Simulation

In this web appendix, we present more details about the simulations in the main manuscript as well as more simulations related to our method. For each simulation, we use models introduced in 3.1. In our main manuscript, the Frobenius errors of different methods are presented in figures without exact numbers. In this section, all the errors are displayed in the table with their uncertainties, which is the difference of 75% quantile and 25% quantile of 200 replicates.

Clustering-based exemplar algorithm

In this part, we show the exact errors and uncertainties in 1. The result is as 1 shows.

[Table 1 about here.]

Comparison with corShrink

In this part, we did 4 simulations in order to compare the behavior of corShrink (**?**) with other covariance matrix estimators. The procedure is as the following 2 settings. In each setting, we generate true covariance matrices $\mathbf{\Sigma}$ as 6 models in 3.1. Sample size n is taken as 20 and 100.

In first setting, we compare the estimation of covariance matrix $\mathbf{\Sigma}$. For corShrink, covariance matrix is estimated as $\widehat{\mathbf{\Sigma}} = \text{diag}(\widehat{SD})\widehat{\boldsymbol{R}}\text{diag}(\widehat{SD})$ where $\widehat{SD}$ are sample standard deviations. The result is presented in 2 and **??**.

[Table 2 about here.]

[Table 3 about here.]

In the second setting, we compare the estimation of correlation matrix $\boldsymbol{R}$. For other covari-

ance matrix estimators, $\widehat{\boldsymbol{R}} = \text{diag}(1/\widehat{SD})\widehat{\boldsymbol{\Sigma}}\text{diag}(1/\widehat{SD})$, where $\widehat{SD}$ are standard deviations derived from $\widehat{\boldsymbol{\Sigma}}$. The result is presented in 4 and 5.

[Table 4 about here.]

[Table 5 about here.]

Misspecification

Our method is based on the assumption that data is generated from multivariate Gaussian distribution. We are also interested on its performance when the data is misspecified. To investigate such case, we generated $\boldsymbol{Y}$ from two non-normal distributions with variance of 1 and let $\boldsymbol{X} = \boldsymbol{L}\boldsymbol{Y}$ where $\boldsymbol{L}$ is Cholesky decomposition of covariance matrix $\boldsymbol{\Sigma}$. In first simulation, $\boldsymbol{Y} \sim U(0,1)/\sqrt{(}1/12)$. In the second simulation, $\boldsymbol{Y}$ was generated from negative binomial distributions with size of 10 and mean of 4, and then was normalized to have variance of 1.

[Table 6 about here.]

[Table 7 about here.]

Large dimension

We are also interested in the case when p is 1000. The averaged error is presented in 8.

[Table 8 about here.]

The result shows that our estimator has substantially better performance even when p has large value.

Table 1: Simulations investigating Clustering-based exemplar algorithm, as described in Section 3.2.

Model	Method	p=30	p=100	p=200
Model1	no cluster	3.51 (0.95)	6.96 (1.04)	11.27 (0.81)
	ratio=2	3.68 (0.96)	7.21 (0.88)	11.54 (0.74)
	ratio=1	3.98 (0.9)	7.36 (0.99)	11.61 (0.77)
	ratio=0.5	4.26 (0.86)	7.84 (0.84)	11.9 (0.79)
	ratio=0.25	4.72 (0.96)	8.99 (1.22)	12.34 (0.97)
Model2	no cluster	5.19 (6.86)	15.75 (19.95)	32.23 (34.18)
	ratio=2	5.42 (7.2)	15.51 (20.15)	32.61 (33.87)
	ratio=1	5.34 (7.03)	15.25 (19.32)	32.06 (33.43)
	ratio=0.5	5.4 (6.75)	15.48 (18.94)	33.58 (34.96)
	ratio=0.25	5.22 (6.59)	16.31 (18.37)	33.03 (34.54)
Model3	no cluster	4.18(2.96)	13.79(8.17)	26.66(16.73)
	ratio=2	4.12(3.06)	13.81(7.99)	16.64(16.51)
	ratio=1	4.13(3.04)	13.83(8.13)	26.53(16.64)
	ratio=0.5	4.23(2.93)	13.95(8.44)	26.46(16.85)
	ratio=0.25	4.54(2.70)	13.74(8.35)	26.56(17.47)
Model4	no cluster	4.87(3.54)	17.52(11.62)	38.63(32.05)
	ratio=2	4.89(3.51)	17.57(11.54)	38.62(31.94)
	ratio=1	4.81(3.48)	17.40(11.66)	38.55(32.05)
	ratio=0.5	4.92(3.43)	17.19(11.99)	38.27(32.65)
	ratio=0.25	5.45(3.27)	16.90(12.01)	38.54(34.55)
Model5	no cluster	5.72 (0.27)	13.02 (0.16)	20.23 (0.25)
	ratio=2	5.71 (0.28)	13.01 (0.16)	20.22 (0.26)
	ratio=1	5.71 (0.25)	13.01 (0.18)	20.24 (0.26)
	ratio=0.5	5.78 (0.29)	13.01 (0.16)	20.25 (0.24)
	ratio=0.25	6.05 (0.39)	13.04 (0.16)	20.27 (0.26)
Model6	no cluster	2.62 (0.21)	3.71 (0.12)	4.76 (0.15)
	ratio=2	2.62 (0.2)	3.7 (0.12)	4.74 (0.16)
	ratio=1	2.63 (0.21)	3.71 (0.11)	4.75 (0.16)
	ratio=0.5	2.66 (0.2)	3.71 (0.11)	4.77 (0.16)
	ratio=0.25	2.77 (0.19)	3.77 (0.13)	4.77 (0.18)

