Julia Syntax and Algorithm

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THE MOST important coding rules

Leggi le leggi

("Read the manuals" in Italian)

Resources on Syntax

- Julia Official Tutorial: `https://julialang.org/learning/tutorials/`
- Wikibook on Introducing Julia: `https://en.wikibooks.org/wiki/Introducing_Julia`
- QuantEcon w/ Julia: `https://julia.quantecon.org/intro.html`
- Julia in 100 Seconds: `https://www.youtube.com/watch?v=JYs_94znYy0`

The REPL

REPL stands for **R**ead, **E**valuate, **P**rint, and **L**oops.

Julia's REPL is the best I have ever seen, includes

- Unicode transformation: type `\alpha` and `tab` leads to α
- Package management: type `]` to enter `Pkg` mode to `add` packages
- Manual query: type `?` to enter `help` mode & find function manual
- Tab completion: type `\al` and `tab` gives you possible commands
- $\uparrow$ / $\downarrow$: up/down arrow key cycle through executed command history

The Language: Good and Bad

Julia language is designed with scientific computing in mind, and thus

- Unicode variable: directly use α as variable, not `alpha`.
- Multiple dispatch: multiple "methods" in one function for input types
- Type system: use `struct` to build custom types ($\approx$ but $\neq$ OOP)

But also have some weird behavior that I am not used to:

- Weird scope: variables defined inside loops (`while`, `for`) are **local**.
- Speed needs discipline: **well-written** code v.s. **sloppy-written** code
- Memory usage: might directly crash the Julia session ($\because$ LLVM?)

Best practice? Still Searching...

Syntax: generating a grid

- Usually the Macro coding starts with the grids of choice variables.
- A grid is a finite sample of continuous choice variable.
- Key to construct a grid is the `collect` and `range` function.
- `range` syntax requires `start` pt, `stop` pt and `length` of this grid
- `collect` then "collect" this `range` object into an array.

```
cnum = 100
lnum = 100
cgrid = collect( range( 0.01, 10.0, length = cnum ) )
lgrid = collect( range( 0.01, 1.0, length = lnum ) )
```

Syntax: Array manipulation

To get one element of a grid, we use `[]` syntax.

```
cval = cgrid[1] # get the first element of cgrid
lval = lgrid[5] # get the fifth element of lgrid
```

To create an array, you can use manual or automatical way.

```
# manually type all the elements
a = [1.0, 2.0, 3.0, 4.0, 5.0 ]
# automatically generate an "empty" array
#                 type    dim empty  row   column
utility = Array{Float64, 2}(undef, cnum, lnum)
utility = zeros(cnum, lnum) # zero array
utility = ones(cnum, lnum) # one array
```

Syntax: `for` loop

To calculate the utility value at each (C, l) bundle, use `for` loop

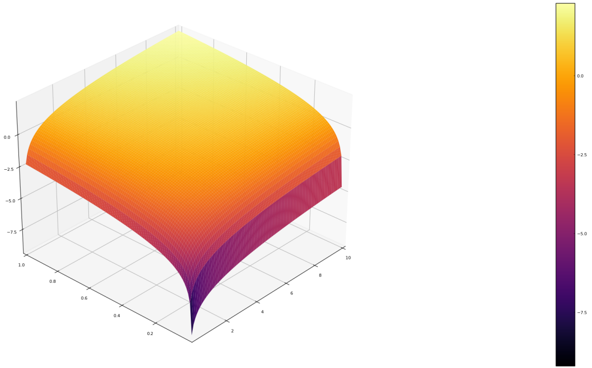
```
utility = Array{Float64, 2}(undef, cnum, lnum)
for indl in 1:1:lnum
    # get the each value in leisure grid
    lval = lgrid[indl]
    for indc in 1:1:cnum
        # get the each value in consumption grid
        cval = cgrid[indc]
        # log utility in both c and l
        utility[indc, indl] = log(cval) + log(lval)
    end
end
```

