Debt Financing, Used Capital Markets and Capital Reallocation[*]

Hui-Jun Chen[†]

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Abstract

This paper studies the business cycle implications of the interaction between secondary capital markets and credit market imperfections in a general equilibrium model with heterogeneous firms. In the model, prices in secondary capital markets affect the economy through two key channels: changes in the cost of investment and fluctuations in the resale value of collateral. The results show that price adjustments in secondary markets amplify the severity and duration of recessions triggered by financial shocks. The rise in investment costs driven by financial shocks deepens the recessionary trough by a factor of three compared to the effect of collateral value fluctuations.

Keywords: Investment irreversibility, Collateral Constraint, Business Cycle

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[†]Department of Economics, The Ohio State University. 1945 North High Street, Columbus, Ohio, United States 43210. Email: chen.9260@buckeyemail.osu.edu

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1 Introduction

Small and young firms play a vital role in the economy by contributing to employment, productivity, and growth (Haltiwanger, Jarmin and Miranda, 2013; Haltiwanger, 2021). One important channel through which they boost the economy is their investment in old capital (Ma, Murfin and Pratt, 2022). Used capital is cheaper to purchase and helps ease the impact of financial constraints on production (Eisfeldt and Rampini, 2006). Consequently, capital reallocation through *secondary markets* is an important source of investment for young firms. At the same time, these firms have limited borrowing capacity which hinders their ability to invest further (Gertler and Gilchrist, 1994). Thus, a second channel arises when the maximum amount that these firms can borrow depends on the *resale value* of the pledged collateral (Banerjee and Blickle, 2021; Ioannidou, Pavanini and Peng, 2022). The interaction between secondary markets and financial frictions demonstrates that the price dynamics in the secondary markets are central to understanding how economic shocks affect firm investments through two distinct channels: the cost of investment and the collateral value of capital. The aggregate impact of the used capital markets depends on the relative strength of each channel. Although existing quantitative models have examined these channels independently (Khan and Thomas, 2013; Lanteri, 2018), they have not adequately assessed how their combined interaction affects aggregate economic outcomes and business cycle dynamics.

In this paper, I develop a dynamic stochastic general equilibrium model with persistent firm heterogeneity, collateral constraints, and a used capital markets. The model features two key frictions that impede capital accumulation: collateral constraints and endogenous capital irreversibility. First, collateral constraints limit investment loans, creating a clear distinction between small and large firms. Second, capital irreversibility arises endogenously through the equilibrium price in the secondary markets. This friction induces firms to follow *(S, s)* investment rules. Following Lanteri (2018), the secondary market prices are determined by the supply and demand for used capital. Large firms supply used investment goods by scaling down their capital stock, while small firms increase their demand for used investment goods as they expand. Furthermore, the price of used investment goods affects firms' borrowing capacity by influencing the valuation of capital in the collateral constraints. As a result, fluctuations in the real sector propagate the shocks to the financial sector, amplifying credit restrictions during downturns.

Through the lens of my model, I find that the secondary capital markets enhance long-run equilibrium by reducing the user cost of capital. Specifically, I compare two steady states: one with secondary markets (*baseline* model) and one without, while keeping the degree of irreversibility constant. This ensures that differences between these two economies stem from the absolute value of the user cost of capital rather than relative differences between purchasing and selling prices. The comparison reveals that the economy without secondary markets experiences lower output and consumption, reduced aggregate capital, and more firms facing binding financial constraints. Although the selling price of capital is lower in the economy with used capital trading, investing firms accept this trade-off in exchange for lower purchasing costs, facilitating current growth (Ma et al., 2022).

Furthermore, the model shows that the overall effects of the used capital markets are driven by the cost of investment channel, with the contribution from the collateral channel being positive but small. To isolate the magnitude of these channels, I fix the selling price of capital in the collateral constraint at the baseline level, effectively shutting down the collateral channel. The resulting economy exhibits only negligible differences from the baseline model, indicating that the size of the collateral channel is small. While the adjustment in collateral valuation improves resource allocation, the cheaper cost of capital reduces the need for external funding, thereby diminishing the overall impact of the collateral channel.

Despite its beneficial role in long-run equilibrium, the secondary capital markets amplify economic downturns by deepening recessions and prolonging recoveries after financial shocks. This occurs when fluctuations in the price of used capital lead to an increasing user cost of capital during recessions. However, as the economy begins to recover, the price of used capital does not immediately rebound to its pre-crisis level. Since the determination of this price depends on the distribution of investing and disinvesting firms, sluggish price adjustments create a persistent drag on investment. As a result, the prices slowly goes back to the pre-crisis level, resulting in a longer recession.

In contrast, I find that the secondary capital markets have little impact when the economy experiences a negative productivity shock, consistent with previous theoretical literature (Hansen, 1985; Khan and Thomas, 2013). Since such shocks do not alter the distribution of firms, used capital prices recover in line with the shock trajectory. As a result, firms adjust investment based on productivity changes rather than secondary market conditions, limiting the markets' role during downturns.

My paper builds on recent empirical literature highlighting the importance of capital reallocation in response to recessions. Eisfeldt and Rampini (2006) present the sales and acquisitions of property, plant, and equipment (PP&E) are procyclical, while the measured benefit from these transactions is countercyclical. Later, Eisfeldt and Rampini (2007) show that financially constrained firms are more likely to purchase used capital and operate on a smaller scale. Recently, Ma et al. (2022) document that young firms prefer older capital due to financial constraints. Eisfeldt and Shi (2018) summarize the findings from Eisfeldt and Rampini (2006) and provide updated stylized facts on business cycle. This body of empirical evidence suggests that capital irreversibility and financial frictions interact, significantly affecting aggregate investment.

I calibrate the elasticity of substitution in the investment technology to the available estimation in the empirical literature. Ramey and Shapiro (2001) analyze data on the closure of aerospace plants and report the resale prices of their physical assets, concluding that significant capital irreversibility exists across sectors and that installing used capital involves adjustment costs. Edgerton (2011) conclude that investment tax credits have a significant effect on the relative prices, and provides a rare estimate on the elasticity of substitution between new and used capital[1]. The level of elasticity of substitution in my model are chosen within the range that Edgerton (2011) estimated.

My model is closely connected to a large theoretical literature on the effects of financial frictions on the real sector. Kiyotaki and Moore (1997) introduce collateral constraints and construct a model of credit cycles to highlight the importance of collateral in driving business cycles. On the household side, Boz and Mendoza (2014) study the effect of credit constraints on a representative household using land as collateral. They assume Bayesian learning and explain substantial increases in debt and land prices given optimal priors. Gavazza and Lanteri (2021) develop a heterogeneous household model to examine the effect of collateral constraints on household decisions to trade or scrap durable goods. On the firm side, Arellano, Bai and Kehoe (2019) show that uncertainty shocks worsen credit conditions. Lanteri and Rampini (2023) characterize the efficiency of a heterogeneous firm model with collateral constraints and propose collateral and distributive externalities. My contribution to this literature is to demonstrate the dynamic role of the used capital markets when the economy faces productivity or financial shocks.

Two papers, both heterogeneous firm models with capital irreversibility, form the foundation of my work. Lanteri (2018) is the first to develop a general equilibrium model in which firms replace capital by trading in the secondary markets, thereby endogenizing capital irreversibility. He introduces a tractable mechanism that incorporates the use of new and used capital into a CES aggregator, allowing the model to track only one asset rather than two when solving the optimization problem. Khan and Thomas (2013) is the first to explore the effect of endogenous total factor productivity (TFP) shocks in a dynamic stochastic general equilibrium setting. They show that credit shocks can generate large and persistent recessions through changes in the distribution of firms. Their model formulation and numerical methods provide the tools I use to incorporate collateral constraints into the framework developed by Lanteri

[1] His analysis is based on existing datasets on aircraft and farm machinery sales and a newly compiled dataset on auction sales of used construction machinery.

(2018).

There are several well-known approaches to financial frictions beyond collateral constraints. Cooley, Marimon and Quadrini (2004) develop a general equilibrium model with limited contract enforceability, concluding that lower enforceability increases macroeconomic volatility. Jermann and Quadrini (2012) proposes a representative firm model with an enforcement constraint that evolves from limited enforceability, linking financial frictions to business cycle fluctuations.

The rest of the paper is described as follows. Section 2 presents the model environment in detail. Section 3 analyzes the model environment. Section 4 shows the calibration target and how well my model matches them. Section 5 illustrate firm-level decision rules and life cycle in the steady state. Section 6 summarizes the long-run equilibrium under different secondary capital markets clearing. Section 7 reports the transitional dynamics under productivity and financial shocks. Section 8 concludes.

2 Model

Time is discrete and infinite. I abstract from aggregate uncertainty and consider stationary equilibrium and transitional dynamics under aggregate, unanticipated exogenous shocks. The economy consists of two types of agents: firms and households. A continuum of heterogeneous firms produces a homogeneous output using labor and predetermined capital stocks. Firms' capital accumulation is affected by two frictions: partial irreversibility and collateral constraints. Partial irreversibility arises endogenously from the equilibrium price in the secondary markets for capital. Collateral constraints are backward-looking, meaning they depend on the *current* stock of capital, valued at the secondary market prices. Households are identical and infinitely lived, financing their consumption with labor income, one-period shares in firms, and one-period non-contingent bonds.

2.1 Firms

2.1.1 Production

Firms produce output y with physical capital k and labor n. The production function is $y = z\varepsilon F(k,n)$, where $F(k,n) = k^{\alpha}n^{\nu}$ with both $\alpha, \nu < 1$ and $\alpha + \nu < 1$. The z is the aggregate TFP shock that is common among firms, while ε is a firm-specific productivity shock. I assume that ε is a Markov chain, i.e., $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, \ldots \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_j | \varepsilon = \varepsilon_i) = \pi^{\varepsilon}_{ij}$, and $\sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{ij} = 1$. Capital accumulation is determined by the law of motion, $k' = (1-\delta)k + I(i_{new}, i_{used})$, where $\delta \in (0,1)$ is the physical capital depreciation rate, and $I(i_{new}, i_{used})$ will be elaborated in section 2.1.2. Meanwhile, firms face a per-period probability of permanent exit $\pi_d \in (0,1)$[2].

