ECON 4002.01 Midterm Exam

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Instruction

Please submit your answer on Carmen Quiz "Midterm Exam".

All numerical answers written on the exam sheet are supposed to **round to the second decimal point**[1] unless otherwise noted.

However, if you choose to use software to calculate the numerical answer, **do not** use the rounded answer from previous question to calculate; use the un-rounded answer to calculate.

You **may** consult any note and textbook, but you **cannot** discuss with your classmate or any other person about the exam.

For numerical answer, you are **recommended** to use software to calculate answers.

The written answer should be recognizable. If I cannot recognize the answer you wrote, then I may not be able to give you credit.

There will be one T/F choice question that is worth 2 points on the answer sheet. The T/F choice question is to confirm: "**I affirm that I have not received or given any unauthorized help on this exam, and that all work is my own.**"

Grades

Question 1 to 40 are worth **2** points, and Question 41 to 44 are worth **5** points. The total grade is **102** points.

[1] round to the second decimal points means that if the third decimal point is a number between 0 to 4, then just get rid of the third decimal point. On the other hand, if the third decimal point is a number between 5 to 9, then round the second decimal point up by adding 1 to the second decimal point number. For example, if the answer you get is 0.534, then round it to 0.53. Yet, if the answer is 0.535, then round it up to 0.54.

Question 1

Considering an one-period general equilibrium model **similar to** (but not exactly the same as) Example in Lecture 08, slide 11 and 12. Also the Experiment 2 from Lecture 07, slide 13 is also a good reference. However, in this model economy, there are two differences:

1. firm rent capital from consumer, and consumers are **endowed** with 2 units of capital ($K^s = 2$).
2. consumer's utility function is $U(C, l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$

1 The competitive equilibrium given $\{G, z, \underline{\ K^s\ }\}$

2 is a set of allocations $\{Y^*, C^*, l^*, N^s, N^d, \pi^*, T^*, \underline{\ K^{d*}\ }\}$

3 and prices $\{w^*, \underline{\ r^*\ }\}$ such that

1. Taken prices and π, T as given, the representative consumer solves

4 $$\max_{\underline{C,l}} U(C, l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$$

5 subject to $C \leq w(h - l) + \underline{\ rK^s\ } + \pi - T$

2. Taken prices as given, the representative firm solves

6 $$\max_{\underline{K^d, N^d}}$$

7 $$z(K^d)^a(N^d)^{1-a} - wN^d - \underline{\ rK^d\ }$$

3. Government collect taxes to balance budget:

8 $\underline{\ T^* = G\ }$

4. Labor market clear means that the equilibrium wage is w^* such that labor supply equals to labor demand:

(9) $\underline{\quad N^s = N^d \quad}$

5. Capital market clear means that the equilibrium rental rate is r^* such that capital supply equals to capital demand:

(10) $\underline{\quad K^s = K^d \quad}$

To solve this model economy, we reformulate the competitive equilibrium into the social planner's problem.

First of all, in social planner's problem, all markets must clear, and thus $N^s = N^d = N$, and $K^s = K^d = K$ (also $K = 2$, but for question 11 to 16, still remain the symbol K).

Through firm's FOC with respect to N and K, we know w and r are

(11) $w = zK^a \underline{\quad (1-a)N^{-a} \quad}$

(12) $r = zN^{1-a} \underline{\quad aK^{a-1} \quad}$

which we can use to retrieve wage and rental rate after solving the social planner's problem.

The social planner problem is given by:

(13) Objective function is the consumer's utility:

$$\max_{C,l,N,Y,\underline{\quad K \quad}} U(C,l) = \frac{C^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$$

subject to

a. Aggregate resource constraint

(14) $C + G = \underline{\quad Y \quad}$

b. production constraint

(15) $Y = z\ \underline{K^a N^{1-a}}$

c. labor constraint

(16) $N = \underline{1 - l}$

d. capital constraint

(17) $K = \underline{2}$

To solve the social planner's problem, we start with substituting the constraints into utility function:

a. Substituting 16 and 17 into 15, we get

(18) $Y = z\ \underline{2^a(1-l)^{1-a}}$

b. Substituting 18 into 14, we get

(19) $C = z\ \underline{2^a(1-l)^{1-a} - G}$

c. Finally, substituting 19 into 13, we get

(20) $\max\limits_{\underline{l}}$

(21) $U(C(l), l) = \frac{(z\ \underline{2^a(1-l)^{1-a} - G}\ )^{1-b}}{1-b} + \frac{l^{1-d}}{1-d}$

Let $z = 1, G = 0, a = \frac{1}{2}, b = 2, d = \frac{3}{2}$ and solve for all unknowns,

(22) $l = \underline{0.67}$

(23) $N = \underline{0.33}$

(24) $w^* =$ $\underline{\quad 1.23 \quad}$

(25) $r^* =$ $\underline{\quad 0.20 \quad}$

The following is the calculation for the answer from 22 to 25:

FOC results in $l^{-d} = z^{-b}2^{-ab}(1-l)^{-b+ab}(1-a)z2^{a}(1-l)^{-a}$

$$l^{-d} = z^{1-b}2^{-ab+a}(1-a)(1-l)^{-a-b+ab}$$

$$l^{-\frac{3}{2}} = \frac{1}{2\sqrt{2}z}(1-l)^{-\frac{3}{2}}$$

$$\left(\frac{1-l}{l}\right)^{\frac{3}{2}} = \frac{1}{2\sqrt{2}z} = \frac{1}{2\sqrt{2}}$$

$$\left(\frac{1-l}{l}\right) = \left(\frac{1}{2\sqrt{2}}\right)^{\frac{2}{3}} \approx 0.4999 \Rightarrow 1-l = 0.4999l \Rightarrow l \approx 0.6666 \approx 0.67$$

$N = 1 - l = 0.33$, $w = (1-a)zK^{a}N^{-a} = 1.231 \approx 1.23$,

$$r = azK^{a-1}N^{1-a} \approx 0.203 \approx 0.20$$

Question 2

Consider a model economy with distorting labor taxes similar to Lecture 11, but having two difference:

1. the production function is Cobb-Douglas requiring only labor input, i.e., $Y = zN^a$, and
2. consumer get "disutility" from working, and the utility function is given by $U(C, N) = \ln C - bN$, where b is a parameter.

