

Lecture 17

The Real Business Cycle Model

Part 4: Formal Examples

Hui-Jun Chen

The Ohio State University

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Credit: Kyle Dempsey

- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
 - Lecture 14: consumer
 - Lecture 15: firm
 - Lecture 16: competitive equilibrium
 - Lecture 17: formal example
 - Lecture 18: application to bring RBC to data

Assumptions

- **consumer**: assume discounting factor $\beta \in (0, 1)$ and utility function is

$$\tilde{U}(C, N, C') = \ln C + \beta \ln C' + \gamma \ln(1 - N),$$

where $\gamma > 0$, and consumer endowed with 1 unit of time.

- we assume no dis-utility in date 1 labor supply to simplify analysis

- **firm**: assume production is Cobb-Douglas in both periods:

$$Y = zK^\alpha N^{1-\alpha} \text{ and } Y' = z'K'^\alpha N'^{1-\alpha},$$

where K is initial capital, TFP $z = 1$, and depreciation $\delta \in (0, 1)$

- **government**: spend G and G' , which is financed by lump-sum taxes T, T' and deficit B

Competitive Equilibrium

Given exogenous quantities $\{G, G', z, z', K\}$, a competitive equilibrium is a set of (1) consumer choices $\{C, C', N_S, N'_S, l, l', S\}$; (2) firm choices $\{Y, Y', \pi, \pi', N_D, N'_D, I, K'\}$; (3) government choices $\{T, T', B\}$, and (4) prices $\{w, w', r\}$ such that

- ① Taken $\{w, w', r, \pi, \pi'\}$ as given, consumer chooses $\{C', N_S, N'_S\}$ to solve

$$\max_{C', N_S, N'_S} \ln \left(wN_S + \pi - T + \frac{w'N'_S + \pi' - T' - C'}{1+r} \right) + \beta \ln C' + \gamma \ln(1 - N_S),$$

where we can back out $\{C, S, l, l'\}$.

- ② Taken $\{w, w', r\}$ as given, firm chooses $\{N_D, N'_D, K'\}$ to solve

$$\max_{N_D, N'_D, K'} zK^\alpha N_D^{1-\alpha} - wN_D - [K' - (1-\delta)K] + \frac{z'(K')^\alpha (N'_D)^{1-\alpha} - w'N'_D + (1-\delta)K'}{1+r},$$

where we can back out $\{Y, Y', \pi, \pi', I\}$.

- ③ Taxes and deficit satisfy $T + \frac{T'}{1+r} = G + \frac{G'}{1+r}$ and $G - T = B$.

- ④ All markets clear: (i) labor, $N_S = N_D$ & $N'_S = N'_D$; (ii) goods, $Y = C + G$ & $Y' = C' + G'$; (iii) bonds at date 0, $S = B$.

Step 0: Result Implied by Assumptions

- ① $N'_S = 1$, since consumer don't value leisure at date 1.
 - If consumer don't value leisure, then choose the highest possible N'_S can expand the budget set without decreasing the utility.
- ② $N'_D = N'_S = 1$, by future labor market clearing.
- ③ The future wage w' is determined by MPN' :

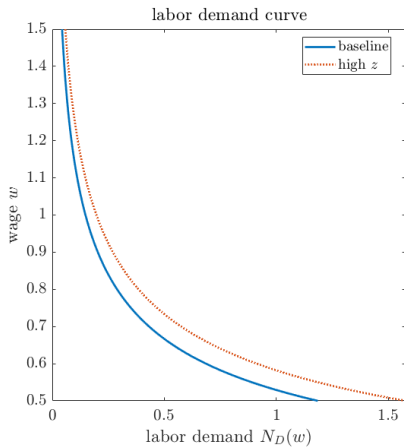
$$MPN' = z(1 - \alpha) \left(\frac{K'}{N'_D} \right)^\alpha,$$

where $N'_D = 1$ leads to

$$w' = z(1 - \alpha)(K')^\alpha.$$

Step 1: Firm's Current Labor Demand

For date 0 labor demand,



$$MPN = z(1 - \alpha) \left(\frac{K}{N_D} \right)^\alpha = w$$

$$\Rightarrow N_D = \left(\frac{z(1 - \alpha)}{w} \right)^{\frac{1}{\alpha}} K$$

- $N_D \downarrow$ in current wage w
- $N_D \uparrow$ in current TFP z (dotted line)
- N_D invariant to interest rate