[2]Exogenous exit shocks, alongside one-for-one replacement with new entrants, are a tractable way of capturing firm lifecycle dynamics observed empirically (Khan and Thomas, 2013).

2.1.2 Capital adjustment process

Firms that invest and those that disinvest adjust their capital stocks through distinct processes. Following the specification of Lanteri (2018), investing firms combine new investment goods i_{new} and used investment goods i_{used} into a single unit of investment goods. The capital accumulation process is given by:

$$\begin{aligned} k' &= (1-\delta)k + I(i_{new}, i_{used}) \\ I(i_{new}, i_{used}) &= \left[\eta^{\frac{1}{s}}(i_{new})^{\frac{s-1}{s}} + (1-\eta)^{\frac{1}{s}}(i_{used})^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}} \end{aligned} \tag{1}$$

where $I(\cdot)$ is the constant elasticity of substitution (CES) aggregator between new and used investments. Inside this CES aggregator, the parameter $\eta \in (0,1)$ determines the average ratio between new and used investment, and $s > 0$ represents the elasticity of substitution between new and used investment. The specification of η and s ensures that the composition $I(\cdot)$ is a constant return to scale technology in both i_{new} and i_{used}. The corresponding price index associated with the composition $I(\cdot)$ is

$$Q(q) = \left[\eta + (1-\eta)q^{1-s}\right]^{\frac{1}{1-s}}, \tag{2}$$

where the cost of a unit of new investment goods in terms of units of output is normalized to 1, and the counterpart of used investment goods is q, the equilibrium price of used investment goods. In any equilibrium with positive trading volume in used investment goods, the price of used investment goods must be lower than the price of new investment goods. Otherwise, firms would demand only new investment goods. This implies $q \leq 1$. I assume a strict inequality holds in equilibrium, such that $q < 1$ leads to $Q < 1$.

Disinvesting firms supply the used investment goods d by downward-adjusting their capital stock. The process is given by

$$k' = (1-\delta)k - d. \tag{3}$$

2.1.3 Partial irreversibility and financial frictions

The partial irreversibility of capital arises from the discrepancy between the selling price of capital in the secondary markets, q, and the purchasing price of capital, Q. Specifically, when a firm invests, the investment bundle $I(\cdot)$ is valued at the purchasing price Q. This implies that the total cost of investment $I(\cdot)$ is $QI(\cdot)$, or equivalently, $Q(k' - (1-\delta)k)$. Similarly, a disinvesting firm can only sell its capital, where $d > 0$, at the selling price q, receiving $qd = q(k' - (1-\delta)k)$ in output goods. Consequently, the secondary market prices endogenously determine the degree of capital irreversibility.

Firms finance their investment by borrowing one-period debt from households, subject to a collateralized borrowing limit. The amount of newly-issued debt, b', is priced at q_b, and cannot exceed ζ fraction of the value of firms' current stock of capital qk; that is, $b' \leq q\zeta k$[3]. The fraction ζ represents the efficiency of the economy's financial sector. I summarize the aggregate states used in transitional equilibrium as $s = (z, \zeta)$.

2.1.4 Firms' Problem

At the beginning of each period, a firm is defined by three states:

1. its predetermined capital stock $k \in \mathbf{K} \subset \mathbb{R}_+$,
2. its level of one-period bond $b \in \mathbf{B} \subset \mathbb{R}$,
3. its realized idiosyncratic productivity $\varepsilon \in \mathbf{E}$.

The distribution of firms is represented by a probability measure $\mu(k, b, \varepsilon)$, defined over the Borel σ-algebra generated by the open sets of the product space $\mathcal{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E}$. The law of motion of the distribution of firms is given by $\mu' = \Gamma(s, \mu)$.

Given all individual states, the firm maximizes dividend and the expected discounted continuation value by choosing current employment level n, future capital stock k', next-period

[3]Here, a larger fraction of ζ results in looser collateral constraints. This assumption stems from the limited enforceability of financial contracts. Brooks and Dovis (2020) demonstrate that the distinction between backward- and forward-looking collateral constraints hinges on whether the debt limit adjusts to profit opportunities.

debt level b', and dividend D. For each unit of labor employed, the firm pays competitive wage $w(s, \mu)$, which depends on aggregate exogenous states and the distribution of firms.

Let $v_0(k, b, \varepsilon; s, \mu)$ denotes the expected discounted value of a firm before the realization of the exogenous exit shock π_d. If the firm does not survive, then it chooses labor demand n, sells out its capital stock, and repays the debt to maximize the downward-adjusting cash on hand x^d. The functional formulation is defined as

$$v_0(k, b, \varepsilon; s, \mu) = \pi_d \max_n [x^d(k, b, \varepsilon; s)] + (1 - \pi_d) v(k, b, \varepsilon; s, \mu), \tag{4}$$

where $x^d(k, b, \varepsilon, s) = z\varepsilon F(k, n) - w(s, \mu)n - b + q(1 - \delta)k$. Conditional on survival, the continuation problem is a discrete choice among two options,

$$v(k, b, \varepsilon; s, \mu) = \max\{v^u(k, b, \varepsilon; s, \mu), v^d(k, b, \varepsilon; s, \mu)\}, \tag{5}$$

where $v^u(\cdot)$ denotes the value of the investing (upward-adjusting) firms, and $v^d(\cdot)$ represents the value of disinvesting (downward-adjusting) firms.

In either case, firms maximize the current dividend D and expected discounted future firm value. Let $d_g(s, \mu)$ be the stochastic discounting factor for the firm's next-period expected value given the current aggregate states are (s, μ). The dynamic problem for those investing firms is

$$v^u(k, b, \varepsilon_i; s, \mu) = \max_{k', b', D} D + d_g(s, \mu) \sum_{j=1}^{N_s} \pi_{ij}^{\varepsilon} v_0(k', b', \varepsilon_j; s', \mu'). \tag{6}$$

subject to

$$0 \leq D \leq x^u(k, b, \varepsilon; s) + q_b b' - Qk' \tag{7}$$

$$x^u(k, b, \varepsilon_i; s) = z\varepsilon_i F(k, n) - w(s, \mu)n - b + Q(1 - \delta)k \tag{8}$$

$$k' \geq (1 - \delta)k \tag{9}$$

$$b' \leq q\zeta k \tag{10}$$

$$\mu' = \Gamma(s;\mu) \tag{11}$$

where $x^u(\cdot)$ is the cash on hand for the upward-adjusting firm,

The downward-adjusting firms are different from the above problem only through (1) the investment must be nonpositive, and (2) firm's capital is evaluated by selling price q rather than purchasing price Q,

$$v^d(k,b,\varepsilon_i;s,\mu) = \max_{k',b',D} D + d_g(s,\mu)\sum_{j=1}^{N_s}\pi^{\varepsilon}_{ij} v_0(k',b',\varepsilon_j;s',\mu'). \tag{12}$$

subject to

$$0 \leq D \leq x^d(k,b,\varepsilon;s) + q_b b' - qk' \tag{13}$$

$$x^u(k,b,\varepsilon_i;s) = z\varepsilon F(k,n) - w(s,\mu)n - b + q(1-\delta)k \tag{14}$$

$$k' \leq (1-\delta)k \tag{15}$$

$$b' \leq q\zeta k \tag{16}$$

$$\mu' = \Gamma(s;\mu) \tag{17}$$

Since all firms are choosing labor demand regardless of continuation or not, given (k,ε), their decision rules on labor $N(k,\varepsilon;s,\mu)$ and output $Y(k,\varepsilon;s,\mu)$ does not depend on the current level of debt. On the contrary, the decisions on next-period capital and bond depend on all state variables, i.e., the decision rule on capital is $K(k,b,\varepsilon;s,\mu)$, and on the bond is $B(k,b,\varepsilon;s,\mu)$.

2.2 Household

The model economy is populated by a unit measure of identical households. Let the flow utility function be $u(c,1-n^h)$, In each period, households maximize their lifetime utility by choosing consumption, c, labor supply, n^h, future firm shareholdings, λ', and future bond holding, a':

$$
\begin{aligned}
V^h(\lambda, a; s, \mu) = & \max_{c, n^h, a', \lambda'} \left\{ u(c, 1-n^h) + \beta V^h(\lambda', a'; s', \mu') \right\} \\
\text{s.t.} \quad & c + q(s,\mu)a' + \int_{\mathbf{S}} \rho_1(k', b', \varepsilon'_j; s', \mu') \lambda(d[k' \times b' \times \varepsilon']), \\
& \leq w(s,\mu)n^h + a + \int_{\mathbf{S}} \rho_0(k, b, \varepsilon; s, \mu)\lambda(d[k \times b \times \varepsilon])
\end{aligned}
\tag{18}
$$

where $\rho_0(\cdot)$ is the dividend-inclusive price of the current share, and $\rho_1(\cdot)$ is the ex-dividend price of the future share.

2.3 Recursive Equilibrium

A *recursive competitive equilibrium* is a set of functions,

$$
w, q, q_b, d_g, \rho_0, \rho_1, v_0, N, K, B, D, I, i_{new}, i_{used}, d, V^h, C^h, N^h, A, \Lambda^h \tag{19}
$$

such that

1. v_0, v^u, v^d, and v solves (4)-(17). The associated policy functions for firms are (N, K, B, D).
2. V^h solves (18), and (C^h, N^h, Λ^h) are the corresponding policy functions for households.
3. $\Lambda^h(k', b', \varepsilon'_j, \lambda, a; s, \mu) = \mu'(k', b', \varepsilon'_j; s, \mu)$ for all $(k', b', \varepsilon'_j) \in \mathbf{S}$.
4. Labor market clears:

$$
N^h(\lambda, \eta; s, \mu) = \int_{\mathbf{S}} [N(k, \varepsilon_i; s, \mu)]\mu(d[k \times b \times \varepsilon]), \tag{20}
$$

5. For upward-adjusting firms, i.e., firms such that $v^u(k, b, \varepsilon; s, \mu) \geq v^d(k, b, \varepsilon; s, \mu)$, the policy function $K(k, b, \varepsilon; s, \mu)$ solves (6)-(11), and the investment $I(k, b, \varepsilon; s, \mu) = K(k, b, \varepsilon; s, \mu) - (1-\delta)k$. Furthermore, the allocation of $i_{used}(k, b, \varepsilon; s, \mu)$ and $i_{new}(k, b, \varepsilon; s, \mu)$ is determined

by the CES expenditure minimization problems,

$$\frac{i_{used}}{i_{new}} = \frac{1-\eta}{\eta}(q(s,\mu)+\gamma)^{-s}, \tag{21}$$

and the corresponding aggregate price index is (2).

