

Problem Set 2

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Problem 1: Macroeconomic Analysis of the 2025 Taiwan Cash Handout

Applied Context: The Policy

In late 2025, the Taiwanese government finalized a plan to distribute a universal cash handout of NTD 10,000 to all eligible citizens and residents. This policy, officially part of the "Special Act for Strengthening Economic and Social Resilience," was designed to share the benefits of strong economic performance and provide relief from global inflationary pressures.

As a junior economist at a local think tank in Hsinchu, you are tasked with analyzing the potential impact of this "helicopter money" on household labor supply. You decide to apply a standard micro-founded macroeconomic model.

Important Note: This is a **partial equilibrium** analysis focusing only on the consumer's decision. It assumes the wage is fixed and does not solve for a general equilibrium where the labor market clears.

The Model

- **Utility Function:** $U(C, l) = \frac{C^{1-\sigma}}{1-\sigma} + b \cdot l$
 - **Budget Constraint:** $C \leq w(h - l) + \pi - T$
 - **Parameters:** $\sigma = 2, b = 5, w = 10, h = 1$
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Part I: Theoretical Foundations

- ① The **Frisch elasticity of labor supply** ($\epsilon_{N,w}^{Frisch}$) measures the percentage change in hours worked (N) in response to a one percent change in the real wage (w), holding the marginal utility of wealth (λ) constant. Mathematically, it is defined as:

$$\epsilon_{N,w}^{Frisch} = \left. \frac{\partial N/N}{\partial w/w} \right|_{\lambda \text{ constant}}$$

Given the utility function in this model, what is the Frisch elasticity of labor supply?

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- (A) 0
- (B) 1
- (C) -1
- (D) Infinite (∞)

Solution

The Frisch elasticity holds λ constant. The optimality condition is $b = \lambda \cdot w$. Since b and λ are both treated as constants in this experiment, if w changes even slightly, one side becomes permanently greater than the other, causing an "all-or-nothing" jump in labor supply from 0 to max hours, which is an infinite response.

- ② The MRS represents the rate at which a consumer is willing to trade consumption for leisure while maintaining the same level of utility. What is the correct formula for the $MRS_{l,C}$ for this model? A

- (A) bC^σ
- (B) C^σ/b
- (C) b/C
- (D) w/b

Solution

The MRS is the ratio of the marginal utility of leisure ($MU_l = b$) to the marginal utility of consumption ($MU_C = C^{-\sigma}$). Therefore, $MRS_{l,C} = \frac{MU_l}{MU_C} = \frac{b}{C^{-\sigma}} = bC^\sigma$.

Part II: The Economy Before the Handout

For this section, assume the baseline scenario where $\pi - T = 0$.

- ③ What is the consumer's optimal consumption (C_0) in this pre-policy environment? C

- (A) 2
- (B) 10
- (C) $\sqrt{2}$
- (D) 5

Solution

The optimal interior choice occurs where $MRS_{l,C} = w$, so $bC^\sigma = w$. Plugging in the parameters: $5 \cdot C_0^2 = 10 \Rightarrow C_0^2 = 2 \Rightarrow C_0 = \sqrt{2} \approx 1.414$.

④ What is the consumer's optimal choice of leisure (l_0)? A

- (A) $1 - \frac{\sqrt{2}}{10}$
- (B) 1
- (C) 0.5
- (D) 0

Solution

Substitute $C_0 = \sqrt{2}$ into the budget constraint: $\sqrt{2} = 10(1 - l_0) + 0 \Rightarrow 10l_0 = 10 - \sqrt{2} \Rightarrow l_0 = 1 - \frac{\sqrt{2}}{10} \approx 0.8586$.

⑤ Given the choice of leisure in the previous question, how many hours does the consumer choose to work ($N_0 = h - l_0$)? B

- (A) $1 - \frac{\sqrt{10}}{2}$
- (B) $\frac{\sqrt{2}}{10}$
- (C) 0.5
- (D) 1

Solution

Labor supply is total time minus leisure: $N_0 = h - l_0 = 1 - (1 - \frac{\sqrt{2}}{10}) = \frac{\sqrt{2}}{10} \approx 0.1414$.

Part III: The Economy After the Handout

The NTD 10,000 handout is now distributed. We normalize this amount so the post-policy value is $\pi - T = 1$.

- ⑥ What is the consumer's new optimal consumption (C_1)? D
- (A) $1 + \sqrt{2}$
 - (B) 11
 - (C) 1
 - (D) $\sqrt{2}$

Solution

For an interior solution, optimal consumption depends only on the wage and preferences ($C = (w/b)^{1/\sigma}$), not non-wage income. Since the wage is constant and the cash handout is not large enough to push the consumer to a corner solution, their target consumption level remains the same.

