Debt Financing, Used Capital Market and Capital Reallocation

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Motivation and Question

How much do financial frictions and endogenous capital irreversibility explain aggregate investment volatility?

- Khan and Thomas (2013): constant irreversibility, $\sigma_I/\sigma_Y = 3.86$.
- Lanteri (2018): endogenous irreversibility, $\sigma_I/\sigma_Y = 3.542$.
- Data: $\sigma_I/\sigma_Y = 2.821$.

Why?

1. cheaper used capital purchasing price $\Rightarrow$ dampens the response.
2. irreversibility $\uparrow$ in recession $\Rightarrow$ exacerbates the response.

What I do

- This paper: financial frictions $\Rightarrow$ purchasing price $\downarrow\downarrow \Rightarrow \sigma_I/\sigma_Y \downarrow\downarrow$.
    - endogenous tightening of collateral constraints harms small firms.
- What I do: collateral constraint + Lanteri (2018) (RBC & used K)
- Contribution: evaluate the joint effect of both frictions
    - Khan and Thomas (2013): predicts countercyclical capital reallocation yet the data is procyclical.
    - Lanteri (2018): explains only 30% of the cyclical volatility of total capital reallocation in data.

Endogenous Collateral Constraint

- In Lanteri (2018), *(S, s)* thresholds move apart in recession $\Rightarrow$ expand inaction region. details
  - downward-adjusting: irreversibility $\uparrow$ $\Rightarrow$ disinvest less.
  - upward-adjusting: expectation on future resale price $\Rightarrow$ invest less.
- In my model, collateral constraint is $b' \leq \zeta k$, so in recession,
  - looser for disinvesting firm: switch from disinvestment to borrowing.
  - tighter for investing firm: invest is relatively preferred $\Rightarrow$ demand $\uparrow$.

purchasing price $\downarrow\downarrow \Rightarrow \sigma_I/\sigma_Y \downarrow\downarrow$.

Empirical Evidence

- Matched by Lanteri (2018)
  - $> 20\%$ share of used capital in four industries in US. table
  - Price of used investment is $2 \sim 4$ times volatile than new one. figure
- My paper is going to match:
  - Firms holding $10 \sim 30\%$ share of used capital based on firm size. table
  - Small firms are buyers in used capital market. table
  - Debt financing is significantly and positively correlated to capital reallocation. table

Overview

I consider a heterogeneous firm model with real and financial friction:

- **Used capital market**: trade price q is determined by the supply (downward-adjust) and the demand (upward-adjust) Def
- **Households:** own firms $\Rightarrow$ firms discount as HH. HH Problem
- **Firms:** idio.: ϵ_i; TFP: z_f; exogenous exit prob π_d.
  - Upward-adjusting firms: buys capital at cost Q.
  - Downward-adjusting firms: sells used investment goods at price q.
  - Collateral constraint: $b' \leq \zeta k$.

Technology

CES cost minimization problem

- K process for upward-adjusting:

$$k' = (1-\delta)k + I(i_{new}, i_{used})$$
$$I(i_{new}, i_{used}) = \left[\eta^{\frac{1}{s}}(i_{new})^{\frac{s-1}{s}} + (1-\eta)^{\frac{1}{s}}(i_{used})^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}, \quad (1)$$

$\eta \in [0,1]$: average ratio; $s > 0$: elasticity of substitution.

- Agg. price index $Q = \left[\eta + (1-\eta)(q+\gamma)^{1-s}\right]^{\frac{1}{1-s}}$, $q + \gamma < 1$, $Q < 1$.
- $\frac{i_{used}}{i_{new}} = \frac{1-\eta}{\eta}(q+\gamma)^{-s}$.
    - needs modification to match share of used capital $\downarrow$ with firm size.
- K process for downward-adjusting: $k' = (1-\delta)k - d$.

Production and Value Function

- firm-level state variables as $\mathbf{s_i} \equiv \{k, b, \epsilon_i\}$.
- Following Khan and Thomas (2013),

$$v_0(\mathbf{s_i}; z_f; \mu) = \pi_d \max_n [x^d(\mathbf{s_i}; z_f)] + (1 - \pi_d) v(\mathbf{s_i}; z_f; \mu), \tag{2}$$

where $x^d(\cdot)$ is the cash-on-hand for downward-adjusting firms. Def

- Conditional on survival, firm chooses upward- or downward-adjusting:

$$v(\mathbf{s_i}, z_f, \mu) = \max\{v^u(\mathbf{s_i}, z_f, \mu), v^d(\mathbf{s_i}, z_f, \mu)\}. \tag{3}$$

Upward-adjusting Firm

$$v^u(\mathbf{s_i}; z_f; \mu) = \max_{k', b', D} D + \sum_{g=1}^{N_z} \pi^z_{fg} d_g(z_f; \mu) \sum_{j=1}^{N_s} \pi^s_{ij} v_0(\mathbf{s_j}'; z'_g; \mu'), \qquad (4)$$

subject to

$$0 \leq D \leq x^u(\mathbf{s_i}; z_f) + q_b b' - Q k', \qquad \text{(Budget: Up)}$$

$$x^u(\mathbf{s_i}; z_f) = z_f \epsilon_i F(k, n) - w(z_f, \mu) n - b + Q(1-\delta)k \qquad \text{(Cash: Up)}$$

$$b' \leq \zeta k, \qquad \text{(Collateral)}$$

$$k' \geq (1-\delta)k, \qquad \text{(K range)}$$

$$\mu' = \Gamma(z_f; \mu), \qquad \text{(Distribution)}$$

q_b: bond price; $d_g(z_f, \mu)$: SDF; ζ: efficiency of financial sector.

