Lecture 16
The Real Business Cycle Model
Part 3: Competitive Equilibrium

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May 8, 2022

Overview

- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

Review: Consumer's Problem

Taken $\{w, w', r, T, T', \pi, \pi'\}$ as given, a representative consumer chooses $\{C', N_S, N'_S\}$ to solve

$$\max_{C', N_S, N'_S} \quad u\left(wN_S + \pi - T + \frac{w'N'_S + \pi' - T' - C'}{1+r}\right), \\ - v(N_S) + u(C') - v(N'_S) \tag{1}$$

which we can back out C, S, l, l'.

Review: Firm's Problem

Taken $\{w, w', r\}$ as given, a representative firm chooses $\{N_D, N'_D, K'\}$ to solve

$$\max_{N_D, N'_D, K'} \quad zF(K, N_D) - wN_D - [K' - (1-\delta)K] + \frac{z'F(K', N'_D) - w'N'_D + (1-\delta)K'}{1+r}, \tag{2}$$

which we can back out Y, Y', π, π', I

Government Budget Constraint

Government behaves exactly the same in two-period model:

- current budget constraint: $G = T + B$
- future budget constraint: $G' + (1 + r)B = T'$
- lifetime budget constraint: $G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$

Taken $\{r\}$ as given, government satisfy lifetime budget constraint by choosing $\{T, T', B\}$.

Market Clear

There are three markets to clear:

1. labor markets clear at each date determines wage:
   - find w such that $N_S = N_D$
   - find w' such that $N'_S = N'_D$
2. goods markets clear at each date determines consumption and investment:
   - date 0 (today): $Y = C + I + G$
   - date 1 (tomorrow): $Y' = C' + I' + G'$
3. bonds market clears at date 0 determines real interest rate:
   - find r such that $S = B$

Competitive Equilibrium: RBC Model

Given exogenous quantities $\{G, G', z, z', K\}$, a **competitive equilibrium** is a set of

1. consumer choices $\{C, C', N_S, N'_S, l, l', S\}$,
2. firm choices $\{Y, Y', \pi, \pi', N_D, N'_D, I, K'\}$,
3. government choices $\{T, T', B\}$, and
4. prices $\{w, w', r\}$

such that

1. consumer solves problems in (1),
2. firm solves problems in (2),
3. government balances its budget, and
4. all three markets clear.

