

Supporting Information for “A Compound Decision Approach to Covariance Matrix Estimation”

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1. Introduction

Here we present more details about the simulations shown in the main manuscript as well as report additional experiments. We used the six models described in Section 4.1 of the main text. We report the medians and interquartile ranges of the Frobenius errors across our replications. We used 200 replications unless otherwise specified.

2. Clustering-based exemplar algorithm

The following table gives exact numerical results corresponding to Figure 1 of the main text.

[Table 1 about here.]

3. Estimation accuracies

The following table gives exact numerical results corresponding to Figure 2 of the main text.

[Table 2 about here.]

4. Correlation matrix estimation

To fairly compare our methods to the CorShrink procedure of Dey and Stephens (2018), which was designed for estimating correlation matrices, we also studied the performance of these methods for estimating correlation matrices under Frobenius norm loss. For each method in Section 4.2, we calculated a correlation matrix estimate by scaling the corresponding covariance matrix estimate such that the estimated variances were equal to one. The results in Table 3 show that CorShrink performs slightly better than our methods except in Model 2, likely because the additional flexibility that our method trades off lower bias for higher variance.

[Table 3 about here.]

5. Model misspecification

Our estimator assumes that data are generated from multivariate Gaussian distribution. To investigate the performance of our methods when the data are non-normal, we generated \mathbf{Y} from either $U(0, 1)$ or a negative binomial distribution with size 10 and mean 4. The \mathbf{Y} were then normalized by their theoretical standard deviations to have unit variance. Finally, we generated the observed data following $\mathbf{X} = \mathbf{LY}$, where \mathbf{L} is the Cholesky decomposition of the desired covariance matrix of \mathbf{Y} . We simulated our data using the covariance matrices defined in Section 4.1. The results in Tables 4 and 5 suggest that MSG and MSGCor still have excellent performance under model misspecification for both continuous and discrete data, though they can be outperformed in Models 5 and 6.

[Table 4 about here.]

[Table 5 about here.]

6. Large dimension

In previous simulations, we took $p = 30, 100$ and 200 . We also studied the performance of the various methods when $p = 1000$. Because of the computational burden, here we only performed 50 replications. The results in Table 6 show that our estimator remains competitive in Models 1 through 4, but can be substantially outperformed in Models 5 and 6.

[Table 6 about here.]

References

Dey, K. K. and Stephens, M. (2018). Corshrink: Empirical bayes shrinkage estimation of correlations, with applications. *bioRxiv* page 368316.

Table 1: Numerical results corresponding to Figure 1

Model	Method	p=30	p=100	p=200
Sparse	Exemplar	3.02 (1.21)	7.35 (1.52)	11.4 (1.73)
	$K = 2p$	3.26 (1.25)	7.51 (1.47)	11.52 (1.78)
	$K = p$	3.38 (1.33)	7.86 (1.53)	11.93 (1.8)
	$K = p/2$	3.59 (1.34)	7.98 (1.69)	11.94 (1.86)
	$K = p/4$	3.95 (1.45)	8.4 (1.77)	11.97 (1.76)
Hypercorrelated	Exemplar	5.19 (6.86)	15.75 (19.95)	32.23 (34.18)
	$K = 2p$	5.42 (7.2)	15.51 (20.15)	32.61 (33.87)
	$K = p$	5.34 (7.03)	15.25 (19.32)	32.06 (33.43)
	$K = p/2$	5.4 (6.75)	15.48 (18.94)	33.58 (34.96)
	$K = p/4$	5.22 (6.59)	16.31 (18.37)	33.03 (34.54)
Dense-0.7-0.7	Exemplar	3.76 (3.81)	12.38 (13.37)	19.93 (21.49)
	$K = 2p$	3.59 (3.79)	12.45 (13.44)	20.07 (21.51)
	$K = p$	3.72 (3.6)	12.27 (12.87)	20.54 (21.65)
	$K = p/2$	3.68 (3.72)	12.57 (13.39)	20.08 (22.77)
	$K = p/4$	3.83 (3.81)	12.9 (12.75)	21.71 (22.82)
Dense-0.7-0.9	Exemplar	4.46 (5.23)	14.63 (21.61)	24.35 (37.1)
	$K = 2p$	4.48 (5.28)	14.83 (22.3)	24.58 (37.05)
	$K = p$	4.65 (5.21)	14.85 (21.99)	24.39 (36.78)
	$K = p/2$	4.48 (5.29)	14.43 (21.76)	24.24 (38.92)
	$K = p/4$	4.49 (5.2)	14.44 (21.0)	25.01 (39.0)
Orthogonal	Exemplar	5.72 (0.27)	13.02 (0.16)	20.23 (0.25)
	$K = 2p$	5.71 (0.28)	13.01 (0.16)	20.22 (0.26)
	$K = p$	5.71 (0.25)	13.01 (0.18)	20.24 (0.26)
	$K = p/2$	5.78 (0.29)	13.01 (0.16)	20.25 (0.24)
	$K = p/4$	6.05 (0.39)	13.04 (0.16)	20.27 (0.26)
Spiked	Exemplar	2.62 (0.21)	3.71 (0.12)	4.76 (0.15)
	$K = 2p$	2.62 (0.2)	3.7 (0.12)	4.74 (0.16)
	$K = p$	2.63 (0.21)	3.71 (0.11)	4.75 (0.16)
	$K = p/2$	2.66 (0.2)	3.71 (0.11)	4.77 (0.16)
	$K = p/4$	2.77 (0.19)	3.77 (0.13)	4.77 (0.18)