Syntax: 3-D plotting

Install `Plots` and `PyPlot` by typing `]` and type `add Plots PyPlot`

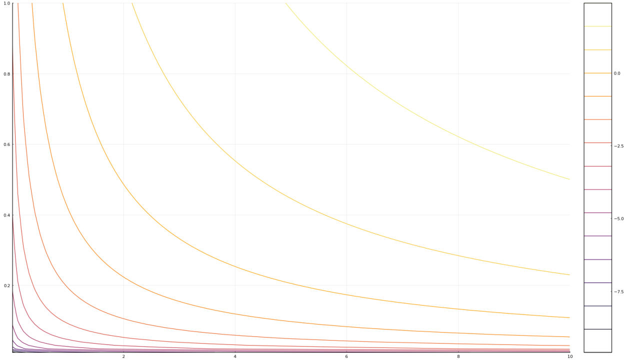
Plot the `utility` array by

```
using Plots; pyplot();
surface(cgrid, lgrid, utility)  # 3-D figure
```



Syntax: contour plotting

```
using Plots; pyplot();
contour(cgrid, lgrid, utility)  # contour figure
```



Syntax: `println` print something out

To show some info inside the `for` loop, `println` is a convenient tool.
If you want to know what (C, l) bundle leads to $U(C, l) = 0.0$,

```
for indl in 1:1:lnum
    for indc in 1:1:cnum
        # the abs of u is close enough to 0.0
        if abs(utility[indc, indl]) < 1e-2
        # '$': string interpolation (IMO inefficient)
            println("U ~ 0 at (C, l) = ($indc, $indl)")
        end
    end
end
```

`println` v.s. `print`: `println` add additional `\n`

Syntax: `while` loop

`while` loop mostly used when iteration only hault in some conditions.

In my experience it is mostly used if something needs convergence.

The following code is **NOT** an efficient way to find minimum location.

(should use `argmin` for `minimum` and `argmax` for `maximum`)

```
dist = 1.0; iter = 0;
while (dist > 1e-2)
    iter += 1 # same as "iter = iter + 1"
    indc = rand(1:1:cnum); indl = rand(1:1:lnum)
    dist = utility[indc, indl] - minimum(utility)
    if (dist < 1e-2)
        println("Find minimum at ($indc, $indl)")
        println("Iterates $iter times")
    end
end
```

Syntax: Rounding

Mostly for exam / standardization purpose.

```
round(pi)                 # 3.0
round(pi, digits = 1)     # 3.1
round(pi, digits = 2)     # 3.14
round(pi, digits = 3)     # 3.142
round(pi, digits = 4)     # 3.1416
round(pi, digits = 5)     # 3.14159
```

Application: Laffer curve

There are going to be two applications for Julia syntax learned:

1. Laffer curve in distorting taxes, and
2. Government spending in CRRA utility function.

Recall that $Y = zN^d$ implies labor supply $N^s(t)$ equals to

$$N^s(t) = 1 - l = \frac{1}{2} - \cancel{\frac{\pi}{2(1-t)}}, \quad (1)$$

$\pi \to 0$

$N^s(t) = \frac{1}{2}$

$t = 1 - \cancel{\sqrt{\pi}} \to 0$

and the total tax revenue is given by

$$R(t) = wtN^s(t). \quad (2)$$

In equilibrium $w = z = 1$, so $\pi = zN^d - wN^d = 0$, so this question is trivial...

Laffer curve in Cobb-Douglas Production Function

Assume $Y = zN^a$, where $a < 1$, so firm's problem leads to

$D_N Y = a \cdot zN^{a-1}$

$$w(N) = MPN = zaN^{a-1}, \tag{3}$$

$w \cdot N = a \cdot z \cdot N^a = aY$

$$\pi(N) = Y - wN = z(1-a)N^a, \tag{4}$$

$(1-a)Y$

and recall $MRS_{l,C} = w(1-t)$ and binding BC $C = w(1-t)N + \pi$, so

$z = 1$

$w = aN^{a-1}$

$$MRS_{l,C} = \frac{C}{l} = \frac{w(1-t)N + \pi}{l} = w(1-t) \tag{5}$$

$$= \frac{w(N)(1-t)N + \pi(N)}{(1-N)} = w(N)(1-t) \tag{6}$$

expands, we get a monster:

$$\frac{zaN^{a-1}(1-t)N + z(1-a)N^a}{1-N} = zaN^{a-1}(1-t) \tag{7}$$

Laffer curve in Cobb-Douglas Production Function (cont.)

