Julia Syntax and Algorithm

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Outline

Leggi le leggi

(“Read the manuals” in Italian)

Resources on Syntax

- Julia Official Tutorial: https://julialang.org/learning/tutorials/
- Wikibook on Introducing Julia: https://en.wikibooks.org/wiki/Introducing_Julia
- QuantEcon w/ Julia: https://julia.quantecon.org/intro.html
- Julia in 100 Seconds: https://www.youtube.com/watch?v=JYs_94znYy0

The REPL

REPL stands for Read, Evaluate, Print, and Loops.

Julia's REPL is the best I have ever seen, includes

- Unicode transformation: type `\alpha` and `tab` leads to α
- Package management: type `]` to enter `Pkg` mode to `add` packages
- Manual query: type `?` to enter `help` mode & find function manual
- Tab completion: type `\al` and `tab` gives you possible commands
- $\uparrow$ / $\downarrow$: up/down arrow key cycle through executed command history

The Language: Good and Bad

Julia language is designed with scientific computing in mind, and thus

- Unicode variable: directly use α as variable, not `alpha`.
- Multiple dispatch: multiple "methods" in one function for input types
- Type system: use `struct` to build custom types ($\approx$ but $\neq$ OOP)

But also have some weird behavior that I am not used to:

- Weird scope: variables defined inside loops (**`while`**, **`for`**) are local.
- Speed needs discipline: well-written code v.s. sloppy-written code
- Memory usage: might directly crash the Julia session ($\cdot\_\cdot$ LLVM?)

Best practice? Still Searching...

Outline

Syntax: generating a grid

- Usually the Macro coding starts with the grids of choice variables.
- A grid is a finite sample of continuous choice variable.
- Key to construct a grid is the `collect` and `range` function.
- `range` syntax requires `start` pt, `stop` pt and `length` of this grid
- `collect` then "collect" this `range` object into an array.

```
cnum = 100
lnum = 100
cgrid = collect( range( 0.01, 10.0, length = cnum ) )
lgrid = collect( range( 0.01, 1.0, length = lnum ) )
```

To get one element of a grid, we use `[]` syntax.

```
cval = cgrid[1] # get the first element of cgrid
lval = lgrid[5] # get the fifth element of lgrid
```

To create an array, you can use manual or automatical way.

```
# manually type all the elements
a = [1.0, 2.0, 3.0, 4.0, 5.0 ]
# automatically generate an "empty" array
#               type    dim empty  row   column
utility = Array{Float64, 2}(undef, cnum, lnum)
utility = zeros(cnum, lnum) # zero array
utility = ones(cnum, lnum) # one array
```

Syntax: `for` loop

To calculate the utility value at each (C, l) bundle, use `for` loop

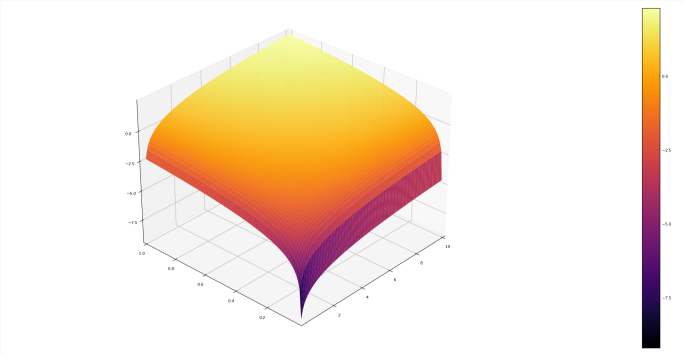
```
utility = Array{Float64, 2}(undef, cnum, lnum)
for indl in 1:1:lnum
    # get the each value in leisure grid
    lval = lgrid[indl]
    for indc in 1:1:cnum
        # get the each value in consumption grid
        cval = cgrid[indc]
        # log utility in both c and l
        utility[indc, indl] = log(cval) + log(lval)
    end
end
```