Table 2: Numerical results corresponding to Figure 2

Model	Method	p=30	p=100	p=200
Sparse	MSG	3.46 (1.39)	7.97 (1.67)	12.18 (1.87)
	MSGCor	3.42 (1.41)	7.75 (1.72)	11.79 (1.9)
	Adap	5.27 (1.53)	14.24 (1.61)	23.04 (1.67)
	Linear	4.91 (1.21)	15.28 (1.19)	28.06 (1.24)
	QIS	4.79 (1.26)	14.45 (1.28)	29.57 (1.48)
	NERCOME	4.94 (1.32)	14.54 (1.34)	26.61 (1.21)
	CorShrink	3.97 (1.49)	8.99 (1.53)	13.53 (1.74)
	Sample	4.98 (1.03)	16.56 (1.52)	33.21 (1.8)
	OracNonlin	3.99 (0.74)	13.5 (0.95)	25.65 (1.01)
	OracMSG	2.03 (0.76)	6.12 (1.35)	10.09 (1.49)
Hypercorrelated	MSG	5.26 (5.04)	17.71 (19.52)	29.12 (42.35)
	MSGCor	5.26 (5.04)	17.71 (19.52)	29.12 (42.35)
	Adap	13.59 (2.5)	55.12 (6.47)	114.28 (13.71)
	Linear	7.3 (3.24)	25.88 (13.03)	44.84 (28.3)
	QIS	7.08 (3.63)	24.74 (13.66)	65.1 (39.25)
	NERCOME	6.97 (4.13)	23.95 (15.33)	46.8 (31.58)
	CorShrink	6.73 (3.96)	23.0 (13.92)	42.03 (30.29)
	Sample	7.36 (3.66)	25.7 (13.44)	46.21 (28.97)
	OracNonlin	4.9 (1.15)	16.35 (2.03)	32.98 (4.15)
	OracMSG	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Dense-0.7	MSG	3.6 (3.96)	12.37 (13.53)	23.42 (28.93)
	MSGCor	3.6 (3.96)	12.37 (13.53)	23.42 (28.93)
	Adap	4.97 (2.99)	18.51 (10.55)	46.22 (32.62)
	Linear	5.09 (3.1)	16.57 (9.56)	33.6 (19.65)
	QIS	5.13 (3.23)	16.14 (10.2)	37.25 (30.02)
	NERCOME	5.01 (3.09)	16.84 (10.99)	30.67 (23.03)
	CorShrink	4.25 (3.16)	14.94 (11.29)	28.68 (24.66)
	Sample	5.03 (2.83)	17.18 (10.01)	32.92 (20.91)
	OracNonlin	3.27 (0.56)	10.83 (1.41)	21.3 (2.44)
	OracMSG	0.01 (0.08)	0.23 (0.61)	0.9 (1.52)
Dense-0.9	MSG	4.64 (5.06)	14.23 (16.91)	30.05 (34.95)
	MSGCor	4.64 (5.06)	14.22 (16.91)	30.05 (34.95)
	Adap	4.77 (4.71)	16.17 (15.34)	33.03 (29.22)
	Linear	4.78 (4.77)	15.18 (15.51)	31.13 (29.03)
	QIS	4.8 (4.76)	16.14 (15.19)	34.61 (32.51)
	NERCOME	4.95 (4.85)	16.42 (16.58)	34.18 (32.12)
	CorShrink	4.72 (4.73)	15.83 (15.48)	33.28 (31.55)
	Sample	4.8 (4.74)	16.14 (14.96)	33.86 (30.36)
	OracNonlin	2.01 (0.43)	6.8 (0.81)	13.81 (1.6)
	OracMSG	0.0 (0.01)	0.0 (0.03)	0.01 (0.11)
Orthogonal	MSG	4.24 (0.19)	8.89 (0.19)	13.85 (0.39)
	MSGCor	4.24 (0.19)	8.89 (0.19)	13.85 (0.39)
	Adap	5.0 (0.19)	9.54 (0.11)	12.93 (0.16)
	Linear	4.12 (0.12)	8.48 (0.09)	11.71 (0.1)
	QIS	4.3 (0.26)	10.19 (0.31)	37.19 (0.48)
	NERCOME	4.09 (0.15)	8.52 (0.06)	11.69 (0.05)
	CorShrink	4.52 (0.22)	9.07 (0.13)	12.71 (0.15)
	Sample	8.15 (0.42)	24.43 (0.68)	51.86 (0.81)
	OracNonlin	3.79 (0.12)	8.34 (0.06)	11.53 (0.03)
	OracMSG	4.13 (0.13)	8.47 (0.07)	11.64 (0.03)
Spiked	MSG	2.61 (0.2)	3.71 (0.09)	4.77 (0.2)
	MSGCor	2.61 (0.2)	3.71 (0.09)	4.77 (0.2)
	Adap	3.59 (0.07)	3.95 (0.06)	4.24 (0.08)
	Linear	2.58 (0.19)	3.52 (0.06)	3.71 (0.05)
	QIS	2.27 (0.26)	4.05 (0.2)	15.17 (0.21)
	NERCOME	2.24 (0.27)	3.25 (0.3)	3.66 (0.07)
	CorShrink	2.68 (0.2)	3.79 (0.09)	4.21 (0.08)
	Sample	3.67 (0.28)	10.72 (0.28)	20.73 (0.31)
	OracNonlin	2.05 (0.23)	2.99 (0.22)	3.4 (0.14)
	OracMSG	2.54 (0.22)	3.51 (0.08)	3.68 (0.04)

