2.2

(a) The events of spinning a wheel each time are independent to each other. No matter how many times watching a roulette or which slot the ball lands on, the probability of landing on a particular color of slot is constant. So the probability of landing on a red slot on the next spin is:

$$P(\text{red}) = \frac{\#red}{\#total} = \frac{18}{38} = \frac{9}{19} \approx 0.474$$

(b) Theoretically, the probability of landing on a red slot should remain the same for each time. As illustrated in (a), the probability of landing on a red slot on the next spin should be 0.474. But the ball landing on a red slot for consecutive 300 times is weird. Maybe there is some other external reason that makes it. So in this case the probability of landing on a red slot maybe higher than 0.474.

(c) I am more confident of my answer for part (a). Because in part (b), for 300 consecutive times the expected times of landing on a red slot should be $300 \times \frac{9}{19} \approx 142$ times if each slot has an equal chance of capturing the ball. But the actual result is 300 times. Maybe there is some other external reason that makes it. In part (a), however, 3 times landing on a red slot for 3 consecutive times can normally happen because of randomness.

2.4

First, for each roll, the events of rolling two dice are independent to each other. So the results do not have any influence on each other. The probability of rolling some number is constant. Second, the probability of rolling two 6s is equal to the probability of rolling two 3s, which is $\frac{1}{36}$. Third, rolling two dice is a random event whose result cannot be certain. So my rolls were just as likely as my friend's. Maybe next time he will roll two 6s. Good luck.

2.5

(a) Flipping a coin 10 times are independent to each other. For each time, $P(\text{H}) = 0.5$, $P(\text{T}) = 0.5$. The probability of getting all tails is $P(\text{allT}) = \text{P(T)}^{10} = 0.5^{10} \approx 0.00098$.

(b) The probability of getting all heads is $P(\text{allH}) = \text{P(H)}^{10} = 0.5^{10} \approx 0.00098$

(c) The probability of getting at least one tail is P(at least one tail) = 1 - P(allH) = 1- 0.00098 = 0.99902

2.6

(a) The probability of getting a sum of 1 is 0. Because the least sum should be 2 by rolling two 1s.

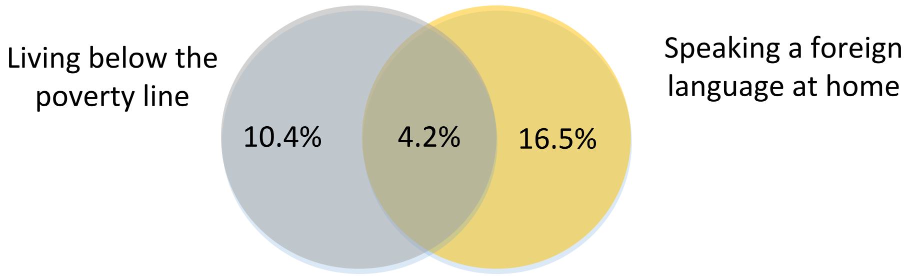
(b) The total number of cases of rolling a pair of dice is 36 (assume the two dice are ordered). There are 4 cases of getting a sum of 5, which are (2,3), (3,2), (4,1) and (1,4). The probability of getting a sum of 5 is $P(\text{sum} = 5) = \frac{4}{36} = \frac{1}{9}$.

(c) There is only one case of getting a sum of 12, which is (6,6). The probability is $P(\text{sum} = 12) = \frac{1}{36}$.

2.8

(a) No, they are not disjoint. There are people both living below the poverty line and speaking a foreign language at home.

(b)



People living below the poverty line but not speaking foreign language at home accounts for 10.4%. People speaking a foreign language at home but not living below the poverty line accounts for 16.5%. People living below the poverty line and speaking a foreign language at home accounts for 4.2%. So there are 14.6% of Americans live below the poverty line, and 20.7% speak a foreign language at home.

(c) Since (14.6% - 4.2% = 10.4%), 10.4% of Americans live below the poverty line and only speak English at home.

(d) Since (14.6% + 20.7% - 4.2% = 31.1%), 31.1% of Americans live below the poverty line or speak a foreign language at home.

(e) Since (1 - 14.6% - 16.5% = 68.9%), 68.9% of Americans live above the poverty line and only speak English at home.

(f) $P(\text{poverty}) \times \mathrm{P}(\text{foreign}) = 14.6\% \times 20.7\% = 3.0222\%$, which does not equal P(poverty and speak foreign language) = 4.2%, so the events are dependent.

2.10

(a) For the five questions, the events of getting the right answers are independent on the basis of random guess. P(getting right for a question) = 1/4 = 0.25. P(getting wrong for a question) = 3/4 = 0.75.

P(the first question she gets right is the 5th question) = P(the first four get wrong and the 5th gets right)

= $\left(\frac{3}{4}\right)^4 \times \frac{1}{4} \approx 0.0791 = 7.91\%$

(b) P(she gets all of the questions right) = $\left(\frac{1}{4}\right)^5 \approx 0.0977\%$.

(c) P(she gets at least one question right) = 1 - P(she gets no question right)

= $1 - \left(\frac{3}{4}\right)^5 \approx 1 - 0.2373 = 0.7627 = 76.27\%$

2.12

(a) P(doesn't miss any days of school due to sickness this year) = 1 - 25% - 15% - 28% = 32%.

