Lecture 14
The Real Business Cycle Model
Part 1: Consumer

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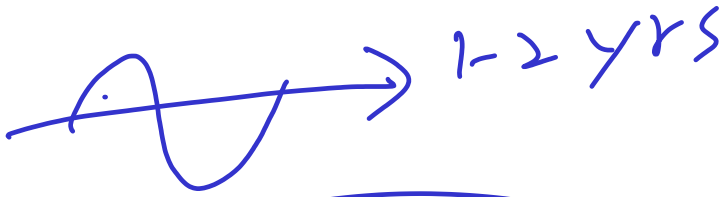
Overview

- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

Real Business Cycle Model

longer term : 3 - 30 - 50 growth

- One of the workhorse frameworks in modern macroecnomics
- Real: not about *money* and *inflation*



- Business Cycle: mainly explain the short- and medium-term economics fluctuation ("business cycle frequency")
- Three agents: representative consumer, representative firm, and government
- All agents make **static and dynamic** decisions
- Larger "scale" model (i.e., more endogenous variables), but build upon the technique learned before

Consumer: Constraints

There are 11 variables associated with the representative consumer:

- choice variables: consumption (C, C') and labor supply (N_S, N_S')
  - leisure follows labor choice: $l = h - N_S$, and $l' = h - N_S'$
- owns the firm and get profits (π, π') and pays taxes (T, T')
- taken the equilibrium price as given (w, w', r)

Saving (S) at date 0 to construct lifetime budget constraint:

$$\text{today:} \quad C + S = wN_S + \pi - T$$

$$\text{tomorrow:} \quad C' = w'N_S' + \pi' - T' + (1+r)S$$

$$\text{lifetime constraint:} \quad C + \frac{C'}{1+r} = \underbrace{wN_S + \pi - T}_{\approx Y \text{ in last lecture}} + \frac{\overbrace{w'N_S' + \pi' - T'}^{\approx Y' \text{ in last lecture}}}{1+r}$$

$$C + S = wN_s + \pi - T$$

$$C' = w'N_s' + \pi' - T' + (1+r)\cdot S$$

$$\Rightarrow S = \frac{C'}{1+r} - \frac{w'N_s' + \pi' - T'}{1+r}$$

$$\Rightarrow C + \frac{C'}{1+r} = wN_s + \pi - T + \frac{w'N_s' + \pi' - T'}{1+r}$$

Consumer: Preference

In general, utility fcn across consumption and labor choice can be mixed:

- e.g. mix C and N_S: GHH preferences
- e.g. mix current and future: Epstein–Zin preferences

Here, we are making simplified assumption: **additive for both direction**:

$$U(C, C', N_S, N_S') = u(C) - v(N_S) + u(C') - v(N_S'). \tag{1}$$

To see why additive can simplify analysis, recall the MRS in both intratemporal (w/i period) and intertemporal (b/w period) substitution:

$$MRS_{l,C} = -MRS_{N_S,C} = \frac{v'(N_S)}{u'(C)}, \text{ and } MRS_{C,C'} = \frac{u'(C)}{u'(C')}$$

Representative Consumer's Problem

$$\begin{aligned} \max_{C,C',N_S,N'_S} \quad & u(C) - v(N_S) + u(C') - v(N'_S) \\ \text{subject to} \quad & C + \frac{C'}{1+r} = wN_S + \pi - T + \frac{w'N'_S + \pi' - T'}{1+r} \end{aligned} \tag{2}$$

- Hard to analyze in graph, $\because$ 4 choices variables $\Rightarrow$ 4-dim problem!
- Yet, usual procedure in Calculus still works!
- Why? Because partial derivatives only looks the optimality in 1-dim
- Each FOC is optimal for 1-dim $\Rightarrow$ solution satisfies ALL FOCs

Consumer's Optimality Conditions

- Step 1: substitute C by budget constraint,

$$\max_{C',N_S,N_S'} u\left(wN_S+\pi-T+\frac{w'N_S'+\pi'-T'-C'}{1+r}\right) - v(N_S)+u(C')-v(N_S')$$

- Step 2: find FOCs for C', N_S, and N_S':

$$[C']: \quad u'(C')-\frac{1}{1+r}u'(C)=0 \Rightarrow u'(C')=\frac{1}{1+r}u'(C)$$

$$[N_S]: \quad wu'(C)-v'(N_S)=0 \Rightarrow wu'(C)=v'(N_S)$$

$$[N_S']: \quad \frac{w'}{1+r}u'(C)-v'(N_S')=0 \Rightarrow \frac{w'}{1+r}u'(C)=v'(N_S')$$

Consumer's Optimality Conditions (Cont.)