But not too bad, because you realize:

$$\text{Common } N: \quad \frac{zaN^{a-1}(1-t)N + z(1-a)N^a}{1-N} = zaN^{a-1}(1-t) \tag{8}$$

$$\text{Common } zN^a: \quad \frac{zaN^a(1-t) + z(1-a)N^a}{1-N} = zaN^{a-1}(1-t) \tag{9}$$

w·N

w(N)

$$\text{Erase } zN^{a-1}: \quad \frac{zN^a[a(1-t)+1-a]}{1-N} = za(1-t)N^{a-1} \tag{10}$$

→ distortion

w

$\frac{l}{1-l}$

$$\text{Divide } [\cdot]: \quad \frac{N[a(1-t)+1-a]}{1-N} = a(1-t) \tag{11}$$

$$\boxed{\frac{N}{1-N}} = \boxed{\frac{a(1-t)}{a(1-t)+1-a}} \equiv A(t) \tag{12}$$

$$N = A(1-N) = A - AN \tag{13}$$

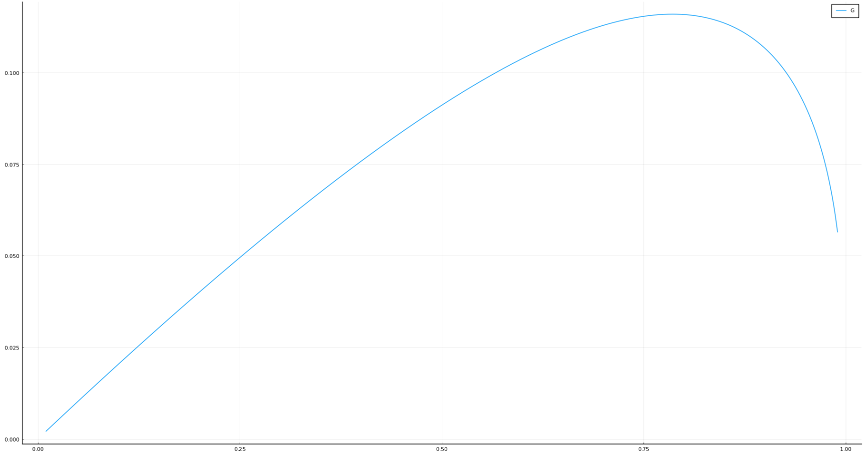
$$(1+A)N = A \Rightarrow N(t) = \boxed{\frac{A(t)}{1+A(t)}} \tag{14}$$

Laffer curve in Julia

```
a = 0.33; tnum = 1000
tgrid = collect( range(0.01, 0.99, length = tnum) )
Gvec = Array{Float64, 1}(undef, tnum)
for indt = 1:1:tnum
    t = tgrid[indt]
    A = ( a*(1-t) ) / ( a*(1-t) + 1 - a )
    N = A / (1 + A)
    w = a*N^(a-1)
    Gvec[indt] = w * t * N
end
Gmax = maximum(Gvec); tmax = tgrid[argmax(Gvec)];
println("G* = $Gmax; t* = $tmax")
```

Laffer curve in Julia (cont.)

```julia
using Plots; pyplot()
plot(tgrid, Gvec, label = "G")
```



Grid search

Just calculate value **on** the grid points! Like for loop slide

Recall the formula with gov spending:

$$\max_{l} \quad \frac{(z(1-l)^{1-\alpha}-G)^{1-b}}{1-b}+\frac{l^{1-d}}{1-d}. \tag{15}$$

We want to solve $l(z,G)$, but how to choose the `Ggrid`?