6. For downward-adjusting firms, i.e., $v^u(k,b,\varepsilon;s,\mu) < v^d(k,b,\varepsilon;s,\mu)$, the policy function $K(k,b,\varepsilon;s,\mu)$ solves (12)-(17), and $d(k,b,\varepsilon;s,\mu) = (1-\delta)k - K(k,b,\varepsilon;s,\mu)$.

7. Good markets clear:

$$\begin{aligned} C(s,\mu) = \int_{\mathbf{S}} \Big\{ & z\varepsilon F(k, N(k,\varepsilon;s,\mu)) \\ & - (1-\pi_d)Q(s,\mu)I(k,b,\varepsilon;s,\mu) \\ & + (1-\pi_d)q(s,\mu)d(k,b,\varepsilon;s,\mu) \\ & + \pi_d[q(s,\mu)(1-\delta)k - Q(s,\mu)k_0] \Big\} \mu(d[k\times b\times \varepsilon]) \end{aligned}, \tag{22}$$

where k_0 is the initial capital stock. I assume k_0 for each entering firm is a fixed χ fraction of the long-run aggregate capital stock, i.e.,

$$k_0 = \chi \int k\tilde{\mu}(d[k\times b\times\varepsilon]), \tag{23}$$

where $\tilde{\mu}$ represents the steady-state distribution.

8. The used investment price $q(s,\mu)$ clears the secondary markets of capital

$$\int_{\mathbf{S}} d(k,b,\varepsilon;s,\mu)\mu(d[k\times b\times\varepsilon]) = \int_{\mathbf{S}} i_{used}(k,b,\varepsilon;s,\mu)\mu(d[k\times b\times\varepsilon]). \tag{24}$$

9. The distribution of firms in steady state, $\tilde{\mu}(k,b,\varepsilon)$, is a fixed point of Γ function. $\Gamma(s,\mu)$ is consistent with policy functions (K,B) and law of motion of ε.

10. Bond market clear condition

$$A(s,\mu) = \int_{\mathbf{S}} B(k,b,\varepsilon;s,\mu)\mu(d[k\times b\times\varepsilon]) \tag{25}$$

satisfies Walras' law, where A is households' policy functions for bonds.

3 Analysis

Before solving the recursive competitive equilibrium, I reformulate the firms' problem by exploiting the optimality conditions implied by the households' problem. In equilibrium, the wage $w(s,\mu)$ equals the marginal rate of substitution between leisure and consumption:

$$w(s,\mu) = \frac{D_2u(C,1-N)}{D_1u(C,1-N)}, \tag{26}$$

Similarly, the bond price q equals the inverse of the expected real interest rate. As there is no aggregate uncertainty in the economy, the expected real interest rate is $\frac{1}{\beta}$. The stochastic discounting factor $d_g(s,\mu)$ equals to household's discounting factor,

$$d_g(s,\mu) = \beta\frac{D_1u(c',1-n^{h'})}{D_1u(c,1-n^h)}. \tag{27}$$

Without the loss of generality, I assign $p(s,\mu)$ to be the household's marginal utility of consumption. The p function represents the output price in terms of households' marginal utility. It allows firms to discount their current dividend and payment by households' subjective discounting factor. I can rewrite (26)-(27) as

$$w(s,\mu) = \frac{D_2u(C,1-N)}{p(s,\mu)}, \tag{28}$$

$$q_b(s,\mu) = \beta\frac{p(s',\mu')}{p(s,\mu)}, \tag{29}$$

After incorporating the household's optimality condition into the prices that firms face, I define a new value V as the product of $p(s,\mu)$ and v, and rewrite dynamic problem (4)-(17) as

$$V_0(k,b,\varepsilon;s,\mu) = \pi_d \max_n [p(s,\mu)x^d(k,b,\varepsilon;s)] + (1-\pi_d)V(k,b,\varepsilon;s,\mu), \tag{30}$$

where

$$V(k,b,\varepsilon;s,\mu) = \max\{V^u(k,b,\varepsilon;s,\mu), V^d(k,b,\varepsilon;s,\mu)\}. \tag{31}$$

The dynamic problem for upward-adjusting firms is

$$V^u(k,b,\varepsilon;s,\mu) = \max_{k',b',D} p(s,\mu)D + \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} V_0(k',b,\varepsilon_j';s',\mu') \tag{32}$$

subject to

$$0 \le D \le x^u(k,b,\varepsilon;s) + q_b b' - Qk' \tag{33}$$

$$x^u(k,b,\varepsilon;s) = z\varepsilon F(k,n) - w(s,\mu)n - b + Q(1-\delta)k \tag{34}$$

$$k' \ge (1-\delta)k \tag{35}$$

$$b' \le q\zeta k \tag{36}$$

$$\mu' = \Gamma(s;\mu) \tag{37}$$

and the dynamic problem for downward-adjusting firms is

$$V^d(k,b,\varepsilon;s,\mu) = \max_{k',b',D} p(s,\mu)D + \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} V_0(k',b',\varepsilon_j';s',\mu'). \tag{38}$$

subject to

$$0 \le D \le x^d(k,b,\varepsilon;s) + q_b b' - qk' \tag{39}$$

$$x^d(k,b,\varepsilon;s) = z\varepsilon F(k,n) - w(s,\mu)n - b + q(1-\delta)k \tag{40}$$

$$k' \leq (1-\delta)k \tag{41}$$

$$b' \leq q\zeta k \tag{42}$$

$$\mu' = \Gamma(s,\mu) \tag{43}$$

To characterize a firm's intertemporal decisions, I divide firms into two groups *unconstrained* and *constrained*, based on whether their investment decision is influenced by collateral constraints[4]. *Unconstrained* firms have enough financial savings so that collateral constraints never affect their investment decision in all possible current and future states. As a result, these firms do not care whether they pay dividends to households or keep savings within the firm. Following Khan and Thomas (2013), I settle this choice by applying a *minimum saving policy* to unconstrained firms, requiring them to focus on paying dividends while keeping the minimum amount of financial savings needed to stay unconstrained. More details on this policy are in Section 3.2.

3.1 Decisions among unconstrained firms

Let W represent the value function of an unconstrained firm. The start-of-period value before exit shocks are realized, denoted W_0, is given by:

$$W_0(k,b,\varepsilon;s,\mu) = \pi_d \max_n \left[p(s,\mu)x^d(k,b,\varepsilon;s)\right] + (1-\pi_d)W(k,b,\varepsilon;s,\mu), \tag{44}$$

whereas the value function W is:

$$W(k,b,\varepsilon;s,\mu) = \max\left\{W^u(k,b,\varepsilon;s,\mu), W^d(k,b,\varepsilon;s,\mu)\right\}, \tag{45}$$

[4]This classification is first proposed by Khan and Thomas (2013), and become a standard way to resolve indeterminacy between dividend and financial savings (Jo and Senga, 2019; Jo, 2024).

with $W^u(\cdot)$ denoting the value for investing (upward-adjusting) firms and $W^d(\cdot)$ for disinvesting (downward-adjusting) firms. For unconstrained firms, bond choices b' are independent of capital choices k'. Thus, I express $W(k,b,\varepsilon;s,\mu) = W(k,0,\varepsilon;s,\mu) - pb$ and $W_0(k,b,\varepsilon;s,\mu) = W_0(k,0,\varepsilon;s,\mu) - pb$.

Using these expressions, the value functions for investing and disinvesting firms become:

$$W^u(k,b,\varepsilon;s,\mu) = p(s,\mu)x^u(k,b,\varepsilon;s) + \max_{k' \geq (1-\delta)k} \left[-pQk' + \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} W_0(k',0,\varepsilon_j;s,\mu) \right], \tag{46}$$

where $x^u(k,b,\varepsilon;s) = z\varepsilon F(k,n) - w(s,\mu)n - b + Q(1-\delta)k$, and:

$$W^d(k,b,\varepsilon;s,\mu) = p(s,\mu)x^d(k,b,\varepsilon;s) + \max_{k' \leq (1-\delta)k} \left[-pqk' + \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} W_0(k',0,\varepsilon_j;s,\mu) \right], \tag{47}$$

where $x^d(k,b,\varepsilon;s) = z\varepsilon F(k,n) - w(s,\mu)n - b + q(1-\delta)k$.

The optimal capital targets for upward and downward adjustments, $k_u^*(\varepsilon;s,\mu)$ and $k_d^*(\varepsilon;s,\mu)$, are the solutions to (46) and (47), respectively. Consequently, the capital decision rule for unconstrained firms follows an (S,s) decision rules,

$$K^w(k,\varepsilon;s,\mu) = \begin{cases} k_u^*(\varepsilon;s,\mu) & \text{if } k_u^*(\varepsilon;s,\mu) > (1-\delta)k, \\ (1-\delta)k & \text{if } k_d^*(\varepsilon;s,\mu) \leq (1-\delta)k \leq k_u^*(\varepsilon;s,\mu), \\ k_d^*(\varepsilon;s,\mu) & \text{if } k_d^*(\varepsilon;s,\mu) < (1-\delta)k, \end{cases} \tag{48}$$

Following Jorgenson (1963), the user cost of capital represents the net cost of buying one unit of capital at time t and selling it at time $t+1$, accounting for depreciation and discounting. This is expressed as:

$$c(q) = Q(q) - \beta(1-\delta)q = \left[\eta + (1-\eta)q^{1-s} \right]^{\frac{1}{1-s}} - \beta(1-\delta)q \tag{49}$$

When q sufficiently close to one, $c(q)$ is decreasing in q, and $c(1) = 1 - \beta(1-\delta)$. In section 5, I will show the effects of secondary market prices on firms' capital and bond decisions.