Table 3: Estimation errors for correlation matrices.

Model	Method	p=30	p=100	p=200
Sparse	MSG	1.5 (0.44)	3.59 (0.55)	5.8 (0.66)
	MSGCor	1.49 (0.46)	3.54 (0.52)	5.68 (0.69)
	Adap	1.98 (0.58)	5.81 (0.61)	9.63 (0.78)
	Linear	2.58 (0.28)	8.21 (0.45)	14.85 (0.45)
	QIS	2.32 (0.34)	7.68 (0.4)	17.44 (0.46)
	NERCOME	2.35 (0.37)	7.76 (0.4)	14.12 (0.4)
	CorShrink	1.38 (0.4)	3.35 (0.55)	5.08 (0.62)
	Sample	2.78 (0.3)	9.75 (0.48)	19.83 (0.46)
	OracNonlin	2.19 (0.36)	7.58 (0.44)	14.29 (0.39)
	OracMSG	0.89 (0.39)	2.78 (0.64)	4.56 (0.68)
Hypercorrelated	MSG	1.0 (0.61)	2.28 (1.47)	4.34 (2.9)
	MSGCor	1.0 (0.61)	2.25 (1.43)	4.36 (2.92)
	Adap	6.51 (1.15)	26.75 (2.07)	56.71 (2.84)
	Linear	2.21 (0.43)	7.38 (1.09)	14.86 (2.2)
	QIS	1.73 (0.52)	5.66 (1.19)	23.33 (5.42)
	NERCOME	1.66 (0.45)	5.54 (1.09)	11.01 (2.02)
	CorShrink	2.01 (0.31)	6.53 (0.59)	12.91 (1.07)
	Sample	2.31 (0.35)	7.65 (0.75)	15.35 (1.25)
	OracNonlin	1.52 (0.37)	5.17 (0.77)	10.3 (1.44)
	OracMSG	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Dense-0.7	MSG	0.76 (0.74)	1.97 (2.2)	4.74 (4.55)
	MSGCor	0.76 (0.7)	2.01 (2.22)	4.64 (4.56)
	Adap	1.4 (0.48)	5.16 (2.51)	13.69 (10.33)
	Linear	1.5 (0.82)	4.94 (2.3)	9.88 (4.98)
	QIS	1.87 (1.02)	6.82 (2.99)	16.29 (5.92)
	NERCOME	1.74 (0.71)	7.04 (2.67)	14.37 (5.81)
	CorShrink	0.64 (0.83)	1.82 (2.37)	4.53 (4.44)
	Sample	1.39 (0.42)	4.55 (1.12)	9.41 (2.44)
	OracNonlin	1.5 (0.25)	6.41 (0.42)	13.66 (0.58)
	OracMSG	0.01 (0.08)	0.21 (0.57)	0.73 (1.89)