Downward-adjusting Firm

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$$v^d(\mathbf{s_i}; z_f; \mu) = \max_{k', b', D} D + \sum_{g=1}^{N_z} \pi_{fg}^z d_g(z_f; \mu) \sum_{j=1}^{N_s} \pi_{ij}^s v_0(\mathbf{s_j}'; z_g'; \mu'), \qquad (5)$$

subject to

$$0 \leq D \leq x^d(\mathbf{s_i}; z_f) + q_b b' - q k', \qquad \text{(Budget: Down)}$$

$$x^d(\mathbf{s_i}; z_f) = z_f \epsilon_i F(k, n) - w(z_f, \mu) n - b + q(1-\delta)k \qquad \text{(Cash: Down)}$$

$$b' \leq \zeta k, \qquad \text{(Collateral)}$$

$$k' \leq (1-\delta)k, \qquad \text{(K range)}$$

$$\mu' = \Gamma(z_f; \mu), \qquad \text{(Distribution)}$$

Definition of *recursive equilibrium*, Rewrite (2), (3), (4), (5) in terms of $p(z_f; \mu)$

Firms' Problem with Zero Dividend Policy detail

$$V_0(\mathbf{s_i}, z_f, \mu) = \pi_d p x^d(z_f, \mu) + (1-\pi_d) V(\mathbf{s_i}, z_f, \mu),$$

$$V(\mathbf{s_i}, z_f, \mu) = \max\{V^u(\mathbf{s_i}, z_f, \mu), V^d(\mathbf{s_i}, z_f, \mu)\},$$

$$V^u(k, b, \epsilon_i, z_f, \mu) = \max_{k' \in \Omega^u(\mathbf{s_i})} \beta \sum_{g=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi^z_{fg} \pi^\epsilon_{ij} V_0(k', b'_u(k'), \epsilon_j, z_g, \mu'),$$

$$\text{s.t.} \quad b'_u(k') = \frac{Qk' - x^u}{q_b},$$

$$V^d(k, b, \epsilon_i, z_f, \mu) = \max_{k' \in \Omega^d(\mathbf{s_i})} \beta \sum_{g=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi^z_{fg} \pi^\epsilon_{ij} V_0(k', b'_d(k'), \epsilon_j, z_g, \mu'),$$

$$\text{s.t.} \quad b'_d(k') = \frac{qk' - x^d}{q_b},$$

$$\Omega^u(\mathbf{s_i}) = [(1-\delta)k, \bar{k}_u(\mathbf{s_i})]; \quad \Omega^d(\mathbf{s_i}) = [0, \min\{(1-\delta)k, \bar{k}_d(\mathbf{s_i})\}],$$

$$\bar{k}_u = \frac{q_b \zeta k + x^u}{Q}; \quad \bar{k}_d = \frac{q_b \zeta k + x^d}{q}.$$

Progress, To Do and Anticipated Result

- Replicate Lanteri (2018) (✓) details; Solve proposed model (X) details
- To Do:
  - More realistic collateral: $b' \leq q\zeta k$.
  - Modify capital accumulation process to match firm size ↑, used capital share ↓: $\gamma(k)$ as an increasing function of k.
- Anticipated result of time-varying financial friction:
  - generate higher cyclical volatility of total capital reallocation.
  - negative effects on agg. productivity propagate over time.
  - match the volatility of agg. investment better.

Appendix

References I

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Table: Lanteri (2018)

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TABLE 1—SHARES OF ASSET TYPES IN US EQUIPMENT STOCK

Type	Aircraft	Ships	Autos and trucks	Construction	Total
Share of equipment (%)	6.11	1.33	11.86	3.51	22.81

Source: Bureau of Economic Analysis Asset Tables 2015, author's calculations

Figure: Lanteri (2018)

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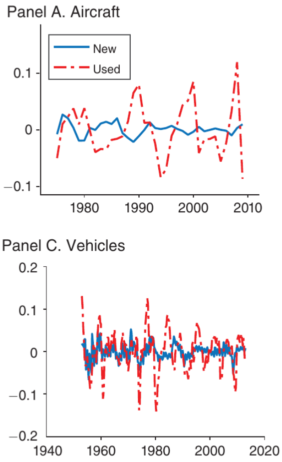

FIGURE 2. PRICES OF NEW AND USED CAPITAL (*Cyclical Components*)

Notes: Log-deviations from trend of price index of new capital and price index of used capital for the following types of capital: Aircraft, Ships, Vehicles, Construction equipment. Data definitions and elaboration are explained under Table 2. More details on data sources and construction are in online Appendix A.

Table: Eisfeldt and Shi (2018)

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Table 1 Cyclical properties of reallocation and productivity dispersion; deviations from trend are computed using an annual HP filter (Hodrick & Prescott 1997)

	Correlation with GDP	Unconditional mean	Boom mean	Recession mean
Panel *a*: Capital reallocation turnover rate				
Total reallocation turnover	0.5752*** (0.1454)	1.96%	2.30%***	1.61%
Sales of PP&E turnover	0.3455* (0.1680)	0.40%	0.43%**	0.36%
Acquisition turnover	0.5861*** (0.1413)	1.56%	1.87%***	1.25%
Panel *b*: Benefits to reallocation				
Standard deviation of Tobin's q (firm level, $0 \leq q \leq 5$)	−0.0580 (0.2250)	0.77	0.77	0.77
Standard deviation of TFP growth rates (3-digit NAICS level)	−0.1463 (0.3003)	3.79	3.56	3.99
Standard deviation of capacity utilization (3-digit NAICS level)	−0.4948*** (0.1650)	5.20	4.69	5.64
Panel *c*: Labor reallocation				
Job creation rate	0.6180*** (0.1540)	16.69%	17.65%	15.68%
Job destruction rate	−0.3760 (0.2391)	14.71%	14.51%	14.93%
Excess job reallocation rate	−0.1030 (0.3153)	14.42%	14.51%	14.32%