3.2 Minimum saving policy

The minimum saving policy, $B^w(k, \varepsilon; s, \mu)$, is derived recursively by searching over all possible largest debt levels, $\tilde{B}(K^w(\cdot), \varepsilon'; s', \mu')$, which ensures firms maintain an unconstrained status entering the next period. If the firm chooses its future debt level $B^w(\cdot)$ as the minimum of all possible $\tilde{B}(\cdot)$, then firms are paying the maximum amount of dividends without risking their unconstrained status in the future:

$$B^w(k, \varepsilon; s, \mu) = \min_{\{\varepsilon_j | \pi^{\varepsilon}_{ij} > 0\}} \tilde{B}(K^w(k, \varepsilon_i; s, \mu), \varepsilon_j; s', \mu') \tag{50}$$

where $\tilde{B}(k, \varepsilon_i; s, \mu)$ is defined as the highest current debt that a firm can take without violating the collateral constraints:

$$\begin{aligned} \tilde{B}(k, \varepsilon; s, \mu) = {} & z\varepsilon F(k, N(k, \varepsilon)) - wN(k, \varepsilon) \\ & + q_b \min\{B^w(k, \varepsilon; s, \mu), q\zeta k\} \\ & + \mathcal{J}(K^w(k, \varepsilon) - (1-\delta)k)[K^w(k, \varepsilon; s, \mu) - (1-\delta)k] \end{aligned} \tag{51}$$

where $\mathcal{J}(x) = Q$ if $x \geq 0$, and $\mathcal{J}(x) = q$ if $x < 0$. Given the decisions on bond and capital, the unconstrained firms' dividend payments can be retrieved as:

$$D^w(k, b, \varepsilon; s, \mu) = \begin{cases} x^u(k, b, \varepsilon_i; s) - QK^w(k, \varepsilon) & \\ \qquad\qquad + q_b \min\{B^w(k, \varepsilon; s, \mu), q\zeta k\} & \text{if } K^w(k, \varepsilon) \geq (1-\delta)k \\ x^d(k, b, \varepsilon_i; s) - qK^w(k, \varepsilon) & \\ \qquad\qquad + q_b \min\{B^w(k, \varepsilon; s, \mu), q\zeta k\} & \text{if } K^w(k, \varepsilon) < (1-\delta)k \end{cases} \tag{52}$$

3.3 Decisions among constrained firms

Constrained firms also face exogenous exits.

$$V_0(k,b,\varepsilon;s,\mu) = \pi_d \max_n [p(s,\mu)x^d(k,b,\varepsilon;s)] + (1-\pi_d)V(k,b,\varepsilon;s,\mu) \tag{53}$$

Conditional on their survival, they adopt unconstrained firms' decision rules and gain unconstrained status if feasible, i.e., they can implement the minimum saving policy and pay positive dividend, $D^w(k,b,\varepsilon;s,\mu)$:

$$V(k,b,\varepsilon;s,\mu) = \begin{cases} W(k,b,\varepsilon;s,\mu) & \text{if } D^w(k,b,\varepsilon;s,\mu) \geq 0 \\ V^c(k,b,\varepsilon;s,\mu) & \text{otherwise} \end{cases} \tag{54}$$

If they cannot achieve $W(\cdot)$, then they borrow up to the collateral constraint and approach the efficient capital level as closely as possible by choosing either upward or downward capital adjustment:

$$V^c(k,b,\varepsilon;s,\mu) = \max\{V^u(k,b,\varepsilon;s,\mu), V^d(k,b,\varepsilon;s,\mu)\} \tag{55}$$

Since by definition constrained firms prioritize enhancing their status to unconstrained by accumulating financial wealth and achieving efficient capital levels, they pay no dividends. Thus, their objective is to maximize the expected future value of the firm, and their debt level is determined by their capital decision with a binding budget constraint:

$$\begin{aligned} V^u(k,b,\varepsilon;s,\mu) &= \max_{k' \in \Omega^u(k,b,\varepsilon)} \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} V_0(k', b'_u(k'), \varepsilon_j; s', \mu') \\ \text{subject to } b'_u(k') &= \frac{Qk' - x^u(k,b,\varepsilon;s)}{q_b} \end{aligned} \tag{56}$$

and

$$
\begin{aligned}
V^d(k,b,\varepsilon_i;s,\mu) = &\max_{k'\in\Omega^d(k,b,\varepsilon)} \beta \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{ij} V_0(k', b'_d(k'), \varepsilon_j; s', \mu') \\
&\text{subject to } b'_d(k') = \frac{qk' - x^d(k,b,\varepsilon;s)}{q_b}
\end{aligned}
\tag{57}
$$

If the debt decision, $b'_u(\cdot)$ or $b'_d(\cdot)$, reaches the collateral constraint, this defines the endogenous maximum affordable capital stock for each option:

$$
\begin{aligned}
b'_u(k') = q\zeta k = \frac{Qk' - x^u(k,b,\varepsilon;s)}{q_b} &\Rightarrow \bar{k}_u = \frac{q_b q\zeta k + x^u}{Q} \\
b'_d(k') = q\zeta k = \frac{qk' - x^d(k,b,\varepsilon;s)}{q_b} &\Rightarrow \bar{k}_d = \frac{q_b q\zeta k + x^d}{q}
\end{aligned}
\tag{58}
$$

Thus, the endogenous limits, $\bar{k}_u$ and $\bar{k}_d$, define the choice sets, $\Omega^u(k,b,\varepsilon)$ and $\Omega^d(k,b,\varepsilon)$, for the upward- and downward-adjustment problems (56) and (57):

$$
\begin{aligned}
\Omega^u(k,b,\varepsilon) &= [(1-\delta)k, \bar{k}_u(k,b,\varepsilon)] \\
\Omega^d(k,b,\varepsilon) &= [0, \max\{0, \min\{(1-\delta)k, \bar{k}_d(k,b,\varepsilon)\}\}]
\end{aligned}
\tag{59}
$$

Let the solutions for (56) and (57) be $\hat{k}_u(k,b,\varepsilon)$ and $\hat{k}_d(k,b,\varepsilon)$. The policy function for capital is:

$$
K^c(k,b,\varepsilon;s,\mu) = \begin{cases} \hat{k}_u(k,b,\varepsilon) & \text{if } V(\cdot) = V^u(\cdot) \\ \hat{k}_d(k,b,\varepsilon) & \text{if } V(\cdot) = V^d(\cdot) \end{cases}
\tag{60}
$$

and the corresponding policy function for debt is:

$$
B^c(k,b,\varepsilon;s,\mu) = \begin{cases} \frac{Q\hat{k}_u(k,b,\varepsilon) - x^u(k,b,\varepsilon;s)}{q_b} & \text{if } V(\cdot) = V^u(\cdot) \\ \frac{q\hat{k}_d(k,b,\varepsilon) - x^d(k,b,\varepsilon;s)}{q_b} & \text{if } V(\cdot) = V^d(\cdot) \end{cases}
\tag{61}
$$

4 Calibration

Table 1 lists the parameter values for baseline model. Table 2 summarizes the matching of calibrated moments. The length of a period is set to one year. The steady-state total factor productivity z is normalized to one. I assume the functional form of the representative household's utility is $u(c, l) = \log c + \psi l$, following the specification in Rogerson (1988). The production function is Cobb-Douglas: $z\varepsilon F(k, n) = z\varepsilon k^{\alpha} n^{\nu}$. The model incorporates exogenous entry and exit to prevent all firms from eventually outgrowing the collateral constraint. Entrants are endowed with initial capital k_0 as a fraction of steady-state aggregate capital, as specified in (23). The average leverage ratio of entrants, ζ_0, is set to 0.41 to match the Kauffman Firm Survey data. The household's discount factor β is set to 0.96, implying a 4 percent annual interest rate. The disutility of labor parameter ψ is calibrated to generate hours worked equal to one-third of available time. The capital depreciation rate δ is set to 0.064, corresponding to an investment-capital ratio of approximately 6.9 percent. The labor share ν is set to 60 percent, consistent with US postwar data.

To precisely assess the effect of the secondary markets on investment, it is necessary for the model to reproduce firm-level evidence on investment dynamics. To begin, I assume the idiosyncratic productivity shock ε follows a log AR(1) process with persistence ρ_ε and innovation standard deviation $\sigma_{\eta_\varepsilon}$: $\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \eta'_\varepsilon$, where $\eta'_\varepsilon \sim N(0, \sigma^2_{\eta_\varepsilon})$. I calibrate three parameters, ρ_ε, $\sigma_{\eta_\varepsilon}$, and η, to match three moments from the establishment-level investment distribution documented in Cooper and Haltiwanger (2006). Specifically, I set $\rho_\varepsilon = 0.74$ to match the serial correlation of investment rates, $\sigma_{\eta_\varepsilon} = 0.1$ to match the standard deviation of investment rates, and $\eta = 0.9$ to match the fraction of establishments undertaking negative investment. With these parameters determined, I use the Rouwenhorst (1995) method to discretize firms' log-normal idiosyncratic productivity process with 7 values ($N_\varepsilon = 7$) to obtain $\{\varepsilon_i\}_{i=1}^{N_\varepsilon}$ and $(\pi^\varepsilon_{ij})_{i,j=1}^{N_\varepsilon}$. Although not targeted in the calibration, the average investment rate generated by my model is 0.107, close to the empirical counterpart of 0.122. Additionally, the fraction of firms in my model that undertake lumpy investment is 14.3%, also close to the empirical value of 18.6%. In the data, approximately 1.8% of firms undertake lumpy disinvestment, while the model predicts 5.1%.

The elasticity of substitution in the investment technology, s, determines the slope of demand in the secondary markets. Figure 1 illustrates the demand and supply in the secondary markets for different values of s, holding the wage at its equilibrium level. When the degree of substitutability is high ($s = 7$), new and used investment goods are close substitutes, resulting in a downward-sloping demand curve in the secondary markets. However, when s is sufficiently low, the demand curve becomes upward-sloping due to the role of capital as collateral in the model. With low substitutability, new and used investment goods act as close complements. In this case, an increase in the secondary market price q raises both the cost of acquiring capital (through higher $Q(q)$) and the available credit line (through higher $q\zeta k$). In equilibrium, the income effect from an expanded credit line dominates, leading to higher demand for used investment goods despite the price increase. In light of this, I choose the baseline value of elasticity of substitution equals to 7 to keep the downward-sloping secondary market demand[5].

[5] Edgerton (2011) uses the prices of used investment goods to estimate industry-level production functions that employ a CES aggregator over both new and used capital across four industries. The estimated elasticity of substitution ranges from 1 to 10.

