Lecture 15
The Real Business Cycle Model
Part 2: Firm

Hui-Jun Chen

The Ohio State University

July 7, 2022

Overview

- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

Demand for Consumption Goods

Ultimately, 3 markets will have to clear in the current period (date 0):

1. labor (like static model)
2. credit (like dynamic model)
3. consumption goods (implied in each case by Walras' Law)

3 mkts ⇒ only need to clear 2

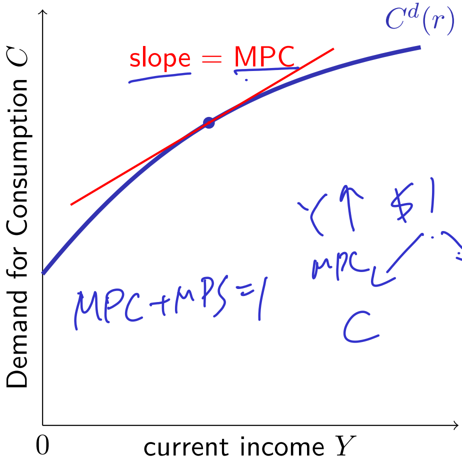
Recall our insights from last classes. Primary determinants of consumption:

- over lifetime: permanent income / lifetime wealth
- across periods: interest rate, current vs future income

Based on this, we'll construct a demand curve for current consumption goods that depends on lifetime wealth and the interest rate

Current Goods Demand and Current Income

Figure 11.4 Consumer's Current Demand for Consumption Goods Increases with Income



MPC+MPS=1

Y ↑ $1

MPC

MPS

C

S

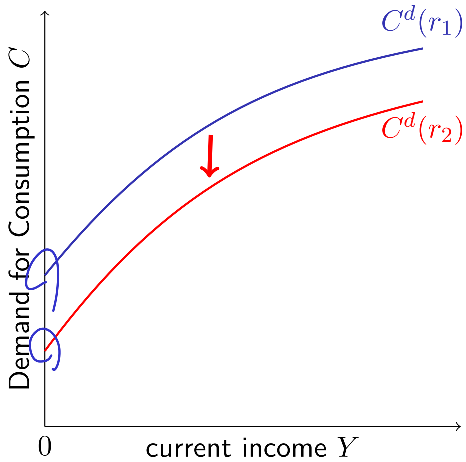
Assumption C1: demands for goods ↑ in income

- Recall pure income effect
- Slope of tangent line is **marginal propensity to consume** (MPC)
  - what fraction of $Y \uparrow$ goes to C?

    $Y: \$1 \uparrow \rightarrow C: \$MPC \uparrow$
  - $MPC = dC_D/dY$
- normal goods: both C and $C' \uparrow$, so saving $S \uparrow$
  - usually $MPC < 1$, i.e., not all $Y \uparrow$ goes to C.

Current Goods Demand and Real Interest Rate

Figure 11.5 Real Interest Rate ↑ Shifts the Demand for Consumption Goods Down



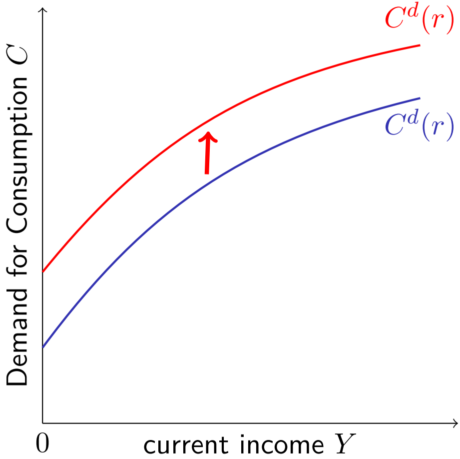
Assumption C2: demands for goods ↓ in real interest rate

- Recall both income and substitution effect (from dynamic model)
- Income effect: ambiguous (for borrowers and lenders)
- Substitution effect: always negative (for borrowers and lenders)
- **C2** assumes substitution effect dominates

Current Goods Demand and Lifetime Wealth

$$\uparrow C+\frac{C'}{1+r} = \overset{\text{lifetime wealth}}{\boxed{y+\frac{y'}{1+r}}}\uparrow$$

Figure 11.6 An Increase in Lifetime Wealth Shifts the Demand for Consumption Goods Up



