Final exam review

① 2 period dynamic general equilibrium model.
representative consumer
firms
gov.

Go to Lecture 16, 17, and 18.

② Lucas human capital accumulation (1988 JME)

2 period

household accumulate human capital by spend time in school.

wn

HH utility: $u(c)+u(c')$ (Doesn't value leisure)

endowed with H of human capital at date 0

H law of motion: $H' = H + (1-\phi)H = (1+1-\phi)H$

$\Rightarrow (2-\phi)H$

$1-\phi$: fraction of time spent in school.

$\in(0,1)$

HH endowed with K unit of physical capital; and

$K' = (1-\delta)K + I$

firms rent physical capital from HH with rent r.

$$Y = K^{\alpha}(\phi H)^{1-\phi} \qquad Y' = (K')^{\alpha}(\phi' H')^{1-\alpha}$$

No gov.

Consumer's budget constraint:

labor income → renting capital.

$$C \le w\phi H + rK - I + \pi$$ ← dividend.

where $\pi = Y - w\phi H - rK$

Can we solve in Social Planner's Problem?

$$C \le \cancel{w\phi H} + \cancel{rK} - I + Y - \cancel{w\phi H} - \cancel{rK}.$$

$\Rightarrow C \le Y - I$ date 0 resource constraint.

$C' \le Y'$ ∵ no third period. ($I' = 0$)

date 1

Social planner's Problem:

$$\max_{C, C', K', H', \phi} u(C) + u(C')$$

s.t.

$$C = Y - I \Rightarrow C = K^{\alpha}(\phi H)^{1-\alpha} - [K' - (1-\delta)K]$$

$$C' = Y' \Rightarrow C' = (K')^{\alpha}(\phi' H')^{1-\alpha}$$

$$\Downarrow$$

$$C' = (K')^{\alpha}(\phi'(2-\phi)H)^{1-\alpha}$$

$$Y = K^{\alpha}(\phi H)^{1-\alpha}$$

$$Y' = (K')^{\alpha}(\phi' H')^{1-\alpha}$$

$$H' = (2-\phi)H$$

$$K' = (1-\delta)K + I \Rightarrow I = K' - (1-\delta)K$$

$$\max_{K', \phi} u(\underbrace{K^{\alpha}(\phi H)^{1-\alpha} - [K' - (1-\delta)K]}_{C})$$

$$+ u(\underbrace{(K')^{\alpha}(\cancel{\phi'}^{\,1}(2-\phi)H)^{1-\alpha}}_{C'})$$

$1-\phi'$: fraction of time that HH devote to education at date 1 (tomorrow.)

work

No third period.

$$\Downarrow$$

$\phi' = 1 \Rightarrow$ devote all time endowment to work!

$$\max_{K', \phi} u(K^{\alpha}(\phi H)^{1-\alpha} - [K' - (1-\delta)K]) + u((K')^{\alpha}((2-\phi)H)^{1-\alpha})$$

$$[K']: -u'(C) + u'(C') \cdot ((2-\phi)H)^{1-\alpha} \alpha (K')^{\alpha-1} = 0$$

$$\Rightarrow \frac{u'(C)}{u'(C')} = \alpha (K')^{\alpha-1}((2-\phi)H)^{1-\alpha}$$

$$[\phi]: u'(C) \cdot K^{\alpha} \cdot (1-\alpha)(\phi H)^{-\alpha} \cdot H$$

$$+ u'(C') \cdot (K')^{\alpha}(1-\alpha)((2-\phi)H)^{-\alpha} \cdot (-H) = 0$$

$$\Rightarrow u'(C) K^{\alpha} \cancel{(1-\alpha)} \underbrace{(\phi H)^{-\alpha}}_{\phi^{-\alpha}\cancel{H^{-\alpha}}} \cancel{H} = u'(C')(K')^{\alpha} \cancel{(1-\alpha)} \underbrace{((2-\phi)H)^{-\alpha}}_{(2-\phi)^{-\alpha}\cancel{H^{-\alpha}}} \cancel{H}$$

$$\Rightarrow u'(c)\,K^{\alpha}\phi^{-\alpha} = u'(c')\,(K')^{\alpha}(2-\phi)^{-\alpha}$$

$$\Rightarrow \frac{u'(c)}{u'(c')} = \left(\frac{K'}{K}\right)^{\alpha}\left(\frac{2-\phi}{\phi}\right)^{-\alpha}$$

$$\alpha(K')^{\alpha-1}\left((2-\phi)H\right)^{1-\alpha} = \left(\frac{K'}{K}\right)^{\alpha}\left(\frac{2-\phi}{\phi}\right)^{-\alpha}$$