- Step 3: Compute multiple MRSs:

$$[C']: \quad MRS_{C,C'} = \frac{u'(C)}{u'(C')} = 1 + r$$

$$[N_S]: \quad -MRS_{N_S,C} = MRS_{l,C} = \frac{v'(N_S)}{u'(C)} = w$$

$$[N'_S]: \quad MRS_{l',C} = \frac{v'(N'_S)}{u'(C)} = \frac{w'}{1+r}$$

- Step 4: Get C by putting C', N_S and N'_S back to budget constraint.

Knowledge Gain from Consumer's Problem

We have derived 4 optimality conditions for 4 choice variables:

$$[C']: \quad MRS_{C,C'} = \frac{u'(C)}{u'(C')} = 1 + r$$

$$[N_S]: \quad -MRS_{N_S,C} = MRS_{l,C} = \frac{v'(N_S)}{u'(C)} = w$$

$$[N'_S]: \quad MRS_{l',C} = \frac{v'(N'_S)}{u'(C)} = \frac{w'}{1+r}$$

$$\text{budget constraint}: \quad C = wN_S + \pi - T + \frac{w'N'_S + \pi' - T' - C'}{1+r}$$

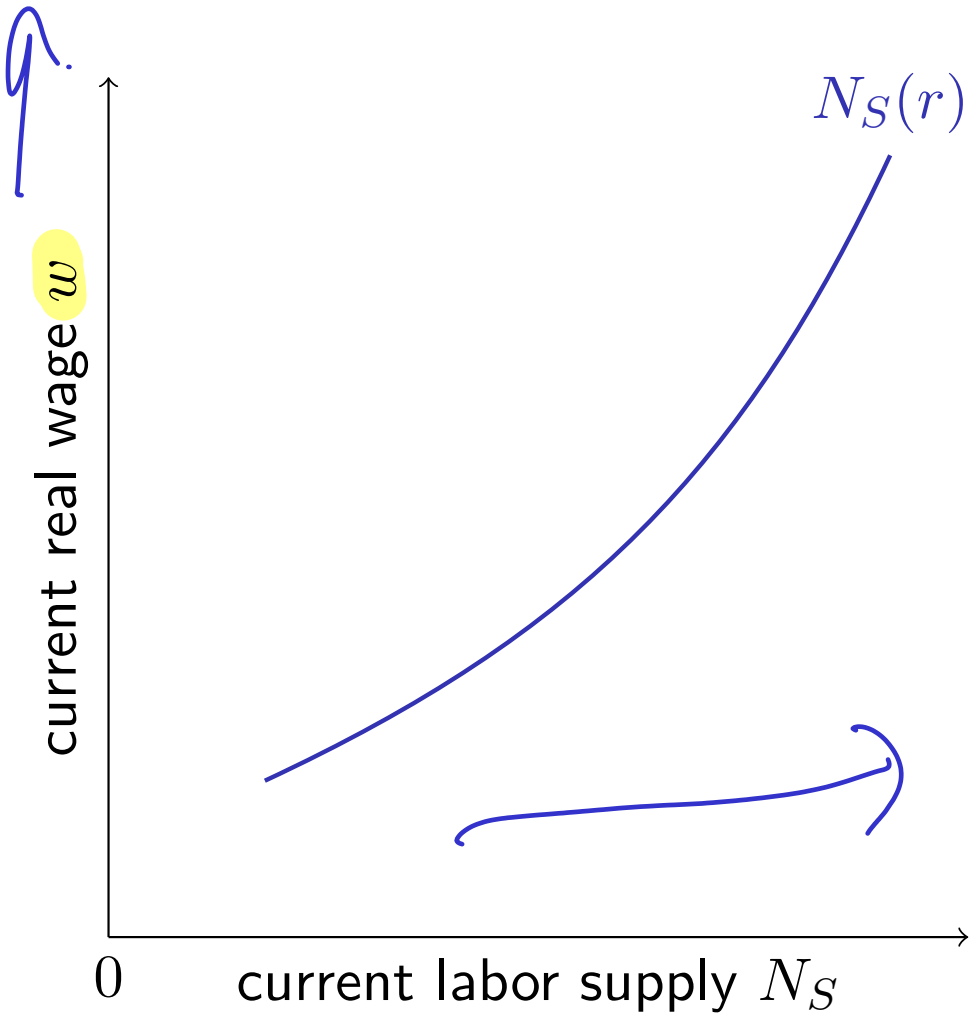
Recall that there are 11 variables, so still 7 variables remain. They are:

- 3 endogenous prices: w, w', r
- 4 endogenous quantities that shift lifetime wealth: π, π', T, T'

Need to know how consumer response to endogenous quantities!

Current Labor Supply and Current Wage

Figure 11.1 The Representative Consumer's Current Labor Supply Curve

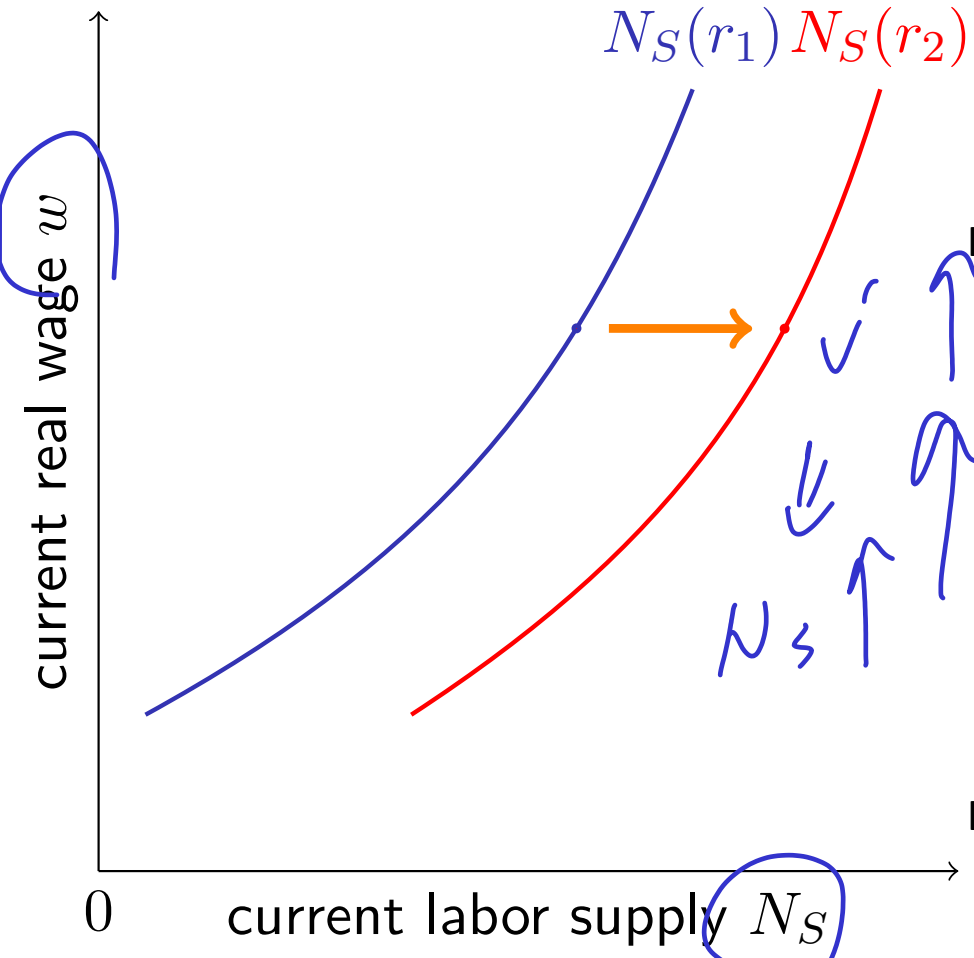


Assumption N1: current labor supply ↑ in current wage

- Recall two effects of wage on labor:
  - income (I): $l \uparrow, N_S \downarrow$
  - substitution (S): $l \downarrow, N_S \uparrow$
- **N1** suggests that substitution effect $>$ income effect
- data: (I) and (S) cancel out in long-run, while RBC focus on short- and medium run!

