

14

attendance < 20

Lecture 16

The Real Business Cycle Model Part 3: Competitive Equilibrium

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- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
 - Lecture 14: consumer
 - Lecture 15: firm
 - Lecture 16: competitive equilibrium
 - Lecture 17: formal example
 - Lecture 18: application to bring RBC to data

Review: Consumer's Problem

prices *vars not decided by consumer*

Taken $\{w, w', r, T, T', \pi, \pi'\}$ as given, a representative consumer chooses $\{C', N_S, N'_S\}$ to solve

$$\max_{C', N_S, N'_S} u \left(wN_S + \pi - T + \frac{w'N'_S + \pi' - T' - C'}{1+r} \right), \quad (1)$$

$$- v(N_S) + u(C') - v(N'_S)$$

which we can back out C, S, l, l' .

$$\begin{aligned} C &= C(C') \\ S &= \text{income} - C \\ l &= 1 - N_S \\ l' &= 1 - N'_S \end{aligned}$$

Review: Firm's Problem

prices

Taken $\{w, w', r\}$ as given, a representative firm chooses $\{N_D, N'_D, K'\}$ to solve

$$\max_{N_D, N'_D, K'} \underbrace{zF(K, N_D) - wN_D - [K' - (1 - \delta)K]}_{\pi} + \frac{\underbrace{z'F(K', N'_D) - w'N'_D + (1 - \delta)K'}_{\pi'}}{1 + r}, \quad (2)$$

which we can back out Y, Y', π, π', I

$$Y = zF(K, N_D)$$

$$Y' = z'F(K', N'_D)$$

$$\pi, \pi'$$

$$I = K' - (1 - \delta)K$$

Government Budget Constraint

Government behaves exactly the same in two-period model:

- current budget constraint: $G = T + B$
- future budget constraint: $G' + (1+r)B = T' \Rightarrow B = \frac{T'}{1+r} - \frac{G'}{1+r}$
- lifetime budget constraint: $G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$

Taken $\{r\}$ as given, government satisfy lifetime budget constraint by choosing $\{T, T', B\}$.

Market Clear

There are three markets to clear:

① labor markets clear at each date determines wage:

- find w such that $N_S = N_D$

- find w' such that $N'_S = N'_D$

Wallas' law.

② goods markets clear at each date determines consumption and investment:

no need to clear.

- date 0 (today): $Y = C + I + G$

- date 1 (tomorrow): $Y' = C' + I' + G'$

③ bonds market clears at date 0 determines real interest rate:

- find r such that $S = B$

supply = demand the credit

Competitive Equilibrium: RBC Model

Given exogenous quantities $\{\underline{G}, \underline{G'}, \underline{z}, \underline{z'}, \underline{K}\}$, a **competitive equilibrium** is a set of

① consumer choices ✓

$\{C, C', N_S, N'_S, l, l', S\}$,

② firm choices ✓

$\{Y, Y', \pi, \pi', N_D, N'_D, I, K'\}$,

③ government choices $\{T, T', B\}$,

and

④ prices $\{w, w', r\}$

such that

① consumer solves problems in (1),

② firm solves problems in (2),

③ government balances its budget,
and

④ all three markets clear.

$$\begin{array}{l} w \rightarrow N_S = N_D \\ w' \rightarrow N'_S = N'_D \end{array} \quad r \rightarrow S = B$$

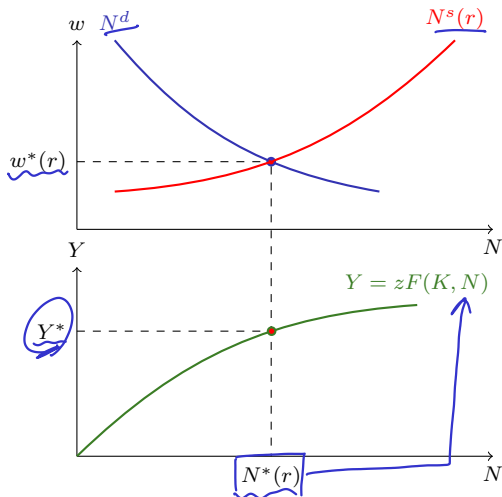
Plan to analyze the Model

In the following slides, we are going to use graphical analysis on **two markets** in **current period**:

- ① **current labor market**: **interaction of firm and consumer today**
 - similar to static model: **labor supply** and **labor demand** curves
 - what's new: both curves reflect the **dynamic tradeoff through interest rate**
labor demand & labor supply
- ② **current goods market**: **interaction of firm, consumer and government today**
 - new: **construct and analyze output supply** and **output demand** curves