◂ Gov spending

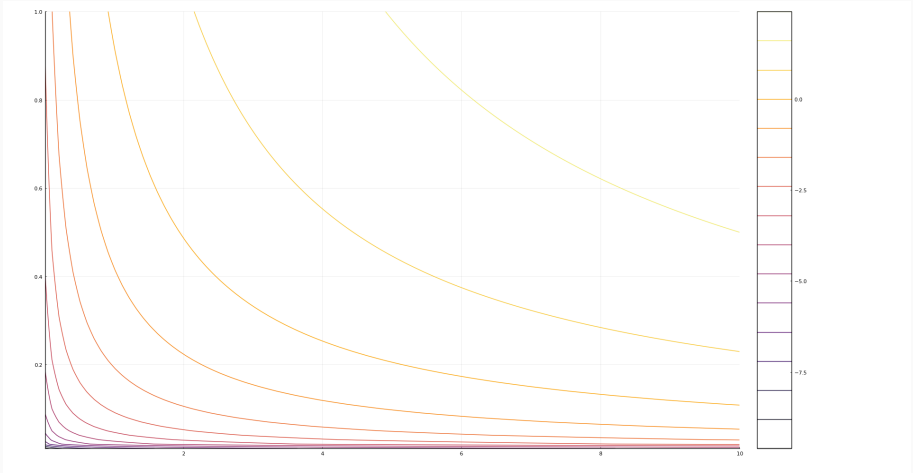
Install `Plots` and `PyPlot` by typing `]` and type `add Plots PyPlot`

Plot the `utility` array by

```
using Plots; pyplot();
surface(cgrid, lgrid, utility)  # 3-D figure
```



```
using Plots; pyplot();
contour(cgrid, lgrid, utility)  # contour figure
```



Syntax: `println` print something out

To show some info inside the `for` loop, `println` is a convenient tool.

If you want to know what (C, I) bundle leads to $U(C, I) = 0.0$,

```
    for indl in 1:1:lnum
        for indc in 1:1:cnum
            # the abs of u is close enough to 0.0
            if abs(utility[indc, indl]) < 1e-2
            # '$': string interpolation (IMO inefficient)
                println("U ~ 0 at (C, l) = ($indc, $indl)")
            end
        end
    end
```

`println` v.s. `print`: `println` add additional `\n`

Syntax: `while` loop

`while` loop mostly used when iteration only hault in some conditions.

In my experience it is mostly used if something needs convergence.

The following code is NOT an efficient way to find minimum location.

(should use `argmin` for `minimum` and `argmax` for `maximum`)

```
dist = 1.0; iter = 0;
while (dist > 1e-2)
    iter += 1 # same as "iter = iter + 1"
    indc = rand(1:1:cnum); indl = rand(1:1:lnum)
    dist = utility[indc, indl] - minimum(utility)
    if (dist < 1e-2)
        println("Find minimum at ($indc, $indl)")
        println("Iterates $iter times")
    end
end
```

Mostly for exam / standardization purpose.

```
round(pi)                # 3.0
round(pi, digits = 1)    # 3.1
round(pi, digits = 2)    # 3.14
round(pi, digits = 3)    # 3.142
round(pi, digits = 4)    # 3.1416
round(pi, digits = 5)    # 3.14159
```

Outline

Application: Laffer curve

There are going to be two applications for Julia syntax learned:

1. Laffer curve in distorting taxes, and
2. Government spending in CRRA utility function.

Recall that $Y = zN^d$ implies labor supply $N^s(t)$ equals to

$$N^s(t) = 1 - l = \frac{1}{2} - \frac{\pi}{2(1-t)}, \tag{1}$$

and the total tax revenue is given by

$$R(t) = wtN^s(t). \tag{2}$$

In equilibrium $w = z = 1$, so $\pi = zN^d - wN^d = 0$, so this question is trivial...