Dense-0.9	MSG	0.36 (0.27)	0.96 (0.86)	1.98 (2.25)
	MSGCor	0.32 (0.27)	0.94 (0.88)	1.88 (2.19)
	Adap	0.49 (0.17)	1.63 (0.56)	3.38 (1.56)
	Linear	0.76 (0.47)	2.59 (1.63)	5.49 (4.09)
	QIS	0.7 (0.45)	2.75 (1.33)	6.28 (2.55)
	NERCOME	0.72 (0.34)	2.95 (1.32)	6.51 (3.49)
	CorShrink	0.29 (0.32)	0.84 (1.04)	1.91 (2.17)
	Sample	0.49 (0.17)	1.63 (0.55)	3.36 (1.37)
	OracNonlin	0.6 (0.05)	2.78 (0.1)	6.01 (0.15)
	OracMSG	0.0 (0.02)	0.01 (0.11)	0.03 (0.35)
Orthogonal	MSG	1.54 (0.07)	3.64 (0.09)	5.34 (0.14)
	MSGCor	1.54 (0.07)	3.64 (0.09)	5.34 (0.14)
	Adap	1.72 (0.03)	3.68 (0.0)	4.6 (0.0)
	Linear	1.5 (0.04)	3.47 (0.04)	4.52 (0.04)
	QIS	1.55 (0.09)	4.17 (0.14)	14.51 (0.1)
	NERCOME	1.48 (0.06)	3.48 (0.03)	4.51 (0.01)
	CorShrink	1.51 (0.05)	3.47 (0.03)	4.5 (0.02)
	Sample	2.95 (0.14)	9.98 (0.15)	20.03 (0.15)
	OracNonlin	1.39 (0.06)	3.41 (0.02)	4.46 (0.01)
	OracMSG	1.5 (0.05)	3.46 (0.03)	4.5 (0.01)
Spiked	MSG	1.99 (0.15)	3.35 (0.08)	4.53 (0.17)
	MSGCor	1.99 (0.15)	3.35 (0.08)	4.53 (0.17)
	Adap	2.69 (0.03)	3.31 (0.0)	3.49 (0.0)
	Linear	1.98 (0.14)	3.16 (0.05)	3.48 (0.05)
	QIS	1.72 (0.2)	3.7 (0.17)	14.78 (0.13)
	NERCOME	1.68 (0.2)	2.89 (0.22)	3.43 (0.06)
	CorShrink	2.02 (0.14)	3.16 (0.06)	3.46 (0.04)
	Sample	2.93 (0.16)	9.98 (0.14)	20.04 (0.14)
	OracNonlin	1.53 (0.22)	2.69 (0.18)	3.19 (0.13)
	OracMSG	1.93 (0.17)	3.14 (0.07)	3.46 (0.04)

Table 4: Estimation errors for covariance matrices from uniformly distributed data.

