Midterm Exam I

Macroeconomics I
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Problem 1: The calculation of Lecture 8

Remember the Example in Lecture 8.

Consumer: $\max_{C,l} \ln C + \ln l$ subject to $C \leq w(1-l) + \pi$

$$\text{FOC} \quad \frac{C}{l} = w \tag{1}$$

$$\text{Binding budget constraint} \quad C = w(1-l) + \pi \tag{2}$$

$$\text{Time constraint} \quad N^s = 1 - l \tag{3}$$

Firm: $\max_{N^d} (N^d)^{\frac{1}{2}} - wN^d$

$$\text{FOC} \quad \frac{1}{2}(N^d)^{-\frac{1}{2}} = w \tag{4}$$

$$\text{Output definition} \quad Y = (N^d)^{\frac{1}{2}} \tag{5}$$

$$\text{Profit definition} \quad \pi = Y - wN^d \tag{6}$$

Market clear:

$$N^s = N^d \tag{7}$$

Fill the following blanks for the step-by-step guide for algebraic calculation:

Step 1: Impose Market clear condition, so shrink all 7 equations to __Q1__ equations

1 __C__

(A) 4 (B) 5 (C) 6 (D) 7

Consumer: $\max_{C,l} \ln C + \ln l$ subject to $C \leq w(1-l) + \pi$

$$\text{FOC} \quad \frac{C}{l} = w \tag{8}$$

$$\text{Binding budget constraint} \quad C = w(1-l) + \pi \tag{9}$$
$$\text{Time constraint} \quad N = 1 - l \tag{10}$$

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

$$\text{FOC} \quad \frac{1}{2}(N)^{-\frac{1}{2}} = w \tag{11}$$
$$\text{Output definition} \quad Y = (N)^{\frac{1}{2}} \tag{12}$$
$$\text{Profit definition} \quad \pi = Y - wN \tag{13}$$

Step 2: replace l in terms of N using $l = 1 - N$

Consumer: $\max_{C,l} \ln C + \ln l \quad$ subject to $\quad C \leq w(1-l) + \pi$

$$\text{FOC} \quad \frac{C}{(\underline{\quad \textbf{Q2} \quad})} = w \tag{14}$$
$$\text{Binding budget constraint} \quad C = w(\underline{\quad \textbf{Q3} \quad}) + \pi \tag{15}$$

2 A

(A) $1 - N$ (B) $2 - N$ (C) $N - 1$ (D) $2 - N$

3 D

(A) $N - 3$ (B) $N - 2$ (C) $N - 1$ (D) N

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

$$\text{FOC} \quad \frac{1}{2}(N)^{-\frac{1}{2}} = w \tag{16}$$
$$\text{Output definition} \quad Y = (N)^{\frac{1}{2}} \tag{17}$$
$$\text{Profit definition} \quad \pi = Y - wN \tag{18}$$

Step 3: replace π and Y as N

Consumer: $\max_{C,l} \ln C + \ln l \quad$ subject to $\quad C \leq w(1-l) + \pi$

$$\text{FOC} \quad \frac{C}{(\underline{\quad \textbf{Q2} \quad})} = w \tag{19}$$

$$\text{Binding budget constraint} \quad C = w(\underline{\quad \textbf{Q3} \quad}) + \pi \tag{20}$$

Firm: $\max_N (N)^{\frac{1}{2}} - wN$

$$\text{FOC} \quad \frac{1}{2}(N)^{-\frac{1}{2}} = w \tag{21}$$

$$\text{Profit definition} \quad \pi = (\underline{\quad \textbf{Q4} \quad}) - wN \tag{22}$$

4 C

(A) N (B) $2N$ (C) $N^{\frac{1}{2}}$ (D) $N^{\frac{1}{4}}$

Step 4: Substitute $\pi(N)$ into Binding budget constraint and get

$$C = (\underline{\quad \textbf{Q5} \quad}) \tag{23}$$

5 B

(A) N (B) $N^{\frac{1}{2}}$ (C) $N^{\frac{1}{4}}$ (D) $N^{\frac{1}{8}}$

Step 5: With consumer's FOC and firm's FOC both equate to w, we can get another expression of C:

$$C = (\underline{\quad \textbf{Q2} \quad}) \times (\underline{\quad \textbf{Q6} \quad}) \tag{24}$$

