

# Lecture 7

## Representative Firm

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# Overview: Lecture 4 - 7

Provide **micro-foundation** for the **macro implication** (**Lucas critique**)

## ■ Representative Consumer:

- Lecture 4: **preference**, **constraints**
- Lecture 5: **optimization**, **application**
- Lecture 6: Numerical Examples

## ■ Representative Firm:

- Lecture 7: **production**, **optimization**, **application**

# Production Function

**Production function** describes the technology possibility for converting inputs into outputs.

Representative firm produces output  $Y$  with production function

$$Y = zF(K, N^d) \quad (1)$$

- $Y$ : output (consumption goods)
- $z$ : total factor productivity (TFP) (productivity for the economy)
- $K$ : capital (fixed for now,  $\therefore$  1-period model)
- $N^d$ : labor demand (chose by firm,  $d$  represents demand)

# Properties of Production Function: Marginal Product

- **Marginal product:** how much  $Y \uparrow$  by one unit of  $K \uparrow$  or  $N^d \uparrow$ .
  - **Marginal product of capital (MPK):**  $zD_K F(K, N^d)$
  - **Marginal product of labor (MPN):**  $zD_N F(K, N^d)$
- Marginal product is **positive** and **diminishing**:
  - **Positive MP:**  $Y \uparrow$  if either  $K \uparrow$  or  $N^d \uparrow$ 
    - more inputs result in more output
  - **Diminishing MP:** MPK  $\downarrow$  as  $K \uparrow$ ; MPN  $\downarrow$  as  $N^d \uparrow$ 
    - the **rate/speed** of output increasing is decreasing
- **Increasing marginal cross-products:**
  - e.g. MPK  $\uparrow$  as  $N \uparrow$ ; MPN  $\uparrow$  as  $K \uparrow$

# Properties of Production Function: Return to Scale

- **Return to scale:** how  $Y$  will change when both  $K$  and  $N$  increase
- **Constant return to scale (CRS):**  $xzF(K, N^d) = zF(xK, xN^d)$ 
  - small firms are **as efficient as** large firms
- **Increasing return to scale (IRS):**  $xzF(K, N^d) > zF(xK, xN^d)$ 
  - small firms are **less efficient than** large firms
- **Decreasing return to scale (DRS):**  $xzF(K, N^d) < zF(xK, xN^d)$ 
  - small firms are **more efficient than** large firms

## Example: Cobb-Douglas Production Function

- **Cobb-Douglas:**  $zF(K, N) = zK^\alpha N^{1-\alpha}$ ,  $\alpha$  is the share of capital contribution to output

- **Positive MPK & MPN:**

- $MPK = D_K zF(K, N) = z\alpha K^{\alpha-1} N^{1-\alpha} = z\alpha \left(\frac{K}{N}\right)^{\alpha-1} > 0$
- $MPN = D_N zF(K, N) = z(1-\alpha)K^\alpha N^{-\alpha} = z(1-\alpha) \left(\frac{K}{N}\right)^\alpha > 0$

- **Diminishing MP:**

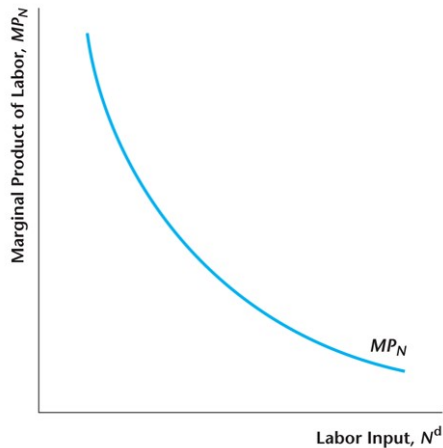
- For  $K$ ,  $D_K (z\alpha K^{\alpha-1} N^{1-\alpha}) = z\alpha(\alpha-1)K^{\alpha-2} N^{1-\alpha} < 0$
- For  $N$ ,  $D_N (z(1-\alpha)K^\alpha N^{-\alpha}) = z(1-\alpha)(-\alpha)K^\alpha N^{-\alpha-1} < 0$

- **Increasing marginal cross-product:**

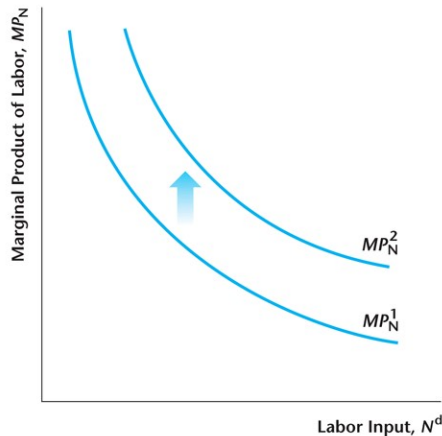
- For MPK,  $D_N (z\alpha K^{\alpha-1} N^{1-\alpha}) = z\alpha(1-\alpha)K^{\alpha-1} N^{-\alpha} > 0$
- For MPN,  $D_K (z(1-\alpha)K^\alpha N^{-\alpha}) = z(1-\alpha)\alpha K^{\alpha-1} N^{-\alpha} > 0$