Table 2: Simulations for covariance matrix estimation. $n = 20$.

Model	Method	p=30	p=100	p=200
Model1	msg_sgrid_km	13.52 (11.74)	37.46 (36.3)	79.16 (84.97)
	msg_km_cor	13.38 (11.74)	37.31 (36.57)	78.69 (85.28)
	adap.thrsd	49.06 (5.94)	169.58 (6.0)	340.46 (3.24)
	linear	17.97 (9.37)	51.27 (24.15)	111.08 (62.51)
	QIS	16.44 (8.7)	61.8 (42.86)	124.46 (87.81)
	NERCOME	17.25 (9.2)	55.19 (33.34)	109.63 (61.05)
	corShrink	17.23 (9.47)	49.85 (27.95)	105.53 (64.96)
	SCM	17.16 (8.15)	54.59 (24.63)	110.25 (57.64)
	oracle_nonlin	11.42 (3.19)	37.61 (7.56)	76.69 (16.73)
	oracle_gmleb	4.42 (5.14)	14.64 (11.01)	30.1 (18.77)
Model2	msg_sgrid_km	5.9 (4.82)	16.68 (13.9)	32.96 (22.78)
	msg_km_cor	5.84 (4.86)	16.37 (14.09)	32.48 (23.17)
	adap.thrsd	5.46 (5.23)	15.19 (15.64)	29.23 (27.79)
	linear	5.56 (4.9)	15.35 (15.25)	28.44 (26.14)
	QIS	5.44 (5.29)	15.06 (15.71)	27.28 (24.79)
	NERCOME	5.31 (5.56)	15.71 (13.22)	27.71 (24.15)
	corShrink	5.26 (5.36)	14.62 (16.06)	27.72 (25.8)
	SCM	5.32 (5.26)	14.97 (15.82)	28.48 (25.29)
	oracle_nonlin	2.03 (0.41)	6.78 (0.83)	13.65 (1.68)
	oracle_gmleb	3.95 (1.58)	11.43 (4.27)	22.62 (7.98)
Model3	msg_sgrid_km	9.81 (7.5)	28.47 (21.43)	58.51 (52.1)
	msg_km_cor	9.81 (7.51)	28.47 (21.43)	58.51 (52.1)
	adap.thrsd	34.48 (2.64)	110.59 (1.92)	222.71 (1.38)
	linear	12.25 (7.11)	35.9 (17.73)	73.51 (47.42)
	QIS	11.16 (5.76)	35.97 (27.67)	81.57 (54.53)
	NERCOME	11.98 (7.71)	34.05 (20.35)	75.18 (52.94)
	corShrink	9.91 (6.75)	29.74 (21.54)	60.75 (47.44)
	SCM	11.53 (5.78)	35.19 (17.44)	70.63 (42.23)
	oracle_nonlin	7.52 (2.06)	23.98 (4.9)	45.88 (10.56)