6 A

(A) $\frac{1}{2}N^{-\frac{1}{2}}$ (B) $\frac{1}{2}N$ (C) $\frac{1}{4}N^{-\frac{1}{2}}$ (D) $\frac{1}{2}N$

Step 6: Let (23) equate (24) and we get N as

$$N = (\underline{\quad \textbf{Q7} \quad}) \tag{25}$$

7 C

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Step 7: Trace back to all unknowns given the value of N, we get

$$C = (\underline{\quad \text{Q8} \quad}) \tag{26}$$

$$l = (\underline{\quad \text{Q9} \quad}) \tag{27}$$

$$Y = (\underline{\quad \text{Q10} \quad}) \tag{28}$$

$$\pi = (\underline{\quad \text{Q11} \quad}) \tag{29}$$

$$w = (\underline{\quad \text{Q12} \quad}) \tag{30}$$

8 A

(A) $\sqrt{\frac{1}{3}}$ (B) $\sqrt{\frac{1}{4}}$ (C) $\sqrt{\frac{1}{5}}$ (D) $\sqrt{\frac{1}{6}}$

9 D

(A) $\frac{3}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{2}{3}$

10 A

(A) $\sqrt{\frac{1}{3}}$ (B) $\sqrt{\frac{1}{4}}$ (C) $\sqrt{\frac{1}{5}}$ (D) $\sqrt{\frac{1}{6}}$

11 B

(A) $\sqrt{\frac{1}{3}} - \frac{1}{3}\sqrt{3}$ (B) $\sqrt{\frac{1}{3}} - \frac{1}{6}\sqrt{3}$ (C) $\sqrt{\frac{1}{6}} - \frac{1}{3}\sqrt{3}$ (D) $\sqrt{\frac{1}{6}} - \frac{1}{6}\sqrt{3}$

12 C

(A) $\frac{1}{3}\sqrt{3}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{1}{2}\sqrt{3}$ (D) $\frac{1}{2}\sqrt{2}$

Problem 2: Macroeconomic Analysis of H-1B Visa Fee Policy

Scenario

A new administration proposes a substantial fee, $t = \$100,000$, that should be paid by firms on every H-1B worker hired. We will analyze this policy using our one-period competitive equilibrium model.

Model Setup

- **Household**: Utility $U(C, l) = \ln C + \beta \ln l$; Time endowment $h = 1$.
- **Firm**: Production $Y = zK^{\alpha}N^{1-\alpha}$, where N is the total labor demand.
- **Government**: Balances its budget, $G = T + tN_H$.
- **Parameters:** $z = 1$, $K = 1$, $\alpha = 1/3$, $\beta = 2$.

Part I: Conceptual Foundations

13 The administration considers a massive \$100,000 fee per H-1B worker. According to the Lucas critique, why is a micro-founded model essential for analyzing such a large policy shift? __A__

(A) Because a large fee is a major policy change, it will fundamentally alter how firms decide to hire, making old statistical data unreliable.

(B) Historical data on visa fees is likely inaccurate and cannot be trusted for forecasting.

(C) Micro-founded models are the only models that can account for government spending.

(D) The Lucas critique states that only small, incremental policies can be accurately modeled.

Solution

The Lucas critique's core argument is that rational agents (like firms) will change their behavior and decision rules in response to a major policy change. A $100,000 fee is a regime shift, not a small tweak, so older models based on historical data from a low-fee regime will likely fail to predict the outcome accurately.

14. A U.S. software firm has $10M in revenue. It pays $2M for intermediate goods, $4M in domestic wages, $1M in H-1B wages, and $500,000 in H-1B fees. Using the income approach , what is this firm's contribution to GDP? B

(A) $7,500,000

(B) $8,000,000

(C) $2,500,000

(D) $5,500,000

Solution

The firm's value added is Revenue - Intermediate Goods = $10M - $2M = $8M. The income approach states that this value added must be fully distributed as income. The components are: Wages ($4M + $1M = $5M), Taxes on Production (the $0.5M fee), and Profits ($10M - $2M - $5M - $0.5M = $2.5M). The total contribution to Gross Domestic Income is the sum of all these income streams: $5M (Wages) + $2.5M (Profits) + $0.5M (Govt Income/Taxes) = $8M.

Part II: The Perfect Substitutes Case

Assume H-1B (N_H) and domestic (N_D) workers are perfect substitutes, so total labor demand is $N = N_D + N_H$. The production function is $Y = N^{\frac{2}{3}}$. The domestic household's labor supply is perfectly inelastic at $N^s = \frac{1}{3}$.