Data: Compustat

Table: Eisfeldt and Rampini (2007)

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Table 1
Ratio of used capital expenditures to total capital expenditures across asset, employment, and sales deciles

Decile	By assets				By employment		By sales	
	Decile cutoff (millions)	Used capital (%)	Used structures (%)	Used equipment (%)	Decile cutoff (thousands)	Used capital (%)	Decile cutoff (millions)	Used capital (%)
1st	0	27.79	28.77	26.21	0	30.27	0	20.38
2nd	0.10	20.17	21.69	17.32	0.01	17.86	0.53	23.28
3rd	0.36	18.51	21.43	15.36	0.03	16.31	2.05	18.93
4th	1.04	17.13	20.20	14.46	0.07	13.54	5.97	16.79
5th	2.94	16.14	20.08	12.97	0.18	11.69	13.65	16.40
6th	7.55	15.07	19.04	12.44	0.52	11.92	27.40	14.86
7th	16.89	12.69	16.15	10.64	0.67	10.52	51.15	13.21
8th	34.46	12.16	15.80	9.72	0.92	10.85	94.93	12.67
9th	69.24	11.22	15.33	9.18	1.45	10.33	186.51	11.81
10th	186.55	10.10	13.04	8.34	3.09	9.23	490.25	9.94

Data: Vehicle Inventory and Use Survey (VIUS) and Annual Capital Expenditures Survey (ACES)

Table: Eisfeldt and Shi (2018)

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Table 2 Reallocation versus productivity dispersion and financial flows; deviations from trend are computed using an annual HP filter (Hodrick & Prescott 1997)

	Total reallocation turnover	Sales of PP&E turnover	Acquisition turnover
Panel *a*: Correlation with benefit of reallocation			
Standard deviation of Tobin's q (F) ($0 \leq q \leq 5$)	−0.0732 (0.2454)	0.1464 (0.2951)	−0.0922 (0.2363)
Standard deviation of TFP growth rates (I)	0.1437 (0.3416)	0.0261 (0.3047)	0.1488 (0.3490)
Standard deviation of capacity utilization (I)	−0.5646*** (0.1218)	−0.2920 (0.1647)	−0.5778*** (0.1207)
Panel *b*: Correlation with financial variables			
Debt financing	0.6590*** (0.1530)	0.4507* (0.2205)	0.6581*** (0.1526)
Equity financing	−0.1661 (0.4199)	0.0766 (0.3439)	−0.1876 (0.4180)
Total financing	0.5261** (0.2114)	0.4768** (0.2029)	0.5122** (0.2144)
VIX	−0.0691 (0.3377)	0.2176 (0.2913)	−0.1082 (0.3287)
Uncertainty shock	0.1744 (0.3183)	0.3433 (0.2194)	0.1518 (0.3247)

(S, s) threshold in Lanteri (2018)

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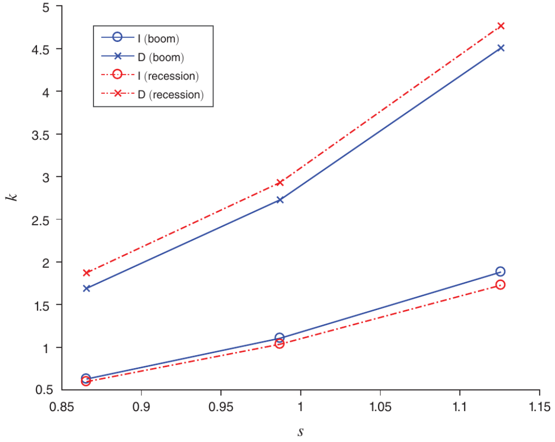

FIGURE 7. THRESHOLDS FOR INVESTMENT AND DISINVESTMENT

Notes: x-axis: idiosyncratic productivity s. y-axis: capital level k. Blue solid lines represent investment (I) and disinvestment (D) thresholds before the aggregate negative shock, while red dashed-dotted lines represent the thresholds after the aggregate negative shock hits.

Calibration Result in Lanteri (2018)

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TABLE 5—BUSINESS-CYCLE STATISTICS: BASELINE MODEL (*HP-Filter* $\lambda = 6.25$)

Statistic	Y	C	I	K	N	r	q	q/Q	reall
mean	0.613	0.509	0.103	1.574	0.336	0.041	0.918	0.933	0.042
$\sigma(\cdot)/\sigma(Y)$	(1.51)	0.482	3.679	0.247	0.534	0.074	0.187	0.133	2.972
corr$(\cdot, Y)$	1	0.983	0.99	−0.335	0.986	0.866	0.986	0.987	0.986
autocorr	0.085	0.144	0.062	0.504	0.061	−0.045	0.184	0.184	0.033

Notes: Rows: mean, standard deviation relative to standard deviation of output, autocorrelation. Columns: output, consumption, investment, capital, hours, real interest rate, price of used capital, degree of irreversibility (price of used capital relative to marginal cost of expanding a firm), capital reallocation (value in terms of output good).