	oracle_gmleb	5.86 (2.21)	17.53 (6.44)	34.77 (14.87)
Model4	msg_sgrid_km	11.21 (9.66)	38.2 (26.76)	69.4 (63.59)
	msg_km_cor	11.17 (9.66)	38.18 (26.81)	69.36 (63.63)
	adap.thrsd	41.24 (13.46)	141.77 (7.6)	286.09 (2.37)
	linear	11.18 (10.4)	37.04 (31.39)	66.57 (63.18)
	QIS	10.71 (9.78)	37.12 (31.8)	70.91 (71.17)
	NERCOME	12.34 (11.45)	39.17 (34.12)	73.0 (80.04)
	corShrink	10.46 (10.25)	34.86 (29.18)	64.61 (67.97)
	SCM	10.69 (10.21)	35.87 (28.58)	65.04 (67.17)
	oracle_nonlin	4.96 (1.46)	15.76 (3.95)	30.91 (6.41)
	oracle_gmleb	7.17 (3.27)	24.3 (10.34)	44.98 (22.76)
Model5	msg_sgrid_km	7.86 (0.54)	16.42 (0.88)	26.81 (1.03)
	msg_km_cor	7.86 (0.54)	16.42 (0.88)	26.81 (1.03)
	adap.thrsd	13.4 (0.81)	27.43 (0.58)	39.85 (0.48)
	linear	7.84 (0.67)	16.56 (1.1)	26.5 (1.59)
	QIS	11.85 (1.13)	53.22 (3.75)	114.43 (6.82)
	NERCOME	7.65 (0.34)	14.7 (0.11)	21.26 (0.36)
	corShrink	8.67 (0.57)	16.85 (0.48)	24.23 (0.64)
	SCM	17.64 (1.75)	57.45 (3.33)	114.9 (4.36)
	oracle_nonlin	7.07 (0.19)	14.35 (0.06)	20.74 (0.03)
	oracle_gmleb	7.49 (0.2)	14.57 (0.06)	20.9 (0.03)
Model6	msg_sgrid_km	3.56 (0.28)	5.1 (0.4)	8.04 (0.58)
	msg_km_cor	3.56 (0.28)	5.1 (0.4)	8.04 (0.59)
	adap.thrsd	6.26 (0.44)	10.44 (0.23)	14.44 (0.19)
	linear	3.49 (0.26)	5.03 (0.69)	7.75 (0.75)
	QIS	5.97 (0.65)	23.34 (1.67)	47.51 (2.17)
	NERCOME	3.42 (0.16)	3.81 (0.16)	3.95 (0.4)
	corShrink	3.98 (0.27)	5.1 (0.28)	6.15 (0.37)
	SCM	8.3 (0.98)	24.51 (1.48)	47.15 (1.59)
	oracle_nonlin	3.1 (0.27)	3.59 (0.08)	3.68 (0.02)
	oracle_gmleb	3.37 (0.18)	3.67 (0.05)	3.72 (0.02)

Table 3: Simulations for covariance matrix estimation. $n = 100$.