Assumption C3: demands for goods ↑ in lifetime wealth

- similar to pure income effect

Note: consumer's demand is only one part of the GDP:

$$Y = C + I + G.$$

We'll discuss I and G in next lecture

$K_2 = 4$

$(1-\delta) K_2 = 4-1$

$\boxed{\delta K = 1}$

$(1-\delta) K = 2$

$\boxed{I = 2}$

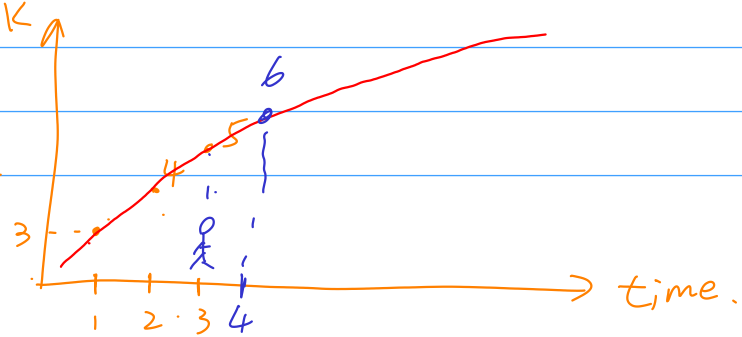
$K_2 = 2+2$

$K_3 = (\underline{\underline{4}} - 1) + 2$

$\overset{4}{K}_4 = (\overset{3}{5} - 1) + 2$

Concept: "stock" v.s. "flow"

stock: accumulate, take across time.



K : stock.

I : flow variable "<u>decision</u>"

n $\Downarrow$

N decision made period by period.

C

Overview: Firm Decision

- **production**: needs both capital K and labor N, $Y = zF(K, N)$
- **endowment**: firm is endowed with initial capital K ← given exogenus.
- **firm decision**:
  - both dates: labor (N), profit (π), and output (Y) by production

$$Y = zF(K, N) \text{ and } Y' = z'F(K', N')$$

  - date 0 (today): **investment** (I) determines future capital K' given initial capital K and depreciation rate $\delta \in [0, 1]$,

$$\underbrace{K'}_{\text{tmw's capital stock.}} = \underbrace{(1-\delta)K}_{\text{undepreciated capital}} + \underbrace{I}_{\text{investment.}} \rightarrow Y$$

- **Assumptions**:
  1. investment made in consumption goods
  2. remaining capital $(1-\delta)K'$ liquidates tomorrow ($\because$ model ends)

$$\underline{\underline{Y}} = C + \underline{\underline{I}} + G$$

Firm's Optimization Problem

Firm maximizes the discounted present value of profits:

capital law of motion

$$\max_{N_D,N_D',K',I} \quad V = \boxed{\pi} + \frac{\boxed{\pi'}}{1+r} \quad \text{subject to} \quad K' = (1-\delta)K + I,$$

investment is fully reversible

where $\boxed{\pi} = Y - wN - I$, and $\boxed{\pi'} = Y' - w'N' + \boxed{\underbrace{(1-\delta)K'}_{\text{liquidate}}}$.

Notice: since we assume that consumer owns the firm, so firm calculates present value using real interest rate r, i.e., how consumer discounts.

By substituting π, π', Y, Y' and I into above problem, we get

$$\begin{aligned}\max_{N_D,N_D',K'} \quad & \boxed{zF(K,N_D) - wN_D - \overbrace{[K'-(1-\delta)K]}^{I}} \quad \pi \\ & + \frac{\boxed{z'F(K',N_D') - w'N_D' + (1-\delta)K'}}{1+r} \quad \pi' \qquad (1)\end{aligned}$$

Firm's Optimality Conditions

$$\begin{cases}[N_D]: & zD_NF(K,N_D)=w\\ [N'_D]: & z'D_NF(K',N'_D)=w'\end{cases}$$

$$[K']: \quad \underbrace{-1}_{\text{cost}} + \underbrace{\frac{\boxed{z'D_{K'}F(K',N'_D)+(1-\delta)}}{1+r}}_{\text{benefit}} = 0$$

- FOCs on current and future labor are the same as static model!
  - Why? Since labor choice is static: choose labor for current production
- FOC on future capital equalize the marginal cost and benefit of investment
  - cost: loss in current consumption (incurred today)
  - benefit: ↑ in marginal production + liquidating K' (incurred tomorrow)

$$[K'] \quad \frac{MPK' + 1 - \delta}{1+r} = 1$$

$$MPK' + \cancel{1} - \delta = \cancel{1} + r$$

$$MPK' - \delta = r$$

Optimal Investment Schedule: Derivation

Solve for $[K']$, we get

$$z' D_{K'} F(K', N'_D) + 1 - \delta = 1 + r \Rightarrow r = MPK' - \delta$$

For consumer, there are 2 assets to undertake intertemporal substitution:

1. saving in credit market (supply in credit mkt; demand in bond mkt)
2. capital held by the firm for production

Investing in capital means giving up (net) return r for (net) return $MPK' - \delta$: optimal investment rule means both must offset, WHY?

- if $r > MPK' - \delta$: consumer will save more for bond ⇒ supply in credit market ↑, r ↓

- if $r < MPK' - \delta$: consumer asks firm to invest more capital ⇒ MPK' ↓

To sum up, $r = MPK' - \delta$ in equilibrium: "optimal" investment rule!