(b) P(misses no more than one day) = P(misses 0 day) + P(misses 1 day) = 32% + 25% = 57%.

(c) P(misses at least one day) = 1- P(misses 0 day) = 1- 32% = 68%.

(d) P(neither kid will miss any school) = P(one misses 0 day) * P(another misses 0 day) = 32%*32% = 10.24%. The necessary assumption is that the two kids not missing the school is independent.

(e) P(both kids will miss school at least one day) = P(misses at least one day)^2 = 46.24%. The necessary assumption is that the two kids missing school is independent.

(f) The assumptions in both (d) and (e) are not reasonable because the two kids are living together. They are influenced by each other's habits, physical conditions and others. So whether they miss school is dependent.

2.14

(a) P(has excellent health and doesn't have health coverage) = $\frac{459}{20000} = 0.02295 = 2.295\%$

(b) P(has excellent health or doesn't have health coverage)

= P(has excellent health) + P(doesn't have health coverage) – P(has excellent health and doesn't have health coverage)

$= \frac{4657}{20000} + \frac{2524}{20000} - \frac{459}{20000} = \frac{6722}{20000} = 0.3361 = 33.61\%$

2.16

P(likes jelly|likes peanut butter) = $\frac{P(\text{jelly\& peanut})}{P(\text{peanut})} = \frac{78\%}{80\%} = 97.5\%$

2.18

(a) Being in excellent health and having health coverage are not mutually exclusive, since P(being in excellent health and having health coverage) = 0.2099 $\neq 0$.

(b) P(has excellent health) = 0.2329

(c) P(has excellent health given that he has health coverage)

= P(excellent health & health coverage) / P(has health coverage)

= 0.2099 / 0.8738 ≈ 0.2402

(d) P(has excellent health given that he doesn't have health coverage)

= P(excellent health & not health coverage) / P(not health coverage)

= 0.0230 / 0.1262 ≈ 0.1823

(e) P(having excellent health and having health coverage) = 0.2099

P(excellent health) * P(health coverage) = 0.2329 * 0.8738 = 0.2035

They are not equal so they are not independent. Based on the results of (c) and (d), the outcomes are different. There is some dependency between excellent health and having health coverage.

2.22

$\because$ P(predisposition) = 3%, P(positive | predisposition) = 99%, P(negative | not predisposition) = 98%

$\therefore$ P(predisposition | positive)

$$= \frac{P(\text{predisposition\& positive})}{P(\text{positive})}$$

$$= \frac{P(\text{predisposition}) \times \text{P}(\text{positive} \mid \text{predisposition})}{P(\text{predisposition}) \times \text{P}(\text{positive} \mid \text{predisposition}) + P(\text{not\_predisposition}) \times \text{P}(\text{positive} \mid \text{not\_predisposition})}$$

$$= \frac{P(\text{predisposition}) \times \text{P}(\text{positive} \mid \text{predisposition})}{P(\text{predisposition}) \times \text{P}(\text{positive} \mid \text{predisposition}) + (1 - P(\text{predisposition})) \times (1 - \text{P}(\text{negative} \mid \text{not\_predisposition}))}$$

$$= \frac{0.03 \times 0.99}{0.03 \times 0.99 + 0.97 \times 0.02}$$

$$= \frac{0.0297}{0.0297 + 0.0194} \approx 0.605 = 60.5\%$$

2.24

We may set S stands for voting in favor of Scott Walker, and C stands for having a college degree.

Then it can be described that P(S) = 0.53, P(C|S) = 0.37, P(C|S^C) = 0.44. The question is to ask what is the value of P(S|C).

$$\text{P(S|C)} = \frac{P(\text{SC})}{P(\text{C})} = \frac{P(\text{S}) * \text{P}(\text{C} \mid \text{S})}{P(\text{S}) * \text{P}(\text{C} \mid \text{S}) + P(\text{S}^C)\,\text{P}(\text{C} \mid \text{S}^C)}$$

$$= \frac{P(\text{S}) * \text{P}(\text{C} \mid \text{S})}{P(\text{S}) * \text{P}(\text{C} \mid \text{S}) + (1 - P(\text{S}))\,\text{P}(\text{C} \mid \text{S}^C)}$$

$$= \frac{0.53 * 0.37}{0.53 * 0.37 + 0.47 * 0.44} \approx 0.4867 = 48.67\%$$

2.26

We may set I stands for identical twins, and F stands for fraternal twins. Then it can be described that

P(I) = 0.3, P(F) = 0.7, P(both_male | I) = P(both_female | I) = 0.5

P(both_male | F) = P(both_female | F) = 0.25, P (male&female | F) = 0.5

The question is to ask the value of P(I | both_female).

$$P(\mathrm{I} \mid \mathrm{both\_female}) = \frac{P(\mathrm{I}\&\,\mathrm{both\_female})}{P(\mathrm{both\_female})} = \frac{P(\mathrm{I}) * \mathrm{P}(\mathrm{both\_female} \mid \mathrm{I})}{P(\mathrm{I}) * \mathrm{P}(\mathrm{both\_female} \mid \mathrm{I}) + P(\mathrm{F}) * \mathrm{P}(\mathrm{both\_female} \mid \mathrm{F})}$$

$$= \frac{0.3 * 0.5}{0.3 * 0.5 + 0.7 * 0.25} \approx 0.462 = 46.2\%$$