15. What is the firm's labor demand curve, $N(w)$? D

(A) $N = \left(\frac{3w}{2}\right)^3$

(B) $N = \left(\frac{2}{3w}\right)^2$

(C) $N = \left(\frac{3}{2w}\right)^{\frac{3}{2}}$

(D) $N = \left(\frac{2}{3w}\right)^{3}$

Solution

The MPN is $\frac{dY}{dN} = \frac{2}{3}N^{-1/3}$. The firm hires until $MPN = w$. So, $w = \frac{2}{3}N^{-1/3}$. Solving for N: $N^{1/3} = 2/(3w) \implies N = (2/3w)^3$.

(16) Assuming that the US does not have any foreign labor supply, only the domestic ones. What is the equilibrium wage w^*? C

(A) $\frac{2}{3}$

(B) $2 \times 3^{2/3}$

(C) $2 \times 3^{-2/3}$

(D) $\frac{3}{2}$

Solution

In equilibrium, $N^s = N^d$. So, $1/3 = (2/3w)^3 = 8/(27w^3)$. Rearranging gives $27w^3 = 24 \implies w^3 = 24/27 = 8/9$. So $w = \sqrt[3]{8/9} = 2/9^{1/3} = 2/3^{2/3} = 2 \times 3^{-2/3}$.

(17) Now, the US government announce the H-1B program and allow foreign workers to enter with a fee $t > 0$. The firm must pay a fee of t for each H-1B worker. The market real wage is w. What is the firm's profit-maximization condition for hiring an H-1B worker? C

(A) $MPN = w - t$

(B) $MPN = w$

(C) $MPN = w + t$

(D) $MPN = t$

Solution

The marginal benefit of hiring an H-1B worker (their MPN) must equal the full marginal cost, which is the wage paid to them (w) plus the fee paid to the government (t).

(18) In this perfect substitutes model, what is the equilibrium level of H-1B employment, N_H^*? __A__

(A) 0

(B) $1/3$

(C) It depends on the size of the fee t.

(D) It is negative.

Solution

Since domestic workers provide the exact same productivity at a strictly lower cost ($w < w + t$), the profit-maximizing firm will never hire an H-1B worker. Employment of H-1B workers drops to zero.

(19) The analysis in previous question suggests firms would hire zero H-1B workers. In reality, firms still hire them. What is the most plausible economic reason our simple model misses? __B__

(A) Firms are not actually profit-maximizers.

(B) Domestic and H-1B workers are not perfect substitutes; H-1B workers may possess unique skills that command a higher effective MPN.

(C) The government forces firms to hire H-1B workers.

(D) The real wage for H-1B workers is secretly lower than for domestic workers.

Solution

This is the most realistic answer. If an H-1B worker has specialized skills such that their MPN_H is significantly higher than a domestic worker's MPN_D, a firm might be willing to pay the extra fee if $MPN_H > w + t$.

Part III: The Cobb-Douglas Production Case

Now assume a more realistic scenario where the two labor types are imperfect substitutes. Let the production function be $Y = N_D^{1/2} N_H^{1/2}$. The supply of domestic labor is perfectly inelastic at $N_D = 1/4$, and the supply of H-1B labor is perfectly inelastic at $N_H = 1/4$.

(20) What is the Marginal Product of a domestic worker, MPN_D? B

(A) $\frac{1}{2}\sqrt{\frac{N_D}{N_H}}$

(B) $\frac{1}{2}\sqrt{\frac{N_H}{N_D}}$

(C) $\sqrt{\frac{N_H}{N_D}}$

(D) $\sqrt{\frac{N_D}{N_H}}$

Solution

The MPN_D is the partial derivative of Y with respect to N_D: $\frac{\partial Y}{\partial N_D} = (\frac{1}{2}N_D^{-1/2})N_H^{1/2} = \frac{1}{2}\sqrt{N_H/N_D}$.

(21) Before the fee ($t = 0$), what is the equilibrium wage for domestic workers, w_D? D

(A) 2

(B) 1

(C) 4

(D) 1/2

Solution

The wage equals the marginal product. Given $N_D = 1/4$ and $N_H = 1/4$, we have $w_D = MPN_D = \frac{1}{2}\sqrt{(1/4)/(1/4)} = \frac{1}{2}\sqrt{1} = 1/2$.