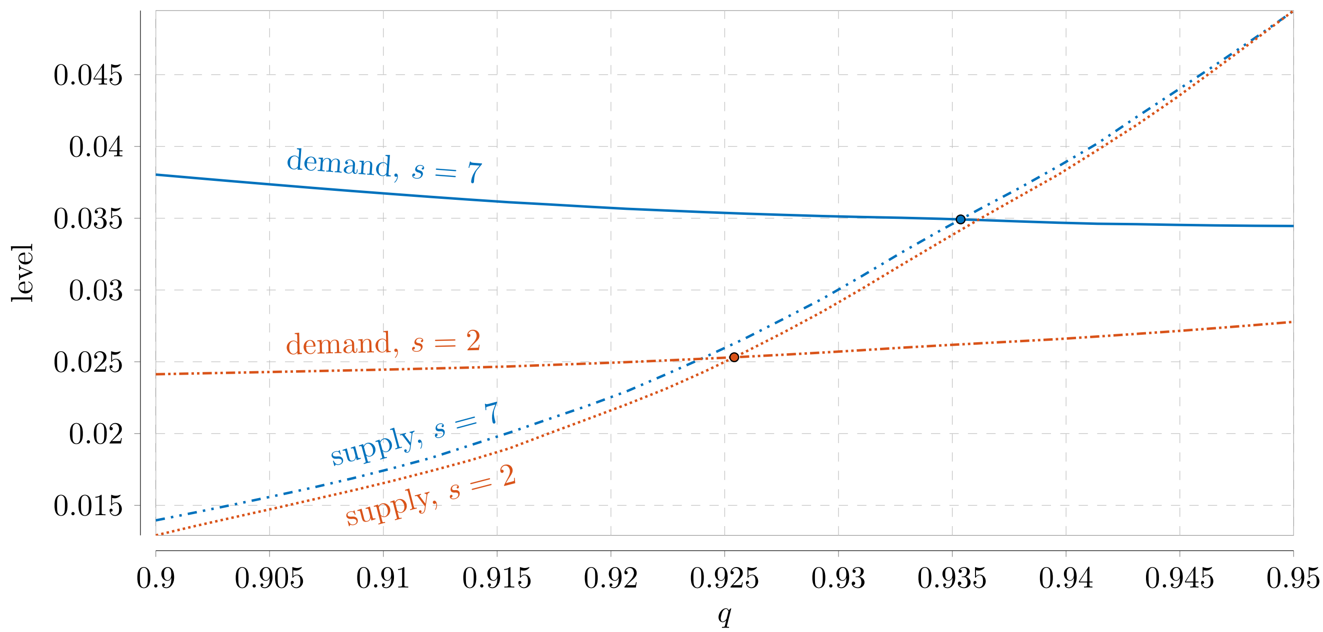
Table 1: Parameter values, baseline model

Parameter	Description	Value
Preferences and technology		
β	Subjective discount factor	0.960
ψ	Disutility from working	2.150
α	Capital share	0.270
ν	Labor share	0.600
δ	Depreciation rate	0.064
Shocks		
ρ_ε	Persistence idiosyncratic productivity shock	0.740
$\sigma_{\eta_\varepsilon}$	Volatility idiosyncratic productivity shock	0.100
Firm characteristic		
ζ	efficiency of the financial sector	1.250
π_d	exogenous exit probability	0.085
χ	relative size of entrants	0.100
ζ_0	entrants leverage	0.410
Investment technology		
η	new investment ratio	0.900
s	elasticity of substitution between new and used investment	7.000

Table 2: Calibrated Moments

	model	data
First moments		
Capital/Output, K/Y	2.3	2.3
Debt/Capital, B/K	0.353	0.370
Labor share, wN/Y	0.6	0.6
Investment/Capital, I/K	0.069	0.069
Second moments		
standard deviation of investment rate, $\sigma(i/k)$	0.338	0.337
serial correlation of investment rate, $\rho(i/k)$	0.043	0.058
frequency of negative investment	0.117	0.104
Untargeted moments		
average investment rate, $\mu(i/k)$	0.107	0.122
frequency of inaction region ($abs(i/k) < 1\%$)	0.504	0.081
frequency of lumpy investment ($i/k > 20\%$)	0.143	0.186
frequency of lumpy disinvestment ($i/k < -20\%$)	0.051	0.018

Figure 1: Partial Equilibrium Analysis: Secondary Markets with different CES parameter



5 Steady State

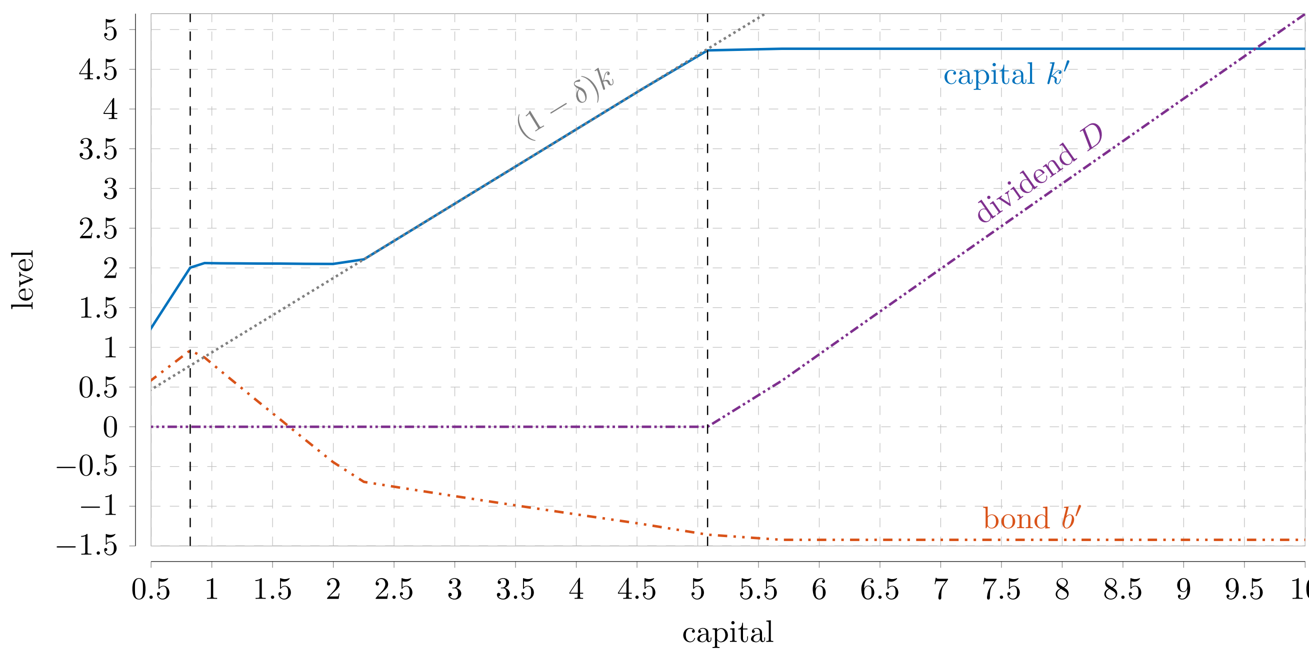
5.1 Decision rules

Figure 2 characterizes the effects of both partial irreversibility and collateral constraints on the decision rules for future capital, k', debt, b', and dividends, D, at zero current debt and median productivity. Two vertical lines separate firms' decisions into three regions.

From left to right, firms in the first region have a low stock of capital and have to borrow to finance their investment as they grow. Once they reach the first (S, s) threshold, $k_u^*(\varepsilon; s, \mu)$, they adopt the capital decision rules for unconstrained firms and gradually deleverage. However, when they enter the inaction region, where $k' = (1-\delta)k$ due to partial irreversibility, the speed of deleveraging slows down, prolonging the transition to unconstrained status. Only upon reaching the second (S, s) threshold, $k_d^*(\varepsilon; s, \mu)$, can firms fully adopt both the capital and bond decision rules for unconstrained firms and begin paying dividends.

This highlights the interdependence between capital and bond decisions. Initially, the lack of savings distorts firms' capital accumulation. Later, partial irreversibility further slows the accumulation of financial savings, potentially exposing firms to fluctuations in credit market inefficiency.

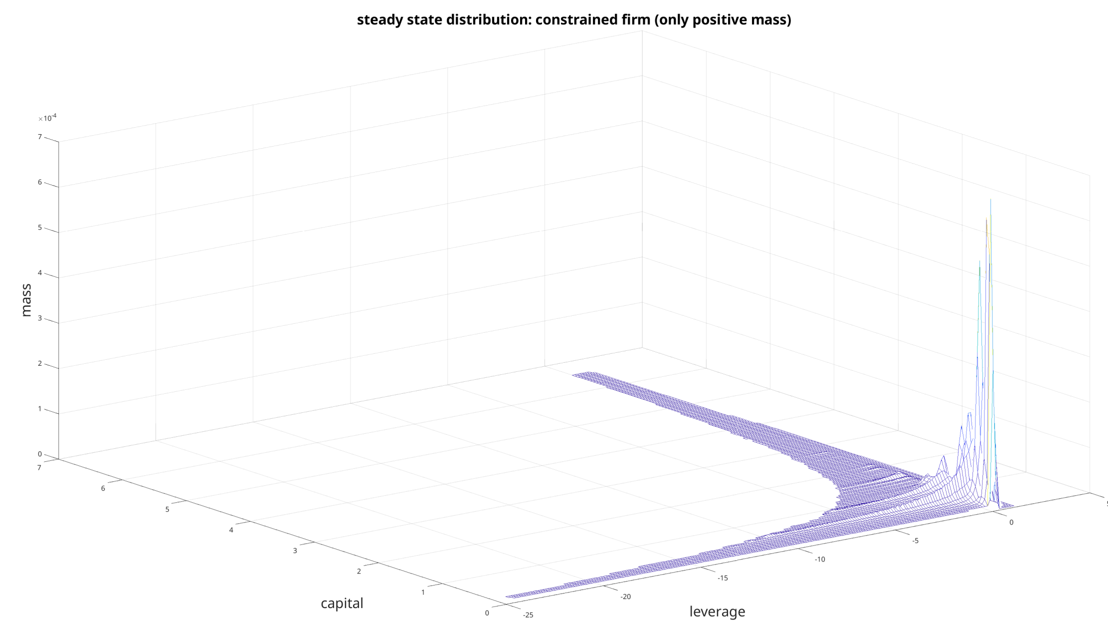
Figure 2: Decision rules at zero debt and median productivity ($\varepsilon = 1.11666$)