TABLE 7—BUSINESS-CYCLE STATISTICS: US ANNUAL DATA (*HP-Filter with* $\lambda = 6.25$)

Statistic	Y	C	I	K	N	w	r	TFP	reall	SPPE only
$\sigma(\cdot)/\sigma(Y)$	(1.44)	0.529	2.86	0.977	1.209	0.568	0.828	0.498	11.022	5.208
corr$(\cdot, Y)$	1	0.81	0.792	0.573	0.894	0.184	0.049	0.402	0.712	0.305
autocorr	0.177	0.27	0.265	0.393	0.276	0.172	0.044	0.177	0.199	0.192

Notes: US business-cycle statistics 1947–2015. Rows: standard deviation relative to standard deviation of GDP, correlation with GDP, autocorrelation. Columns: real GDP, consumption (personal consumption expenditures on nondurables and services, deflated with GDP deflator), investment (fixed private investment and personal consumption expenditures on durables, deflated with GDP deflator), capital (fixed private assets and stock of consumer durables, deflated with GDP deflator), hours (all persons, nonfarm business sector), real wage (real compensation per hour, nonfarm business sector), real interest rate (three-month T-bill, net of ex post GDP-deflator inflation), aggregate TFP (constructed as in the model, i.e., $\log(\text{GDP}) - \alpha \log(K) - \nu \log(N)$), capital reallocation (SPPE + Acquisitions) and SPPE (1971–2011), deflated with GDP deflator.
Sources: BEA, BLS, Board of Governors of the Federal Reserve System, Compustat, author's calculations.

CES Cost Minimization Problem I

Back: What I do, Back: Tech

The CES cost minimization problem to at least achieve $\bar{I}$ level of investment is given by

$$\begin{aligned} \min_{i_{new}, i_{used}} \quad & c_{new} i_{new} + c_{used} i_{used} \\ \text{s.t.} \quad & \left[\eta^{\frac{1}{s}} (i_{new})^{\frac{s-1}{s}} + (1-\eta)^{\frac{1}{s}} (i_{used})^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} \geq \bar{I} \end{aligned}. \tag{6}$$

Note that constraint must bind, so we can denote

$$\bar{I}^{\frac{s-1}{s}} = \left[\eta^{\frac{1}{s}} (i_{new})^{\frac{s-1}{s}} + (1-\eta)^{\frac{1}{s}} (i_{used})^{\frac{s-1}{s}} \right]. \tag{7}$$

CES Cost Minimization Problem II

Back: What I do, Back: Tech

Let the Lagrangian multiplier be λ, the FOC w.r.t. i_{new} and i_{used} are

$$\begin{aligned} [i_{new}]: \quad c_{new} &= \lambda \eta^{\frac{1}{s}} i_{new}^{-\frac{1}{s}} \bar{I}^{\frac{1}{s}} \\ [i_{used}]: \quad c_{used} &= \lambda (1-\eta)^{\frac{1}{s}} i_{used}^{-\frac{1}{s}} \bar{I}^{\frac{1}{s}} \end{aligned}, \tag{8}$$

Rearrange (8) w.r.t. investment,

$$\begin{aligned} i_{new} &= \eta \bar{I} (\frac{c_{new}}{\lambda})^{-s} \\ i_{used} &= (1-\eta) \bar{I} (\frac{c_{used}}{\lambda})^{-s} \end{aligned}. \tag{9}$$

Thus, we get ratio of i_{used} and i_{new} as in (**??**).

CES Cost Minimization Problem III

Back: What I do, Back: Tech

Substitute (9) back to binding constraint and solve for Lagrangian multiplier λ,

$$\lambda = \left[\eta c_{new}^{1-s} + (1-\eta) c_{used}^{1-s}\right]^{\frac{1}{1-s}}, \tag{10}$$

which we define the RHS as Q.

Model: Household Problem

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Representative households maximize their lifetime utility by choosing consumption (c), labor supply (n^h), future firm share holding (λ') and future bond holding (ϕ'):

$$
\begin{aligned}
V^h(\lambda, \phi; z_f, \mu) = \max_{c, n^h, \phi', \lambda'} & \left\{ u(c, 1-n^h) + \beta \sum_{g=1}^{N_z} \pi^z_{fg} V^h(\lambda', \phi'; z'_g, \mu') \right\} \\
\text{s.t.} \quad & c + q(z_f;\mu)\phi' + \int_{\mathbf{s}} \rho_1(\mathbf{s_j}, z'_g; \mu')\lambda(d[k' \times b' \times \epsilon']) \\
& \leq w(z_f;\mu)n^h + \phi + \int_{\mathbf{s}} \rho_0(\mathbf{s_i}, z_f; \mu)\lambda(d[k \times b \times \epsilon])
\end{aligned}
\tag{11}
$$

where $\rho_0(\cdot)$ is the dividend-inclusive price of the current share, and $\rho_1(\cdot)$ is the ex-dividend price of the future share.

Recursive Equilibrium I

Back: Overview, Back: Downward adjusting

A *recursive competitive equilibrium* is a set of function,

$$w, q, q_b, \{d_g\}_{g=1}^{N_z}, \rho_0, \rho_1, v_0, N, K, B, D, I, I_{new}, I_{used}, d, V^h, C^h, N^h, \Phi^h, \Lambda^h \tag{12}$$

such that

1. v_0 solves (2)-(5), and N is the corresponding policy functions for exiting firms, and (N, K, B, D) are the corresponding policy functions for continuing firms.
2. V^h solves (11), and (C^h, N^h, Λ^h) are the corresponding policy functions for households.
3. $\Lambda^h(\mathbf{s_j}', \lambda, \phi; z_f, \mu) = \mu'(\mathbf{s_j}'; z_f, \mu)$ for all $(k', b', \epsilon_j) \in \mathbf{S}$.

