Lecture 11
Distorting Taxes and the Welfare Theorems

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Overview

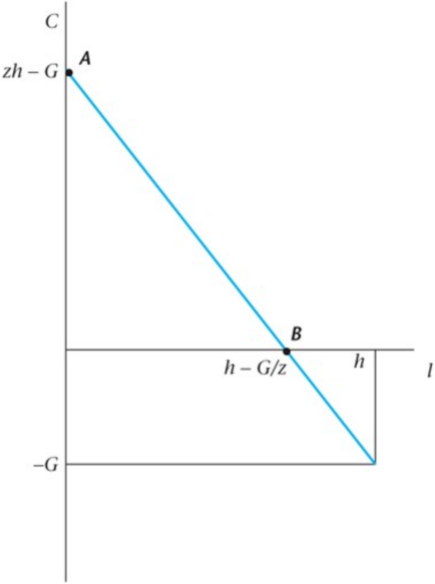
In previous lectures, all the taxes we are discussing is lump-sum tax.

- pure income effect, no change to consumption-leisure allocation
- satisfy both welfare theorems

In this lecture, the distorting taxes will include substitution effect, and thus

- creating "wedges" to distort consumption-leisure choice
- violate the welfare theorems (CE $\neq$ SPP)

SPP in Simplified Model



Assume production is labor-only technology:

$$Y = zN^d$$

So PPF is

$C = Y - G$

$$C = z(h - l) - G$$

Thus, SPP is

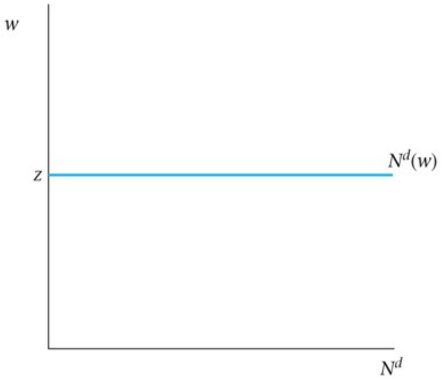
$$\max_l U(z(h-l) - G, l)$$

FOC: $$\frac{D_l U(C,l)}{D_C U(C,l)} = MRS_{l,C}$$

$$= MRT_{l,C} = z = MPN$$

Labor Demand in Simplified Model

Figure 5.15 The Labor Demand Curve in the Simplified Model



$$\max_{N^d} zN^d - wN^d$$

FOC would be $z = w$ (horizontal line)

- if $z < w$: negative profit for every worker hired, choose $N^d = 0$
- if $z > w$: positive profit for every worker hired, choose $N^d = \infty$
- only $z = w$ possible, $\therefore$ linear PPF in previous slide
  - "infinitely elastic" N^d

Competitive Equilibrium w/ Distorting Tax

A competitive equilibrium, with $\{z, G\}$ exogenous, is a list of endogenous prices and quantities $\{C, l, N^s, N^d, Y, \pi, w, t\}$ such that:

1. taking $\{w, \pi\}$ as given, the consumer solves

tax rate

$$\max_{C,l,N^s} U(C,l) \quad \text{subject to} \quad C = w(1-t)N^s + \pi \quad \text{and} \quad N^s + l = h$$

2. taking w as given, the firm solves:

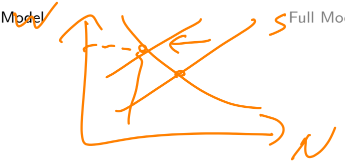
$$\max_{N^d,Y,\pi} \pi \quad \text{subject to} \quad \pi = Y - wN^d \quad \text{and} \quad Y = zN^d$$

3. the government spends $G = \boxed{wtN^s}$

$wt \cdot N^s$

4. the labor market clears at the equilibrium wage, i.e. $N^s = N^d$

Effect of Distorting Tax



Since the tax is imposed on consumers/workers, it distorted the consumption-leisure decision:

$N^S \downarrow$

$$MRS_{l,C} = w(1-t)$$

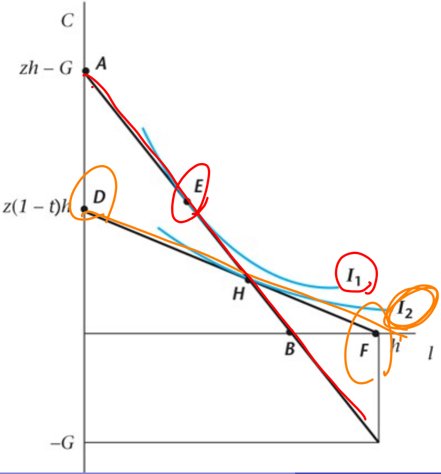
So in the equilibrium, it deviates from SPP:

$$\boxed{MRS_{l,C} = w(1-t)} < w = z = MPN = MRT_{l,C}$$

Result: CE and SPP lead to different allocation!

Graphical Representation

Figure 5.16 Competitive Equilibrium in the Simplified Model with a Proportional Tax on Labor Income



SPP solution lies at point E:

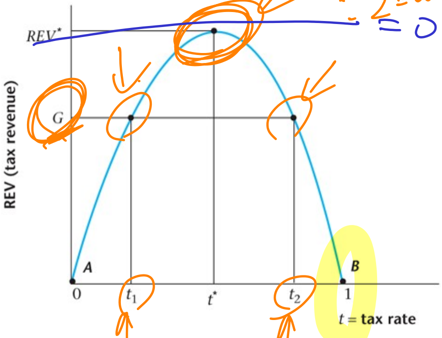
- $\overline{AB}$: PPF, slope $-z$ $= -w$.
- can reach indifference curve I_1

CE solution lies at point H:

- $\overline{DF}$: consumer's budget line
- can only reach I_2
- proportional tax $\Rightarrow N^s \downarrow$
- $N^s\downarrow \Rightarrow Y\downarrow$, but still need to meet G, so $C\downarrow$: gov't budget critical!

How Much Tax Revenue can be Generated?

Figure 5.17 A Laffer Curve



equilibrium wage: $w = z$, implies total tax revenue by solve consumer problem:

$R(t) = wtl^s$

$\because z = w,\ l^s = 1 - l$

$$R(t) = tz(h - l^*(t)),$$

$= zt(1-l)$

$= 0$

What t maximizes? Solve

$$\max_t R(t) = \max_t tz(h - l^*(t)),$$

- not just $t = 1$! tax rate vs tax base
- $t = 0$: no revenue because no tax
- $t = 1$: no revenue because no incentive to work

Back

Full Model Elaboration

$w = MPN = z = 1$

Let $U(C, l) = \ln C + \ln l$, and $h = z = 1$, by firm's problem we know $w = z = 1$. Consumer has some non-labor income denoted as $x > 0$. FOC leads to

$\frac{D_l U}{D_c U} = \frac{1/l}{1/C}$ $\quad MRS = w(1-t)$

N^s

$$
\begin{aligned}
MRS_{l,C} &= \frac{C}{l} \\
&= \boxed{\frac{(1-t)(1-l)+\pi}{l} = 1-t} < 1 = MRT_{l,C} \\
&\Rightarrow (1-t)(1-l) + \pi = (1-t)l \\
&\Rightarrow 1 - l + \frac{\pi}{1-t} = l \Rightarrow 2l = 1 + \frac{\pi}{(1-t)} \\
&\Rightarrow l = \frac{1}{2} + \frac{\pi}{2(1-t)} \\
&\Rightarrow \boxed{N^s(t)} = 1 - l = \frac{1}{2} - \frac{\pi}{2(1-t)}
\end{aligned}
$$