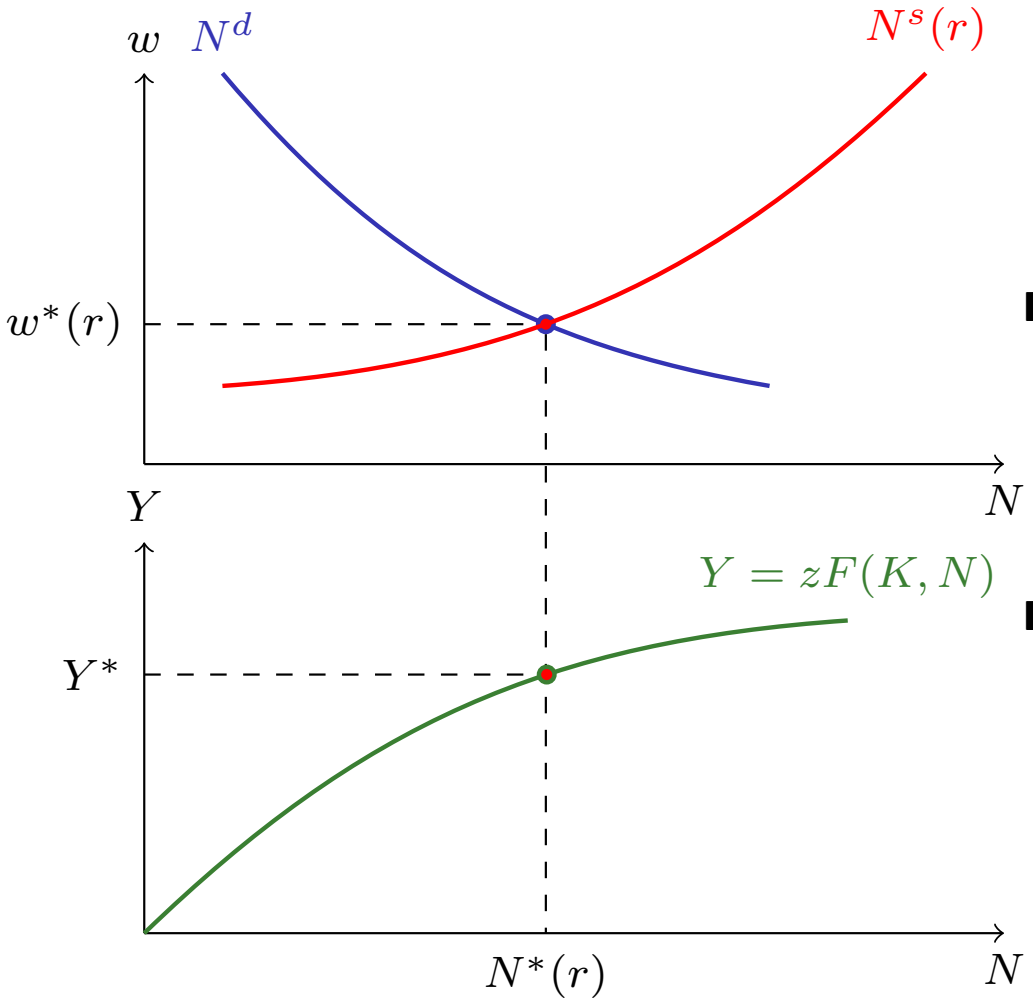
Plan to analyze the Model

In the following slides, we are going to use graphical analysis on two markets in current period:

1. current labor market: interaction of firm and consumer today
   - similar to static model: labor supply and labor demand curves
   - what's new: both curves reflect the dynamic tradeoff through interest rate
2. current goods market: interaction of firm, consumer and government today
   - new: construct and analyze output supply and output demand curves

The Current Labor Market

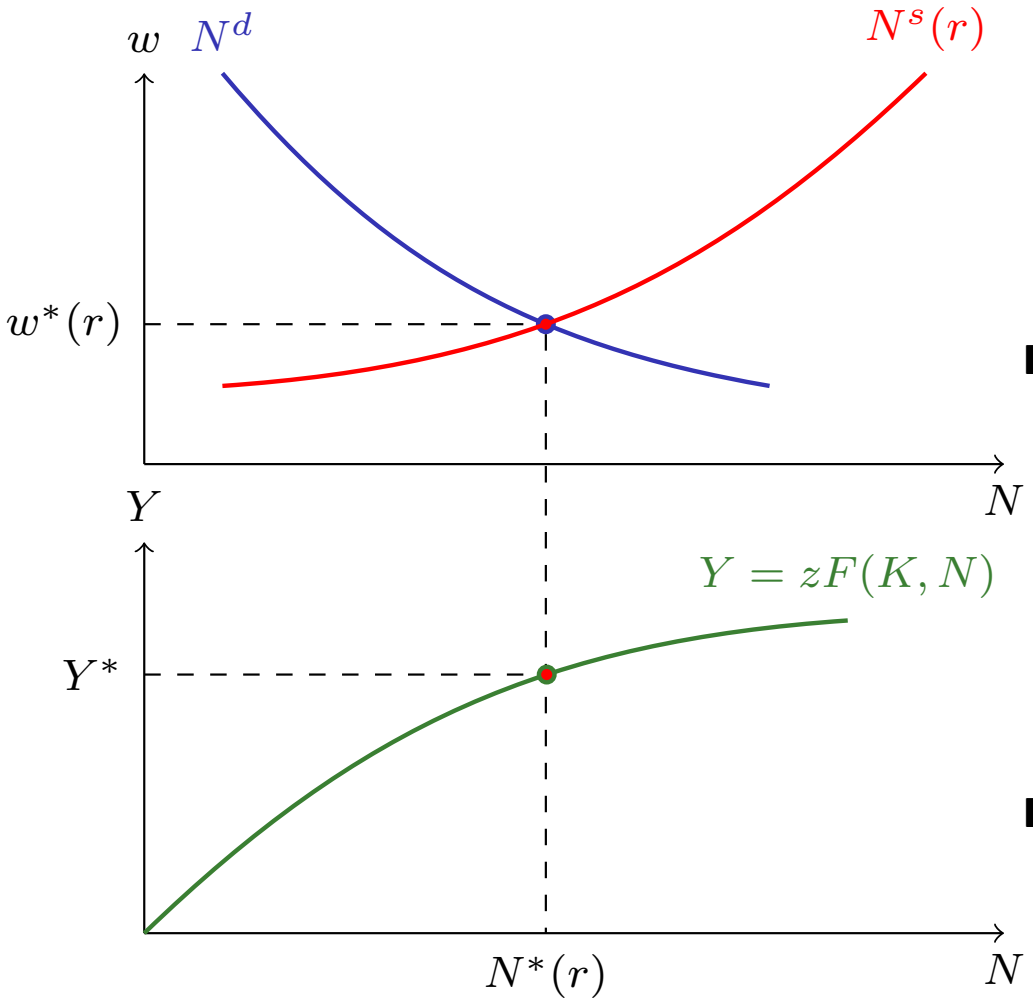
Figure 11.14 Determination of Equilibrium in the Labor Market Given the Real Interest Rate r