Recursive Equilibrium II

Back: Overview, Back: Downward adjusting

4. Labor market clears:

$$N^h(\lambda, \phi; z_f, \mu) = \int_{\mathbf{S}} [N(k, \epsilon_i; z_f, \mu)]\mu(d[k \times b \times \epsilon]), \tag{13}$$

5. For upward-adjusting firms, i.e., firms such that $v^u(\mathbf{s_i}, z_f, \mu) \geq v^d(\mathbf{s_i}, z_f, \mu)$, the policy function $K(\mathbf{s_i}, z_f, \mu)$ solves (4), and the investment $I(\mathbf{s_i}, z_f, \mu) = K(\mathbf{s_i}, z_f, \mu) - (1-\delta)k$. Furthermore, the allocation of $I_{used}(\mathbf{s_i}, z_f, \mu)$ and $I_{new}(\mathbf{s_i}, z_f, \mu)$ is (**??**) and the corresponding aggregate price index is (**??**).

6. For downward-adjusting firms, i.e., $v^u(\mathbf{s_i}, z_f, \mu) < v^d(\mathbf{s_i}, z_f, \mu)$, the policy function $K(\mathbf{s_i}, z_f, \mu)$ solves (5), and $d(\mathbf{s_i}, z_f, \mu) = (1-\delta)k - K(\mathbf{s_i}, z_f, \mu)$.

Recursive Equilibrium III

Back: Overview, Back: Downward adjusting

7. Good markets clear:

$$C(z_f,\mu) = \int_{\mathbf{S}} \Big\{ z_f \epsilon_i F(k, N(k,\epsilon_i; z_f,\mu)) \\ - (1-\pi_d) Q(z_f,\mu) I(\mathbf{s_i}, z_f, \mu) \\ + (1-\pi_d) q(z_f,\mu) d(\mathbf{s_i}, z_f, \mu) \\ + \pi_d [q(z_f,\mu)(1-\delta)k - k_0] \Big\} \mu(d[k \times b \times \epsilon]) \quad , \tag{14}$$

where k_0 is the initial capital stock. We assume k_0 for each entering firm is a fixed χ fraction of the long-run aggregate capital stock, i.e.,

$$k_0 = \chi \int k \tilde{\mu}(d[k \times b \times \epsilon]). \tag{15}$$

Recursive Equilibrium IV

Back: Overview, Back: Downward adjusting

8. The used investment price $q(z_f, \mu)$ clears the market of used capital:

$$\int_{\mathbf{S}} d(\mathbf{s_i}, z_f, \mu)\mu(d[k \times b \times \epsilon]) = \int_{\mathbf{S}} i_{used}(\mathbf{s_i}, z_f, \mu)\mu(d[k \times b \times \epsilon]). \quad (16)$$

9. Evolution of distribution $\Gamma(\mathbf{S}, \mu)$ is defined by

$$\begin{aligned}\mu'(A, \epsilon_i) = (1 - \pi_d) \int_{\{(k,b,\epsilon_i)|K(\mathbf{s_i},z_f;\mu),B(\mathbf{s_i},z_f;\mu)\in A\}} \mu(d[k \times b \times \epsilon]) \\ + \pi_d \chi(k_0) H(\epsilon_j)\end{aligned}, \quad (17)$$

where $\chi(k_0) = 1$ if $(k_0, 0) \in A$, and 0 otherwise.

Recursive Equilibrium V

Back: Overview, Back: Downward adjusting

10. Bond market clear condition

$$\Phi^h(z_f;\mu) = \int_{\mathbf{s}} B(\mathbf{s}, z_f, \mu)\mu(d[k \times b \times \epsilon]) \tag{18}$$

is satisfying Walras's law, where Φ^h is household's policy functions for bond.

Analysis I

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Let $u(c, 1-n^h) = \frac{1}{c}$, and $F(k,n) = k^\alpha n^\nu$, $\alpha + \nu < 1$.
In households' problem, the following three conditions ensure that good market, labor market and bond market clear in this economy:

$$p(z_f;\mu) = D_1 u(c, 1-n^h) = \frac{1}{c} \tag{19}$$

$$w(z_f;\mu) = \frac{D_2 u(c, 1-n^h)}{D_1 u(c, 1-n^h)} = \frac{\psi}{p(z_f;\mu)} \tag{20}$$

$$q_b(z_f;\mu) \equiv \beta \sum_{g=1}^{N_z} \pi^z_{fg} \frac{D_1 u(c_g, 1-n^h_g)}{D_1 u(c, 1-n^h)} = \beta \sum_{g=1}^{N_z} \pi^z_{fg} \frac{p(z_g;\mu')}{p(z_f;\mu)}, \tag{21}$$

where $p(z_f;\mu)$ is the output price when firms current dividends is discounted using households' subjective discount factor.

