Midterm Review

Disclaimer: The questions in the midterm review may be similar but not necessary the same as what will appear in the midterm exam. Use the materials here with caution.

Similar to Lecture 08, slide 11 and 12 and Experiment 2 from Lecture 07, slide 13.

Two difference:

- firm rent capital from consumer, and consumer are endowed with 2 unit of capital ($K^s = 2$)
- consumer's utility function is $U(C, l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$

1 GE with capital endowment

The competitive equilibrium given a set of exogenous variables $\{G, z, K^s\}$, is a set of allocations $\{Y^*, C^*, l^*, N^{s*}, N^{d*}, \pi^*, T^*, K^{d*}\}$ and prices $\{w^*, r^*\}$ such that

1. Taken prices and _ _ as given, consumers solves

$$\max_{C,l} \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d} \tag{1}$$

subject to

$$C \leqslant w(h-l) + rK^s + \pi - T \tag{2}$$

2. Taken prices and _ _ as given, firm solves

$$\max_{K^d, N^d} z\,(K^d)^a\,(N^d)^{1-a} - wN^d - rK^d \tag{3}$$

3. Government budget balance,

$$T^* = G \tag{4}$$

4. The equilibrium wage w^* will clear the labor market:

$$N^s = N^d \tag{5}$$

5. The equilibrium rent r^* will clear the capital market clear:

$$K^s = K^d \tag{6}$$

Questions are

1. $w = \text{MPN} = (1-a)\,z\,(K^d)^a(N^d)^{-a}$

   a. $\max_{K^d, N^d} z\,(K^d)^a\,(N^d)^{1-a} - wN^d - rK^d$

   FOC$=\frac{\partial(z\,(K^d)^a\,(N^d)^{1-a} - wN^d)}{\partial N^d} = 0 \Rightarrow (1-a)\,z\,(K^d)^a(N^d)^{-a} - w = 0 \Rightarrow (1-a)\,z\,(K^d)^a(N^d)^{-a} = w$

2. $r = \text{MPK} = z(N^d)^{1-a}\, a\, (K^d)^{a-1}$

   $\frac{\partial z\, (K^d)^a\, (N^d)^{1-a}}{\partial K^d} = a\, z(N^d)^{1-a}(K^d)^{a-1}$

3. Social planner's problem is

$$\begin{aligned} & \max_{C,l} && \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d} \\ & s.t. && C = Y - G \\ & && Y = zK^a N^{1-a} \\ & && N = 1 - l \\ & && K = 2 \end{aligned}$$

$$\begin{aligned} & \max_{l} && \frac{(zK^a (1-l)^{1-a} - G)^{1-b}}{1-b} + \frac{l^{1-d}}{1-d} \\ & s.t. && K = 2 \end{aligned}$$

   $\frac{(zK^a\, (1-l)^{1-a} - G)^{1-b}}{1-b} \Rightarrow (1-b)\, \frac{(zK^a\, (1-l)^{1-a} - G)^{1-b-1}}{1-b} \Rightarrow (zK^a\, (1-l)^{1-a} - G)^{-b}$

   $zK^a\, (1-l)^{1-a} \Rightarrow (1-a) zK^a\, (1-l)^{1-a-1} \Rightarrow z(K^a)\, ((1-l)^{-a})$

   $(1-l) \Rightarrow -1$

$$l^{-d} - z(K^a)\, ((1-l)^{-a})((z\, (K^a)\, (1-l)^{1-a} - G))^{-b}\, (1-a) = 0 \tag{7}$$

4. Solve for l get

$$l^{-d} = z(K^a)\, ((1-l)^{-a})((z\, (K^a)\, (1-l)^{1-a} - G))^{-b}\, (1-a) \tag{8}$$

   $l^{-d} = zK^a(1-l)^{-a}\, z^{-b}\, K^{-ab}\, (1-l)^{-b(1-a)}(1-a)$

   $\frac{l^{-d}}{(1-l)^{-a-b(1-a)}} = (1-a)\, z^{1-b}\, K^{a-ab}$

5. $z = 1, G = 0, a = \frac{1}{2}, b = 2, d = \frac{3}{2},$ what is l, N, w, r?

2 Labor tax

Similar to Lecture 11 but with two difference:

1. Cobb-Douglas production function: $Y = zN^a$

2. Hansen (1985) utility function: $U(C, N) = \ln C - bN$

So we can start to solve this model by

1. $D_C U(C, N) = \frac{1}{C}$

2. $D_N U(C, N) = -b$

3. $\text{MRS}_{N,C} = \frac{D_N U}{D_C U} = -\frac{b}{1/C} = -bC$

   $N = 1 - l \Rightarrow \text{MRS}_{l,C} = -\text{MRS}_{N,C} = bC$

4. $\text{MRS}_{N,C} = \text{After-tax wage rate} = w(1-t)$

5. $w = \text{MPN} = azN^{a-1} \Rightarrow wN = azN^a$

6. $\pi = Y - wN = zN^a - azN^a = (1-a)\,zN^a$

7. $\text{MRS}_{l,C} = -\text{MRS}_{N,C} = bC = w(1-t) = \text{After-tax wage}$

   $C = w(1-t)\,N + \pi$

$$\begin{aligned}
bC &= w(1-t)\\
b[w(1-t)\,N + \pi] &= w(1-t)\\
b[azN^a(1-t) + (1-a)\,zN^a] &= azN^{a-1}(1-t)\\
zN^a\,b[a(1-t) + (1-a)\,] &= azN^{a-1}(1-t)\\
Nb[a(1-t) + (1-a)\,] &= a(1-t)\\
N &= \frac{a(1-t)}{b[a(1-t) + (1-a)\,]}
\end{aligned}$$

8. $w(t) = azN^{a-1} = az\left(\frac{a(1-t)}{b[a(1-t)+(1-a)\,]}\right)^{a-1}$

9. $G = w(t)\,tN(t) = az\left(\frac{a(1-t)}{b[a(1-t)+(1-a)\,]}\right)^{a-1} t\,\frac{a(1-t)}{b[a(1-t)+(1-a)\,]}$

10. if $t = 0.5, a = 0.33, b = 2.15,$ $G = wtN = 0.0751$

11. Another t that generates the same G is

12. Want to maximize G, the optimal $t =$

13. optimal tax revenue $G =$