ECON 4002.01 Midterm Exam

Hui-Jun Chen

Instruction

Please submit your answer on Carmen Quiz "Midterm Exam". All numerical answers are supposed to **round to the second decimal point**,[1] and all algebraic answers are

- the power has to form with bracket, i.e., if want to write K^a, then you should type `K^{a}`

You **may** consult any note and textbook, but you **cannot** discuss with your classmate or any other person about the exam.

There will be one T/F choice question that worth 2 points in the Carmen Quiz "Midterm Exam". The T/F choice question is to confirm: "**I affirm that I have not received or given any unauthorized help on this exam, and that all work is my own.**"

Question 1

Considering an one-period general equilibrium model similar to Example in Lecture 08, slide 11 and 12. Also the Experiment 2 from Lecture 07, slide 13 is also a good reference. However, in this model economy, there are two differences:

1. firm rent capital from consumer, and consumers are **endowed** with 2 units of capital ($K^s = 2$).
2. consumer's utility function is $U(C, l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$

1 The competitive equilibrium given $\{G, z, \underline{\ K^s\ }\}$

[1] round to the second decimal points means that if the third decimal point is a number between 0 to 4, then just get rid of the third decimal point. On the other hand, if the third decimal point is a number between 5 to 9, then round the second decimal point up by adding 1 to the second decimal point number. For example, if the answer you get is 0.534, then round it to 0.53. Yet, if the answer is 0.535, then round it up to 0.54.

(2) is a set of allocations $\{Y^*, C^*, l^*, N^s, N^d, \pi^*, T^*, \underline{\quad K^{d*} \quad}\}$

(3) and prices $\{w^*, \underline{\quad r^* \quad}\}$ such that

1. Taken prices and π, T as given, the representative consumer solves

(4) $$\max_{\underline{C,l}} U(C,l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$$

(5) subject to $C \leq w(h-l) + \underline{\quad rK^s \quad} + \pi - T$

2. Taken prices as given, the representative firm solves

(6) $$\max_{\underline{K^d, N^d}}$$

(7) $zK^a N^{1-a} - wN^d - \underline{\quad rK^d \quad}$

3. Government collect taxes to balance budget:

(8) $\underline{\quad T^* = G \quad}$

4. Labor market clear means that the equilibrium wage is w^* such that labor supply equals to labor demand:

(9) $\underline{\quad N^s = N^d \quad}$

5. Capital market clear means that the equilibrium rental rate is r^* such that capital supply equals to capital demand:

(10) $\underline{\quad K^s = K^d \quad}$

To solve this model economy, we reformulate the competitive equilibrium into the social planner's problem.

First of all, in social planner's problem, all markets must clear, and thus $N^s =$

$N^d = N$, and $K_s = K^d = K = 2$.

Through firm's FOC with respect to N and K, we know w and r are

(11) $w = zK^a$ $\underline{\ (1-a)N^{-a}\ }$

(12) $r = zN^{1-a}$ $\underline{\ aK^{a-1}\ }$

which we can use to retrieve wage and rental rate after solving the social planner's problem.

The social planner problem is given by:

(13) Objective function is the consumer's utility:

$$\max_{C,l,N,Y,\underline{\ K\ }} U(C,l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$$

subject to

a. Aggregate resource constraint

(14) $C + G = \underline{\ Y\ }$

b. production constraint

(15) $Y = \underline{\ zK^aN^{1-a}\ }$

c. labor constraint

(16) $N = \underline{\ 1-l\ }$

d. capital constraint

(17) $K = \underline{\ 2\ }$

To solve the social planner's problem, we start with substituting the constraints into utility function:

a. Substituting 23 and 17 into 15, we get

(18) $Y = \underline{\quad z2^a(1-l)^{1-a} \quad}$

b. Substituting 18 into 14, we get

(19) $C = \underline{\quad z2^a(1-l)^{1-a} - G \quad}$

c. Finally, substituting 19 into 13, we get

(20) $\max_{\underline{l}}$

(21) $U(C(l), l) = \underline{\quad \frac{(z2^a(1-l)^{1-a}-G)^{1-b}}{1-b} + \frac{l^{1-d}}{1-d} \quad}$

Let $z = 1, G = 0, a = \frac{1}{2}, b = 2, d = \frac{3}{2}$ and solve for all unknowns,

(22) $l = \underline{\quad 0.72 \quad}$

(23) $N = \underline{\quad 0.28 \quad}$

(24) $w^* = \underline{\quad 2.67 \quad}$

(25) $r^* = \underline{\quad 0.19 \quad}$

The following is the calculation for the answer from 22 to 25:

FOC results in $l^{-d} = z^{-b}2^{-ab}(1-l)^{-b+ab}(1-a)z(1-l)^{-a}$

$l^{-d} = z^{1-b}2^{-ab}(1-a)(1-l)^{-a-b+ab}$

$l^{-\frac{3}{2}} = \frac{1}{4z}(1-l)^{-\frac{3}{2}}$

$\left(\frac{1-l}{l}\right)^{\frac{3}{2}} = \frac{1}{4z} = \frac{1}{4}$

$\left(\frac{1-l}{l}\right) = \left(\frac{1}{4}\right)^{\frac{2}{3}} \approx 0.3968 \Rightarrow 1 - l = 0.3968l \Rightarrow l \approx 0.7158 \approx 0.72$

Question 2

Answers

1	K^s
2	K^{d*}
3	r^*
	C, l
5	rK^s
	K^d, N^d
	rK^d
8	$T^* = G$
9	$N^s = N^d$
10	$K^s = K^d$
11	$(1-a)N^{-a}$
12	aK^{a-1}
	K
14	Y
15	zK^aN^{1-a}
16	$1-l$
17	2
18	$z2^a(1-l)^{1-a}$
19	$z2^a(1-l)^{1-a} - G$
	l
21	$\frac{(z2^a(1-l)^{1-a}-G)^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$
22	0.72
23	0.28
24	2.67
25	0.19

FOC results in $l^{-d} = z^{-b}2^{-ab}(1-l)^{-b+ab}(1-a)z(1-l)^{-a}$

$l^{-d} = z^{1-b}2^{-ab}(1-a)(1-l)^{-a-b+ab}$

$l^{-\frac{3}{2}} = \frac{1}{4z}(1-l)^{-\frac{3}{2}}$

$\left(\frac{1-l}{l}\right)^{\frac{3}{2}} = \frac{1}{4z} = \frac{1}{4}$

$\left(\frac{1-l}{l}\right) = \left(\frac{1}{4}\right)^{\frac{2}{3}} \approx 0.3968 \Rightarrow 1-l = 0.3968l \Rightarrow l \approx 0.7158 \approx 0.72$