Model	Method	p=30	p=100	p=200
Model1	msg_sgrid_km	4.13 (0.77)	7.64 (0.96)	12.19 (0.93)
	msg_km_cor	3.91 (0.91)	7.26 (1.06)	11.76 (0.94)
	adap.thrsd	4.12 (1.09)	10.07 (1.16)	16.64 (1.19)
	linear	4.88 (0.92)	13.57 (0.81)	24.67 (0.67)
	QIS	4.56 (0.94)	12.84 (0.83)	26.65 (0.86)
	NERCOME	4.67 (0.98)	13.01 (0.8)	23.97 (0.71)
	corShrink	3.38 (1.07)	7.0 (1.25)	10.84 (1.31)
	SCM	5.22 (0.92)	16.05 (0.78)	32.14 (1.13)
	oracle_nonlin	3.95 (0.71)	12.33 (0.7)	23.34 (0.81)
	oracle_gmleb	3.63 (0.76)	6.74 (0.78)	10.3 (0.85)
Model2	msg_sgrid_km	5.52 (5.47)	16.59 (16.08)	34.58 (40.06)
	msg_km_cor	5.52 (5.47)	16.59 (16.08)	34.58 (40.05)
	adap.thrsd	13.59 (2.89)	54.68 (5.96)	116.37 (15.91)
	linear	7.45 (3.89)	23.91 (11.67)	50.65 (26.53)
	QIS	7.19 (4.36)	23.1 (12.04)	69.21 (54.8)
	NERCOME	7.29 (4.28)	23.01 (13.47)	48.57 (30.61)
	corShrink	6.75 (4.39)	22.26 (11.45)	46.82 (28.74)
	SCM	7.45 (3.93)	24.36 (11.11)	50.47 (28.47)
	oracle_nonlin	4.91 (0.99)	16.44 (2.06)	32.86 (3.94)
	oracle_gmleb	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Model3	msg_sgrid_km	4.31 (3.41)	13.83 (10.19)	25.41 (17.42)
	msg_km_cor	4.31 (3.41)	13.79 (10.24)	25.3 (17.41)
	adap.thrsd	4.81 (3.38)	17.81 (10.68)	39.11 (21.77)
	linear	4.78 (3.05)	15.8 (8.85)	29.33 (15.16)
	QIS	4.65 (3.23)	15.33 (9.41)	40.79 (28.32)
	NERCOME	4.74 (3.47)	14.92 (10.12)	30.14 (16.05)
	corShrink	4.21 (3.54)	13.58 (11.03)	26.01 (19.31)
	SCM	4.76 (3.12)	15.61 (9.22)	30.68 (16.19)
	oracle_nonlin	3.3 (0.74)	10.37 (1.37)	20.45 (2.22)
	oracle_gmleb	3.09 (1.04)	9.32 (2.68)	17.89 (4.98)
