - benefit:  $\uparrow$  in marginal production + liquidating  $K'$  (incurred tomorrow)

# Optimal Investment Schedule: Derivation

Solve for  $[K']$ , we get

$$z' D_{K'} F(K', N'_D) + 1 - \delta = 1 + r \Rightarrow r = MPK' - \delta$$

For consumer, there are 2 assets to undertake **intertemporal substitution**:

- ① saving in credit market (supply in credit mkt; demand in bond mkt)
- ② capital held by the firm for production

Investing in capital means giving up (net) return  $r$  for (net) return

$MPK' - \delta$ : **optimal investment rule means both must offset, WHY?**

- if  $r > MPK' - \delta$ : consumer will save more for bond  $\Rightarrow$  supply in credit market  $\uparrow$ ,  $r \downarrow$
- if  $r < MPK' - \delta$ : consumer asks firm to invest more capital  $\Rightarrow MPK' \downarrow$

To sum up,  $r = MPK' - \delta$  in equilibrium: “optimal” investment rule!

# Labor Demand is decreasing in $w$ and increasing in $z, K$

Figure 11.7 The Demand Curve for  $N$  Is the Firm's MPL Schedule

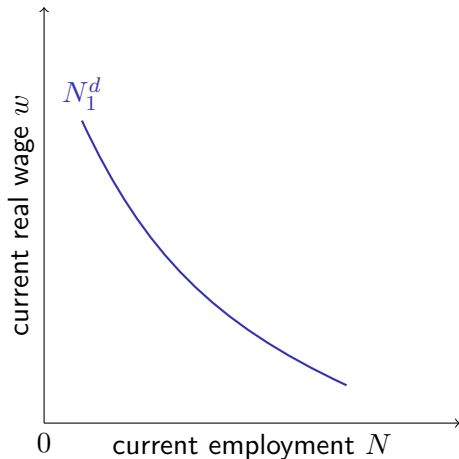
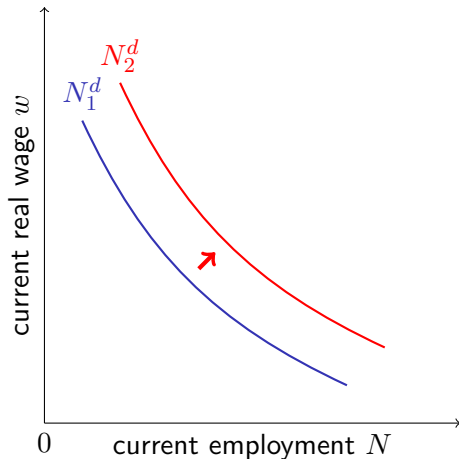
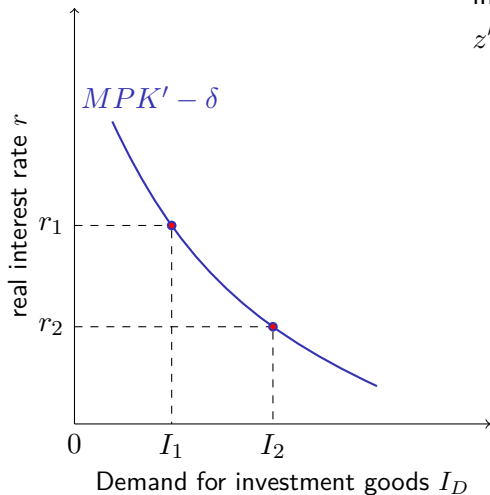


Figure 11.8 The Current Demand Curve for Labor Shifts Due to Changes in  $z$  and  $K$



# Optimal Investment Schedule: Graphical Representation

Figure 11.9 Optimal Investment Schedule for the Representative Firm



Put capital accumulation process into MPK and get

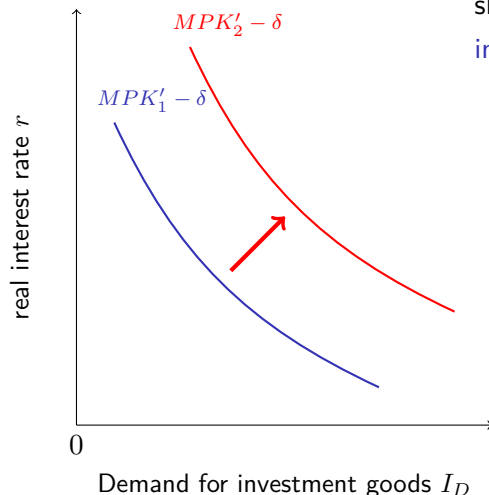
$$z' D_{K'} F((1 - \delta)K + I_D, N'_D) = r + \delta$$

- as  $r \uparrow$ , need less  $K'$  for optimal investment schedule to hold.
  - why? diminishing MPK
- $K' \uparrow$  in  $I$ , so  $r \uparrow$  also means less investment  $\Rightarrow$  downward slope
- i.e., higher opportunity cost of investing

# Optimal Investment Schedule: Effect of $K$ and $z'$

Figure 11.10 The Optimal Investment Schedule

Shifts to the Right if  $K \downarrow$  or expecting  $z' \uparrow$



The optimal investment schedule shifts to the right, i.e., **demand for investment rises** if

- current capital  $K$  decreases:

$$\frac{dI_D}{dK} < 0$$

- Intuition: need to invest more for less endowment

- (expected) future TFP

increases:  $\frac{dI_D}{dz'} > 0$

- Intuition: investment is more productive