5.2 Distribution

Figure 3 illustrate the distribution of constrained firms across capital and bond, given median productivity. For figure 3, the distributions of capital and leverage at other productivity levels exhibit similar patterns. The spike in figure 3 represents the 8.5% exogenous entrants to the economy. Their idiosyncratic productivity follows the ergodic distribution $(\pi_i^\varepsilon)_{i=1}^{N_\varepsilon}$, and they enter with a leverage ratio of 0.41 and an initial capital endowment equivalent to 10% of steady-state aggregate capital. Upon entering production, these firms take on debt to accumulate capital as in figure 2. In the absence of collateral constraints, entrants would immediately take on a large debt to reach their efficiency level of capital, determined by their productivity draw. However, due to borrowing constraints, entrants face limited access to credit, forcing them to gradually accumulate capital as they age. This smooth capital accumulation is reflected in the series of smaller spikes following the initial entrant spike along the k-dimension. Firms that survive long enough eventually reach their targeted capital level and reduce their debt or accumulate financial savings. This behavior generates the extended long tails toward negative leverage at each of the small spikes in figure 3. Ultimately, 6.5% of firms accumulate sufficient savings to transition into the unconstrained status, while the remaining 93.4% of firms remain constrained. The average capital among unconstrained firms is 2.15, while the average capital among constrained firms is 1.25, with 28 percent of firms facing binding collateral constraint.

Figure 3: Constrained firm steady-state distribution: median productivity



5.3 Life cycle

The life cycle aspects of this economy can be seen from figure 4a, which displays the average capital and bond choices. I simulate 50,000 firms over 100 periods to get a large panel for established firms as seen in Compustat data. In the initial 6 periods, firms are accumulating capital by raising debt. Starting at period 7, they begin to reduce their debt yet still accumulate capital. By period 16, the firm has become a net saver, and eventually reach its desired leverage level at period 35.

Figure 4b compares the firms' life cycle between the baseline model with the *low q* model. Low q model lowers the secondary market price q by 10% in partial equilibrium, holding the parameters and the equilibrium wage w the same as in the baseline. From the induction of user cost of capital in equation (49), lower q leads to higher cost of capital $c(q)$ to firms. This results in the lower aggregate capital and slower accumulation of savings.

6 Long-run effects of secondary markets

I examine the long-run effects of two channels associated with the secondary markets for capital: the cost of investment channel and the collateral channel. I provide an overview of the aggregate outcomes of the baseline model and counterfactual experiments, and assess the magnitude of both channels and their impacts on aggregate outcomes.

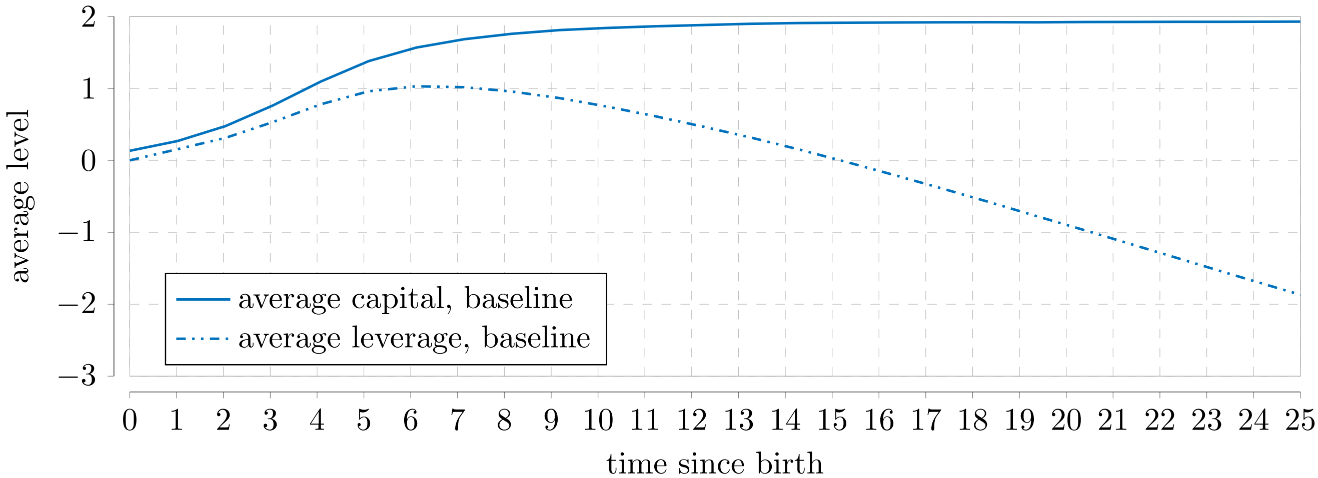
6.1 Overview of counterfactual experiments

In my counterfactual experiments, I compare the aggregate outcomes of the baseline model with the new steady states under different conditions of collateral value and secondary markets clearing. In particular, the aggregates of counterfactual experiments are expressed as a percentage of the baseline outcomes.

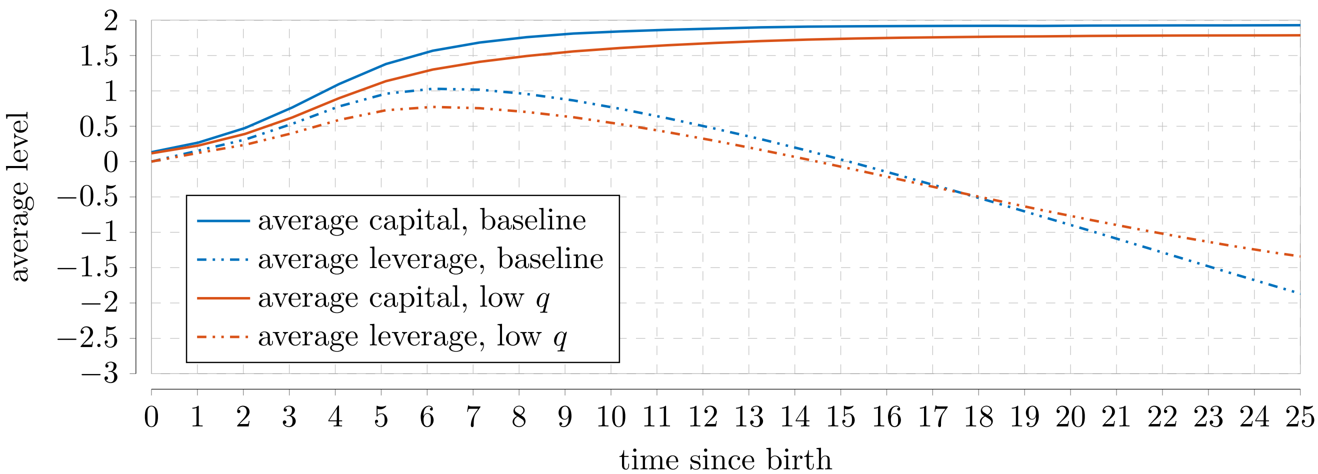
I conduct two counterfactual experiments: (1) *Fix irreversibility* model has the same degree of capital irreversibility, q/Q, as the baseline model, but holds the purchasing price of capital at one. Specifically, I assign the capital selling price q as capital irreversibility in the baseline model and set the capital purchasing price Q to one without clearing the secondary markets. This model allows me to compare the baseline model with the partial irreversibility literature[6].

[6] For example, Veracierto (2002), Khan and Thomas (2013), Lanteri (2018), Lanteri, Medina and Tan (2023)

Figure 4: Life cycle simulation



(a) Cohort in steady state



(b) Comparing baseline economy with the economy with low q

Note: Baseline model follows the specification in section 4, and solves for equilibrium wage w and secondary market price q. Low q model uses the same set of parameters and w as in the baseline model, but lower the secondary market price q by 10% to show the effects of used capital price fluctuation in firms' life cycle.

(2) *Cost channel* model isolates the cost of investment channel by fixing the selling price q in the collateral constraint at the baseline value, and resolve the model with secondary markets clearing. This essentially shuts down the collateral channel.

6.2 Aggregate results from counterfactual experiments

Table 3 summarizes the experimental results. Compared to the *baseline* model, the *fix irreversibility* model exhibits declines across all aggregates, with consumption decreasing by 0.108% and output falling by 1.2%. Additionally, both constrained and unconstrained firms hold less aggregate capital in the *fix irreversibility* model, and a greater number of firms face binding collateral constraints.

These outcomes can be attributed to differences in the user cost of capital, as defined in equation (49). The *baseline* model has $c(q) = 0.1514$, whereas the *fix irreversibility* model results in a higher cost of $c(q) = 0.1526$. This suggests that both the relative degree of irreversibility and the absolute scale of investment costs play crucial roles in shaping firm behavior and aggregate outcomes.

In my model, investing firms take into account the lower future selling price q while benefiting from a purchasing price Q that is below one unit of output goods. Nonetheless, they accept a lower future selling price in exchange for current expansion (Eisfeldt and Rampini, 2007; Ma et al., 2022), as a lower purchasing price Q alleviates collateral constraints in absolute terms. Consequently, a cheaper purchasing price of capital incentivizes greater investment by constrained firms, enabling them to expand their capital stock despite collateral constraints. This mechanism highlights the significance of investment costs in determining firm-level and aggregate equilibrium dynamics.