- ⑦ What is the consumer's new optimal leisure (l_1)? B
- (A) It remains unchanged.
 - (B) $1 - \frac{\sqrt{2}-1}{10}$
 - (C) 1
 - (D) $1 - \frac{\sqrt{2}}{10}$

Solution

Plug the new parameters into the budget constraint: $\sqrt{2} = 10(1-l_1)+1 \Rightarrow \sqrt{2}-1 = 10-10l_1 \Rightarrow 10l_1 = 11-\sqrt{2} \Rightarrow l_1 = \frac{11-\sqrt{2}}{10} \approx 0.9586$.

- ⑧ What is the new number of hours worked ($N_1 = h - l_1$)? A
- (A) $\frac{\sqrt{2}-1}{10}$
 - (B) $\frac{\sqrt{2}}{10}$

- (C) 0
- (D) 0.1

Solution

$$N_1 = h - l_1 = 1 - \left(\frac{11-\sqrt{2}}{10}\right) = \frac{10-11+\sqrt{2}}{10} = \frac{\sqrt{2}-1}{10} \approx 0.0414.$$

- ⑨ What is the approximate percentage change in hours worked as a result of the cash handout? C
- (A) -29.3%
 - (B) +70.7%
 - (C) -70.7%
 - (D) +29.3%

Solution

The percentage change is $\frac{N_1 - N_0}{N_0} = \frac{(\sqrt{2}-1)/10 - \sqrt{2}/10}{\sqrt{2}/10} = \frac{\sqrt{2}-1-\sqrt{2}}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \approx -0.707$, or -70.7%.

Part IV: Analysis and Interpretation

- ⑩ The cash handout represents a pure... C
- (A) Substitution Effect
 - (B) Technology Shock
 - (C) Income Effect
 - (D) Preference Shock

Solution

A cash transfer that does not depend on hours worked is a pure increase in non-wage income. This shifts the budget constraint outward without changing its slope (the wage), which is the definition of a pure income effect.

- ⑪ The model's prediction that leisure increases when non-wage income rises implies that leisure is what kind of good? D
- (A) An inferior good
 - (B) A Giffen good
 - (C) A public good
 - (D) A normal good

Solution

A normal good is any good for which demand increases when income increases. Since the consumer "buys" more leisure (i.e., works less) when their non-wage income rises, leisure is a normal good.

- ⑫ This partial equilibrium analysis focuses only on the household labor supply decision. If this were the only effect, what would the model predict is the immediate impact of the cash handout on GDP? B
- (A) GDP would increase, because consumption rises.
 - (B) GDP would decrease, because total hours worked in the economy fall.
 - (C) GDP would remain unchanged, because the money is just a transfer.
 - (D) The impact cannot be determined.

Solution

In this simple model, GDP is a function of total labor input. Since the policy causes the representative consumer to work less ($N_1 < N_0$), the total hours worked in the economy would fall, leading to a decrease in output (GDP).

Part V: Reconciling Theory and Application

- ⑬ Question 1 established that the Frisch elasticity is infinite, implying an "all-or-nothing" labor response to wage changes. Yet, your answer to Question 5 shows an **interior solution** for labor supply. Why does the model predict the average person works *some* of the time? C

- (A) The income effect from the wage is large enough to offset the infinite substitution effect.
- (B) The Frisch elasticity is only a theoretical concept and does not apply to this model.
- (C) In the full optimization, the consumer adjusts their consumption, which in turn adjusts their marginal utility of wealth, allowing them to find an interior balance.
- (D) The specific parameter values ($b = 5, w = 10$) are uniquely chosen to create an interior solution.

Solution

This is the key conceptual point. The infinite Frisch elasticity comes from an artificial experiment where the marginal utility of wealth (λ) is held constant. In the full optimization, the consumer is free to adjust their consumption level (C). By choosing $C = \sqrt{2}$, they implicitly choose a marginal utility of wealth ($\lambda = C^{-\sigma} = (\sqrt{2})^{-2} = 0.5$) that makes them perfectly willing to work for the going wage, leading to an interior solution.

- ⑭ At the benchmark optimum, what is the marginal utility of an additional dollar of income ($\lambda = MU_C$)? A

- (A) 0.5
- (B) 2
- (C) 1
- (D) 10

Solution

The marginal utility of wealth (λ) is equal to the marginal utility of consumption (MU_C) at the optimum. $\lambda = MU_C = C_0^{-\sigma} = (\sqrt{2})^{-2} = 1/2 = 0.5$.

- ⑮ Which of the following is a significant limitation of applying this simple model to the real-world policy? B

- (A) The model incorrectly assumes people like leisure.
- (B) The model uses a fixed wage, ignoring that a widespread drop in labor supply could cause wages to rise, creating a **general equilibrium effect**.
- (C) The model's utility function is too complex to be realistic.
- (D) The model does not account for the government's budget.

Solution

This is a core concept of general vs. partial equilibrium. The model's assumption of a fixed wage is its biggest weakness. In reality, if a large portion of the population reduces their labor supply, firms would likely have to increase wages to attract workers, creating a general equilibrium effect that would counteract some of the initial decline in work.