The Current Labor Market

Figure 11.14 Determination of Equilibrium in the Labor Market Given the Real Interest Rate r



■ **consumer optimality:** ceteris paribus, $N^s \uparrow$ in $w \uparrow$

- N1: $\frac{dN^s}{dw} > 0$, substitution dominates income effect

■ **firm optimality:** $N^d \downarrow$ in $w \uparrow$

- $MPN = w$, \therefore diminishing MPN

■ account for **multiple markets:** intersect at $N^*(r)$

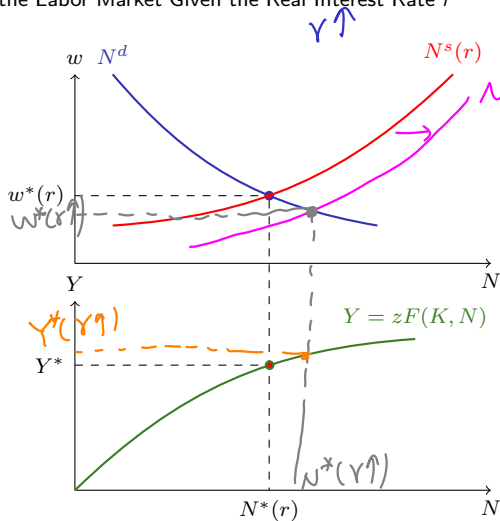
- note: top figure is given r, π
- labor mkt clearing w is $w^*(r)$

The Current Labor Market (Cont.)

$$r \uparrow \Rightarrow Y^* \uparrow$$

Figure 11.14 Determination of Equilibrium in the Labor Market Given the Real Interest Rate r

■ r increases?



• **N2** (consumer): $N^s(r) \uparrow$ in $r \uparrow$
 $N^s(r \uparrow) \Rightarrow w \downarrow, N^*(r) \uparrow$

• **firm**: $\because MPN = w$, same

■ consumer wealth increases?

• **N3** (consumer):
 $N^s(r) \downarrow \Rightarrow w \uparrow, N^*(r) \downarrow$

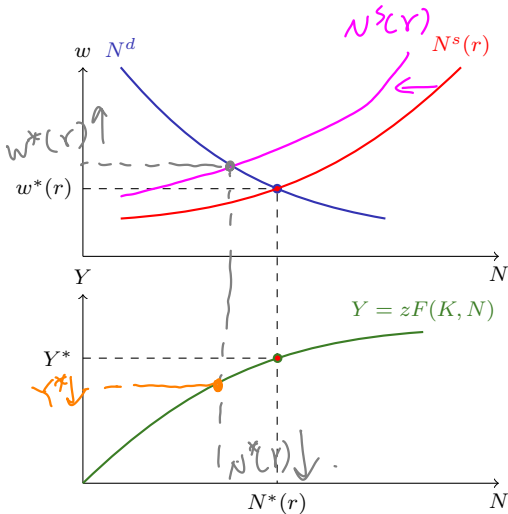
• **firm**: nothing

■ Bottom chart: $N^*(r) \rightarrow Y^*(r)$,
 output supply!

The Current Labor Market (Cont.)

net worth $\uparrow \Rightarrow Y^* \downarrow$.

Figure 11.14 Determination of Equilibrium in the Labor Market Given the Real Interest Rate r



■ r increases?

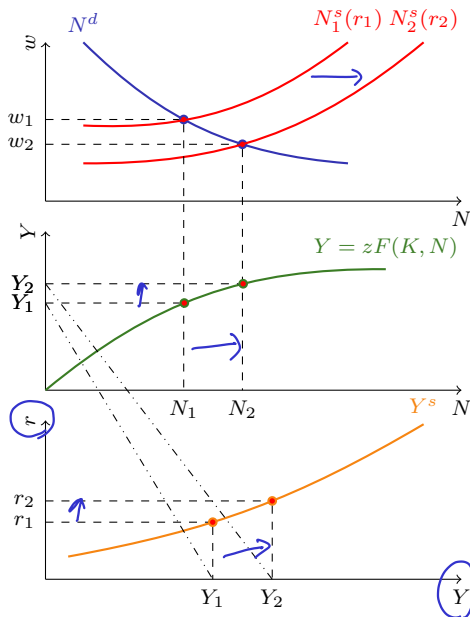
- **N2** (consumer): $N^s(r) \uparrow$ in $r \Rightarrow w \downarrow, N^*(r) \uparrow$
- **firm**: $\because MPN = w$, same

■ consumer wealth increases?