Model	Method	p=30	p=100	p=200
Sparse	MSG	3.39 (1.58)	7.64 (1.64)	11.92 (1.81)
	MSGCor	3.36 (1.58)	7.48 (1.79)	11.58 (1.81)
	Adap	5.17 (1.4)	13.71 (1.43)	22.21 (1.54)
	Linear	4.84 (1.22)	15.15 (1.23)	28.15 (1.04)
	QIS	4.73 (1.32)	14.34 (1.29)	29.63 (1.47)
	NERCOME	4.82 (1.44)	14.49 (1.19)	26.71 (1.15)
	CorShrink	3.89 (1.52)	8.91 (1.42)	13.51 (1.78)
	Sample	4.97 (1.25)	16.46 (1.56)	33.05 (1.79)
	OracNonlin	4.01 (0.81)	13.42 (1.01)	25.66 (0.93)
	OracMSG	1.9 (0.91)	6.01 (1.16)	10.07 (1.64)
Hypercorrelated	MSG	4.18 (3.87)	14.2 (16.03)	28.91 (31.48)
	MSGCor	4.18 (3.87)	14.2 (16.03)	28.91 (31.48)
	Adap	12.14 (2.75)	50.89 (5.48)	110.43 (9.1)
	Linear	6.54 (2.74)	22.22 (9.85)	47.03 (20.36)
	QIS	6.21 (2.86)	21.13 (10.67)	60.5 (42.01)
	NERCOME	6.3 (2.66)	21.98 (10.21)	43.9 (21.92)
	CorShrink	5.83 (2.71)	20.68 (10.95)	42.38 (21.83)
	Sample	6.64 (2.4)	23.08 (10.13)	47.14 (20.15)
	OracNonlin	4.8 (0.85)	16.2 (2.04)	33.0 (3.52)
	OracMSG	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Dense-0.7	MSG	3.4 (3.29)	10.07 (11.07)	21.37 (25.63)
	MSGCor	3.4 (3.29)	10.05 (11.07)	21.37 (25.63)
	Adap	4.75 (2.23)	15.17 (6.74)	31.85 (16.09)
	Linear	4.68 (2.49)	15.11 (7.61)	30.87 (15.31)
	QIS	4.53 (2.48)	14.63 (7.85)	37.31 (30.31)
	NERCOME	4.54 (2.39)	14.44 (6.97)	30.14 (18.25)
	CorShrink	3.96 (2.54)	12.19 (8.2)	26.07 (19.72)
	Sample	4.73 (2.09)	15.08 (6.66)	30.69 (16.01)
	OracNonlin	3.17 (0.66)	10.79 (1.4)	21.13 (2.43)
	OracMSG	0.0 (0.02)	0.07 (0.31)	0.34 (0.83)
Dense-0.9	MSG	3.23 (3.58)	11.32 (11.95)	20.44 (24.74)
	MSGCor	3.23 (3.59)	11.32 (11.95)	20.44 (24.74)
	Adap	3.72 (3.08)	13.4 (10.75)	24.37 (19.79)
	Linear	3.7 (3.15)	13.61 (11.23)	24.28 (20.68)
	QIS	3.72 (3.13)	13.42 (10.95)	24.46 (18.67)
	NERCOME	4.0 (2.99)	13.49 (10.66)	24.32 (19.74)
	CorShrink	3.75 (2.9)	13.0 (10.73)	24.31 (20.56)
	Sample	3.86 (2.79)	13.4 (10.41)	25.16 (19.93)
	OracNonlin	2.03 (0.39)	6.89 (0.94)	13.97 (1.4)
	OracMSG	0.0 (0.0)	0.0 (0.01)	0.0 (0.01)
Orthogonal	MSG	4.14 (0.14)	8.68 (0.13)	13.11 (0.23)
	MSGCor	4.14 (0.14)	8.68 (0.13)	13.11 (0.23)
	Adap	4.79 (0.13)	9.23 (0.09)	12.37 (0.1)
	Linear	4.07 (0.12)	8.46 (0.08)	11.7 (0.09)
	QIS	4.15 (0.24)	10.09 (0.28)	37.24 (0.41)
	NERCOME	4.04 (0.17)	8.51 (0.07)	11.68 (0.05)
	CorShrink	4.23 (0.15)	8.72 (0.11)	12.13 (0.1)
	Sample	7.92 (0.43)	24.31 (0.49)	51.82 (0.7)
	OracNonlin	3.74 (0.15)	8.34 (0.05)	11.53 (0.02)
	OracMSG	4.09 (0.13)	8.49 (0.07)	11.64 (0.03)
Spiked	MSG	2.6 (0.21)	3.61 (0.08)	4.44 (0.1)
	MSGCor	2.6 (0.21)	3.61 (0.08)	4.44 (0.1)
	Adap	3.55 (0.06)	3.8 (0.05)	3.94 (0.04)
	Linear	2.59 (0.22)	3.51 (0.06)	3.71 (0.05)
	QIS	2.26 (0.28)	4.0 (0.18)	15.18 (0.2)
	NERCOME	2.25 (0.28)	3.22 (0.27)	3.65 (0.09)
	CorShrink	2.64 (0.22)	3.63 (0.07)	3.9 (0.04)
	Sample	3.63 (0.23)	10.65 (0.22)	20.7 (0.28)
	OracNonlin	2.02 (0.26)	2.98 (0.2)	3.4 (0.17)
	OracMSG	2.53 (0.23)	3.48 (0.09)	3.66 (0.03)

