Asset Pricing in Production Economy

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Overview

How to value a firm?

- We extend Lucas (1978) to production economy $\Rightarrow$ **firms**
- firms are active player in macro: **investment v.s. GDP volatility**
  - corporate finance: firm debt? capital investment?
  - human resource: hiring / lay off employee?
  - international economics: multi-nation enterprise? FDI?
- To be able to reach some conclusion, we need simplification:
  - similar setting as Lucas (1978), representative HH & firm
  - firm pays dividend $\Leftarrow$ firm are **DRS**
  - labor-only technology $\Rightarrow$ no other intertemporal asset other than share.

Firm Problem

Dividend and Wage

- Production function: $y = zn^{\alpha}$, where z is TFP shock, and $\alpha \in (0, 1)$.
- Firm's profit maximization problem: $\max_{n} zn^{\alpha} - wn$
  - FOC: $w = \alpha z n^{\alpha - 1}$
- Wage bill: $wn = \alpha z n^{\alpha} = \alpha y$
- Assume firm all profits as dividend, $d = y - wn = (1 - \alpha) y$

Household's Problem

Household Problem

Assume HH value leisure, and thus

$$V(s,z) = \max_{c \geq 0, s' \geq 0, n \geq 0} \log c + \psi(1-n) + \beta \mathbb{E}_{z'|z}[V(s',z')] \tag{1}$$

$$\text{s.t.} \quad c + ps' \leq (d+p)s + wn \tag{2}$$

We know in equilibrium / steady state, three markets need to clear:

1. find w such that labor demand = labor supply
2. find p such that $s = 1$
3. by Walras' law, goods market clear, implying $c = y$.

Solve Household Problem

Using the same solution technique,

$$V(s,z) = \max_{s',c,n} \log c + \psi(1-n) + \beta \mathbb{E}_{z'|z}[\log c' + \psi(1-n')] \tag{3}$$
$$+ \beta^2 \mathbb{E}_{z'|z}[V(s'',z'')] \tag{4}$$

subject to

$$c + ps' \leq (d+p)s + wn \tag{5}$$
$$c' + p's'' \leq (d'+p')s' + w'n' \tag{6}$$

Replace c and c' and get

$$V(s,z) = \max_{s',n} \log((d+p)s + wn - ps') + \psi(1-n) \tag{7}$$
$$+ \beta \mathbb{E}_{z'|z}[\log((d'+p')s' + w'n' - p's'') + \psi(1-n')] \tag{8}$$
$$+ \beta^2 \mathbb{E}_{z'|z}[V(s'',z'')] \tag{9}$$

First Order Condition

$$V(s,z) = \max_{s',n} \log((d+p)s + wn - ps') + \psi(1-n) \tag{10}$$
$$+ \beta \mathbb{E}_{z'|z}[\log((d'+p')s' + w'n' - p's'') + \psi(1-n')] \tag{11}$$
$$+ \beta^2 \mathbb{E}_{z'|z}[V(s'', z'')] \tag{12}$$

FOC:

$$[n]: \quad \frac{w}{c} = \psi \tag{13}$$
$$[s']: \quad \frac{1}{c} \cdot p = \beta \mathbb{E}_{z'|z}\left[\frac{1}{c'} \cdot (d'+p')\right] \tag{14}$$

Equilibrium Outcome

Optimality Conditions

$$
\begin{aligned}
[n]: &\quad \frac{w}{c} = \psi \Rightarrow w = \psi c && (15)\\
[s']: &\quad \frac{1}{c} \cdot p = \beta \mathbb{E}_{z'|z}\left[\frac{1}{c'} \cdot (d' + p')\right] && (16)\\
[\text{Firm}]: &\quad w = \alpha z n^{\alpha-1} && (17)
\end{aligned}
$$

$w = w$, (15) equals to (17), and $c = y$ yields

$$
\begin{aligned}
\psi y = \alpha \frac{y}{n} \Rightarrow n = \frac{\alpha}{\psi} \Rightarrow y = z n^{\alpha} = z\left(\frac{\alpha}{\psi}\right)^{\alpha} && (18)\\
\Rightarrow w = \alpha z n^{\alpha-1} = \alpha z \left(\frac{\alpha}{\psi}\right)^{\alpha-1} && (19)\\
\Rightarrow d = (1-\alpha) y = (1-\alpha) z \left(\frac{\alpha}{\psi}\right)^{\alpha} && (20)
\end{aligned}
$$

Share Euler Equation

Focus on (16), we can use $c' = y'$ as well as $d' = (1-\alpha)y'$ to simplify:

$$\frac{p}{y} = \beta \mathbb{E}_{z'|z}\left[\frac{p'}{y'} + \frac{(1-\alpha)y'}{y'}\right] \tag{21}$$

$$= \beta(1-\alpha) + \beta \mathbb{E}_{z'|z}\left[\frac{p'}{y'}\right] \tag{22}$$

Somehow you got a prophecy from the spirit and his/her voice tells you to guess $\frac{p}{y} \equiv \Lambda$, a constant over time regardless of TFP shock. Is that true?

$$\Lambda = \beta(1-\alpha) + \beta \mathbb{E}_{z'|z}[\Lambda] = \beta(1-\alpha) + \beta\Lambda \tag{23}$$

$$\Lambda = \frac{\beta(1-\alpha)}{1-\beta} \tag{24}$$

It true ✌✌✌

Intepretation

Stock price to GDP ratio, $\frac{p}{y}$, is constant over time, which implies

1. stock price is procyclical: ⬆ and ⬇ with TFP z,
2. the percentage std of stock price matches percentage std of dividend,
3. stock is risky: $p = \dfrac{\beta(1-\alpha)}{1-\beta} y \Rightarrow$ requires $(+)$ risk premium
   - $e(z, z') = \dfrac{d' + p'}{p} = \dfrac{(1-\alpha)y' + \Lambda y'}{\Lambda y} = \dfrac{\frac{1-\alpha}{1-\beta} y'}{\frac{\beta(1-\alpha)}{1-\beta} y} = \dfrac{1}{\beta} \dfrac{y'}{y}$
   - SDF $= \dfrac{\beta u'(c')}{u'(c)} = \beta \dfrac{y}{y'}$
   - Risk premium $= \dfrac{\mathbb{E}_t[e(z,z') - R_t]}{R_t} = -cov_t\left[SDF, e(z,z')\right] > 0$

The very times firm shares pay high is when your consumption is low!

Appendix

References I

Lucas, Robert E. (1978) "Asset Prices in an Exchange Economy," *Econometrica*, 46 (6), 1429, 10.2307/1913837.