Lecture 5
Representative Consumer Optimization and Application

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August 29, 2022

Overview: Lecture 4 - 7

Provide micro-foundation for the macro implication (Lucas critique)

- **Representative Consumer**:
  - Lecture 4: preference, constraints
  - Lecture 5: optimization, application
  - Lecture 6: Numerical Examples
- **Representative Firm**:
  - Lecture 7: production, optimization, application

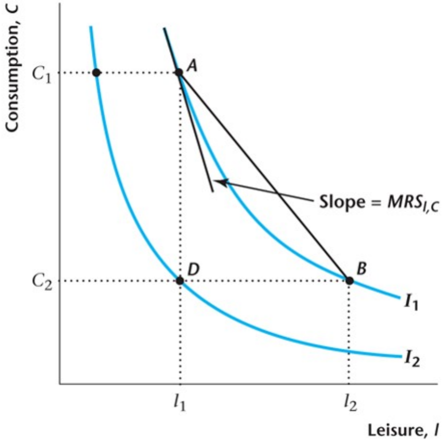
Review: MRS

- Normality: Marginal Rate of Substitution
  - **Marginal**: for arbitrary small change in x-axis (leisure in this case)
  - **rate of substitution**: the amount on y-axis has to be sacrificed (consumption in this case)

$$MRS_{l,C} = \frac{D_l U(C,l)}{D_C U(C,l)}, \qquad (1)$$

where $D_x U(\cdot)$ is derivative of U w.r.t. x

Figure 4.2 MRS



Consumer's Problem

The consumer choose consumption and leisure bundle to achieve highest indifference curve, while still satisfying budget constraint

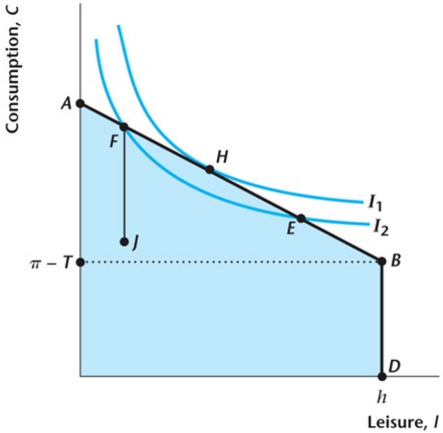
$$\begin{aligned} \max_{C,l} \quad & U(C,l) \\ \text{subject to} \quad & C \leq w(h-l)+\pi-T \end{aligned} \tag{2}$$

- **Rational behavior**: decision is made given preference & constraints
- **Analysis**: both graphically and algebraically

Graphical Analysis: Interior Solution

- **Interior**: sol. at middle of budget set, not end pts
- MRS must equal to real wage ($MRS_{l,C} = w$), WHY?
    - sacrificed consumption comes from the decrease of labor income
- Sol. at indifference curve tangent to budget set
- **Convexity**: E v.s. H & F v.s. H

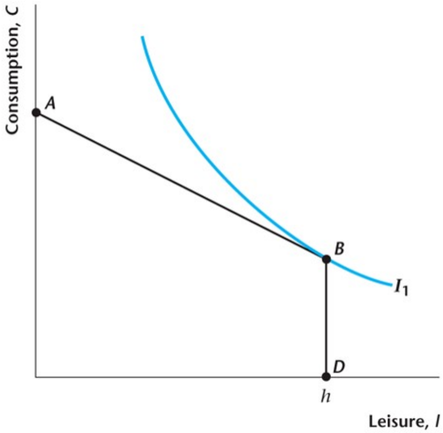
Figure 4.5 Interior Solution



Graphical Analysis: Corner Solution

- **Corner**: sol. at end pts of budget set
- MRS NOT equal to real wage ($MRS_{l,C} \neq w$), WHY?
  - working limited to total h hours, "kink"
- Sol. is NOT tangent to indifference curve

Figure 4.6 Corner Solution



Algebraic Analysis: Interior Solution

Recall consumer's problem:

$$\begin{aligned} \max_{C,l} \quad & U(C,l) \\ \text{subject to} \quad & C \leq w(h-l) + \pi - T \end{aligned} \tag{3}$$

- Calculus is about derivative: not defined at "kink" $\Rightarrow$ only interior sol.
- Sol. at the border of budget set $\Rightarrow$ budget constraint is "$=$" (binding)

Plug the budget constraint into utility function to replace C, we get

$$\max_{l} \quad U(w(h-l) + \pi - T, l) \tag{4}$$

Algebraic Analysis: Interior Solution (Cont.)