Table 5: Estimation errors for covariance matrices from negative binomial data.

Model	Method	p=30	p=100	p=200
Sparse	MSG	3.56 (1.39)	8.37 (2.14)	12.75 (1.86)
	MSGCor	3.5 (1.41)	8.2 (2.25)	12.45 (1.98)
	Adap	5.49 (1.62)	14.8 (1.69)	24.13 (1.98)
	Linear	5.03 (1.28)	15.44 (1.36)	28.33 (1.12)
	QIS	4.84 (1.29)	14.65 (1.29)	29.6 (1.65)
	NERCOME	4.98 (1.37)	14.85 (1.36)	26.82 (1.18)
	CorShrink	3.93 (1.47)	9.37 (1.65)	14.08 (1.73)
	Sample	5.04 (1.36)	16.65 (1.59)	33.02 (2.07)
	OracNonlin	3.96 (0.78)	13.68 (0.89)	25.83 (1.12)
	OracMSG	2.07 (0.89)	6.41 (1.46)	10.29 (1.55)
Hypercorrelated	MSG	5.13 (5.59)	15.17 (21.1)	38.68 (45.87)
	MSGCor	5.13 (5.59)	15.16 (21.13)	38.68 (45.87)
	Adap	14.02 (3.18)	56.02 (9.97)	121.63 (26.82)
	Linear	7.44 (3.8)	23.47 (13.98)	52.9 (33.53)
	QIS	7.13 (4.24)	22.34 (16.16)	60.51 (52.2)
	NERCOME	6.94 (4.38)	22.88 (14.15)	50.61 (36.64)
	CorShrink	6.83 (4.17)	21.38 (15.8)	49.5 (35.57)
	Sample	7.36 (4.08)	23.61 (14.1)	52.89 (34.0)
	OracNonlin	4.83 (0.99)	16.34 (2.41)	32.87 (4.19)
	OracMSG	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Dense-0.7	MSG	3.68 (4.3)	12.52 (11.77)	22.57 (27.91)
	MSGCor	3.68 (4.3)	12.52 (11.8)	22.56 (27.91)
	Adap	5.42 (3.46)	21.91 (15.03)	58.59 (41.83)
	Linear	4.83 (2.82)	17.7 (9.35)	32.7 (18.85)
	QIS	4.85 (2.96)	16.81 (9.72)	36.33 (31.04)
	NERCOME	4.95 (3.53)	15.86 (9.61)	31.35 (21.77)
	CorShrink	4.53 (3.3)	14.91 (10.05)	28.64 (22.39)
	Sample	5.07 (2.94)	17.19 (8.85)	32.61 (19.27)
	OracNonlin	3.23 (0.64)	10.66 (1.6)	20.95 (2.89)
	OracMSG	0.02 (0.13)	0.52 (1.08)	1.54 (2.48)
Dense-0.9	MSG	4.66 (6.0)	17.77 (20.8)	34.8 (37.69)
	MSGCor	4.65 (6.0)	17.77 (20.81)	34.79 (37.69)
	Adap	5.09 (5.39)	21.68 (21.41)	41.3 (36.24)
	Linear	5.01 (5.01)	19.26 (18.01)	36.06 (35.59)
	QIS	5.07 (5.2)	19.89 (18.7)	35.65 (34.73)
	NERCOME	5.46 (5.44)	19.21 (19.57)	35.68 (37.16)
	CorShrink	4.87 (5.24)	18.97 (19.15)	36.71 (35.06)
	Sample	4.93 (5.19)	19.25 (19.16)	37.16 (34.44)
	OracNonlin	2.02 (0.49)	6.93 (1.21)	14.01 (1.99)
	OracMSG	0.0 (0.02)	0.01 (0.2)	0.05 (0.5)
Orthogonal	MSG	4.34 (0.3)	9.13 (0.28)	14.66 (0.48)
	MSGCor	4.34 (0.3)	9.13 (0.28)	14.66 (0.48)
	Adap	5.18 (0.32)	10.25 (0.51)	14.93 (0.74)
	Linear	4.13 (0.13)	8.49 (0.08)	11.7 (0.09)
	QIS	4.31 (0.27)	10.2 (0.28)	37.35 (0.61)
	NERCOME	4.09 (0.18)	8.53 (0.07)	11.7 (0.05)
	CorShrink	4.66 (0.27)	9.29 (0.19)	13.04 (0.19)
	Sample	8.25 (0.49)	24.45 (0.75)	52.11 (0.99)
	OracNonlin	3.79 (0.13)	8.35 (0.06)	11.54 (0.03)
	OracMSG	4.13 (0.13)	8.49 (0.07)	11.64 (0.03)
Spiked	MSG	2.66 (0.23)	3.82 (0.11)	5.13 (0.23)
	MSGCor	2.66 (0.23)	3.82 (0.11)	5.13 (0.23)
	Adap	3.64 (0.1)	4.33 (0.24)	5.22 (0.38)
	Linear	2.6 (0.23)	3.52 (0.06)	3.71 (0.06)
	QIS	2.31 (0.26)	4.09 (0.21)	15.18 (0.24)
	NERCOME	2.28 (0.27)	3.28 (0.24)	3.66 (0.07)
	CorShrink	2.75 (0.24)	3.91 (0.1)	4.39 (0.11)
	Sample	3.71 (0.28)	10.73 (0.33)	20.75 (0.39)
	OracNonlin	2.05 (0.27)	3.01 (0.2)	3.4 (0.13)
	OracMSG	2.6 (0.27)	3.53 (0.08)	3.7 (0.05)