- **consumer optimality**: ceteris paribus, $N^s \uparrow$ in w
  - **N1**: $\frac{dN^S}{dw} > 0$, substitution dominates income effect
- **firm optimality**: $N^d \downarrow$ in w
  - $MPN = w$, $\because$ diminishing MPN
- account for multiple markets: intersect at $N^*(r)$
  - note: top figure is given r, π
  - labor mkt clearing w is $w^*(r)$

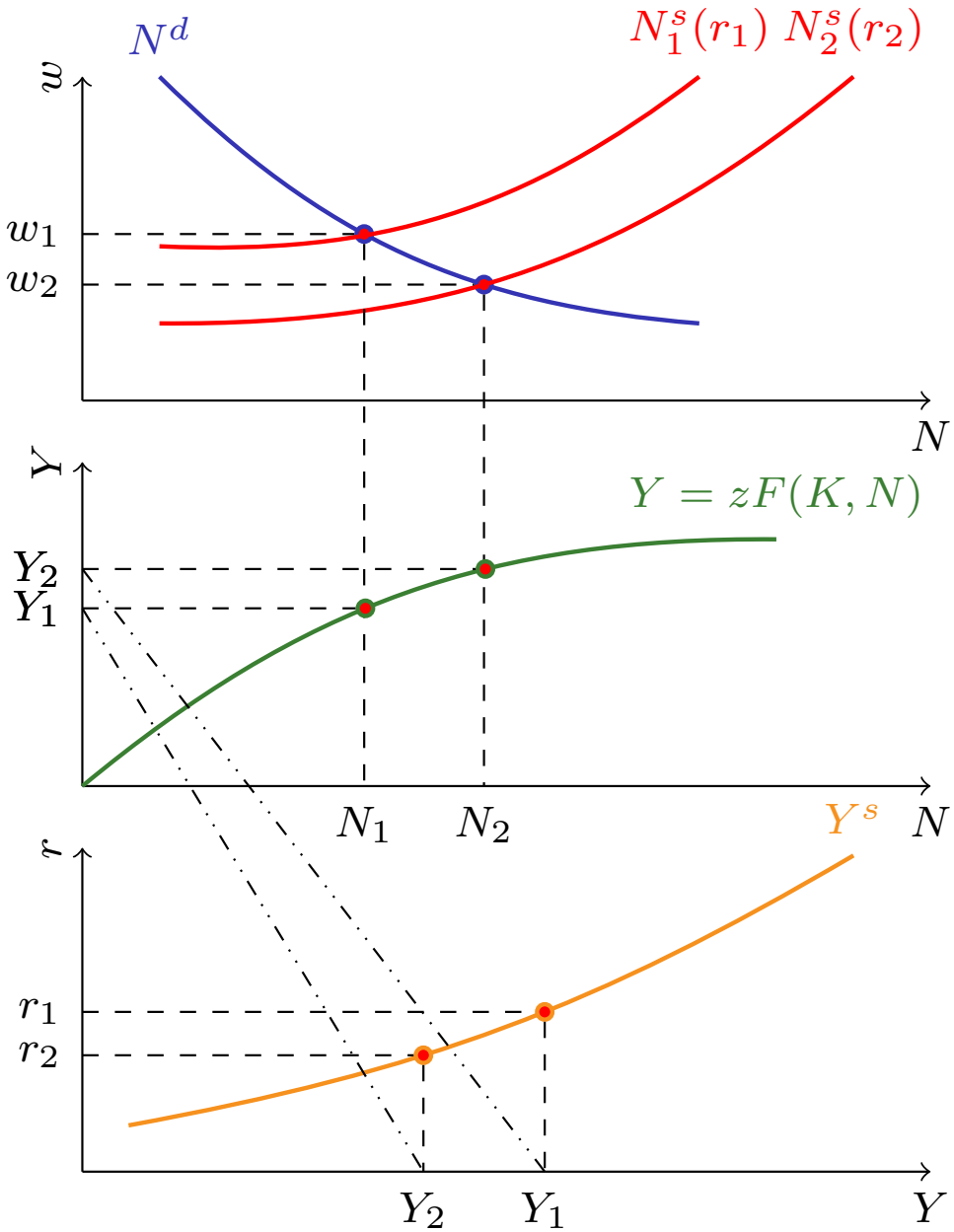
The Current Labor Market (Cont.)

