Lecture 17
The Real Business Cycle Model
Part 4: Formal Examples

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Overview

- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

Assumptions

- **consumer**: assume discounting factor $\beta \in (0,1)$ and utility function is
$$\tilde{U}(C, N, C') = \ln C + \beta \ln C' + \gamma \ln(1 - N),$$
where $\gamma > 0$, and consumer endowed with 1 unit of time.
  - we assume no dis-utility in date 1 labor supply to simplify analysis

- **firm**: assume production is Cobb-Douglas in both periods:
$$Y = zK^{\alpha}N^{1-\alpha} \text{ and } Y' = z'K'^{\alpha}N'^{1-\alpha},$$
where K is initial capital, TFP $z = 1$, and depreciation $\delta \in (0,1)$

- **government**: spend G and G', which is financed by lump-sum taxes T, T' and deficit B

Competitive Equilibrium

Given exogenous quantities $\{G, G', z, z', K\}$, a competitive equilibrium is a set of (1) consumer choices $\{C, C', N_S, N_S', l, l', S\}$; (2) firm choices $\{Y, Y', \pi, \pi', N_D, N_D', I, K'\}$; (3) government choices $\{T, T', B\}$, and (4) prices $\{w, w', r\}$ such that

1. Taken $\{w, w', r, \pi, \pi'\}$ as given, consumer chooses $\{C', N_S, N_S'\}$ to solve
$$\max_{C', N_S, N_S'} \ln\left(wN_S + \pi - T + \frac{w'N_S' + \pi' - T' - C'}{1+r}\right) + \beta \ln C' + \gamma \ln(1 - N_S),$$
where we can back out $\{C, S, l, l'\}$.

2. Taken $\{w, w', r\}$ as given, firm chooses $\{N_D, N_D', K'\}$ to solve
$$\max_{N_D, N_D', K'} zK^\alpha N_D^{1-\alpha} - wN_D - [K' - (1-\delta)K] + \frac{z'(K')^\alpha (N_D')^{1-\alpha} - w'N_D' + (1-\delta)K'}{1+r},$$
where we can back out $\{Y, Y', \pi, \pi', I\}$.

3. Taxes and deficit satisfy $T + \frac{T'}{1+r} = G + \frac{G'}{1+r}$ and $G - T = B$.

4. All markets clear: (i) labor, $N_S = N_D$ & $N_S' = N_D'$; (ii) goods, $Y = C + G$ & $Y' = C' + G'$; (iii) bonds at date 0, $S = B$.

Step 0: Result Implied by Assumptions

1. $N'_S = 1$, since consumer don't value leisure at date 1.
   - If consumer don't value leisure, then choose the highest possible N'_S can expand the budget set without decreasing the utility.

2. $N'_D = N'_S = 1$, by future labor market clearing.

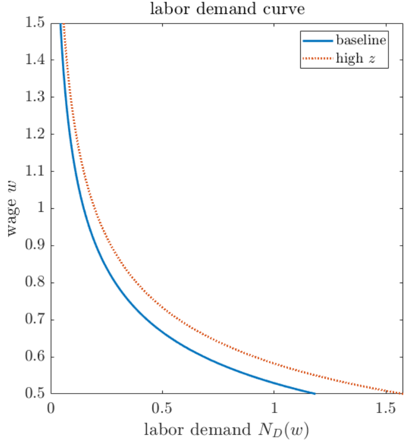
3. The future wage w' is determined by MPN':

$$MPN' = z(1-\alpha)\left(\frac{K'}{N'_D}\right)^{\alpha},$$

where $N'_D = 1$ leads to

$$w' = z(1-\alpha)(K')^{\alpha}.$$

Step 1: Firm's Current Labor Demand



For date 0 labor demand,

$$MPN = z(1-\alpha)\left(\frac{K}{N_D}\right)^{\alpha} = w$$

$$\Rightarrow N_D = \left(\frac{z(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$$

- $N_D \downarrow$ in current wage w
- $N_D \uparrow$ in current TFP z (dotted line)
- N_D invariant to interest rate

Step 2: Consumer & Current Labor Supply

- labor supply at date 0:

$$MRS_{l,C} = -MRS_{N,C} = -\frac{D_N \tilde{U}(\cdot)}{D_C \tilde{U}(\cdot)}$$
$$= -\frac{-\gamma/(1-N_S)}{1/C} = \frac{\gamma C}{1-N_S} = w$$

- Saving at date 0:

$$MRS_{C,C'} = \frac{1/C}{b/C'} = \frac{C'}{bC} = 1 + r$$

- Recall $N'_S = 1$, we can denote the x notation to be the part of the income that is NOT directly affected by consumer choice:

$$x = \pi - T \quad \text{and} \quad x' = w' + \pi' - T'$$

Step 2: Consumer & Current Labor Supply (Cont.)

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Appendix

References I