Problem 2

Credit: Sungmin Park

Suppose that the only consumption good in the state of Ohio is soybeans, and suppose that Ohio's output Y of number of soybeans is determined by the following production function

$$F(K, L) = zK^{\frac{1}{3}}L^{\frac{2}{3}}, \quad (1)$$

where z is TFP, K is its current stock of capital, and L is the size of its current labor force. Suppose also that the current technological level is $z = 1$, the current stock of capital is 1,000, and the current labor force is 8,000.

(a) At the current technological level, capital stock, and labor force, what is the total output of soybeans? What is the marginal product of capital? What is the marginal product of labor?

$$MPK = \underline{\text{Q13}}$$

$$MPN = \underline{\text{Q14}}$$

- (A) 0.333 (B) 1.333 (C) 2.333 (D) 3.333

17 A

- (A) 0.333 (B) 1.333 (C) 2.333 (D) 3.333

Suppose that the markets for capital and labor are competitive—that is, there are many firms (soybean farms) in Ohio, so that they are price-takers.

(b) What is the real rental rate of capital? What is the real wage? How many soybeans go to the capitalists? (i.e. what is the capital income?) How many soybeans go to the workers? (i.e. what is the labor income?) What is the total income?

$$\text{rental rate } R = \underline{\text{Q15}}$$

$$\text{wage } w = \underline{\text{Q16}}$$

$$\text{Capital income } R \times K \underline{\text{Q17}}$$

$$\text{Labor income } w \times L \underline{\text{Q18}}$$

$$\text{Total income } R \times K + w \times L \underline{\text{Q19}}$$

18 D

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{3}$ (D) $\frac{4}{3}$

19 A

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{3}$ (D) $\frac{4}{3}$

20 B

- (A) $\frac{2000}{3}$ (B) $\frac{4000}{3}$ (C) $\frac{6000}{3}$ (D) $\frac{8000}{3}$

21 D

- (A) $\frac{2000}{3}$ (B) $\frac{4000}{3}$ (C) $\frac{6000}{3}$ (D) $\frac{8000}{3}$

22) B

- (A) 2000 (B) 4000 (C) 6000 (D) 8000

Now suppose that the demand for soybeans in Ohio are as follows. First, the demand for the consumption of soybeans is given as

$$C = 0.5(Y - T) + 500, \quad (2)$$

where $T = 400$ is the number of soybeans taxed by the Ohio state government. Second, the demand for investing soybeans for future production is given as

$$I = 1500 - 100r, \quad (3)$$

where r is the real interest rate in percent. Finally, the demand for soybeans from the Ohio state government is as the government collect the same amount as taxes T and spend it on running the Ohio State University.

(c) What are the equilibrium interest rate, investment, and consumption?

$$r = \underline{\text{Q20}}$$

$$I = \underline{\text{Q21}}$$

$$C = \underline{\text{Q22}}$$

23) A

- (A) 2 (B) 3 (C) 4 (D) 5

24) C

- (A) 1500 (B) 1400 (C) 1300 (D) 1200

25) C

- (A) 2100 (B) 2200 (C) 2300 (D) 2400

(d) Suppose the Ohio state government reduces the taxation to $T = 200$ while maintaining the same spending of $G = 400$, as a part of COVID-19 stimulus package. What is the government surplus/deficit? What is the equilibrium interest rate, investment, and consumption after this government policy?

government surplus/deficit: $T - G = \underline{\text{Q23}}$

$r = \underline{\text{Q24}}$

$I = \underline{\text{Q25}}$

$C = \underline{\text{Q26}}$

26) C

- (A) 200 (B) 400 (C) -200 (D) -400

27) B

- (A) 2 (B) 3 (C) 4 (D) 5

28) D

- (A) 1500 (B) 1400 (C) 1300 (D) 1200

29) D

- (A) 2100 (B) 2200 (C) 2300 (D) 2400

Suppose instead that Ohio state residents' consumption demand also depends on the real interest rate:

$$C = 0.5(Y - T) + 600 - 50r. \quad (4)$$

That is, for every percentage point increase in the interest rate, the Ohio state resident would rather not consume 50 soybeans and save them instead.

(e) Analyze again the effects of reduced taxation to $T = 200$, using the new consumption demand but using everything else the same as (d)

government surplus/deficit: $T - G =$ Q27

$$r = \underline{\text{Q28}}$$

$$I = \underline{\text{Q29}}$$

$$C = \underline{\text{Q30}}$$

30) C

- (A) 200 (B) 400 (C) -200 (D) -400

31) B

- (A) 1.667 (B) 2.667 (C) 1.333 (D) 2.333

32) C

- (A) 1537 (B) 3212 (C) 1233 (D) 2367

33) D

- (A) 1537 (B) 3212 (C) 1233 (D) 2367