Analysis II

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Following Khan and Thomas (2013), we can rewrite equations (2)-(5) as

$$V_0(\mathbf{s_i}, z_f, \mu) = \pi_d \max_n [p(z_f,\mu)x^d(\mathbf{s_i}, z_f)] + (1-\pi_d)V(\mathbf{s_i}, z_f, \mu), \tag{22}$$

where

$$V(\mathbf{s_i}, z_f, \mu) = \max\{V^u(\mathbf{s_i}, z_f, \mu), V^d(\mathbf{s_i}, z_f, \mu)\}. \tag{23}$$

The dynamic problem for upward-adjusting firms is

$$\begin{aligned} V^u(\mathbf{s_i}; z_f; \mu) = &\max_{k',b',D} p(z_f,\mu)D + \beta \sum_{g=1}^{N_z}\sum_{j=1}^{N_s} \pi^z_{fg}\pi^s_{ij} V_0(\mathbf{s_j}'; z'_g; \mu') \\ \text{s.t. } & 0 \le D \le x^u(\mathbf{s_i}; z_f) + q_b b' - Qk' \\ & x^u(\mathbf{s_i}; z_f) = z_f s_i F(k,n) - w(z_f,\mu)n - b + Q(1-\delta)k \\ & k' \ge (1-\delta)k; \quad b' \le \zeta k; \quad \mu' = \Gamma(z_f; \mu) \end{aligned}, \tag{24}$$

Analysis III

Back

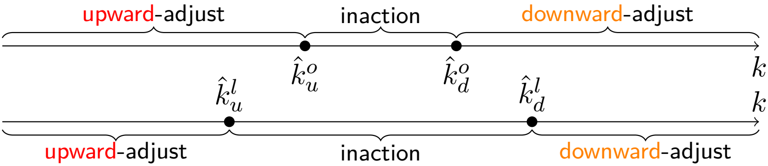
and the dynamic problem for downward-adjusting firms is

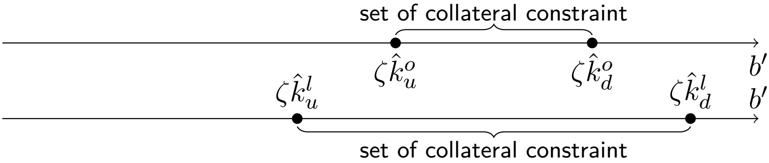
$$
\begin{aligned}
V^d(\mathbf{s_i}; z_f; \mu) = & \max_{k', b', D} p(z_f, \mu)D + \beta \sum_{g=1}^{N_z} \sum_{j=1}^{N_s} \pi^z_{fg} \pi^s_{ij} V_0(\mathbf{s_j}'; z'_g; \mu') \\
\text{s.t. } & 0 \leq D \leq x^d(\mathbf{s_i}; z_f) + q_b b' - q k' \\
& x^d(\mathbf{s_i}; z_f) = z_f s_i F(k, n) - w(z_f, \mu)n - b + q(1-\delta)k \\
& k' \leq (1-\delta)k; \quad b' \leq \zeta k; \quad \mu' = \Gamma(z_f; \mu)
\end{aligned} \quad . \tag{25}
$$

Mechanism: $k \Rightarrow b$

Back

First line: ordinary (o); Second line: recession (l)





Zero Dividend Policy: Detailed Explanation I

Back

All firms assumed to have $D = 0 \Rightarrow$ budget constraints binding.

- constrained firms cannot issue dividend: $D = 0$.
- unconstrained firms are indifferent: assume $D = 0$.

Benefit: if we know the decision on k', we know the decision on b'.

Rewrite the models in terms of $p(z_f, \mu)$,

$$V_0(\mathbf{s_i}, z_f, \mu) = \pi_d p x^d(z_f, \mu) + (1 - \pi_d) V(\mathbf{s_i}, z_f, \mu), \tag{26}$$

where

$$V(\mathbf{s_i}, z_f, \mu) = \max\{V^u(\mathbf{s_i}, z_f, \mu), V^d(\mathbf{s_i}, z_f, \mu)\}. \tag{27}$$

Zero Dividend Policy: Detailed Explanation II

Back

For upward-adjusting firm, the dynamic problem is

$$\begin{aligned} V^u(k,b,\epsilon_i,z_f,\mu) &= \max_{k'\in\Omega^u(\mathbf{s_i})} \beta \sum_{g=1}^{N_z}\sum_{j=1}^{N_\epsilon} \pi^z_{fg}\pi^\epsilon_{ij} V_0(k',b'_u(k'),\epsilon_j,z_g,\mu') \\ \text{s.t.} \quad b'_u(k') &= \frac{Qk'-x^u}{q_b} \end{aligned}, \quad (28)$$

and for downward-adjusting firm,

$$\begin{aligned} V^d(k,b,\epsilon_i,z_f,\mu) &= \max_{k'\in\Omega^d(\mathbf{s_i})} \beta \sum_{g=1}^{N_z}\sum_{j=1}^{N_\epsilon} \pi^z_{fg}\pi^\epsilon_{ij} V_0(k',b'_d(k'),\epsilon_j,z_g,\mu') \\ \text{s.t.} \quad b'_d(k') &= \frac{qk'-x^d}{q_b} \end{aligned}. \quad (29)$$