Model4	msg_sgrid_km	5.9 (4.82)	16.68 (13.9)	32.96 (22.78)
	msg_km_cor	5.84 (4.86)	16.37 (14.09)	32.48 (23.17)
	adap.thrsd	5.46 (5.23)	15.19 (15.64)	29.23 (27.79)
	linear	5.56 (4.9)	15.35 (15.25)	28.44 (26.14)
	QIS	5.44 (5.29)	15.06 (15.71)	27.28 (24.79)
	NERCOME	5.31 (5.56)	15.71 (13.22)	27.71 (24.15)
	corShrink	5.26 (5.36)	14.62 (16.06)	27.72 (25.8)
	SCM	5.32 (5.26)	14.97 (15.82)	28.48 (25.29)
	oracle_nonlin	2.03 (0.41)	6.78 (0.83)	13.65 (1.68)
	oracle_gmleb	3.95 (1.58)	11.43 (4.27)	22.62 (7.98)
Model5	msg_sgrid_km	5.72 (0.28)	13.03 (0.18)	20.24 (0.24)
	msg_km_cor	5.72 (0.28)	13.03 (0.18)	20.24 (0.24)
	adap.thrsd	8.2 (0.13)	15.31 (0.09)	21.7 (0.1)
	linear	5.63 (0.23)	12.93 (0.12)	19.61 (0.1)
	QIS	4.86 (0.36)	13.39 (0.29)	35.44 (0.51)
	NERCOME	4.84 (0.36)	12.65 (0.17)	19.54 (0.07)
	corShrink	5.82 (0.26)	13.31 (0.14)	20.12 (0.1)
	SCM	7.64 (0.56)	25.29 (0.7)	50.46 (0.88)
	oracle_nonlin	4.37 (0.34)	12.27 (0.14)	19.21 (0.07)
	oracle_gmleb	5.64 (0.19)	12.94 (0.12)	19.66 (0.07)
Model6	msg_sgrid_km	2.62 (0.19)	3.71 (0.11)	4.76 (0.2)
	msg_km_cor	2.62 (0.19)	3.71 (0.11)	4.76 (0.2)
	adap.thrsd	3.6 (0.07)	3.95 (0.07)	4.23 (0.08)
	linear	2.6 (0.2)	3.52 (0.06)	3.71 (0.06)
	QIS	2.29 (0.27)	4.07 (0.22)	15.19 (0.25)
	NERCOME	2.28 (0.29)	3.24 (0.29)	3.66 (0.08)
	corShrink	2.72 (0.17)	3.79 (0.08)	4.2 (0.07)
	SCM	3.68 (0.26)	10.72 (0.29)	20.77 (0.32)
	oracle_nonlin	2.05 (0.25)	3.0 (0.19)	3.4 (0.14)
	oracle_gmleb	2.56 (0.22)	3.5 (0.08)	3.68 (0.03)

Table 4: Simulations for correlation matrix estimation. $n = 20$.