Labor Demand is decreasing in w and increasing in z, K

Figure 11.7 The Demand Curve for N Is the Firm's MPL Schedule

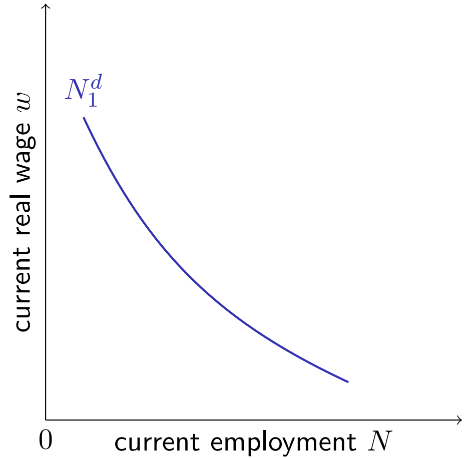
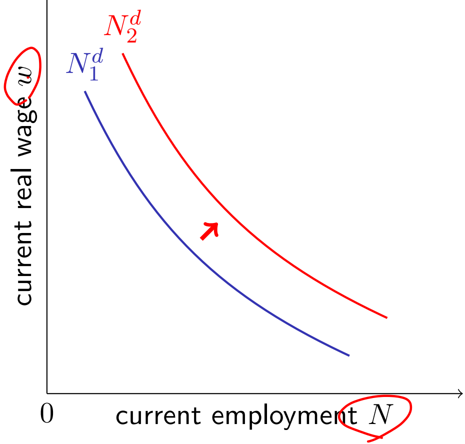


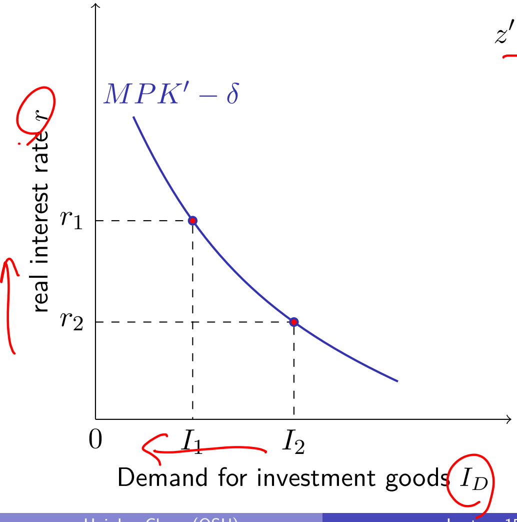
Figure 11.8 The Current Demand Curve for Labor Shifts Due to Changes in z and K

$z\uparrow F(K\uparrow, \underline{\underline{N}})\uparrow$



Optimal Investment Schedule: Graphical Representation

Figure 11.9 Optimal Investment Schedule for the Representative Firm



Put capital accumulation process into MPK and get

$$z' D_{K'} F((1-\delta)K + I_D, N'_D) = r + \delta$$

- as $r \uparrow$, need less K' for optimal investment schedule to hold.
  - why? diminishing MPK
- $K' \uparrow$ in I, so $r \uparrow$ also means less investment $\Rightarrow$ downward slope
- i.e., higher opportunity cost of investing

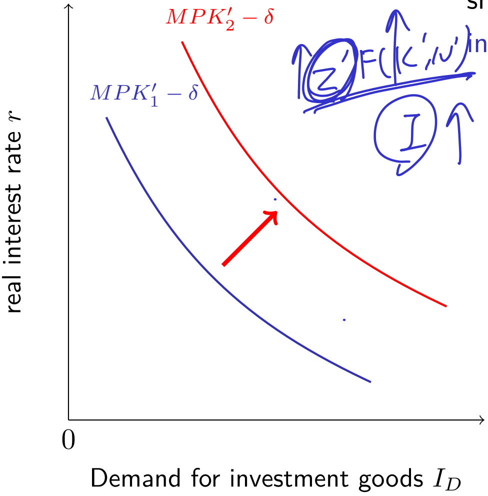
Optimal Investment Schedule: Effect of K and z'

$K' = (1-\delta)K\downarrow + I\,\uparrow$

Figure 11.10 The Optimal Investment Schedule Shifts to the Right if $K\downarrow$ or expecting $z'\uparrow$

$\uparrow z' F(K'\uparrow, N')$

$I\uparrow$



The optimal investment schedule shifts to the right, i.e., demand for investment rises if

- current capital K decreases: $\frac{dI_D}{dK} < 0$
  - Intuition: need to invest more for less endowment
- (expected) future TFP increases: $\frac{dI_D}{dz'} > 0$
  - Intuition: investment is more productive