Step 2: Consumer & Current Labor Supply

- labor supply at date 0:

$$\begin{aligned} MRS_{l,C} &= -MRS_{N,C} = -\frac{D_N \tilde{U}(\cdot)}{D_C \tilde{U}(\cdot)} \\ &= -\frac{-\gamma/(1-N_S)}{1/C} = \frac{\gamma C}{1-N_S} = w \end{aligned}$$

- Saving at date 0:

$$MRS_{C,C'} = \frac{1/C}{\beta/C'} = \frac{C'}{\beta C} = 1+r \Rightarrow C' = \beta(1+r)C$$

- Recall $N'_S = 1$, we can denote the x notation to be the part of the income that is NOT directly affected by consumer choice:

$$x = \pi - T \quad \text{and} \quad x' = w' + \pi' - T'$$

Step 2: Consumer & Current Labor Supply (Cont.)

Recall consumer budget constraint,

$$C + \frac{C'}{1+r} = wN_S + \pi - T + \frac{w'N'_S + \pi' - T'}{1+r}$$

$$C + \frac{\beta(1+r)C}{1+r} = wN_S + x + \frac{x'}{1+r}$$

$$C = \frac{1}{1+\beta} \left(wN_S + x + \frac{x'}{1+r} \right)$$

plug back to labor supply condition:

$$w(1 - N_S) = \gamma C$$

$$w(1 - N_S) = \frac{\gamma}{1+\beta} \left(wN_S + x + \frac{x'}{1+r} \right)$$

$$wN_S \left(\frac{\gamma}{1+\beta} + 1 \right) = w - \frac{\gamma}{1+\beta} \left(x + \frac{x'}{1+r} \right)$$

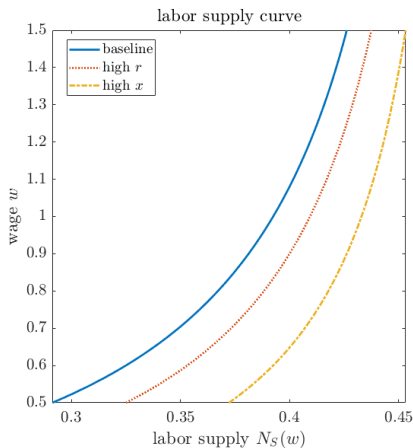
$$N_S = \frac{1+\beta}{1+\beta+\gamma} - \frac{1}{w} \frac{\gamma}{1+\beta+\gamma} \left(x + \frac{x'}{1+r} \right)$$

Check: Labor Supply Assumptions

yellow dotted line is supposed to label as "low x "

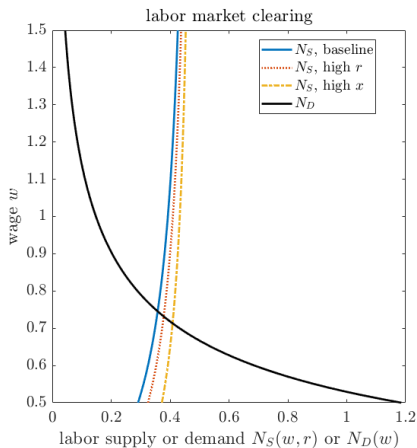
Recall **N1-N3** assumptions,

- **N1**: labor supply \uparrow in wage, $dN_S/dw > 0$ (all lines)
- **N2**: labor supply \uparrow in real interest rate, $dN_S/dr > 0$ (red v.s. blue)
- **N3**: labor supply \downarrow in lifetime wealth, $dN_S/d(x + x') < 0$ (yellow v.s. blue)



Check: Labor Market Clearing

yellow dotted line is supposed to label as
"low x "



higher interest rate (**N2**), lower
lifetime wealth (**N3**) both shifts out
labor supply curve:

- wage $w^*(r)$ decreases
- equilibrium quantity of labor
 $N^*(r)$ increases

Next: construct output supply curve

Step 3: Output Supply Curve

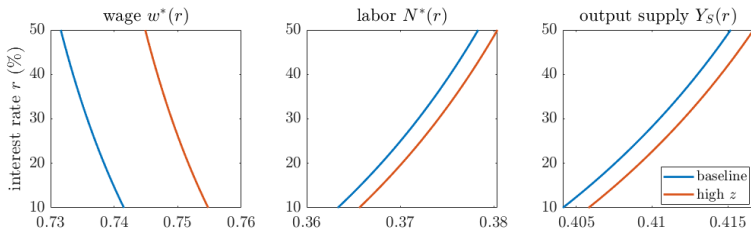
Labor market clearing requires:

$$N_S = \frac{1 + \beta}{1 + \beta + \gamma} - \frac{1}{w} \frac{\gamma}{1 + \beta + \gamma} \left(x + \frac{x'}{1 + r} \right) = \left(\frac{z(1 - \alpha)}{w} \right)^{\frac{1}{\alpha}} K = N_D.$$