I demonstrate that fluctuations in the resale value of pledged collateral have a negligible impact on the aggregate economy in steady state. The *cost channel* model isolates the effect of cheaper used capital while holding the selling price in collateral constraints fixed at its baseline value. As shown in Table 3, long-run aggregates in the *cost channel* model are virtually identical to those in the *baseline* model. Specifically, restricting price adjustments in collateral constraints has a minor negative effect[7]. Despite small in magnitude, the adjustment of the resale value

[7]For example, output is reduced by only -0.0000062%

Table 3: Steady state comparison

	Description	Baseline	Fix irreversibility	Cost channel
Prices				
w	wage	1.024	1.023	1.024
q	selling price	0.935	0.943	0.935
Q	purchasing price	0.992	1.000	0.992
q/Q	capital irreversibility	0.943	0.943	0.943
Aggregates (in percentage of baseline)				
Y	output	(0.567)	-1.202	-0.000
C	consumption	(0.476)	-0.108	-0.000
N	labor	(0.332)	-1.095	-0.000
K	capital	(1.311)	-2.004	-0.000
I	investment	(0.228)	-2.079	-0.000
$B > 0$	debt	(0.464)	-1.704	-0.000
$\hat{z}$	measured TFP	(1.021)	-0.002	-0.000
Distribution				
μ_{unc}	unconstrained firm mass	0.066	0.094	0.066
μ_{con}	constrained firm mass	0.934	0.906	0.934
$\mu_{\text{unc}}K$	unconstrained capital	2.156	2.057	2.156
$\mu_{\text{con}}K$	constrained capital	1.251	1.204	1.251
μ_{binding}	firms with binding $q\zeta k$	0.280	0.284	0.280

Note: *Baseline* is the model with both capital secondary market and labor market clearing, and *Fix irreversibility* is the model that clears only the labor market with exogenously impose the value of q and Q such that degree of irreversibility is the same as baseline model. *Cost channel* is the model that isolate the cost of investment channel. This model clears both markets, but hold the q in the collateral constraints at baseline level.

of collateral is beneficial to the long-run equilibrium. This positive yet negligible effects of collateral channel persists in transitional dynamics, reinforcing the limited role of resale value fluctuations in shaping macroeconomic outcomes.

7 Business cycles

I now examine the role of the used capital markets in propagating aggregate shocks. To do so, I compare impulse responses across models under different market-clearing scenarios. This analysis helps isolate the extent to which each channel amplifies or dampens macroeconomic fluctuations.

7.1 Impulse response: credit shock

The credit shock in my model is characterized by a decline in the efficiency of the financial sector, ζ. I consider a transitional equilibrium in which the baseline economy is hit by a credit shock while holding exogenous TFP fixed. Specifically, the value of ζ drops by 40% from its baseline level to match a 26% reduction in debt, consistent with empirical observations[8]. The shock persists for four periods before gradually recovering, with a persistence parameter of $\rho_\zeta = 0.3125$.

Figure 5 illustrates the dynamics of aggregate variables following the aforementioned shock. The credit shock induces a persistent recession in the baseline economy (*solid blue line*). In the first panel of Figure 5a, measured TFP declines to a trough of 0.94% below the baseline steady-state level. This suggests significant resource misallocation in the economy due to an inefficient financial sector. Output drops by 3.14%, and my model explains 43% of the observed drop in the data.

Notably, the effects of a large credit shock persist beyond its peak. While the half-life of the shock is under 2 years, its effects on aggregates are long-lasting. GDP takes almost 3 years to reach its half-life, while consumption takes over 5 years to recover. The reason for the slow recovery is twofold. First, the slow adjustment of the capital distribution keeps aggregate

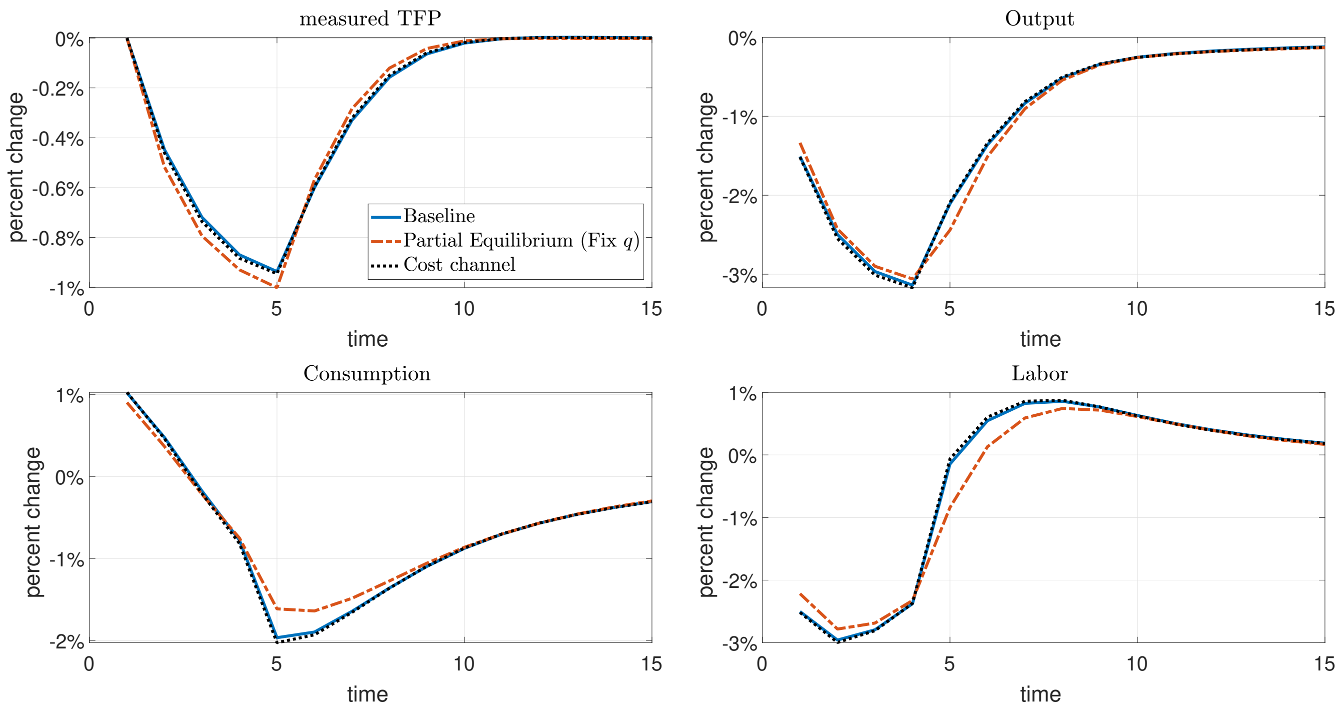
[8] Different measurements on the reduction of business loans during the Great Recession. Ivashina and Scharfstein (2010) estimate an almost 50% reduction in new loans to large borrowers during the peak period of the Great Recession, while Khan and Thomas (2013) shows that real lending issued by commercial banks fell by approximately 26%. I follow the latter in my experiments.

productivity below its steady-state level. Rebuilding the capital stock takes time, with a half-life of roughly 7 years. Second, movements in the used capital price q further exacerbate the aggregate outcome under credit shock. After reaching a trough at period 4, the used capital price remains 0.8% below the baseline level, leading to a higher user cost of capital for investing firms. To put this in perspective, the baseline user cost of capital is $c(q^*) = 0.1515$, whereas in period 5, it rises to $c(q_5) = 0.2042$, reflecting a 34% surge in user cost. The rising user cost significantly slows the recovery of investment, further disrupting capital accumulation.

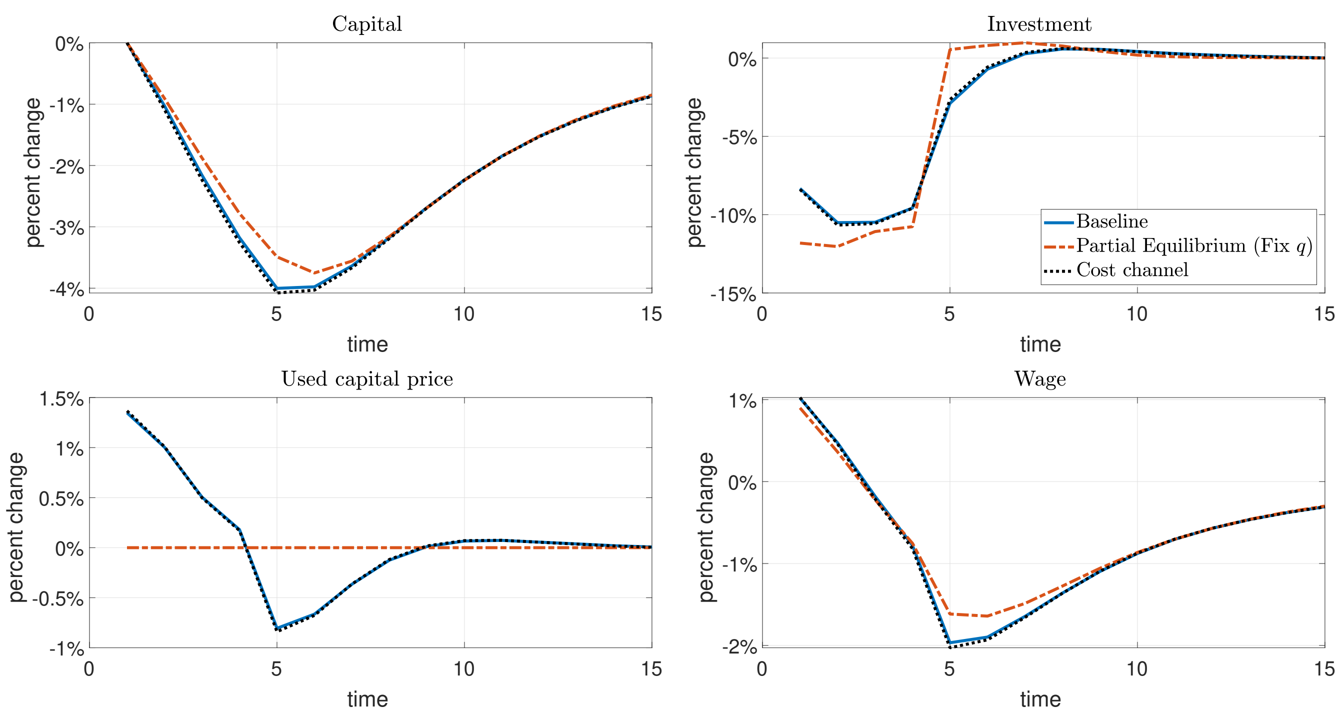
To assess the extent to which the secondary capital markets influence aggregate fluctuations and resource misallocation, I compute the *partial equilibrium* scenario (*dashed orange line*) by fixing the used capital market prices, q and Q, at their steady-state levels while only allowing the labor market clearing. Despite leading to larger initial declines in measured TFP and investment, the partial equilibrium scenario results in a milder trough in output (3.06%) and consumption (1.64%). Once credit efficiency begins to recover in period 5, investment under partial equilibrium immediately surges above its pre-crisis level. In contrast, the recovery of baseline investment is hampered by the declining used capital price, which delays investment recovery until period 7. The fluctuations in the used capital price raise the user cost of capital during a credit crisis, causing sluggish recoveries in baseline investment and prolonging the recession.