Current Labor Supply and Real Interest Rate

Figure 11.2 Real Interest Rate ↑ Shifts the Current Labor Supply Curve to the Right



Assumption N2: current labor supply ↑ as real interest rate ↑

- can substitute intertemporally using **both** consumption and labor
- relative price of future leisure in terms of current leisure:

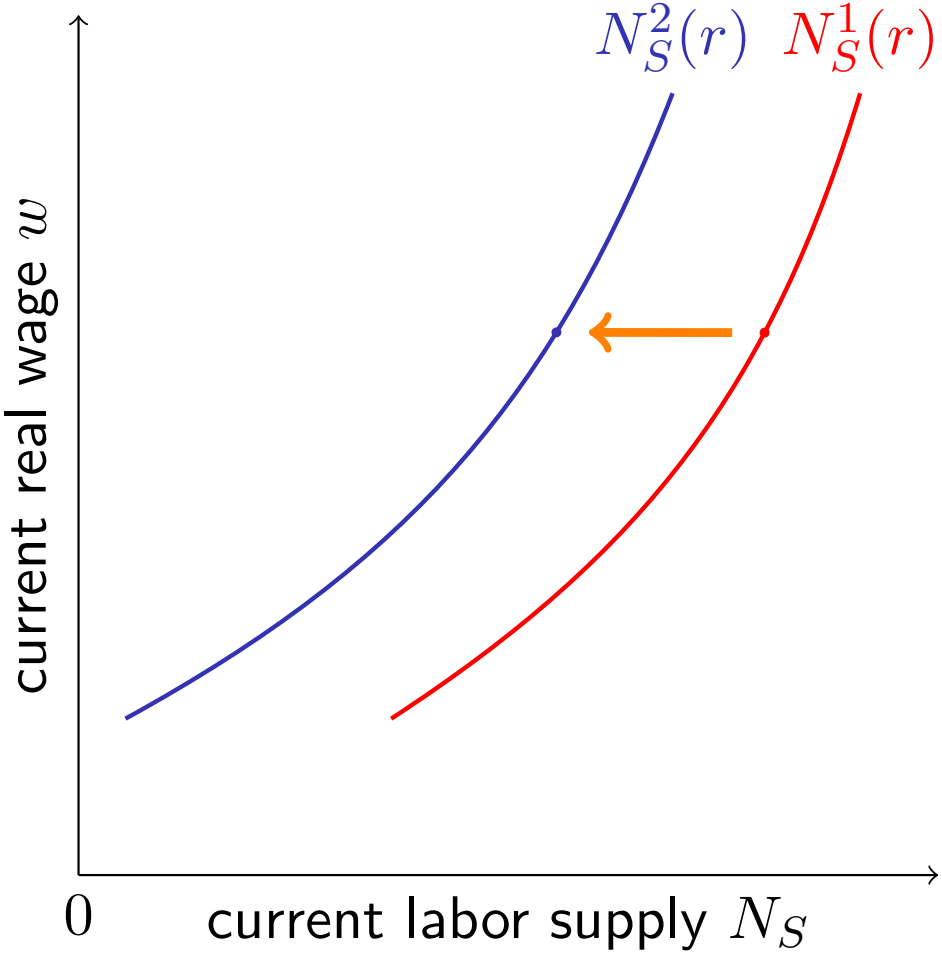
$$\frac{v'(N_S)}{v'(N_S')} = \frac{v'(N_S)}{u'(C)} \times \frac{u'(C)}{u'(C')} \times \frac{u'(C')}{v'(N_S')}$$

$$= w \times (1 + r) \times \frac{1}{w'}$$

- fix w and w', r ↑ makes l become more costly, so N_S ↑

Current Labor Supply and Wealth

Figure 11.3 Effects of an Increase in Lifetime Wealth



Assumption N3: current labor supply ↓ as lifetime wealth ↑

- only pure income effect on normal goods (consumption & leisure), and thus labor decreases

Summary of Effect on Labor Supply

- **Assumption N1**: current labor supply $\uparrow$ in current wage
  - $\frac{dN_S}{dw} > 0$
- **Assumption N2**: current labor supply $\uparrow$ as real interest rate $\uparrow$
  - $\frac{dN_S}{dr} > 0$
- **Assumption N3**: current labor supply $\downarrow$ as lifetime wealth $\uparrow$
  - $\frac{dN_S}{dx} < 0$, where $x = \pi - T$.

All statements are properties about **supply curve**, not equilibrium quantities!