$$\max_{l} \quad U(w(h-l)+\pi-T, l)$$

Remember that now $C = w(h-l)+\pi-T$. Take first order condition w.r.t. l,

$$\overbrace{D_C U(C,l) \times \frac{d[w(h-l)+\pi-T]}{dl}}^{\text{Derivative on } C \text{ direction, chain rule}} + \overbrace{D_l U(C,l)}^{\text{Derivative on } l \text{ direction}} \tag{5}$$

$$D_C U(C,l) \times (-w) + D_l U(C,l) \tag{6}$$

$$w = \frac{D_l U(C,l)}{D_C U(C,l)} = MRS_{l,C} \tag{7}$$

Note: $D_x f(\cdot)$ is a shorthand for $\frac{df(\cdot)}{dx}$, meaning differentiation of $f(\cdot)$ with respect to choice variable x.

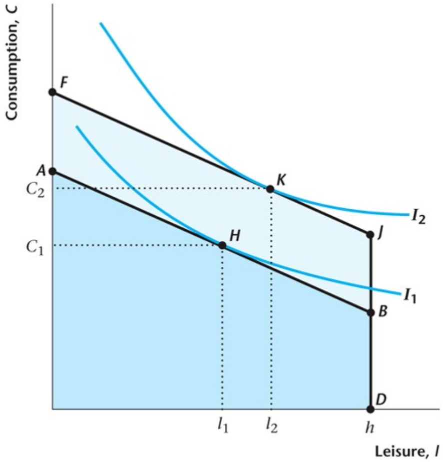
Build model for experiment

- Macroeconomists usually **cannot** do experiment: severe impact
- Yet we still want to know what's the result of changes!
- Lucas critique: need to understand individual behavior
- Consider two experiments:
  1. direct increase in real income (no C and l trade off, pure income effect)
  2. increase in real wage (income + substitution effect)

Experiment 1: Increase in dividends / Decrease in Tax

- **Recall**: C & l are normal goods
- **Income effect**: income $\uparrow \Rightarrow$ normal goods $\uparrow$
- Increase in dividends or decrease in taxes are level shifts up in real income, regardless of actions
- Consumer increases consumption, reduces quantity of labor supplied (increase leisure).

Figure 4.6 $\pi \uparrow / T \downarrow$



Experiment 2: Increase in Real Wage

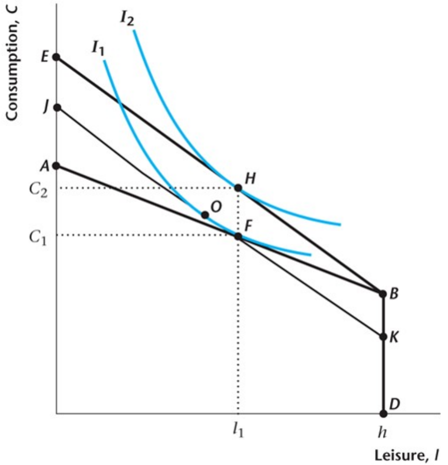
Substitution effect: $w \uparrow$, leisure is costly, sacrifice l for C

- budget line AB to JK, keeps F just affordable
- move along I_1 : new slope of budget line

Income effect: income $\uparrow \Rightarrow$ normal goods $\uparrow$

- budget line JK to EB, actual new budget line
- move up to I_2: higher utility possible

Figure 4.8 $w \uparrow$, both effects canceled out

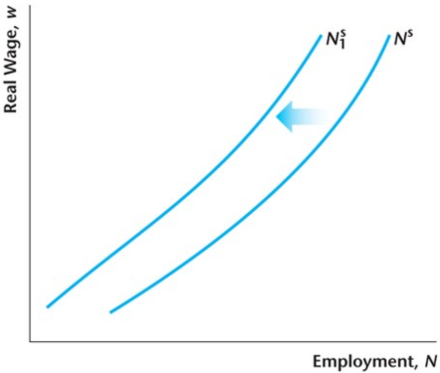


Experiment 1 & 2: Labor Supply

Looking ahead to putting the pieces together in a full model:

- Solution to consumer problem defines the supply curve for the labor market!
- What assumption ensures this is increasing in the wage?
    - Income effect $>$ substitution effect

Figure 4.10, LS on $\pi \uparrow$ / $T \downarrow$



Appendix

Chain rule

Back

In the main slide, we applied chain rule to the C direction of the $U(C, l)$. By binding budget constraints, we know $C(l) = w(h - l) + \pi - T$, i.e., consumption is a function of leisure.

$$\frac{d}{dl}U(C(l)) = \frac{dU(C, l)}{dC} \times \frac{dC(l)}{dl} = D_C U(C, l) \times D_l C(l) \tag{8}$$

where $D_l C(l) = -w$.