- **N3** (consumer): $N^s(r) \downarrow \Rightarrow w \uparrow, N^*(r) \downarrow$
- **firm**: nothing

■ Bottom chart: $N^*(r) \rightarrow Y^*(r)$, output supply!

The Output Supply Curve



Using our insight from labor market, we can repeat our analysis for any real interest rate r

- Top: each r implies a different “labor market equilibrium”, i.e., w & N
- Middle: each $N(r)$ yields production $Y^S(r)$
- Bottom: combined to show $\frac{dY^S}{dr} > 0$

Shifts in the Output Supply Curve

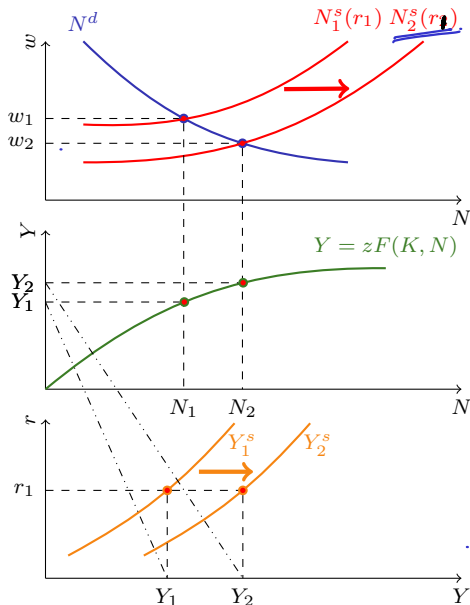
K : endowment
 K' : firm's choice.

How do changes in exogenous variables shift $Y^S(r)$? Consider 2 cases:

- ① shift in lifetime wealth (for example, by gov't spending or taxation)
- ② change in total factor productivity (TFP) or ^{Initial} capital stock
 - recall static model: with K fixed, these have the same effect

In each case, we can start our analysis with the current labor market.

Wealth and Output Supply



Complete the Model

Work with Model

$$we = w \cdot N_s + \pi - T$$

$$we' = w' \cdot N_s' + \pi' - T'$$

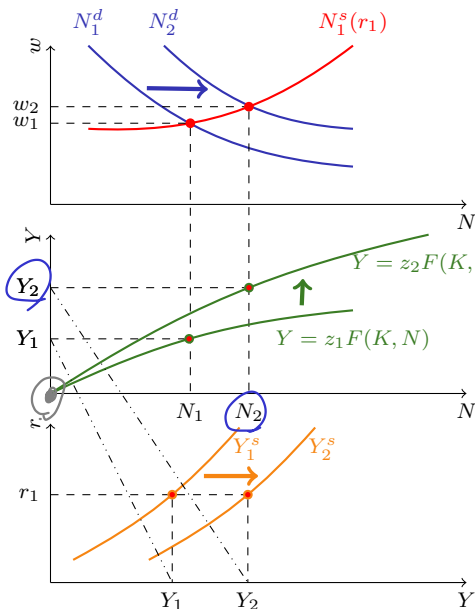
$$T + \frac{T'}{1+r} = G + \frac{G'}{1+r}$$

Suppose $G \uparrow$ or $G' \uparrow$.

- gov. budget: $T \uparrow$ or $T' \uparrow$
- consumer budget: $we \downarrow$
- **N3**: $dN^S/d(we) < 0$, $N^S(r) \uparrow$
(shift to the right, top panel)
- Middle: $N \uparrow \Rightarrow Y^S \uparrow$
- bottom: combine, get rightward shift in output supply

$$G \uparrow \Rightarrow we \downarrow \Rightarrow Y^S \uparrow$$

TFP / Capital and Output Supply



Suppose TFP $z \uparrow$.

■ **firm optimality:**

$$\textcircled{MPN} = \textcircled{z D_N F(\cdot)} \uparrow \Rightarrow \textcircled{N^d} \uparrow$$

■ **Top:** N^d shifts out \Rightarrow
 $w^* \uparrow, N^* \uparrow$

■ **Middle:** production fcn shifts
 up, $\because z \uparrow$

■ **Bottom:** combine, outward shift
 in output supply

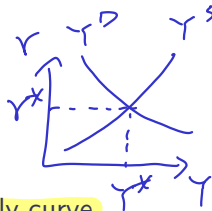
$$z \uparrow \Rightarrow Y^s \uparrow$$

Summary: Current Labor Market

We have constructed most of the model!

