

Problem Set 2

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Instruction

Due at 11:59 PM (Eastern Time) on Sunday, June 14, 2022.

Please answer this problem set on Carmen quizzes “Problem Set 2”. In the following problems, the part that is in **red and bold** are the order of questions that should be answered on Carmen quizzes.

Problem 1

Remember the Example in Lecture 8.

Consumer: $\max_{C,l} \ln C + \ln l$ subject to $C \leq w(1 - l) + \pi$

$$\text{FOC} \quad \frac{C}{l} = w \quad (1)$$

$$\text{Binding budget constraint} \quad C = w(1 - l) + \pi \quad (2)$$

$$\text{Time constraint} \quad N^s = 1 - l \quad (3)$$

Firm: $\max_{N^d} (N^d)^{\frac{1}{2}} - wN^d$

$$\text{FOC} \quad \frac{1}{2}(N^d)^{-\frac{1}{2}} = w \quad (4)$$

$$\text{Output definition} \quad Y = (N^d)^{\frac{1}{2}} \quad (5)$$

$$\text{Profit definition} \quad \pi = Y - wN^d \quad (6)$$

Market clear:

$$N^s = N^d \quad (7)$$

Fill the following blanks for the step-by-step guide for algebraic calculation:

1. Step 1: Impose Market clear condition, so shrink all 7 equations to **6** equations

Consumer: $\max_{C,l} \ln C + \ln l$ subject to $C \leq w(1 - l) + \pi$

FOC $\frac{C}{l} = w$ (8)

Binding budget constraint $C = wN + \pi$ (9)

Time constraint $N = 1 - l$ (10)

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

FOC $\frac{1}{2}(N)^{-\frac{1}{2}} = w$ (11)

Output definition $Y = (N)^{\frac{1}{2}}$ (12)

Profit definition $\pi = Y - wN$ (13)

2. Step 2: replace l in terms of N using $l = 1 - N$

Consumer: $\max_{C,l} \ln C + \ln l$ subject to $C \leq w(1 - l) + \pi$

FOC $\frac{C}{(\underline{1-N})} = w$ (14)

Binding budget constraint $C = w(\underline{N}) + \pi$ (15)

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

FOC $\frac{1}{2}(N)^{-\frac{1}{2}} = w$ (16)

Output definition $Y = (N)^{\frac{1}{2}}$ (17)

Profit definition $\pi = Y - wN$ (18)

3. Step 3: replace π and Y as N

Consumer: $\max_{C,l} \ln C + \ln l$ subject to $C \leq w(1 - l) + \pi$

FOC $\frac{C}{(\underline{1-N})} = w$ (19)

Binding budget constraint $C = w(\underline{N}) + \pi$ (20)

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

FOC $\frac{1}{2}(N)^{-\frac{1}{2}} = w$ (21)

Profit definition $\pi = (\underline{N^{\frac{1}{2}}}) - wN$ (22)

4. Step 4: Substitute $\pi(N)$ into Binding budget constraint and get

$C = (\underline{N^{\frac{1}{2}}})$ (23)

5. Step 5: With consumer's FOC and firm's FOC both equate to w , we can get another expression of C :

$C = (\underline{1 - N}) \times (\underline{\frac{1}{2}N^{-\frac{1}{2}}})$ (24)

6. Step 6: Let (23) equate (24) and we get N as

$N = (\underline{\frac{1}{3}})$ (25)

7. Step 7: Trace back to all unknowns given the value of N , we get

$C = (\underline{\sqrt{\frac{1}{3}}})(0.577)$ (26)

$l = (\underline{\frac{2}{3}})(0.666)$ (27)

$Y = (\underline{\sqrt{\frac{1}{3}}})(0.577)$ (28)

$\pi = (\underline{\sqrt{\frac{1}{3}} - \frac{1}{6}\sqrt{3}})(0.288)$ (29)

$w = (\underline{\frac{1}{2}\sqrt{3}})(0.866)$ (30)