# ECON 4002.01 Problem Set 4
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## Question 1

Consider a model that is **similar to** (not exactly the same!) Lecture 17 RBC model but with several differences:

1. Now consumer values leisure in date $1$. The lifetime utility function is given by
$$U(C, N, C', N') = \ln C + \ln(1 - N) + \ln C' + \ln(1 - N').$$

First, we start by defining the competitive equilibrium:

(1) Given the exogenous quantities ___A___

    (A) $\{G, G', z, z', K\}$             (B) $\{G, G', z, z'\}$

    (C) $\{G, G'\}$                    (D) $\{z, z', K\}$

a competitive equilibrium is a set of

(2) consumer choices ___C___

    (A) $\{C, C', N_S, S\}$            (B) $\{N_S, N'_S, l, l', S\}$

    (C) $\{C, C', N_S, N'_S, l, l', S\}$      (D) $\{C, C', S\}$

(3) firm choices ___B___

    (A) $\{Y, Y', N_D, N'_D, I, K'\}$       (B) $\{Y, Y', \pi, \pi', N_D, N'_D, I, K'\}$

    (C) $\{Y, Y', \pi, \pi', I, K'\}$         (D) $\{\pi, \pi', N_D, N'_D, I, K'\}$

(4) government choices ___D___

(A)  $\{G, G', T, T', B\}$         (B)  $\{G, G', B\}$

(C)  $\{G, G', T, T'\}$         (D)  $\{T, T', B\}$

(5) and prices ___B___

(A)  $\{w, w', q\}$         (B)  $\{w, w', r\}$

(C)  $\{q, q', r\}$         (D)  $\{r, r', q\}$

such that

  1.

(6) Taken ___A___

    (A)  $\{w, w', r, \pi, \pi'\}$         (B)  $\{w, w', r\}$

    (C)  $\{w, w', \pi, \pi'\}$         (D)  $\{r, \pi, \pi'\}$

    as given,

(7) consumer chooses ___D___

    (A)  $\{r', N_S, N'_S\}$         (B)  $\{C', K, K'\}$

    (C)  $\{r', K, K'\}$         (D)  $\{C', N_S, N'_S\}$

    to solve

$$\max_{C', N_S, N'_S} \ln\left(wN_S + \pi - T + \frac{w'N'_S + \pi' - T' - C'}{1 + r}\right).$$
$$+ \ln C' + \ln(1 - N_S) + \ln(1 - N'_S)$$

    where we can back out $\{C, S, l, l'\}$.

  2.

(8) Taken ___B___ as given,

    (A)  $\{w, w', q\}$         (B)  $\{w, w', r\}$

    (C)  $\{q, q', r\}$         (D)  $\{r, r', q\}$

(9) firm chooses __C__

(A) $\{H_D, H_D', K'\}$        (B) $\{N_D, N_D', C'\}$

(C) $\{N_D, N_D', K'\}$        (D) $\{\pi, \pi', K'\}$

to solve

$$\max_{N_D, N_D', K'} zK^\alpha N_D^{1-\alpha} - wN_D - [K' - (1-\delta)K] +$$
$$\frac{z'(K')^\alpha(N_D')^{1-\alpha} - w'N_D' + (1-\delta)K'}{1+r} \ .$$

where we can back out $\{Y, Y', \pi, \pi', I\}$.

3.

(10) Taxes and deficit satisfy __D__

(A) $T + \frac{T'}{1+q} = G + \frac{G'}{1+q}$        (B) $T + \frac{T'}{1+r} = G + \frac{G'}{1+r}$

(C) $T + \frac{T'}{1+w} = G + \frac{G'}{1+w}$        (D) $\pi + \frac{\pi'}{1+r} = G + \frac{G'}{1+r}$

and $G - T = B$.

4. All markets clear: (i) labor, $N_S = N_D$ & $N_S' = N_D'$; (ii) goods, $Y = C + G$ & $Y' = C' + G'$; (iii) bonds at date 0, $S = B$.

After defining the competitive equilibrium, now we are going to solve this model.

Step 1: Labor market

(11) From the lecture, we know that the current marginal product of labor ($MPN$) will equal to current wage. $MPN = $ __D__

(A) $z'(1-\alpha)\left(\frac{K}{N_D}\right)^\alpha$        (B) $z(1-\alpha)\left(\frac{K'}{N_D}\right)^\alpha$

(C) $z'(1-\alpha)\left(\frac{K'}{N_D'}\right)^\alpha$        (D) $z(1-\alpha)\left(\frac{K}{N_D}\right)^\alpha$

(12) and thus the current labor demand $N_D$ given the wage $w$ is __C__

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(A) $N_D = \left(\frac{z'(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$      (B) $N_D = \left(\frac{z(1-\alpha)}{w'}\right)^{\frac{1}{\alpha}} K$

(C) $N_D = \left(\frac{z(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$      (D) $N_D = \left(\frac{z'(1-\alpha)}{w'}\right)^{\frac{1}{\alpha}} K'$

⑬ From the lecture, we know that the future marginal product of labor ($MPN'$) will equal to future wage. $MPN' = \underline{\quad C \quad}$

(A) $z'(1-\alpha)\left(\frac{K}{N_D}\right)^{\alpha}$      (B) $z(1-\alpha)\left(\frac{K'}{N_D}\right)^{\alpha}$