From the FOC we know

$$G=F(l)=z(1-l)^{1-\alpha}-\left[\frac{l^{-d}}{(1-\alpha)z(1-l)^{-\alpha}}\right]^{-\frac{1}{b}}. \tag{16}$$

Our first step starts with generating a TFP grid:

```
znum = 100
zgrid = collect( range( 0.8, 1.2, length = znum ) )
```

Grid search: preperation

We want to find the upper/lower bound of `Ggrid`:

```
a = 1/2; b = 2; d = 3/2;
GovFOC(z, l) = z*(1-l)^(1-a) - # line continuation!
    ( (l^(-d)) / ( (1-a)*z*(1-l)^(-a) ) )^(-1/b)
# upper & lower bound of Ggrid
Gbound = Array{Float64, 2}(undef, znum, 2)
for indz = 1:1:znum
    zval = zgrid[indz]
    Gbound[indz, 1] = GovFOC(zval, 0.99) # lower bound
    Gbound[indz, 2] = GovFOC(zval, 0.01) # upper bound
end
```

Grid search: preperation (cont.)

```julia
# lower bound should higher than 0.01
Glow = max(0.0, minimum(Gbound))
Ghigh = maximum(Gbound)
# build Ggrid
Gnum = 100
Ggrid = collect( range( Glow, Ghigh, length = Gnum ) )
# build lgrid
lnum = 100
lgrid = collect( range( 0.01, 1.0, length = lnum ) )
```

and then find the optimal leisure using the value on this grid:

Grid search: structure

```
a = 1/2; b = 2; d = 3/2;
# define implicit utility function
utility(l, z, G) = ( ( z*(1-l)^(1-a) - G )^(1-b) ) /
                    ( 1-b ) +
                    ( l^(1-d) ) / (1-d)
# Array for storage
## for temporary storage
uvec = Array{Float64, 1}(undef, lnum)
## for optimal utility value
ustar = Array{Float64, 2}(undef, znum, Gnum)
## for optimal leisure given z, G
lstar = Array{Float64, 2}(undef, znum, Gnum)
```

Grid search: structure (cont.)

```
for indG = 1:1:Gnum
    Gval = Ggrid[indG]
    for indz = 1:1:znum
        zval = zgrid[indz]
        for indl = 1:1:lnum
            lval = lgrid[indl]
            cval = zval*(1-lval)^(1-a) - Gval
            uvec[indl] = ( cval < 0.0 ? -Inf :
                                utility(lval, zval, Gval) )
        end
        ustar[indz, indG] = maximum(uvec)
    end
end
```

Grid search: analysis

Notice that in previous slide, I check whether `cval < 0.0`

and also we find the highest utility and the corresponding (z, G) value by

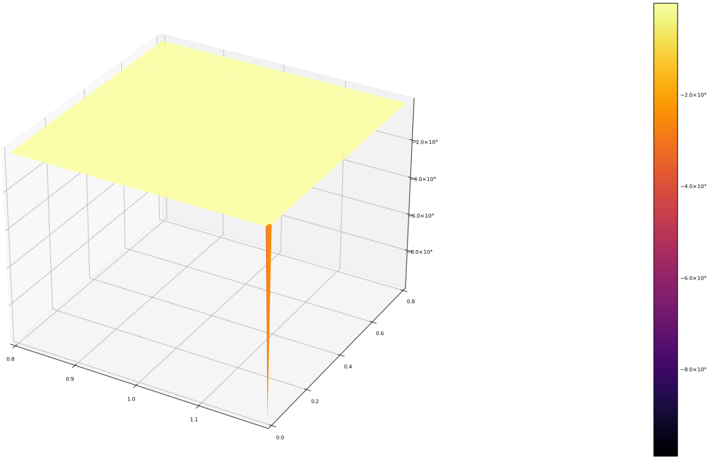
```
umax = maximum(ustar)
zloc = argmax(ustar)[1]
Gloc = argmax(ustar)[2]
zmax = zgrid[zloc]
Gmax = Ggrid[Gloc]
```

But if you plot you will see that the plot is slightly “off”:

```
using Plots; pyplot()
surface(zgrid, Ggrid, ustar)
```

$\because$ large negative point that drag down the scale of every point.