Laffer curve in Cobb-Douglas Production Function

Assume $Y = zN^a$, where $a < 1$, so firm's problem leads to

$$w(N) = MPN = zaN^{a-1}, \tag{3}$$

$$\pi(N) = Y - wN = z(1-a)N^a, \tag{4}$$

and recall $MRS_{l,C} = w(1-t)$ and binding BC $C = w(1-t)N + \pi$, so

$$MRS_{l,C} = \frac{C}{l} = \frac{w(1-t)N + \pi}{l} = w(1-t) \tag{5}$$

$$= \frac{w(N)(1-t)N + \pi(N)}{(1-N)} = w(N)(1-t) \tag{6}$$

expands, we get a monster:

$$\frac{zaN^{a-1}(1-t)N + z(1-a)N^a}{1-N} = zaN^{a-1}(1-t) \tag{7}$$

Laffer curve in Cobb-Douglas Production Function (cont.)

But not too bad, because you realize:

$$\text{Common } N: \quad \frac{zaN^{a-1}(1-t)N + z(1-a)N^a}{1-N} = zaN^{a-1}(1-t) \tag{8}$$

$$\text{Common } zN^a: \quad \frac{zaN^a(1-t) + z(1-a)N^a}{1-N} = zaN^{a-1}(1-t) \tag{9}$$

$$\text{Erase } zN^{a-1}: \quad \frac{zN^a[a(1-t)+1-a]}{1-N} = za(1-t)N^{a-1} \tag{10}$$

$$\text{Divide } [\cdot]: \quad \frac{N[a(1-t)+1-a]}{1-N} = a(1-t) \tag{11}$$

$$\frac{N}{1-N} = \frac{a(1-t)}{a(1-t)+1-a} \equiv A(t) \tag{12}$$

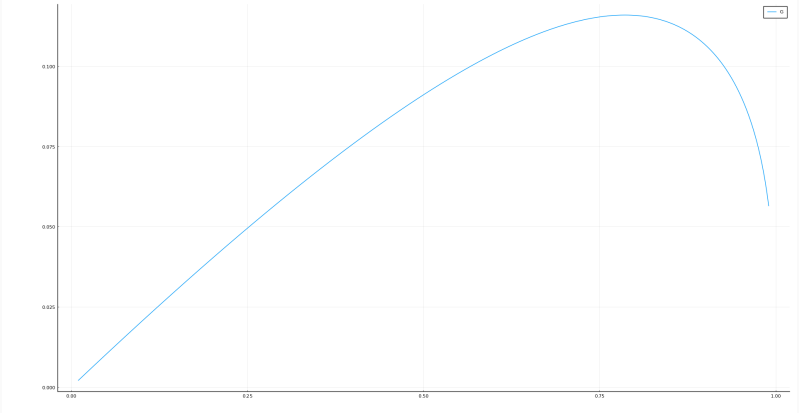
$$N = A(1-N) = A - AN \tag{13}$$

$$(1+A)N = A \Rightarrow N(t) = \frac{A(t)}{1+A(t)} \tag{14}$$

```
a = 0.33; tnum = 1000
tgrid = collect( range(0.01, 0.99, length = tnum) )
Gvec = Array{Float64, 1}(undef, tnum)
for indt = 1:1:tnum
    t = tgrid[indt]
    A = ( a*(1-t) ) / ( a*(1-t) + 1 - a )
    N = A / (1 + A)
    w = a*N^(a-1)
    Gvec[indt] = w * t * N
end
Gmax = maximum(Gvec); tmax = tgrid[argmax(Gvec)];
println("G* = $Gmax; t* = $tmax")
```

Laffer curve in Julia (cont.)

```
using Plots; pyplot()
plot(tgrid, Gvec, label = "G")
```



Outline

Grid search

Just calculate value on the grid points! Like for loop slide

Recall the formula with gov spending:

$$\max_{l} \quad \frac{(z(1-l)^{1-\alpha} - G)^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}. \tag{15}$$

We want to solve $l(z, G)$, but how to choose the `Ggrid`?