Model	Method	p=30	p=100	p=200
Model1	msg_sgrid_km	3.78 (0.74)	9.1 (0.92)	15.01 (0.98)
	msg_km_cor	3.73 (0.75)	9.06 (0.94)	14.84 (1.0)
	adap.thrsd	8.63 (1.0)	18.45 (0.44)	26.94 (0.23)
	linear	5.24 (0.68)	13.59 (0.61)	21.61 (0.67)
	QIS	5.2 (0.69)	22.1 (1.61)	46.66 (2.8)
	NERCOME	5.25 (0.81)	13.56 (0.67)	21.28 (0.43)
	corShrink	3.83 (0.77)	8.84 (0.96)	13.62 (0.93)
	SCM	6.35 (0.64)	22.44 (0.91)	45.27 (1.08)
	oracle_nonlin	4.74 (0.61)	13.11 (0.56)	20.74 (0.39)
	oracle_gmleb	3.4 (0.64)	8.07 (0.95)	12.44 (0.83)
Model2	msg_sgrid_km	2.5 (1.32)	7.28 (4.54)	13.13 (6.89)
	msg_km_cor	2.41 (1.4)	6.61 (4.72)	12.46 (8.64)
	adap.thrsd	15.14 (0.77)	50.57 (0.27)	100.65 (0.49)
	linear	4.5 (1.67)	15.76 (5.0)	32.09 (7.66)
	QIS	4.51 (1.18)	23.83 (3.33)	51.21 (7.74)
	NERCOME	3.78 (1.65)	12.97 (4.37)	25.99 (6.72)
	corShrink	4.16 (0.77)	13.89 (1.91)	27.58 (3.13)
	SCM	5.13 (0.91)	17.67 (2.53)	35.55 (3.62)
	oracle_nonlin	3.33 (1.25)	11.85 (3.06)	24.26 (5.75)
	oracle_gmleb	1.19 (1.35)	5.19 (3.66)	9.79 (4.24)
Model3	msg_sgrid_km	1.53 (1.67)	5.33 (4.51)	10.25 (11.56)
	msg_km_cor	1.48 (1.64)	5.24 (4.28)	10.25 (11.52)
	adap.thrsd	20.54 (1.36)	69.91 (0.89)	140.18 (0.55)
	linear	4.14 (2.81)	13.9 (8.21)	26.88 (17.7)
	QIS	2.6 (1.22)	16.23 (5.01)	34.33 (9.67)
	NERCOME	3.04 (2.66)	9.77 (8.9)	19.18 (16.58)
	corShrink	1.4 (1.68)	4.58 (4.86)	9.34 (12.33)
	SCM	3.07 (1.12)	10.64 (2.81)	21.47 (7.61)
	oracle_nonlin	2.08 (0.65)	7.27 (1.92)	14.38 (3.98)
	oracle_gmleb	0.89 (0.38)	2.96 (0.59)	5.6 (1.05)
Model4	msg_sgrid_km	1.23 (0.69)	3.51 (2.09)	6.54 (3.33)
	msg_km_cor	1.23 (0.86)	3.78 (2.65)	7.51 (5.26)
	adap.thrsd	24.58 (10.24)	89.67 (4.15)	179.98 (3.18)
	linear	2.84 (1.77)	9.66 (6.37)	18.77 (10.64)
	QIS	0.93 (0.48)	6.09 (1.72)	12.71 (2.78)
	NERCOME	0.98 (1.17)	3.32 (4.2)	6.68 (6.68)
	corShrink	0.67 (0.73)	2.07 (2.11)	3.79 (4.44)
	SCM	1.09 (0.48)	3.59 (1.75)	7.36 (2.8)
	oracle_nonlin	0.63 (0.16)	2.25 (0.4)	4.36 (0.73)
	oracle_gmleb	1.38 (0.9)	4.63 (1.66)	8.45 (3.45)
Model5	msg_sgrid_km	3.04 (0.16)	6.49 (0.24)	10.59 (0.42)
	msg_km_cor	3.04 (0.16)	6.49 (0.24)	10.59 (0.42)
	adap.thrsd	5.24 (0.38)	10.94 (0.27)	15.96 (0.21)
	linear	3.04 (0.2)	6.56 (0.35)	10.44 (0.52)
	QIS	4.76 (0.29)	21.29 (0.66)	45.37 (1.32)
	NERCOME	2.96 (0.08)	5.84 (0.03)	8.46 (0.11)
	corShrink	3.0 (0.12)	5.91 (0.15)	8.55 (0.19)
	SCM	6.72 (0.35)	22.76 (0.35)	45.68 (0.36)
	oracle_nonlin	2.81 (0.08)	5.72 (0.03)	8.29 (0.01)
	oracle_gmleb	2.94 (0.07)	5.8 (0.03)	8.35 (0.01)
Model6	msg_sgrid_km	2.67 (0.21)	4.58 (0.33)	7.59 (0.53)
	msg_km_cor	2.67 (0.21)	4.58 (0.33)	7.59 (0.54)
	adap.thrsd	4.93 (0.41)	9.69 (0.26)	13.92 (0.2)
	linear	2.63 (0.21)	4.58 (0.57)	7.29 (0.78)
	QIS	4.92 (0.23)	21.79 (0.65)	45.85 (1.13)
	NERCOME	2.55 (0.13)	3.36 (0.13)	3.66 (0.41)
	corShrink	2.62 (0.15)	3.4 (0.25)	3.5 (0.56)
	SCM	6.7 (0.3)	22.76 (0.36)	45.76 (0.3)
	oracle_nonlin	2.32 (0.19)	3.22 (0.07)	3.45 (0.03)