(C) $z'(1-\alpha)\left(\frac{K'}{N_D'}\right)^{\alpha}$      (D) $z(1-\alpha)\left(\frac{K}{N_D}\right)^{\alpha}$

⑭ and thus the future labor demand $N_D'$ given the future wage $w'$ is $\underline{\quad D \quad}$

(A) $N_D' = \left(\frac{z'(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$      (B) $N_D' = \left(\frac{z(1-\alpha)}{w'}\right)^{\frac{1}{\alpha}} K$

(C) $N_D' = \left(\frac{z(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} K$      (D) $N_D' = \left(\frac{z'(1-\alpha)}{w'}\right)^{\frac{1}{\alpha}} K'$

In the labor supply part, we know that the marginal rate of substitution between leisure and consumption $MRS_{l,C}$ equals to the wage.

⑮ $MRS_{l,C} = \underline{\quad A \quad}$

(A) $\frac{C}{1-N_S}$      (B) $\frac{1-N_S}{C}$

(C) $\frac{N_S}{1-C}$      (D) $\frac{N_S'}{1-N_S}$

In the saving part, we know that the marginal rate of substitution between current and future consumption $MRS_{C,C'}$ equals to the real interest rate $(1+r)$

⑯ $MRS_{C,C'} = \underline{\quad C \quad}$

(A) $\frac{N_S'}{N_S}$      (B) $\frac{C}{C'}$

(C) $\frac{C'}{C}$      (D) $\frac{N_S}{N_S'}$

(17) Solve for $C'$, we get ___B___

(A) $C' = (1+r)N_S$  (B) $C' = (1+r)C$

(C) $C' = (1+r)C'$  (D) $C' = (1+r)N_S'$

Start from now we denote the income that is not directly affected by consumer choice as $x$ and $x'$, similar to Lecture 17.

(18) Substitute $C'$ using your answer in 17 into the budget constraint and solve for $C$, we get ___A___

(A) $C = \frac{1}{2}\left(wN_S + x + \frac{x'}{1+r}\right)$  (B) $C = \frac{1}{1+\beta}\left(wN_S + x + \frac{x'}{1+r}\right)$

(C) $C = \frac{1}{1+\beta}\left(wN_S + C' + \frac{C'}{1+r}\right)$  (D) $C = \frac{1}{2}\left(wN_S + N_S' + \frac{N_S'}{1+r}\right)$

(19) Substitute your answer of 18 into your answer in 15, we can solve the labor supply $N_S = $ ___D___

(A) $\frac{1}{3} - \frac{2}{3w}\left(x + \frac{x'}{1+r}\right)$  (B) $\frac{2}{3} - \frac{w}{3}\left(x + \frac{x'}{1+r}\right)$

(C) $\frac{2}{5} - \frac{5}{3w}\left(x + \frac{x'}{1+r}\right)$  (D) $\frac{2}{3} - \frac{1}{3w}\left(x + \frac{x'}{1+r}\right)$

(20) From 12 we solve for labor demand $N_D$. From 19 we solve for labor supply $N_S$. If for this question we let $\alpha = 1$, then we can solve the wage $w$ as a function of real interest rate $r$ as ___C___

(A) $w^*(r) = x + \frac{x'}{1+r}$  (B) $w^*(r) = \frac{1}{3}\left(x + \frac{x'}{1+r}\right)$

(C) $w^*(r) = \frac{1}{2}\left(x + \frac{x'}{1+r}\right)$  (D) $w^*(r) = zK\left(x + \frac{x'}{1+r}\right)$

For the output demand curve, we know that the optimal investment schedule is given by $MPK' - \delta = r$.

(21) We know that the $MPK'$ is ___B___

(A) $\alpha z K^{\alpha-1}N^{1-\alpha}$  (B) $\alpha z' K'^{\alpha-1}N'^{1-\alpha}$

(C) $(1-\alpha)z'K'^{\alpha}N'^{-\alpha}$  (D) $\alpha z K^{\alpha}N^{-\alpha}$

(22) We can solve the optimal investment schedule and get $K' = \underline{\quad C \quad}$

(A) $\left(\frac{z'\alpha}{q+\delta}\right)^{\frac{1}{1-\alpha}} N'$

(B) $\left(\frac{z'\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} N$

(C) $\left(\frac{z'\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} N'$

(D) $\left(\frac{z'\alpha}{q+\delta}\right)^{\frac{1}{1-\alpha}} N$

(23) and the investment $I_D$ is determined by capital accumulation process $K' - (1 - \delta)K$ and is $\underline{\quad D \quad}$

(A) $\left(\frac{z'\alpha}{q+\delta}\right)^{\frac{1}{1-\alpha}} N' - (1-\delta)K$

(B) $\left(\frac{z'\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} N - (1-\delta)K$

(C) $\left(\frac{z'\alpha}{q+\delta}\right)^{\frac{1}{1-\alpha}} N - (1-\delta)K$

(D) $\left(\frac{z'\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} N' - (1-\delta)K$

(24) Based on your answer in 23, the investment demand $I_D$ is $\underline{\quad A \quad}$ in future labor $N'$.

(A) increasing

(B) no related

(C) decreasing