Lastly, I find that the magnitude of the used capital price fluctuation through collateral channel is positive but small, similar to the steady-state analysis. I do so by holding the q in the collateral constraint at the steady-state level, isolating the *cost channel* and allowing both the secondary capital markets and labor market clearing. Comparing with the *baseline* model, shutting down the adjustment of the collateral value accounts for 0.03 percentage point drop in GDP, and 0.06 percentage point drop in consumption.

Figure 5: Aggregate dynamics following a four-period credit crisis



(a) Measured TFP, Output, Consumption, and Labor



(b) Capital, Investment, Used capital price, and Wage

Table 4: Peak-to-Trough Declines: Credit Shock

	TFP	Y	C	N	I	Debt
Data	2.18	5.59	4.08	6.03	18.98	25.94
Baseline	0.94	3.14	1.97	2.96	10.51	25.63
Partial Equilibrium	1.00	3.06	1.64	2.79	11.97	25.57
Cost channel	0.95	3.17	2.03	3.00	10.67	26.01

7.2 Impulse response: TFP shock

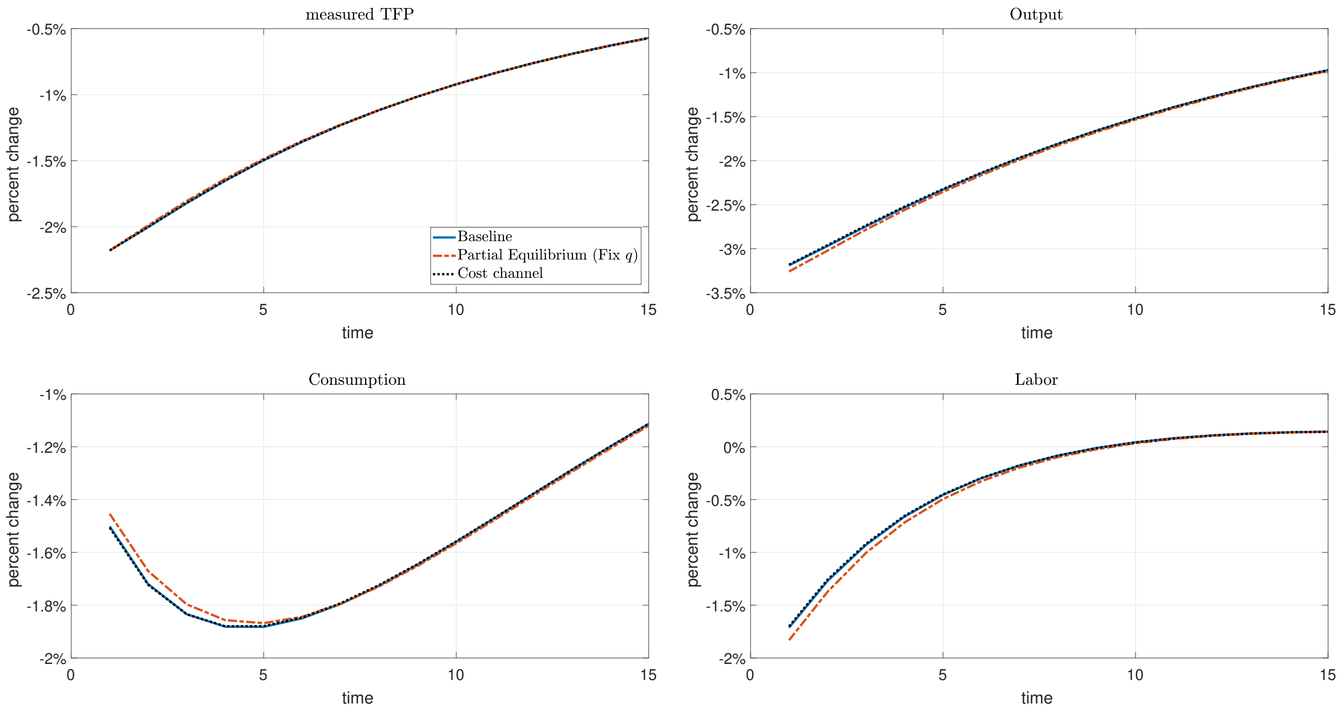
Table 5 reports the peak-to-trough decline in response to a negative 2.18% drop in TFP with a persistence of $\rho_z = 0.909$. Figure 6 illustrates the corresponding transitional dynamics. The shock magnitude is calibrated to match the observed decline in measured TFP during the Great Recession. Output, consumption, and employment decline immediately before gradually recovering to their steady-state levels. While measured TFP closely tracks changes in exogenous TFP over time, other variables experience significantly smaller declines compared to the credit crisis scenario in Figure 5.

Comparing the *baseline* model with either the *partial equilibrium* or isolating the *cost channel*, we see almost no aggregate impact secondary capital markets. As TFP shock affects all firms equally, it does not have distributional effects, hence have negligible impact on resource misallocation.

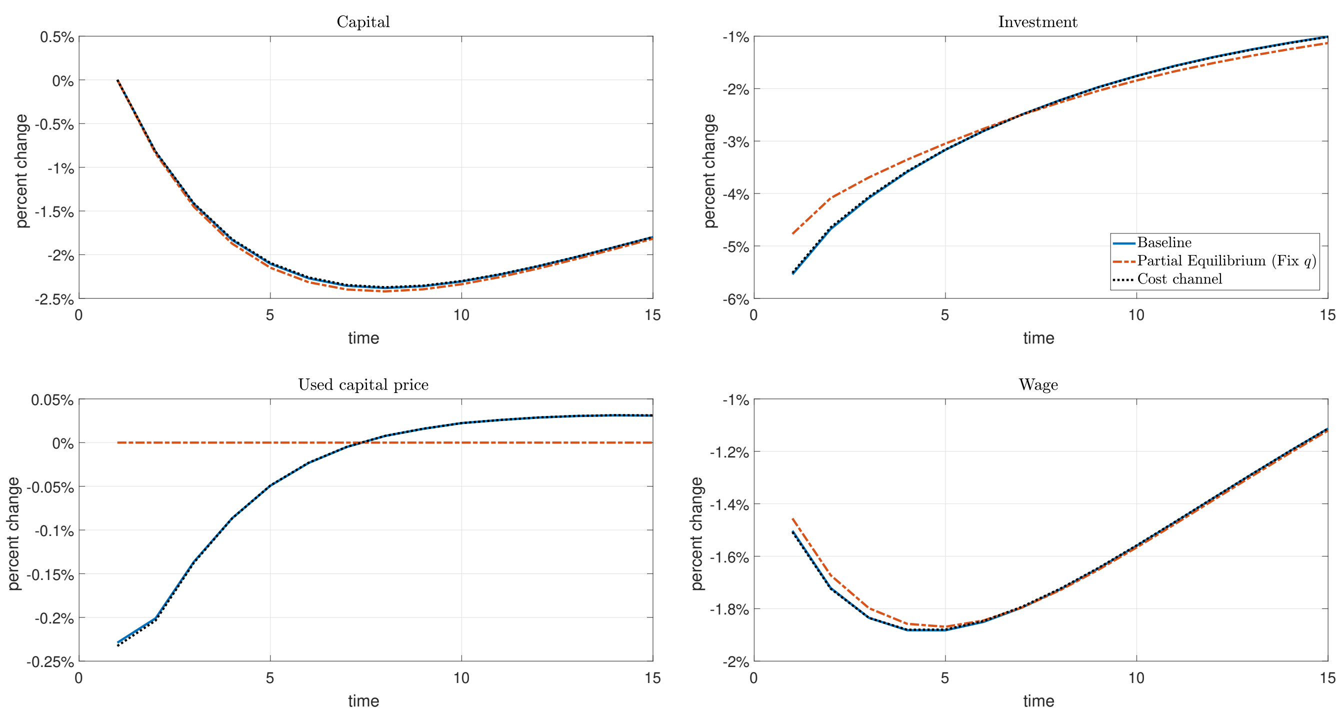
Table 5: Peak-to-Trough Declines: TFP shock

	TFP	Y	C	N	I	Debt
Data	2.18	5.59	4.08	6.03	18.98	25.94
Baseline	2.18	3.19	1.88	1.71	5.54	2.69
Partial Equilibrium	2.18	3.26	1.87	1.83	4.77	2.67
Cost channel	2.18	3.18	1.88	1.70	5.51	2.66

Figure 6: Aggregate dynamics following a persistent TFP shock



(a) Measured TFP, Output, Consumption, and Labor



(b) Capital, Investment, Used capital price, and Wage

8 Concluding Remarks

This paper examines the business cycle implications of the secondary capital markets through two key channels: the cost of investment and the value of collateral. To do so, I develop a dynamic stochastic general equilibrium model featuring persistent idiosyncratic shocks, secondary capital markets that endogenize capital irreversibility, and collateralized borrowing constraints tied to secondary market prices. The model generates a rich distribution of firms over productivity, capital, and debt, which in turn shapes aggregate output and investment.

In my framework, new and used investment goods are imperfect substitutes due to capital specificity, and the price of used capital is determined by the distribution of investing and disinvesting firms. Through the lens of this model, I find that changes in the cost of investment primarily drive aggregate fluctuations, and the secondary capital markets slow economic recovery following a financial shock. A lower cost of investment reduces firms' reliance on external financing, thereby weakening the collateral channel. Unlike productivity shocks, financial shocks distort the firm distribution, reducing demand for used capital and driving down its price. This decline in used capital prices is persistent, as it reflects prolonged distortions in the firm distribution caused by credit constraints.

My analysis focuses on fluctuations driven by the secondary capital markets in the presence of exogenous financial frictions. In an economy with default risk, these financial frictions become endogenous, potentially interacting with used capital fluctuations. The relative importance of these channels and the sign of the collateral channel may differ in such a setting. I leave the quantitative exploration of this mechanism for future research.

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A Additional figures

Figure 7: Unconstrained firm steady-state distribution: median productivity