From the FOC we know

$$G = F(l) = z(1-l)^{1-\alpha} - \left[\frac{l^{-d}}{(1-\alpha)z(1-l)^{-\alpha}}\right]^{-\frac{1}{b}}. \tag{16}$$

Our first step starts with generating a TFP grid:

```
znum = 100
zgrid = collect( range( 0.8, 1.2, length = znum ) )
```

We want to find the upper/lower bound of `Ggrid`:

```
a = 1/2; b = 2; d = 3/2;
GovFOC(z, l) = z*(1-l)^(1-a) - # line continuation!
    ( (l^(-d)) / ( (1-a)*z*(1-l)^(-a) ) )^(-1/b)
# upper & lower bound of Ggrid
Gbound = Array{Float64, 2}(undef, znum, 2)
for indz = 1:1:znum
    zval = zgrid[indz]
    Gbound[indz, 1] = GovFOC(zval, 0.99) # lower bound
    Gbound[indz, 2] = GovFOC(zval, 0.01) # upper bound
end
```

```
    # lower bound should higher than 0.01
    Glow = max(0.0, minimum(Gbound))
    Ghigh = maximum(Gbound)
    # build Ggrid
    Gnum = 100
    Ggrid = collect( range( Glow, Ghigh, length = Gnum ) )
    # build lgrid
    lnum = 100
    lgrid = collect( range( 0.01, 1.0, length = lnum ) )
```

and then find the optimal leisure using the value on this grid:

```
a = 1/2; b = 2; d = 3/2;
# define implicit utility function
utility(l, z, G) = ( ( z*(1-l)^(1-a) - G )^(1-b) ) /
                   ( 1-b ) +
                   ( l^(1-d) ) / (1-d)
# Array for storage
## for temporary storage
uvec = Array{Float64, 1}(undef, lnum)
## for optimal utility value
ustar = Array{Float64, 2}(undef, znum, Gnum)
## for optimal leisure given z, G
lstar = Array{Float64, 2}(undef, znum, Gnum)
```

```
for indG = 1:1:Gnum
    Gval = Ggrid[indG]
    for indz = 1:1:znum
        zval = zgrid[indz]
        for indl = 1:1:lnum
            lval = lgrid[indl]
            cval = zval*(1-lval)^(1-a) - Gval
            uvec[indl] = ( cval < 0.0 ? -Inf :
                           utility(lval, zval, Gval) )
        end
        ustar[indz, indG] = maximum(uvec)
    end
end
```

Grid search: analysis

Notice that in previous slide, I check whether `cval < 0.0`

and also we find the highest utility and the corresponding (z, G) value by

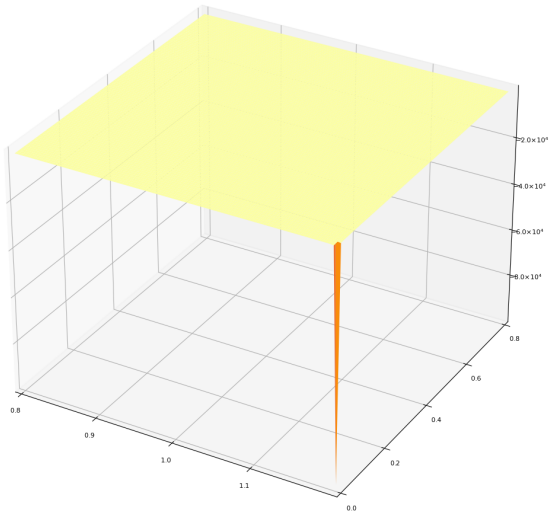
```
umax = maximum(ustar)
zloc = argmax(ustar)[1]
Gloc = argmax(ustar)[2]
zmax = zgrid[zloc]
Gmax = Ggrid[Gloc]
```

But if you plot you will see that the plot is slightly "off":

```
using Plots; pyplot()
surface(zgrid, Ggrid, ustar)
```

$\because$ large negative point that drag down the scale of every point.