...Yeah, it is very difficult to solve it by hand (actually cannot), but notice

- most of the terms are parameters: $\alpha, \beta, \gamma, z, K$,
- or lifetime wealth that needs gov: x and x' .
- Our main goal is to **solve for $w^*(r)$** !
 - solve real wage w as a function of real interest rate r
 - then, back out $N^*(r)$ and $Y_S(r)$
 - get $N^*(r)$ by plug $w^*(r)$ into either N_D or N_S
 - get $Y_S(r)$ by plug $N^*(r)$ into $zK^\alpha(N^*)^{1-\alpha}$

Check: Output Supply Curve



Confirm our intuition:

- $r \uparrow$ leads to $w \downarrow$ and $N^*(r) \uparrow$
- given positive MPN and fixed K , more labor means more production, so output supply shifts up.

Step 4: Output Demand Curve

Recall that the date 0 output demand curve are composite of

- government spending G and G' : exogenous (easy!)
- firm's investment demand $I_D(r)$ (next slide)
- consumer's consumption demand $C_D(r, Y)$:
 - recall **income-expenditure identity**, total income = total demand,

$$C + \frac{C'}{1+r} = wN + \pi - T + \frac{w'N' + \pi' - T'}{1+r}$$

$$\because \pi = Y - wN - I; \pi' = Y' - w'N' + (1-\delta)K'$$

$$(1+\beta)C = Y + \frac{Y'}{1+r} - I + \frac{(1-\delta)K'}{1+r} - \left(T + \frac{T'}{1+r}\right)$$

- given r , we can solve consumption-saving problem.

Firm's Optimal Investment

Recall

- labor market clearing at date 1: $N'_D = N'_S = N' = 1$, and
- MPK at date 1: $MPK' = z'\alpha(K')^{\alpha-1}$.

Thus, according to optimal investment schedule,

$$MPK' - \delta = r$$

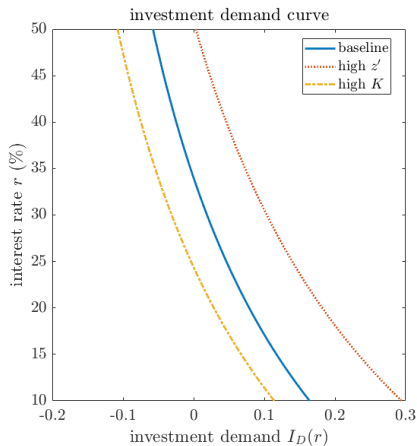
$$z'\alpha(K')^{\alpha-1} = r + \delta$$

$$K' = \left(\frac{z'\alpha}{r + \delta} \right)^{\frac{1}{1-\alpha}}$$

and we can also determine investment by capital accumulation process:

$$I_D = K' - (1 - \delta)K = \left(\frac{z'\alpha}{r + \delta} \right)^{\frac{1}{1-\alpha}} - (1 - \delta)K$$

Check: Investment Demand Assumption

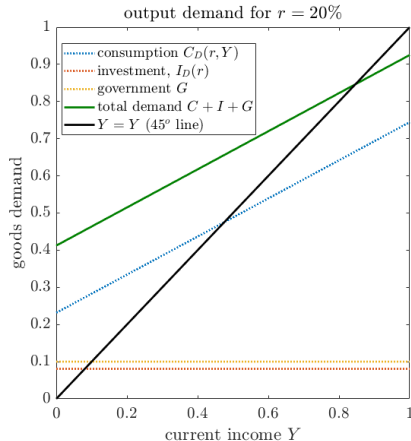


$$I_D = \left(\frac{z' \alpha}{r + \delta} \right)^{\frac{1}{1-\alpha}} - (1 - \delta)K$$

Recall assumptions from Lecture 15:

- $I_D(r) \downarrow$ in r (✓)
- $I_D(r)$ shifts in when $K \uparrow$:
yellow v.s. blue
- $I_D(r)$ shifts out when $z' \uparrow$: red
v.s. blue

Constructing the Output Demand Curve



Aggregate all three components:

- investment (red) and government (yellow) are horizontal
- consumption (blue) increase in income with slope $\approx \frac{1}{1+\beta}$
- total output demand (green) gain the slope from consumption, and is the sum of all three

Constructing the Output Demand Curve (Cont.)

$$r \uparrow \Rightarrow I_D(r) \downarrow \Rightarrow \text{total demand} \downarrow$$