Table 6: Simulations investigating behavior when $p = 1000$.

Method	Sparse	Hypercorrelated	Dense-0.7	Dense-0.9	Orthogonal	Spiked
MSG	39.11(2.51)	210.24(205.80)	111.39(141.12)	176.32(151.84)	49.36(1.07)	16.71(0.45)
MSGCor	37.81(2.48)	210.22(205.80)	111.34(141.08)	176.27(151.84)	49.36(1.07)	16.71(0.45)
Adap	66.52(1.89)	645.60(161.94)	472.83 (321.31)	202.18(154.34)	30.22(0.28)	6.28(0.17)
Linear	97.60(0.61)	275.21(141.56)	152.10(107.99)	172.98(135.58)	28.35(0.21)	4.87(0.18)
QIS	166.11(4.33)	618.28(393.61)	401.43 (206.23)	212.00(213.07)	240.58(2.01)	97.52(0.87)
NERCOME	97.46(0.83)	271.85(183.10)	157.60 (84.65)	173.08(160.65)	27.46(0.01)	3.83(0.01)
CorShrink	33.40(1.50)	256.70(151.64)	138.11(101.14)	190.83(152.22)	29.63(0.12)	5.86(0.11)
Sample	163.29(3.08)	278.28(153.45)	161.04 (88.05)	194.30(149.77)	250.26(1.89)	101.12(0.88)
OracNonlin	96.31(0.73)	167.39(141.56)	105.08(10.96)	68.40(7.33)	27.33(0.01)	3.73(0.01)
OracMSG	31.33(1.75)	0.0(0.0)	7.15(6.41)	0.52(1.16)	27.37(0.01)	3.74(0.00)