Figure 11.14 Determination of Equilibrium in the Labor Market Given the Real Interest Rate r



- r increases?
  - **N2** (consumer): $N^s(r) \uparrow$ in r $\Rightarrow w \downarrow$, $N^*(r) \uparrow$
  - **firm**: $\because MPN = w$, same
- consumer wealth increases?
  - **N3** (consumer): $N^s(r) \downarrow \Rightarrow w \uparrow$, $N^*(r) \downarrow$
  - **firm**: nothing
- Bottom chart: $N^*(r) \to Y^*(r)$, output supply!

The Output Supply Curve



Using our insight from labor market, we can repeat out analysis for any real interest rate r

- Top: each r implies a different "labor market equilibrium", i.e., w & N
- Middle: each $N(r)$ yields production $Y^S(r)$
- Buttom: combined to show $\frac{dY^S}{dr} > 0$

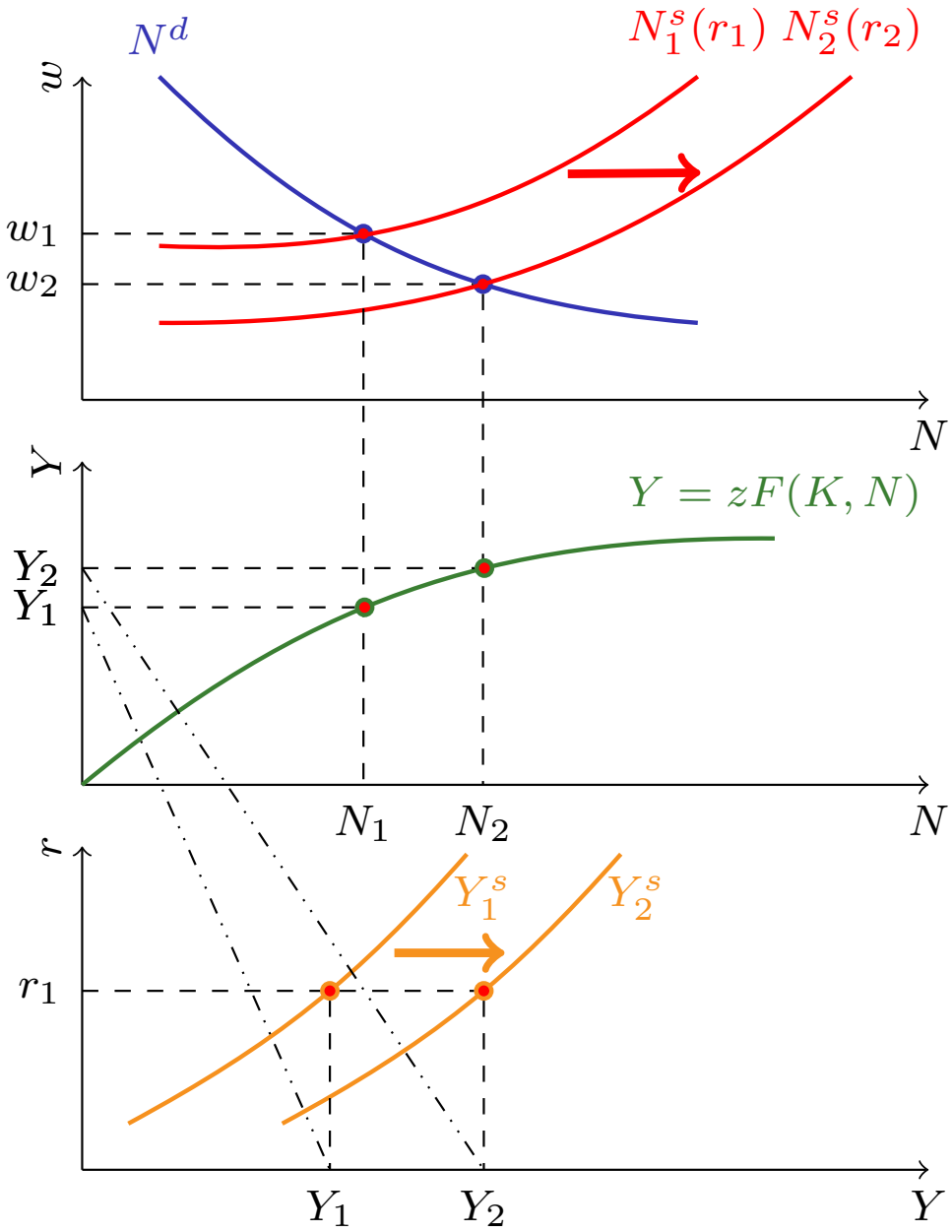
Shifts in the Output Supply Curve

How do changes in exogenous variables shift $Y^S(r)$? Consider 2 cases:

1. shift in lifetime wealth (for example, by gov't spending or taxation)
2. change in total factor productivity (TFP) or capital stock
   - recall static model: with K fixed, these have the same effect

In each case, we can start our analysis with the current labor market.