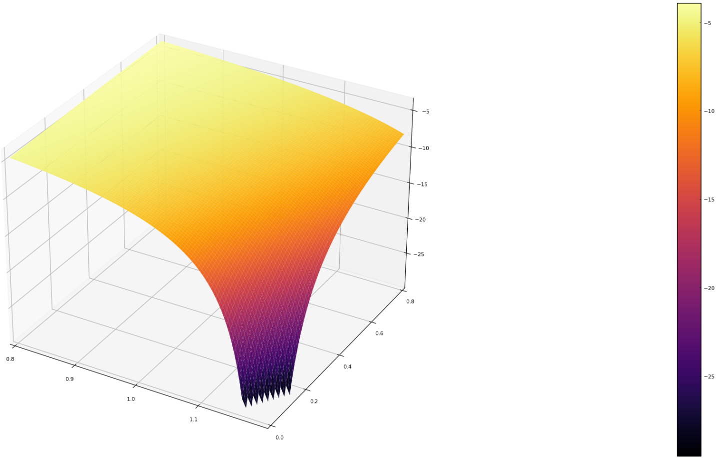
grid search: misleading figure



Grid search: revising (erase `uval < -30.0`)

```
for indG = 1:1:Gnum
    Gval = Ggrid[indG]
    for indz = 1:1:znum
        zval = zgrid[indz]
        for indl = 1:1:lnum
            lval = lgrid[indl]
            cval = zval*(1-lval)^(1-a) - Gval
            uval = utility(lval, zval, Gval)
            uvec[indl] = ( (cval < 0.0 || uval < -30.0)
                                    ? -Inf : uval )
        end
        ustar[indz, indG] = maximum(uvec)
    end
end
```

Grid search: better figure



Grid search: Can we do better?

All of the `-Inf` stuff we are assigning manually is because $C < 0$.
Recall that $C = Y - G$, and thus for $C \geq 0$, $Y - G \geq 0 \Rightarrow Y > G$.

```
ymat = Array{Float64, 2}(undef, znum, lnum)
for indl = 1:1:lnum
    lval = lgrid[indl]
    for indz = 1:1:znum
        zval = zgrid[indz]
        ymat[indz, indl] = zval * (1-lval)^(1-a)
    end
end
ymin = minimum(ymat)
```

You will get $min(y) = 0.081$, which means that if you choose the `Ghigh = 0.08`, $C > 0, \forall z, G$ assigned.

Grid search: do better

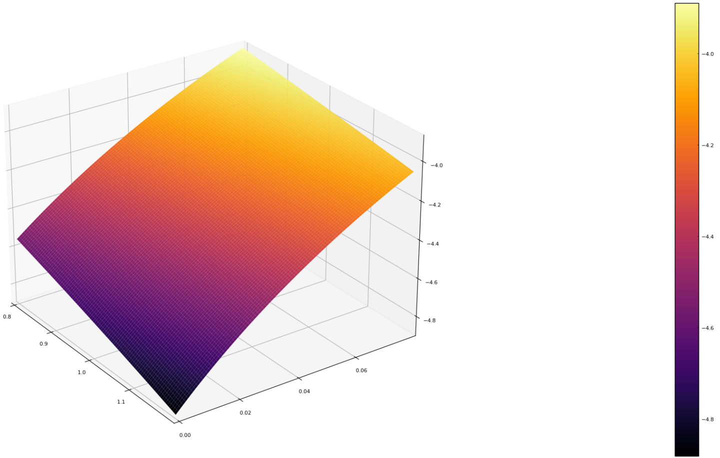
```
# lower bound should higher than 0.01
Glow = 0.0
Ghigh = 0.08
# build Ggrid
Gnum = 100
Ggrid = collect( range( Glow, Ghigh, length = Gnum ) )
# build lgrid
lnum = 100
lgrid = collect( range( 0.01, 0.99, length = lnum ) )
```

and then find the optimal leisure using the value on this grid:

Grid search: do better (cont.)

```
for indG = 1:1:Gnum
    Gval = Ggrid[indG]
    for indz = 1:1:znum
        zval = zgrid[indz]
        for indl = 1:1:lnum
            lval = lgrid[indl]
            cval = zval*(1-lval)^(1-a) - Gval
            uvec[indl] = utility(lval, zval, Gval)
        end
        ustar[indz, indG] = maximum(uvec)
    end
end
```

Grid search: better figure



Grid search method: additional details

Calculate on the grid point $\Rightarrow$ result are correct but speed is slow.

Notice that when you choose the grid points, better to avoid some value:

Example

When I create `cgrid` and `lgrid`, I avoid the start point of 0.0, but 0.01, since $log(0.0) = \infty$.

In general, if theoretical range, say leisure, is $[0, 1]$, then it is safe to build a grid from $[0.01, 0.99]$.