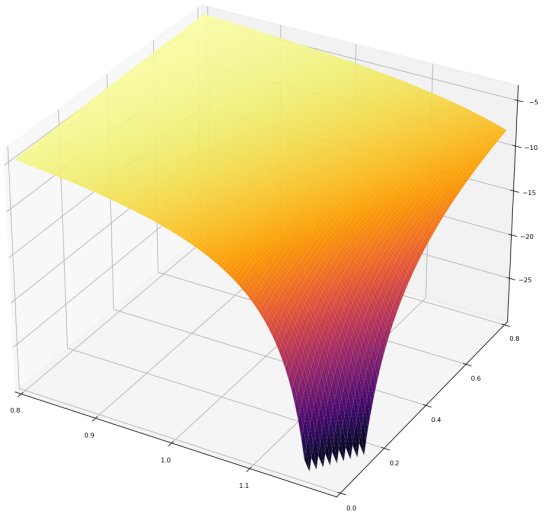
grid search: misleading figure

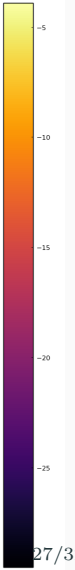


Grid search: revising (erase `uval < -30.0`)

```
for indG = 1:1:Gnum
    Gval = Ggrid[indG]
    for indz = 1:1:znum
        zval = zgrid[indz]
        for indl = 1:1:lnum
            lval = lgrid[indl]
            cval = zval*(1-lval)^(1-a) - Gval
            uval = utility(lval, zval, Gval)
            uvec[indl] = ( (cval < 0.0 || uval < -30.0)
                           ? -Inf : uval )
        end
        ustar[indz, indG] = maximum(uvec)
    end
end
```

Grid search: better figure





Grid search: Can we do better?

All of the `-Inf` stuff we are assigning manually is because $C < 0$.

Recall that $C = Y - G$, and thus for $C \geq 0$, $Y - G \geq 0 \Rightarrow Y > G$.

```
ymat = Array{Float64, 2}(undef, znum, lnum)
for indl = 1:1:lnum
    lval = lgrid[indl]
    for indz = 1:1:znum
        zval = zgrid[indz]
        ymat[indz, indl] = zval * (1-lval)^(1-a)
    end
end
ymin = minimum(ymat)
```

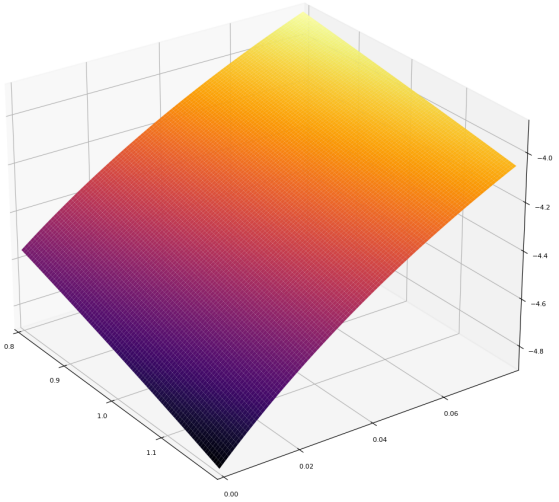
You will get $min(y) = 0.081$, which means that if you choose the `Ghigh = 0.08`, $C > 0, \forall z, G$ assigned.

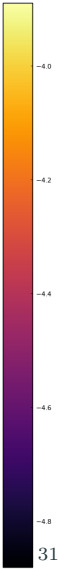
```
# lower bound should higher than 0.01
Glow = 0.0
Ghigh = 0.08
# build Ggrid
Gnum = 100
Ggrid = collect( range( Glow, Ghigh, length = Gnum ) )
# build lgrid
lnum = 100
lgrid = collect( range( 0.01, 0.99, length = lnum ) )
```

and then find the optimal leisure using the value on this grid:

```
for indG = 1:1:Gnum
    Gval = Ggrid[indG]
    for indz = 1:1:znum
        zval = zgrid[indz]
        for indl = 1:1:lnum
            lval = lgrid[indl]
            cval = zval*(1-lval)^(1-a) - Gval
            uvec[indl] = utility(lval, zval, Gval)
        end
        ustar[indz, indG] = maximum(uvec)
    end
end
```

Grid search: better figure





Grid search method: additional details

Calculate on the grid point $\Rightarrow$ result are correct but speed is slow.

Notice that when you choose the grid points, better to avoid some value:

Example
When I create `cgrid` and `lgrid`, I avoid the start point of 0.0, but 0.01, since $log(0.0) = \infty$.

In general, if theoretical range, say leisure, is $[0, 1]$, then it is safe to build a grid from $[0.01, 0.99]$.