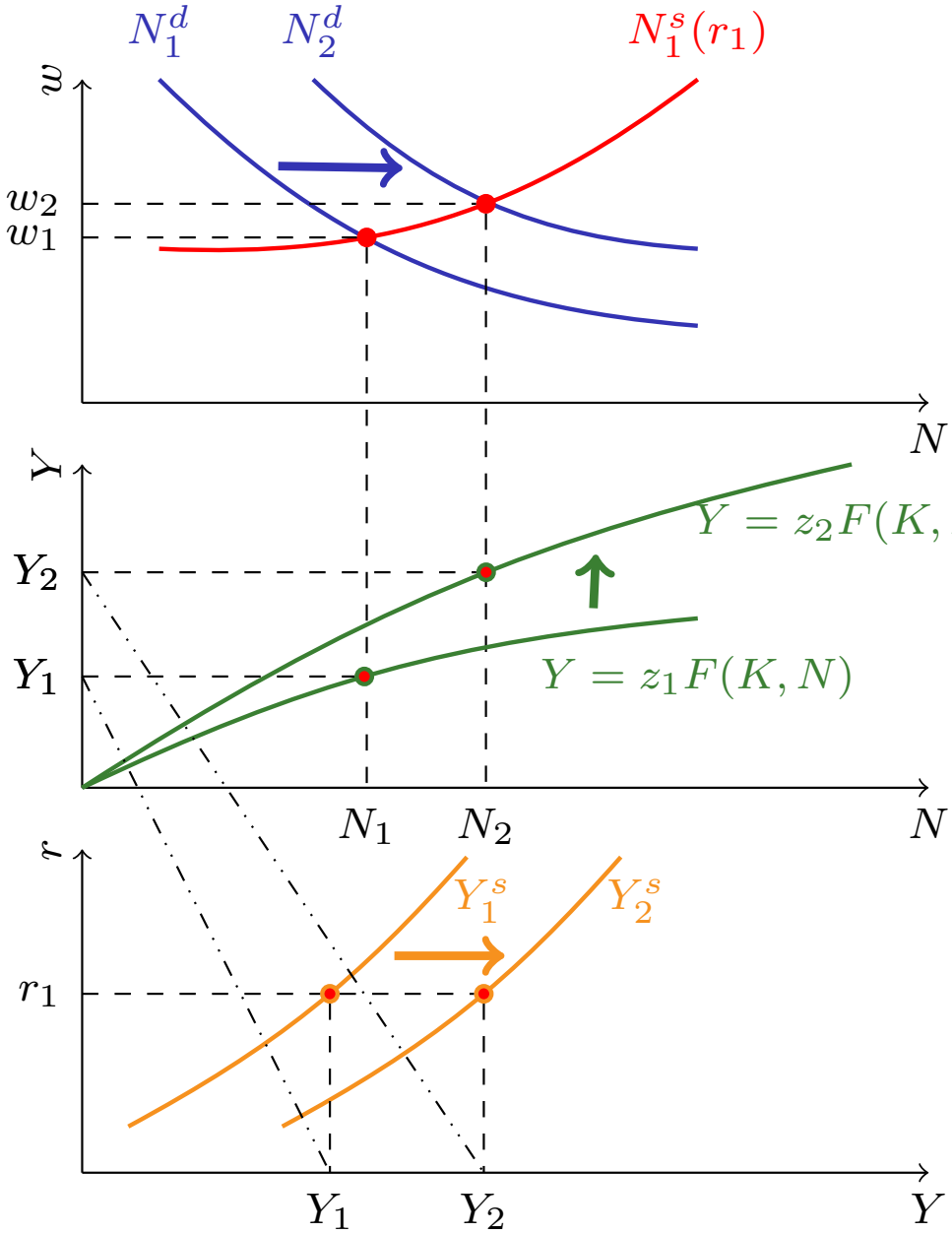
Wealth and Output Supply



Suppose $G \uparrow$ or $G' \uparrow$.

- gov. budget: $T \uparrow$ or $T' \uparrow$
- consumer budget: $we \downarrow$
- **N3**: $dN^S/d(we) < 0$, $N^S(r) \uparrow$ (shift to the right, top panel)
- Middle: $N \uparrow \Rightarrow Y^S \uparrow$
- bottom: combine, get rightward shift in output supply

TFP / Capital and Output Supply



Suppose TFP $z \uparrow$.

- **firm optimality**:
  $MPN = zD_NF(\cdot) \uparrow \Rightarrow N^d \uparrow$