- **labor market clearing**, *conditional on* the interest rate

- trace through production function to get **output supply curve**



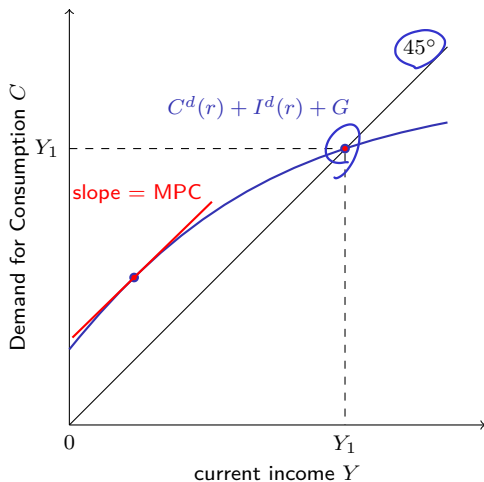
Now we need to determine the **equilibrium interest rate**, r^* .

- pair the **output supply curve** with the **output demand curve**
- who demands goods today, and how much?
 - **consumer**: **consumption** $C^d(r, Y)$
 - **firm**: **investment** $I^d(r)$
 - **government**: **expenditures** G
 - use **GDP accounting** to get aggregate demand for goods

Current Goods Demand
 45°: what has been produced = what has been "needed"

$$D(r, Y) = C^d(r, Y) + I^d(r) + G$$

Figure 11.18 The Demand for Current Goods



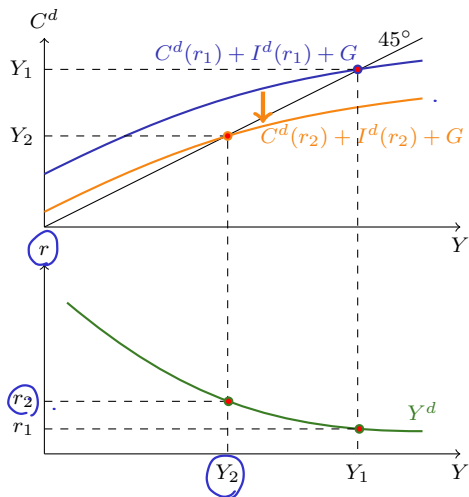
- plot $D(r, Y)$ on y -axis, Y on x -axis

- C^d depends on wealth: $w e = wN + \pi - T + \frac{w'N' + \pi' - T'}{1+r}$, which depends on income.

- Not true for I and G
 - $MPC < 1$: flatter than 45° line
 - MPC diminishing: concave
 - combine: cross 45° line at $Y^d(r)$

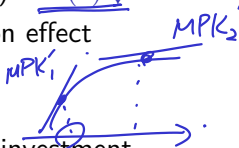
Constructing Output Demand

How different r affect output demand?



$$MPK'_2 < MPK'_1$$

- **C2** (consumer): $C^d(r) \downarrow$ if $r \uparrow$ (substitution effect dominates)



- **firm**: optimal investment schedule ($r = MPK' - \delta$), $r \uparrow \Rightarrow MPK' \uparrow \Rightarrow I^d \downarrow$

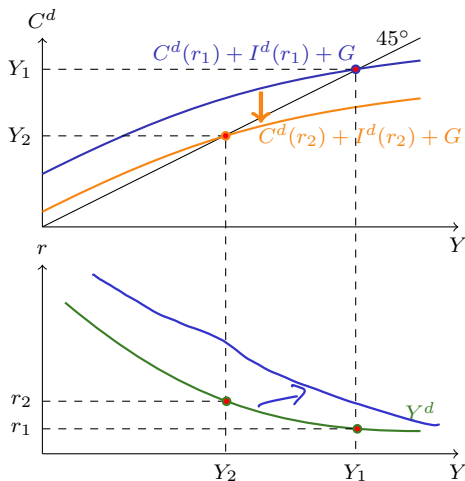
- **gov**: no change, $\therefore G$ exogenous

- Combine:

- ① intersection with 45° line is lower

Constructing Output Demand (Cont.)

Combine:

① intersection with 45° ② output demand curve Y^d
downward sloping $Y^d(r)$ shift to the right if $T + \frac{T'}{1+r}$ ① present value of taxes $\downarrow \Rightarrow C^d \uparrow$
 $w'N'_s, \pi' \uparrow$ ② future income $\uparrow \Rightarrow C^d \uparrow$ ③ future TFP $\uparrow \Rightarrow I^d \uparrow$ ④ current capital $\downarrow \Rightarrow I^d \uparrow$ Other changes (e.g., current TFP)
are ambiguous in general!

$$\uparrow I_d = \overline{K'} - (1-\delta)K \downarrow$$

Competitive Equilibrium

Figure 11.21 The Complete Real Intertemporal Model