Table 5: Simulations for correlation matrix estimation. $n = 100$.

Model	Method	p=30	p=100	p=200
Model1	msg_sgrid_km	2.06 (0.31)	4.4 (0.53)	6.93 (0.5)
	msg_km_cor	2.02 (0.35)	4.33 (0.6)	6.78 (0.56)
	adap.thrsd	1.96 (0.59)	5.92 (0.74)	9.54 (0.75)
	linear	2.64 (0.33)	8.54 (0.42)	15.22 (0.33)
	QIS	2.32 (0.37)	7.83 (0.41)	16.23 (0.46)
	NERCOME	2.36 (0.4)	8.17 (0.4)	15.14 (0.39)
	corShrink	1.41 (0.45)	3.57 (0.52)	5.42 (0.53)
	SCM	2.78 (0.32)	9.77 (0.43)	19.83 (0.41)
	oracle_nonlin	2.19 (0.37)	7.71 (0.36)	14.76 (0.34)
	oracle_gmleb	1.9 (0.35)	3.87 (0.45)	5.78 (0.47)
Model2	msg_sgrid_km	0.97 (0.62)	2.51 (2.0)	4.42 (3.85)
	msg_km_cor	0.97 (0.62)	2.47 (1.99)	4.36 (3.85)
	adap.thrsd	6.37 (1.21)	26.59 (2.15)	56.3 (3.24)
	linear	2.17 (0.42)	7.3 (1.21)	14.79 (1.97)
	QIS	1.69 (0.52)	5.6 (1.46)	23.4 (5.37)
	NERCOME	1.66 (0.47)	5.45 (1.32)	11.02 (2.04)
	corShrink	2.02 (0.29)	6.65 (0.76)	13.21 (1.58)
	SCM	2.3 (0.38)	7.62 (0.81)	15.5 (1.52)
	oracle_nonlin	1.5 (0.44)	5.03 (0.91)	10.28 (1.54)
	oracle_gmleb	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Model3	msg_sgrid_km	0.83 (0.53)	2.95 (1.66)	6.29 (3.23)
	msg_km_cor	0.82 (0.5)	2.72 (1.47)	5.35 (3.03)
	adap.thrsd	1.35 (0.4)	5.29 (2.96)	14.69 (17.18)
	linear	1.39 (0.69)	4.74 (2.0)	9.35 (4.54)
	QIS	1.42 (0.95)	4.49 (2.35)	18.95 (6.43)
	NERCOME	1.25 (0.62)	4.46 (1.81)	8.65 (3.85)
	corShrink	0.62 (0.7)	1.94 (2.24)	4.51 (4.99)
	SCM	1.34 (0.39)	4.53 (1.08)	9.33 (2.31)
	oracle_nonlin	1.12 (0.18)	4.02 (0.36)	8.0 (0.62)
	oracle_gmleb	0.86 (0.2)	2.98 (0.68)	6.07 (1.43)
Model4	msg_sgrid_km	1.09 (0.17)	3.79 (0.35)	7.57 (0.53)
	msg_km_cor	1.13 (0.3)	6.25 (0.98)	17.17 (1.86)
	adap.thrsd	0.5 (0.22)	1.69 (0.64)	3.34 (1.34)
	linear	0.78 (0.54)	2.45 (1.75)	4.91 (2.99)
	QIS	0.61 (0.43)	1.83 (1.17)	7.3 (2.19)
	NERCOME	0.55 (0.3)	1.77 (0.95)	3.71 (1.77)
	corShrink	0.27 (0.37)	0.85 (0.94)	1.73 (2.18)
	SCM	0.5 (0.22)	1.67 (0.6)	3.3 (1.18)
	oracle_nonlin	0.43 (0.05)	1.59 (0.08)	3.14 (0.12)
	oracle_gmleb	1.51 (0.59)	4.8 (1.7)	9.56 (3.4)
Model5	msg_sgrid_km	2.22 (0.1)	5.18 (0.08)	8.07 (0.1)
	msg_km_cor	2.22 (0.1)	5.18 (0.08)	8.07 (0.1)
	adap.thrsd	3.23 (0.04)	5.99 (0.02)	8.47 (0.01)
	linear	2.2 (0.08)	5.15 (0.05)	7.83 (0.03)
	QIS	1.91 (0.17)	5.34 (0.12)	14.2 (0.14)
	NERCOME	1.93 (0.15)	5.04 (0.07)	7.8 (0.03)
	corShrink	2.19 (0.09)	5.15 (0.04)	7.82 (0.02)
	SCM	2.91 (0.18)	9.98 (0.2)	20.02 (0.17)
	oracle_nonlin	1.74 (0.13)	4.9 (0.05)	7.67 (0.03)
	oracle_gmleb	2.2 (0.09)	5.16 (0.05)	7.85 (0.04)
Model6	msg_sgrid_km	1.99 (0.14)	3.35 (0.09)	4.53 (0.18)
	msg_km_cor	1.99 (0.14)	3.35 (0.09)	4.53 (0.18)
	adap.thrsd	2.69 (0.03)	3.31 (0.0)	3.49 (0.0)
	linear	1.97 (0.15)	3.15 (0.05)	3.49 (0.05)
	QIS	1.71 (0.19)	3.7 (0.18)	14.78 (0.1)
	NERCOME	1.69 (0.2)	2.89 (0.22)	3.42 (0.08)
	corShrink	2.0 (0.15)	3.16 (0.05)	3.47 (0.04)
	SCM	2.03 (0.14)	3.33 (0.13)	29.05 (0.14)

Table 6: Simulations result for uniform distributed data.