Other than the two changes mentioned above, the definition of the general equilibrium is exactly the same as slide 5 in Lecture 11. Therefore, this question focus mainly on algebraic calculation.

26 The derivative of utility function with respect to consumption C is $\underline{\quad \frac{1}{C} \quad}$

27 The derivative of utility function with respect to labor N is $\underline{\quad -b \quad}$

28 The marginal rate of substitution between labor and consumption ($MRS_{N,C}$) is $\underline{\quad -bC \quad}$

29 According to the slide 6 and 9, Lecture 11, in equilibrium MRS is going to be equal to the after-tax wage, i.e., $\underline{\quad w(1-t) \quad}$

30 According to the slide 5, Lecture 11, consumer's budget constraint is saying that $C = \underline{\quad w(1-t) \quad} N + \pi$

31 Different from slide 5, Lecture 11, now the production function is $Y = zN^a$, and thus by solving the firm's problem, the equilibrium wage as a function of labor demand N must be $w = z \underline{\quad aN^{a-1} \quad}$

32 Following 31, firm's profit $\pi = z \underline{\quad (1-a)N^a \quad}$

The $MRS_{N,C}$ we calculated above is MRS between labor and consumption. The equilibrium condition that in the lecture 11 is according to the $MRS_{l,C}$, the MRS between leisure and consumption. Recall that $l = 1 - N$, and thus $MRS_{l,C} = -MRS_{N,C}$.

(33) Combining your answer in 28 and 29 together, let $MRS_{l,C} = -MRS_{N,C}$, and substitute consumption from your answer in 30, we get the optimal condition is $b(\underline{\;w(1-t)\;}N + \pi) = w(1-t)$

The following three blanks are corresponds to 34 to 36. Combining your answer in 31 and 32 and substitute w and π into your answer in 33, we get

$$b(\underbrace{z\,\underline{aN^a(1-t)}}_{w \text{ part}} + \underbrace{z\,\underline{(1-a)N^a}}_{\pi \text{ part}}) = \underbrace{z\,\underline{aN^{a-1}}}_{w \text{ part 2}}(1-t)$$

(34) $\underbrace{z\,\underline{aN^a(1-t)}}_{w \text{ part}}$

(35) $\underbrace{z\,\underline{(1-a)N^a}}_{\pi \text{ part}}$

(36) $\underbrace{z\,\underline{aN^{a-1}}}_{w \text{ part 2}}$

(37) Solve for N as a function of t, we get $N = \frac{a(1-t)}{b(\underline{\;a(1-t)\;}+(1-a))}$

Some calculation details:

$b(azN^a(1-t) + (1-a)zN^a) = azN^{a-1}(1-t)$

$\Rightarrow bzN^a(a(1-t) + (1-a)) = azN^{a-1}(1-t)$

$\Rightarrow bN(a(1-t) + (1-a)) = a(1-t)$

$\Rightarrow N = \frac{a(1-t)}{b(a(1-t)+(1-a))}$

(38) Now let $z = 1$, $a = 0.33$, $b = 2.15$ and $t = 0.5$, the N you calculated in 37 is

$\underline{\quad 0.0919 \approx 0.09 \quad}$

(39) after calculate the approximated value for N to the second decimal point, you can also calculate $w = \underline{\quad 1.66 \quad}$

(40) Same for $\pi = \underline{\quad 0.30 \quad}$

(41) For the tax revenue generate by $t = 0.5$, how much government spending G can the government pay off? **For this question, please round to the fourth decimal point** $G = R(t) = wtN = \underline{\quad 0.0746999 \approx 0.0747 \quad}$

(42) According to Laffer curve, there's also another tax rate t_2 such that it can also generate same amount of revenue to pay for the government spending you've calculated 41. What is t_2? $t_2 \approx \underline{\quad 0.9387 \approx 0.94 \quad}$

If you want to solve 42 numerically, you can use the following code chuck and supplement the corresponding N and w:

```
z = 1; a = 0.33; b = 2.15; tnum = 1000
tgrid = collect( range(0.01, 0.99, length = tnum) )
Gvec = Array{Float64, 1}(undef, tnum)
for indt = 1:1:tnum
    t = tgrid[indt]
    N = your answer in 37
    w = z * your answer in 31
    Gtarget = your answer in 41
    G = w*t*N
    if abs(G - Gtarget) < 1e-4
        println("another tax rate is $t at $indt")
    end
    Gvec[indt] = G
end
```

Given that $z = 1, a = 0.33$, and $b = 2.15$, if government wants to maximize the labor tax revenue $R(t) = wtN$,

(43) the optimal tax rate $t^* =$ $\underline{\quad 0.76768 \approx 0.77 \quad}$,

(44) and the optimal labor tax revenue $R(t^*) =$ $\underline{\quad 0.09285 \approx 0.0929 \quad}$ (**For this question, please round to the fourth decimal point**)

If you want to solve the above two questions numerically, you can use the following code chuck (need to let the above code chuck to run first):

```
Gmax = maximum(Gvec)
tmax = tgrid[argmax(Gvec)]
println("optimal t* = $tmax, optimal G* = $Gmax")
```