- Top: N^d shifts out $\Rightarrow$
  $w^* \uparrow, N^* \uparrow$
- Middle: production fcn shifts up, $\because z \uparrow$
- Bottom: combine, outward shift in output supply

Summary: Current Labor Market

We have constructed most of the model!

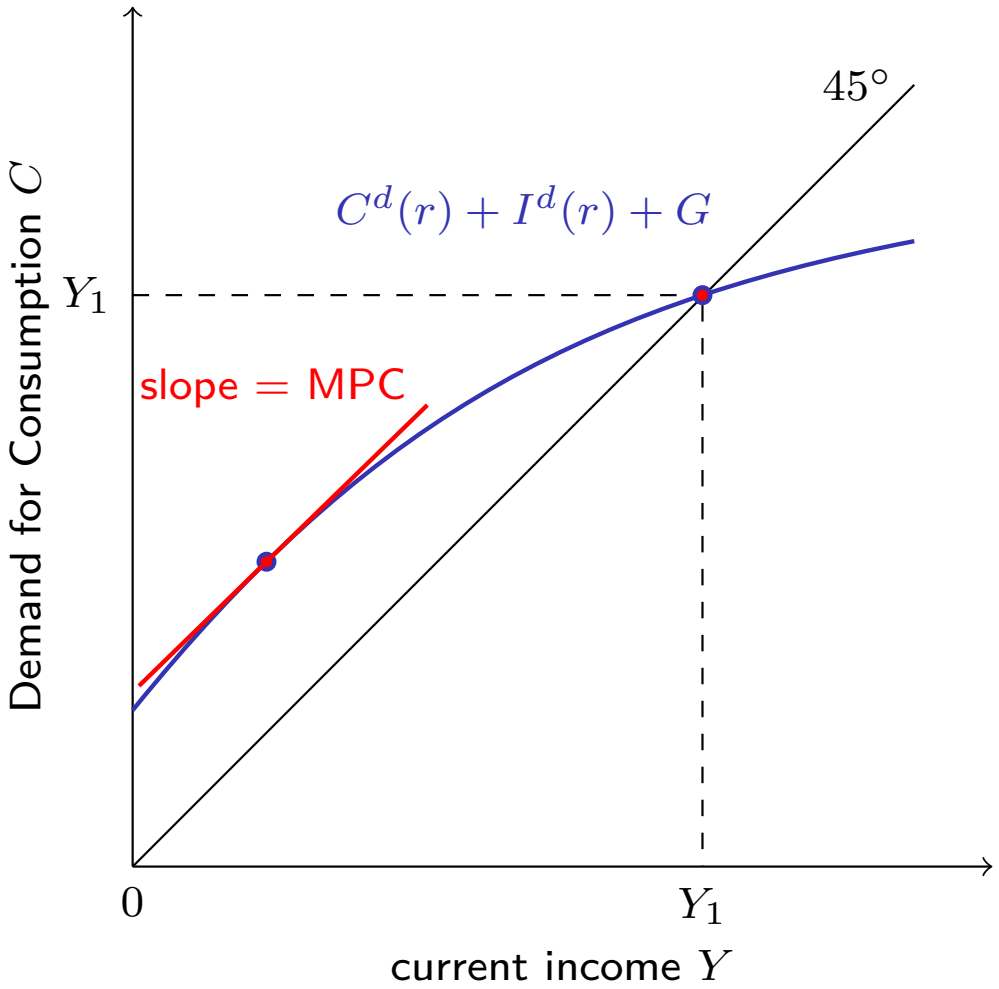
- labor market clearing, *conditional on* the interest rate
- trace through production function to get output supply curve

Now we need to determine the equilibrium interest rate, r^*.