$\times l$

$\div (1-t)$

$t \uparrow \Rightarrow N^s(t) \downarrow$

Maximize Tax Revenue

Total tax revenue is

$\overset{\shortparallel}{w} \cdot t \cdot N^s(t)$

$$R(t) = \underline{tN^s(t)},$$

and thus government's problem is

$$D_t \frac{A(t)}{B(t)} = \frac{DA \cdot B - A \cdot DB}{[B(t)]^2}$$

$$\max_t \frac{1}{2}t - \boxed{\frac{t\pi}{2(1-t)}}.$$

assume gov wants to maximize tax revenue.

$$\Rightarrow \underbrace{t\pi}_{A} \times \underbrace{[2(1-t)]^{-1}}_{B}$$

$$\Downarrow$$

$$DA \times B + A \times DB$$

FOC leads to

$$\frac{1}{2} - \frac{\pi(1-t) + t\pi}{2(1-t)^2} = 0 \Rightarrow \frac{1}{2} - \frac{\pi}{2(1-t)^2} = 0$$

$$\frac{1}{2} = \frac{\pi}{2(1-t)^2} \Rightarrow 1 = \frac{\pi}{(1-t)^2}$$

$$t = 1 - \sqrt{\pi}$$

$$\frac{DA \times B - A \times DB}{[B]^2}$$

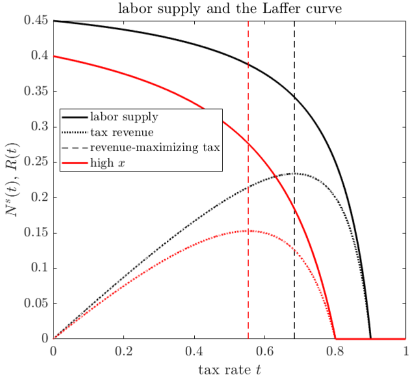
$$D_t\left(\frac{t\pi}{2(1-t)}\right)$$

$$= \frac{\pi \cdot 2(1-t) - t\pi \cdot (-2)}{[2(1-t)]^2}$$

$$= \frac{\cancel{2}\pi - \cancel{2\pi t} + \cancel{2t\pi}}{2\cancel{4}(1-t)^2}$$

$$= \frac{\pi}{2(1-t)^2}$$

Visualization



Consider two cases:

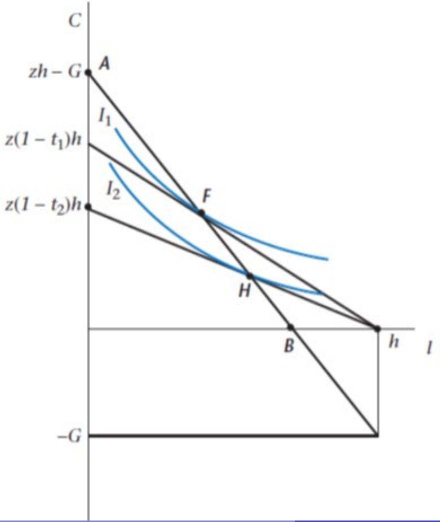
1. consumer is poor (low π)
2. consumer is rich (high π)

For a given after tax-wage , rich consumer supplies less labor

- tax revenue shifts down
- Laffer peak shifts left
- many other conditions also impact this analysis!

Multiple Competitive Equilibria Possible

Figure 5.18 Two Competitive Equilibria



Previous slide logic implies the government can choose 2 tax rates for a given required level of G

- both t_1 and t_2 yield the same revenue
- consumer strictly better off under lower tax rate t_1

Tax Revenue

Conclusion

We've focused on the simple case to keep analysis straightforward, but logic applies more broadly.

- SPP: $MRS_{l,C} = MRT_{l,C} = MPN$, since PPF is $C = zF(K,N) - G$
- CE: same distortion as our simple case:
  - consumer problem implies $MRS_{l,C} = w(1-t)$
  - firm problem implies $MRT_{l,C} = w$
  - same result as simplified model: $MRS_{l,C} \neq MRT_{l,C}$, unlike SPP
  - only difference from simplified model: $MPN = D_N F(K,N) \neq z$