(22) Before the fee ($t = 0$), what is the total output (GDP) of this economy? A

(A) $1/4$

(B) $1/2$

(C) 1

(D) 4

Solution

Total output is $Y = N_D^{1/2} N_H^{1/2} = (1/4)^{1/2}(1/4)^{1/2} = (1/2) \times (1/2) = 1/4$.

23. Now, a fee of $t = 1/4$ is imposed on H-1B workers. What is the new wage paid *to* H-1B workers, w_H? __B__

(A) $1/2$

(B) $1/4$

(C) 0

(D) $-1/4$

Solution

The firm's total cost for an H-1B worker is $w_H + t$. It sets this equal to their marginal product: $w_H + t = MPN_H$. The MPN_H is $\frac{1}{2}\sqrt{N_D/N_H} = \frac{1}{2}\sqrt{(1/4)/(1/4)} = 1/2$. So, $w_H + 1/4 = 1/2 \implies w_H = 1/4$.

24. After the fee is imposed, what are the profits (π) of the representative firm? __A__

(A) 0

(B) $1/8$

(C) $1/4$

(D) $-1/8$

Solution

This production function exhibits constant returns to scale ($1/2 + 1/2 = 1$). A key property of CRS is that if all factors are paid their marginal products, economic profits are zero. Before the policy, $\pi = Y - w_D N_D - w_H N_H = 1/4 - (1/2)(1/4) - (1/2)(1/4) = 0$. After the policy, domestic wages are still $w'_D = 1/2$. $\pi = 1/4 - (1/2)(1/4) - (1/4)(1/4) - (1/4)(1/4) = 1/4 - 1/8 - 1/16 - 1/16 = 0$. Profits remain zero.

25. In this Cobb-Douglas labor model with perfectly inelastic labor supply, who bears the full economic burden of the fee? B

(A) The firm owners (through lower profits).

(B) The H-1B workers (through lower wages).

(C) Domestic workers (through lower wages).

(D) The government.

Solution

The domestic wage did not change ($w_D = 1/2$). The firm's profits remained at zero. The wage paid to H-1B workers fell from $1/2$ to $1/4$. The size of this wage drop is exactly equal to the fee ($t = 1/4$). Therefore, the H-1B workers bear 100% of the economic burden, a classic result when the supply of the taxed factor is perfectly inelastic.

Part IV: General Equilibrium Analysis

Let's return to the general model where the household's labor supply is NOT perfectly inelastic. Production: $Y = zK^\alpha N^{1-\alpha}$. Household utility: $U = \ln(C) + \beta \ln(l)$. The policy is an increase in TFP, z.

26. The production function is $Y = zK^\alpha(N_D^\gamma N_H^{1-\gamma})^{1-\alpha}$. The H-1B fee ($t$) is imposed only on N_H. How does this fee affect the firm's demand for **capital** (K)? B

(A) It has no effect on capital demand because the fee is on labor.

(B) The firm's demand for capital will decrease.

(C) The firm's demand for capital will increase.

(D) The rental rate of capital will fall, but demand will not change.

Solution

The fee reduces the firm's demand for high-skilled labor (N_H). Because capital and labor are complements (i.e., they work together), having less labor makes each unit of capital less productive. This reduces the Marginal Product of Capital (MPK) and causes the firm to demand less capital.

27. The H-1B fee (t) is a lump-sum amount per worker. How does this modify the firm's labor demand curve for H-1B workers? D

(A) It makes the labor demand curve steeper.

(B) It makes the labor demand curve flatter.

(C) It pivots the labor demand curve inward.

(D) It causes a parallel downward shift of the labor demand curve.

Solution

The firm's hiring rule is $MPN = w + t$, or $w = MPN - t$. This means that for any given quantity of labor N_H, the maximum wage the firm is willing to pay is exactly t dollars less than it was before. This is a parallel downward shift of the labor demand curve.

28. The H-1B fees collected are used to fund government spending (G). From the representative household's perspective, how does the policy affect their budget constraint initially, before any wage changes? B

(A) It has no effect.

(B) It decreases their non-wage income ($\pi - T$) because firm profits (π) fall due to the new fee.

(C) It increases their non-wage income ($\pi - T$) because the government lowers other taxes (T).

(D) It increases their wage income (wN^s).