$$\alpha\,\cancel{(K')^{\alpha-1}}\,(2-\phi)^{1-\alpha}H^{1-\alpha} = (K')^{\cancel{\alpha}}(K)^{-\alpha}\frac{1}{(2-\phi)^{\cancel{-\alpha}}}\phi^{\alpha}$$

$$K'^{\,\cancel{\alpha}-\cancel{\alpha}+1} \qquad (2-\phi)^{\cancel{-\alpha}-1+\cancel{\alpha}}$$

$$\Rightarrow \boxed{\frac{\phi^{\alpha}}{2-\phi}}\,\boxed{K'} = \boxed{\alpha\cdot K^{\alpha}H^{1-\alpha}} \rightarrow \text{fixed number.}$$

parameter endowment

① return on human capital

return on physical capital ②

* in equilibrium *

* optimal investment schedule. *

$$r = \underbrace{MPK' - \delta}$$

return on saving

return on capital

③ Solow growth model

(per) Labor productivity: $r > 0 \Rightarrow X_{t+1} = (1+r) X_t$.

$\Rightarrow \frac{X_{t+1}}{X_t} = 1+r$.

Population growth: $n > 0 \Rightarrow L_{t+1} = (1+n) L_t$

Effective labor force $\Rightarrow N_t = X_t L_t$

Production: $Y_t = A K_t^{\alpha} N_t^{1-\alpha}$

$\Rightarrow \begin{cases} C_t = (1-s) Y_t \\ C_t = Y_t - I_t \Rightarrow I_t = s Y_t \\ \delta = 1 \Rightarrow K_{t+1} = I_t \Rightarrow K_{t+1} = s Y_t \end{cases}$

What's the growth rate of N: $\frac{N_{t+1}}{N_t}$

$$\frac{N_{t+1}}{N_t} = \frac{X_{t+1} L_{t+1}}{X_t L_t} = \frac{(1+r)\cdot(1+n) X_t L_t}{X_t L_t} = (1+r)(1+n)$$

"effeciency unit of capital": ① $k_t = \frac{K_t}{N_t}$, ② $k_{t+1} = \frac{K_{t+1}}{N_{t+1}}$

③ $K_{t+1} = s Y_t \xRightarrow{\div N_t} \frac{K_{t+1}}{N_t} = s \frac{Y_t}{N_t}$

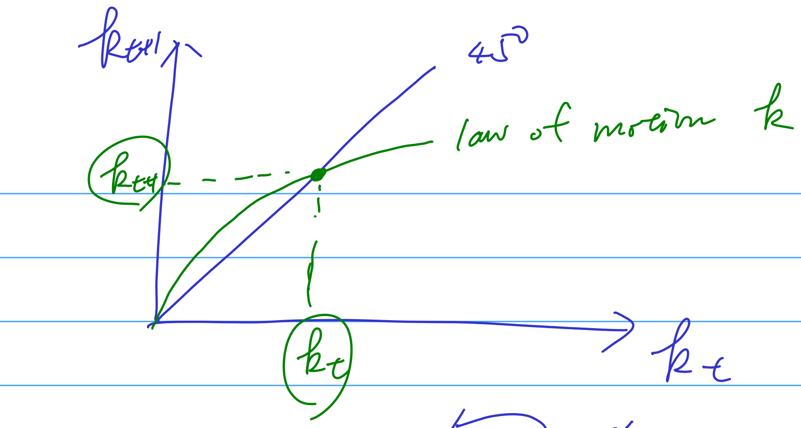
$$\frac{K_{t+1}}{N_t} = s \frac{Y_t}{N_t}$$

$$\underbrace{\frac{K_{t+1}}{N_{t+1}}}_{k_{t+1}} \cdot \frac{N_{t+1}}{N_t}$$

$$k_{t+1} (1+r)(1+n) = s \cdot \frac{A K_t^{\alpha} N_t^{\cancel{1-\alpha}}}{\cancel{N_t}} = s \cdot A \cdot \Big(\underbrace{\frac{K_t}{N_t}}_{k_t}\Big)^{\alpha} = s A k_t^{\alpha}$$

law of motion

effeciency unit of capital



$$k^*(1+r)(1+n) = sA(k^*)^{\alpha}$$

$$(k^*)^{1-\alpha} = \frac{sA}{(1+r)(1+n)} \Rightarrow (k^*) = \left(\frac{sA}{(1+r)(1+n)}\right)^{\frac{1}{1-\alpha}}$$

If there's 2 economy: a & b

b: higher saving rate $s_b > s_a$

higher labor productivity growth: $r_b > r_a$

$a=b$: $\frac{s_a}{1+r_a} = \frac{s_b}{1+r_b}$

$k_a \gtreqless k_b$? $\Rightarrow$ $k_a = k_b$ $\frac{s_a A}{(1+r_a)(1+n)} = \frac{s_b A}{(1+r_b)(1+n)}$