- pair the output supply curve with the **output demand curve**
- who demands goods today, and how much?
  - consumer: consumption $C^d(r, Y)$
  - firm: investment $I^d(r)$
  - government: expenditures G
  - use GDP accounting to get aggregate demand for goods

Current Goods Demand

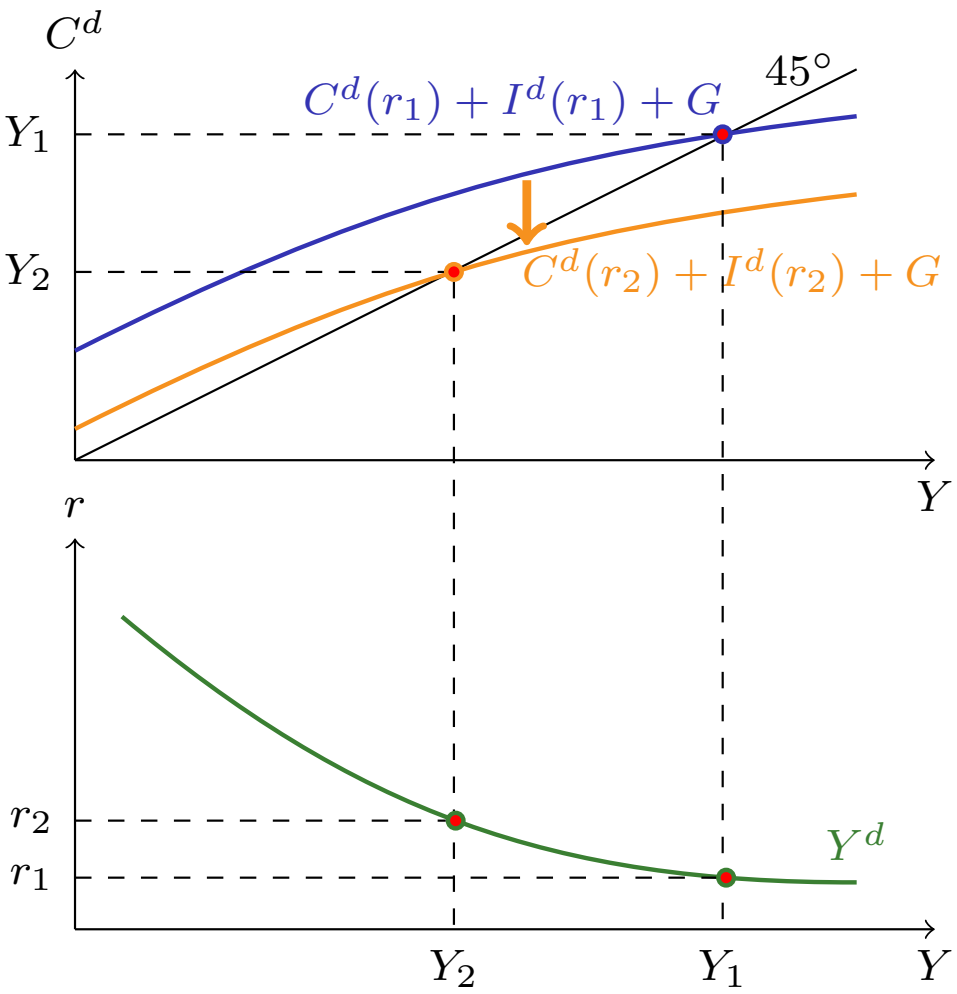
Figure 11.18 The Demand for Current Goods



$$D(r, Y) = C^d(r, Y) + I^d(r) + G$$

- plot $D(r, Y)$ on y-axis, Y on x-axis
- C^d depends on wealth: $we = wN + \pi - T + \dfrac{w'N' + \pi' - T'}{1 + r}$, which depends on income.
- Not true for I and G
  - $MPC < 1$: flatter than $45°$ line
  - MPC diminishing: concave
  - combine: cross $45°$ line at $Y^d(r)$