Zero Dividend Policy: Detailed Explanation III

Back

The search sets for upward and downward adjusting firms are

$$\Omega^u(\mathbf{s_i}) = [(1-\delta)k, \bar{k}_u(\mathbf{s_i})], \tag{30}$$

$$\Omega^d(\mathbf{s_i}) = [0, \min\{(1-\delta)k, \bar{k}_d(\mathbf{s_i})\}], \tag{31}$$

where $\bar{k}_u$ and $\bar{k}_d$ represents the upper bound of the search sets for the upward-adjusting and downward-adjusting firms:

$$\bar{k}_u = \frac{q_b \zeta k + x^u}{Q}, \tag{32}$$

$$\bar{k}_d = \frac{q_b \zeta k + x^d}{q}. \tag{33}$$

Zero Dividend Policy: Detailed Explanation IV

Back

Let the solution for (28) and (29) be $\hat{k}_u(\mathbf{s_i})$ and $\hat{k}_d(\mathbf{s_i})$, the policy function for capital is

$$K(\mathbf{s_i}, z_f, \mu) = \begin{cases} \hat{k}_u(\mathbf{s_i}) & \text{if } V(\cdot) = V^u(\cdot) \\ \hat{k}_d(\mathbf{s_i}) & \text{if } V(\cdot) = V^d(\cdot) \end{cases}, \tag{34}$$

and the corresponding policy function for bond is

$$B(\mathbf{s_i}, z_f, \mu) = \begin{cases} \frac{Q\hat{k}_u(\mathbf{s_i}) - x^u}{q_b} & \text{if } V(\cdot) = V^u(\cdot) \\ \frac{q\hat{k}_d(\mathbf{s_i}) - x^d}{q_b} & \text{if } V(\cdot) = V^d(\cdot) \end{cases}. \tag{35}$$

Difficulties I

Back

- Replicate Lanteri (2018) (✓)
- Difficulties:
  - Firms' next-period debt is overflowed: b/k grid $\in [-25, 1.3]$, yet b'/k' grid $\in [-26, 28]$.
  - For saving ($b/k < 0$) and medium borrowing firms ($b/k \in [0, 0.9051]$), no firm is undertaking downward-adjustment.
  - For high saving firms, $k' > (1-\delta)k$ for inaction firms.
  - For medium saving firms, $k' = (1-\delta)k$ for inaction firms.
- The debt overflow problem might be the key to solve my difficulties.

Difficulties II

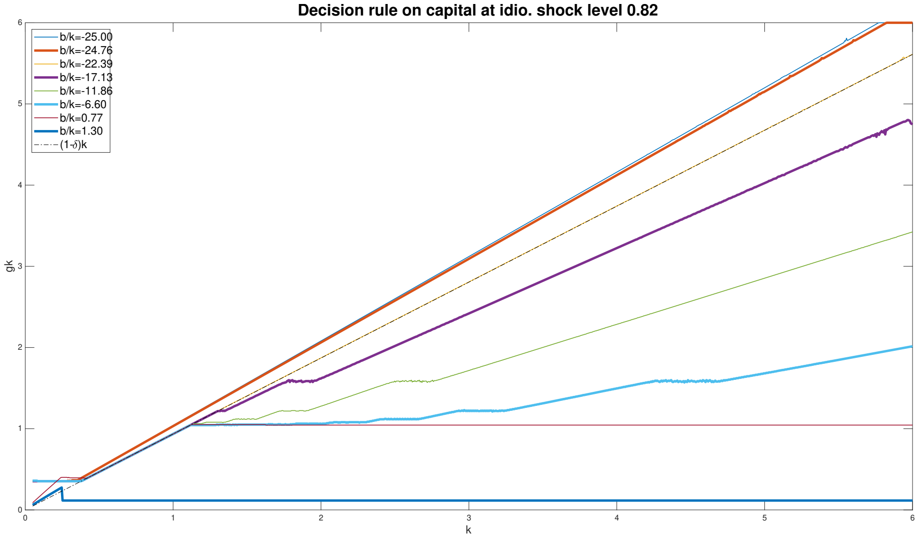
Back

Description on State Space:

- b/k grid $\in [-25, 1.3]$ & k grid $\in [0.05, 6] \Rightarrow b$ grid $\in [-150, 7.8]$.
  - Overflow bond:
    - $b'_u > 7.8$: 15; $b'_u < -150$: 89;
    - $b'_d > 7.8$: 14; $b'_d < -150$: 91.
  - b'_u & $b'_d < -150$: high saving $(b < 0)$ and high capital stock.
  - b'_u & $b'_d > 7.8$: high borrowing $(b > 0)$ and high capital stock.