# Visualization

## Diminishing Marginal Product

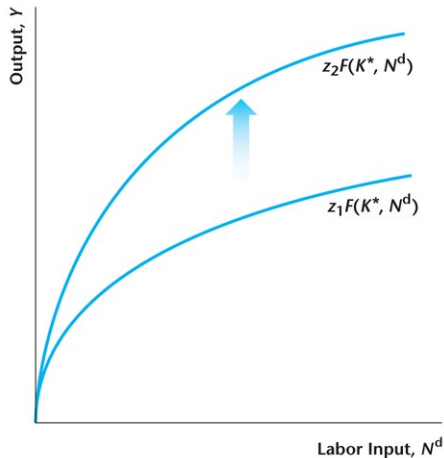


## Increasing Marginal Cross-product

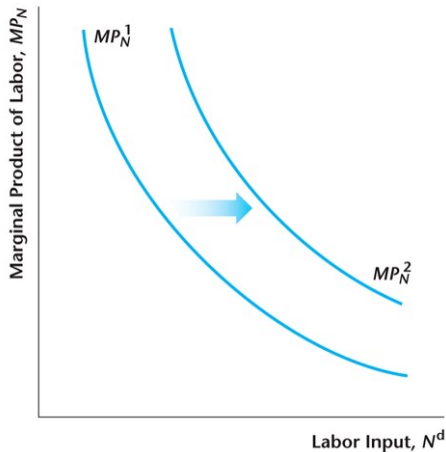


# Visualization: Changes in TFP

TFP shifts up the Production Function



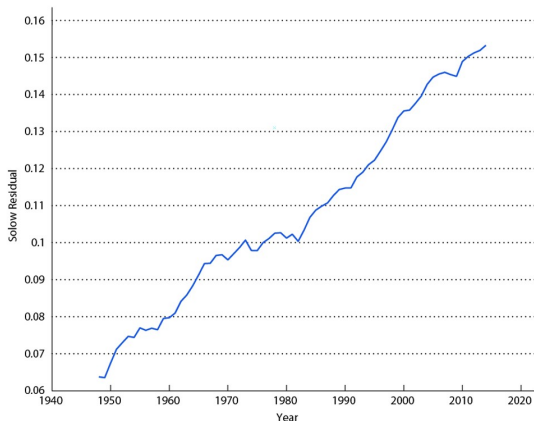
TFP increases MPN





# TFP in Data

## Solow Residual for US



We cannot see TFP, **how to measure it?**

- Assume Cobb-Douglas production function:  
$$Y = zK^{\alpha}N^{1-\alpha}$$
- By data,  $K/Y = 0.3 \Rightarrow \alpha = 0.3$
- Can observe  $K$ ,  $Y$ ,  $N$  in data:

$$z = \frac{Y}{K^{0.3}N^{0.7}}$$

# Firm's Problem: Profit Maximization

Firm maximizes profit ( $\pi$ ), which is the revenue minus the wage bill:

$$\pi = \max_{N^d} zF(K, N^d) - wN^d \quad (2)$$

■ **Constraints:**  $N^d > 0$ , relatively simple!

$$\text{Cobb-Douglas: } zF(K, N^d) = zK^\alpha (N^d)^{1-\alpha} \quad (3)$$

$$\text{FOC: } w = z(1 - \alpha)K^\alpha (N^d)^{-\alpha} \quad (4)$$

$$(N^d)^\alpha = \frac{z(1 - \alpha)K^\alpha}{w} \quad (5)$$

$$\text{Labor demand: } N^d = \left( \frac{z(1 - \alpha)K^\alpha}{w} \right)^{\frac{1}{\alpha}} = \left( \frac{z(1 - \alpha)}{w} \right)^{\frac{1}{\alpha}} K \quad (6)$$

As  $w \uparrow$ ,  $N^d \downarrow \Rightarrow$  **downward-sloping** demand.

## Experiment 1: Payroll Tax

**Payroll tax:** suppose firms have to pay additional per-unit tax  $t > 0$  on the wage bill, then

$$\text{Firm Problem: } \max_{N^d} zK^\alpha (N^d)^{1-\alpha} - w(1+t)N^d \quad (7)$$

$$\text{FOC: } w(1+t) = z(1-\alpha)K^\alpha (N^d)^{-\alpha} \quad (8)$$

$$N^d = K \left( \frac{z(1-\alpha)}{w(1+t)} \right)^{\frac{1}{\alpha}} \quad (9)$$

- **wage**  $\uparrow$ :  $w \uparrow \Rightarrow N^d \downarrow$  (same as benchmark)
- **tax**  $\uparrow$ :  $t \uparrow \Rightarrow N^d \downarrow$
- **capital**  $\uparrow$ :  $K \uparrow \Rightarrow N^d \uparrow \Rightarrow$  what if firm can also choose  $K$ ?

## Experiment 2: Choice of Capital

**Capital rent:** suppose that firm can choose capital level but have to pay  $r$  of per-unit rent.

$$\text{Firm Problem: } \max_{\mathbf{K}, N^d} zK^\alpha (N^d)^{1-\alpha} - rK - wN^d \quad (10)$$

$$\text{FOC on N: } w = z(1 - \alpha)K^\alpha (N^d)^{-\alpha} \quad (11)$$

$$\text{FOC on K: } r = z\alpha K^{\alpha-1} (N^d)^{1-\alpha} \quad (12)$$

$$\text{Divide (11) with (12): } \frac{w}{r} = \frac{(1 - \alpha)}{\alpha} \frac{K}{N^d} \quad (13)$$

$$\text{Capital-Labor ratio: } \frac{K}{N^d} = \frac{w}{r} \frac{\alpha}{1 - \alpha} \quad (14)$$

When firm can choose  $K$ , they choose both capital and labor such that (14) satisfied!