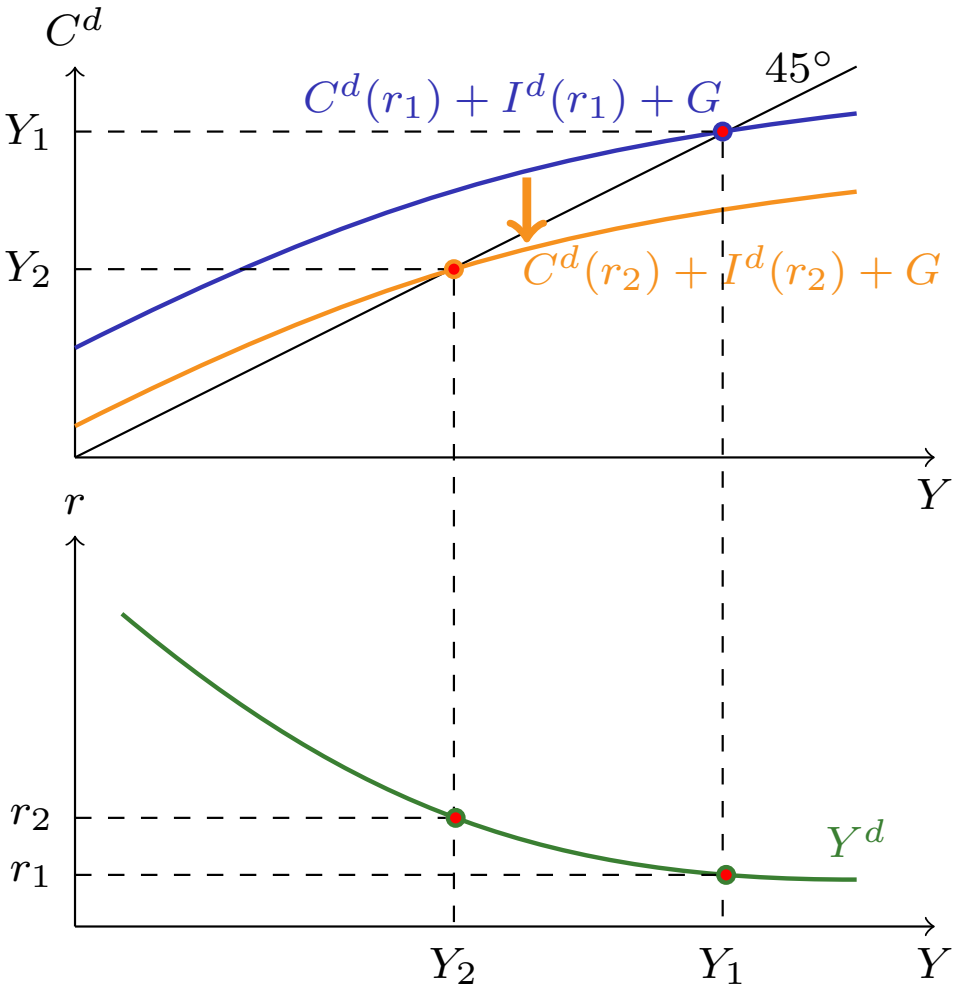
Constructing Output Demand



How different r affect output demand?

- **C2** (consumer): $C^d(r) \downarrow$ if $r \uparrow$(substitution effect dominates)
- **firm**: optimal investment schedule ($r = MPK' - \delta$), $r \uparrow \Rightarrow MPK' \uparrow \Rightarrow I^d \downarrow$
- **gov**: no change, $\because G$ exogenous
- Combine:
  1. intersection with $45°$ line is lower

Constructing Output Demand (Cont.)



Combine:

1. intersection with $45°$ ↓
2. output demand curve Y^d
   downward sloping

$Y^d(r)$ shift to the right if

1. present value of taxes ↓ $\Rightarrow$ C^d ↑
2. future income ↑ $\Rightarrow$ C^d ↑
3. future TFP ↑ $\Rightarrow$ I^d ↑
4. current capital ↓ $\Rightarrow$ I^d ↑

Other changes (e.g., current TFP) are ambiguous in general!

Competitive Equilibrium

Figure 11.21 The Complete Real Intertemporal Model