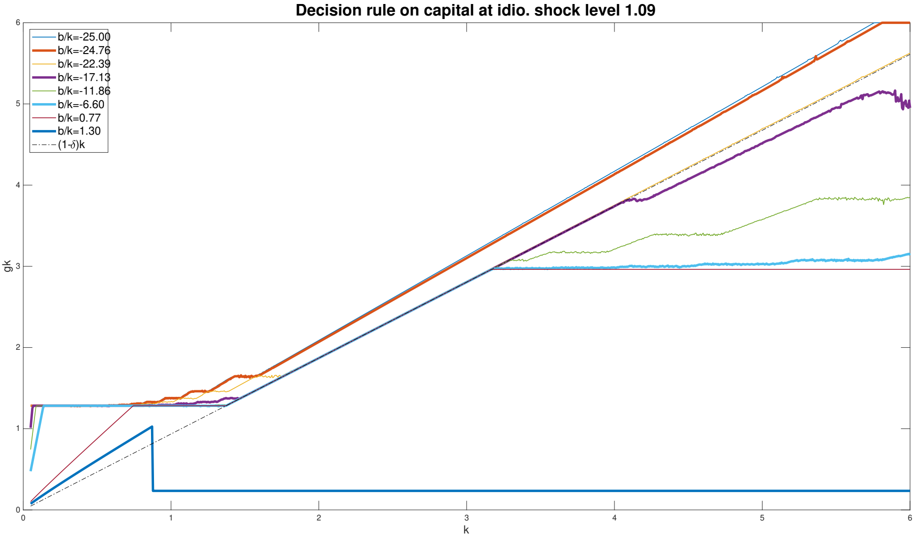
Difficulties III

Back



Difficulties IV

Back



Current Progress: Replicated Lanteri (2018) I

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My code

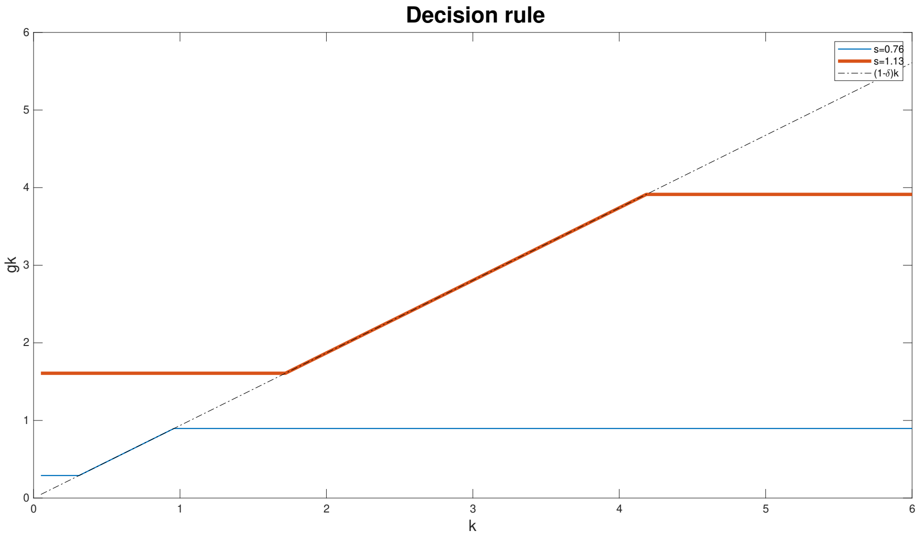
- $\frac{K}{Y} = 2.5497$; $\frac{I_{new}}{K} = 0.0652$
- $mean(I) = 0.1015$; $var(I) = 0.3368$
- inv lumpy freq $= 0.1454$
- disinv lumpy freq $= 0.1033$
- inact freq $= 0.5523$

Lanteri (2018)

- $\frac{K}{Y} = 2.5497$; $\frac{I_{new}}{K} = 0.0652$
- $mean(I) = 0.1015$; $var(I) = 0.3368$
- inv lumpy freq $= 0.1454$.
- disinv lumpy freq $= 0.1033$
- inact freq $= 0.5523$

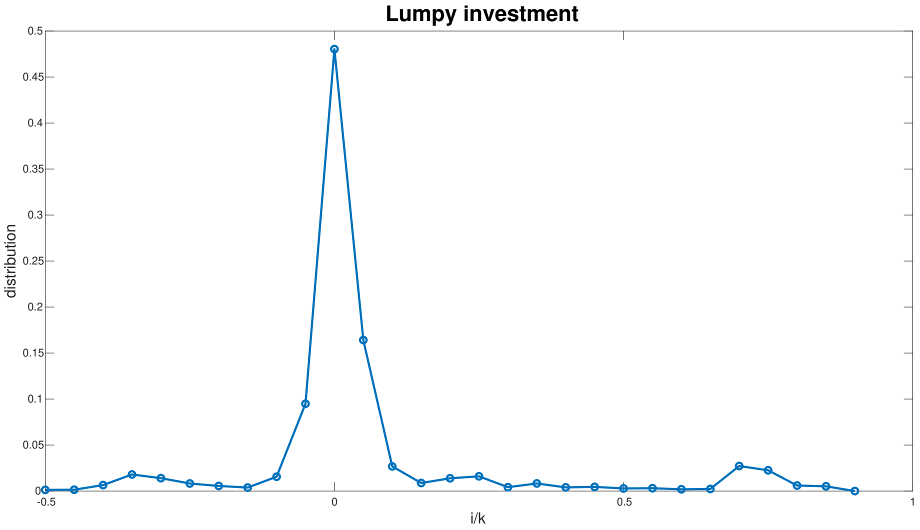
Current Progress: Replicated Lanteri (2018) II

Back



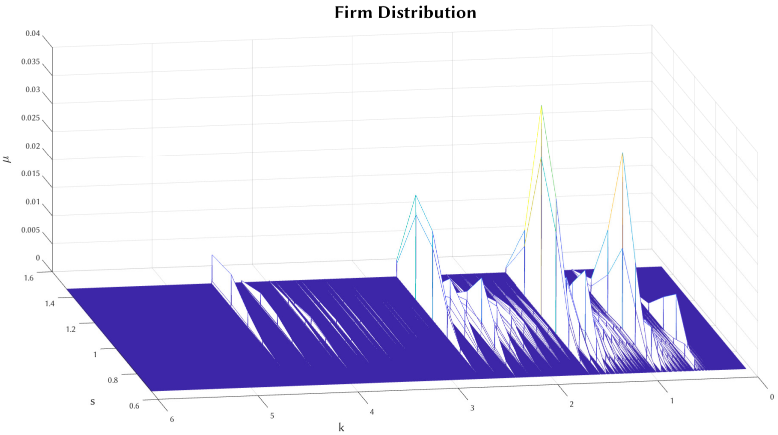
Current Progress: Replicated Lanteri (2018) III

Back



Current Progress: Replicated Lanteri (2018) IV

Back



Mechanism: $b \Rightarrow k$

Expanding credit

$\Downarrow$

lower supply in used capital market

$\Downarrow$

Even lower q and Q