Solution

The firm must pay the fee, which reduces its profits. Since the representative household owns the firm, the dividends (π) they receive will fall, tightening their budget constraint.

29. The reduction in the household's dividend income (π) described in last question is an example of a: B

(A) Pure substitution effect, causing them to work more.

(B) Pure income effect, causing them to work more.

(C) Pure income effect, causing them to work less.

(D) Technology shock, causing them to be less productive.

Solution

This correctly identifies it as a pure income effect. Because the household is poorer, and leisure is a normal good, they will 'buy' less leisure. This means they will work more.

30. The policy shifts the demand for domestic labor to the right (as firms substitute away from H-1B workers), which tends to increase the domestic wage (w). How does this wage increase affect a domestic worker's labor supply? D

(A) It will definitely increase their labor supply.

(B) It will definitely decrease their labor supply.

(C) It has no effect on their labor supply.

(D) The effect is ambiguous, as the income effect (work less) and substitution effect (work more) oppose each other.

Solution

This is the standard textbook result. A higher wage makes you richer, encouraging more leisure (income effect). It also makes leisure more expensive, encouraging less leisure (substitution effect). The net effect on hours worked is theoretically ambiguous.

(31) Assume for the US economy that the substitution effect of a wage change is stronger than the income effect. What is the shape of the labor supply curve? __A__

(A) Upward-sloping

(B) Downward-sloping

(C) Vertical

(D) Horizontal

Solution

If the substitution effect dominates, a higher wage (the reward for working) will always induce people to supply more labor.

(32) Let's assemble the full picture in a competitive equilibrium. The H-1B fee policy causes which two simultaneous shifts in the market for **domestic** labor? __D__

(A) Labor demand shifts right, and labor supply shifts right.

(B) Labor demand shifts left, and labor supply shifts left.

(C) Labor demand shifts right, and labor supply shifts left.

(D) Labor demand for domestic workers shifts right, and the labor supply curve for domestic workers also shifts right.

Solution

This is the correct combination. (1) Firms' demand for domestic workers increases as they are substitutes for now-more-expensive H-1B workers (Demand shifts right). (2) Households are poorer due to lower firm profits, so they want to work more at any given wage to compensate (Supply shifts right).

(33) Given that both the labor demand and labor supply curves for domestic workers shift to the right, what is the predicted effect on the equilibrium for domestic workers? __A__

(A) Employment will increase, but the effect on the wage is ambiguous.

(B) The wage will increase, but the effect on employment is ambiguous.

(C) Both employment and the wage will definitely increase.

(D) Both employment and the wage will definitely decrease.

Solution

This is the standard result of a simultaneous rightward shift in both supply and demand. The quantity (employment) unambiguously increases, but the final price (wage) depends on the relative magnitude of the two shifts.

34 Recall the definition of unemployment rate as the people who are unemployment out of the labor force. What is the unemployment rate **in our equilibrium model**?
C

(A) Unemployment increases.

(B) Unemployment decreases.

(C) Unemployment is always zero.

(D) The effect on unemployment is ambiguous.

Solution

In the basic competitive equilibrium model, the labor market always clear, meaning labor supply equals labor demand. There is no room for involuntary unemployment by definition.

35 What is the most significant weakness of using this one-period (static) model to analyze the H-1B fee policy? C

(A) It cannot account for changes in firm profits.

(B) It cannot account for the consumer's choice between work and leisure.

(C) It ignores the dynamic effects on investment and capital accumulation over time.

(D) It assumes that both consumption and leisure are normal goods.

Solution

This is a crucial limitation. The model cannot analyze how the policy might affect a firm's incentive to invest in new capital (offices, servers, R&D) or a worker's incentive to invest in their own skills over time. These dynamic effects could be very large in the long run.

36. The model uses a representative household that owns the firm. How does this assumption simplify the analysis of the H-1B fee's effect on household income? B

(A) It allows us to ignore the effect on firm profits.

(B) It combines the wage and profit effects into a single household budget, showing that the household ultimately bears the cost of the fee through lower profits.

(C) It assumes that only H-1B workers pay the fee.

(D) It allows us to model workers and firm owners as having conflicting interests.

Solution

This is the key simplification. Instead of tracking separate groups of workers and capital owners, the model shows that the household, as the owner of all factors of production, receives all income (wages and profits). Therefore, any cost imposed on the firm (like the fee) directly reduces the profit portion of the household's income.