Model	Method	p=30	p=100	p=200
Model1	msg_sgrid_km	3.95 (0.7)	7.44 (0.84)	11.92 (0.83)
	msg_km_cor	3.76 (0.75)	7.11 (0.86)	11.47 (0.9)
	adap.thrsd	3.75 (1.04)	9.61 (0.99)	15.76 (1.2)
	linear	4.77 (0.74)	13.64 (0.71)	24.54 (0.58)
	QIS	4.45 (0.74)	12.93 (0.67)	26.59 (0.86)
	NERCOME	4.51 (0.89)	13.01 (0.77)	23.87 (0.68)
	corShrink	3.07 (1.09)	6.84 (0.98)	10.59 (1.17)
	SCM	5.01 (0.76)	15.93 (0.82)	32.12 (1.21)
	oracle_nonlin	3.94 (0.58)	12.32 (0.6)	23.24 (0.64)
	oracle_gmleb	3.53 (0.68)	6.66 (0.74)	10.11 (0.76)
Model2	msg_sgrid_km	4.4 (3.96)	13.45 (16.37)	25.99 (27.39)
	msg_km_cor	4.4 (3.96)	13.45 (16.37)	25.99 (27.39)
	adap.thrsd	12.05 (2.19)	50.63 (5.79)	109.37 (6.96)
	linear	6.82 (2.87)	22.94 (11.02)	44.52 (15.97)
	QIS	6.53 (3.04)	21.83 (12.08)	60.92 (37.96)
	NERCOME	6.64 (3.17)	21.99 (11.44)	41.35 (16.54)
	corShrink	6.15 (2.87)	20.85 (11.67)	39.44 (17.6)
	SCM	6.7 (2.61)	23.25 (10.58)	43.43 (15.48)
	oracle_nonlin	4.84 (0.89)	16.51 (2.05)	32.8 (3.11)
	oracle_gmleb	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Model3	msg_sgrid_km	4.06 (2.69)	12.59 (7.29)	24.83 (16.8)
	msg_km_cor	4.06 (2.69)	12.56 (7.32)	24.72 (16.83)
	adap.thrsd	4.7 (2.62)	14.78 (6.35)	29.68 (15.13)
	linear	4.73 (2.62)	15.14 (6.85)	28.62 (15.54)
	QIS	4.58 (2.76)	14.66 (7.11)	39.29 (25.67)
	NERCOME	4.56 (2.72)	14.09 (6.82)	28.1 (15.08)
	corShrink	3.84 (2.77)	12.28 (7.37)	24.85 (17.61)
	SCM	4.68 (2.5)	14.76 (6.07)	29.65 (14.87)
	oracle_nonlin	3.29 (0.65)	10.31 (1.33)	20.51 (2.33)
	oracle_gmleb	2.86 (0.78)	8.47 (2.07)	16.48 (4.89)
Model4	msg_sgrid_km	4.66 (3.04)	14.34 (8.2)	27.16 (17.92)
	msg_km_cor	4.6 (3.09)	14.02 (8.38)	26.61 (18.29)
	adap.thrsd	4.12 (3.1)	12.34 (9.03)	22.78 (20.32)
	linear	4.11 (3.27)	12.5 (9.02)	23.24 (20.44)
	QIS	4.09 (3.16)	12.4 (9.27)	24.4 (21.89)
	NERCOME	4.12 (3.58)	12.38 (10.43)	22.83 (19.13)
	corShrink	3.95 (3.46)	12.19 (9.64)	21.84 (20.96)
	SCM	4.09 (3.31)	12.57 (9.39)	22.68 (20.14)
	oracle_nonlin	2.09 (0.39)	6.66 (0.75)	13.4 (1.5)
	oracle_gmleb	3.32 (1.18)	10.34 (2.7)	20.02 (6.09)
Model5	msg_sgrid_km	5.64 (0.23)	12.95 (0.12)	19.9 (0.14)
	msg_km_cor	5.64 (0.23)	12.95 (0.12)	19.9 (0.14)
	adap.thrsd	8.12 (0.11)	15.15 (0.07)	21.48 (0.06)
	linear	5.57 (0.22)	12.91 (0.11)	19.59 (0.08)
	QIS	4.77 (0.36)	13.34 (0.25)	35.32 (0.44)
	NERCOME	4.8 (0.3)	12.63 (0.17)	19.54 (0.08)
	corShrink	5.67 (0.22)	13.11 (0.12)	19.86 (0.08)
	SCM	7.53 (0.52)	25.09 (0.6)	50.22 (0.76)
	oracle_nonlin	4.34 (0.33)	12.27 (0.14)	19.2 (0.07)
	oracle_gmleb	5.58 (0.22)	12.93 (0.12)	19.63 (0.06)
Model6	msg_sgrid_km	2.56 (0.2)	3.61 (0.07)	4.43 (0.11)
	msg_km_cor	2.56 (0.2)	3.61 (0.07)	4.43 (0.11)
	adap.thrsd	3.54 (0.06)	3.8 (0.04)	3.94 (0.04)
	linear	2.55 (0.21)	3.51 (0.06)	3.71 (0.05)
	QIS	2.22 (0.28)	3.98 (0.17)	15.17 (0.2)
	NERCOME	2.19 (0.28)	3.21 (0.25)	3.65 (0.08)
	corShrink	2.58 (0.22)	3.62 (0.07)	3.91 (0.04)
	SCM	3.63 (0.22)	10.64 (0.23)	20.69 (0.28)
	oracle_nonlin	1.99 (0.25)	2.97 (0.17)	3.39 (0.14)

Table 7: Simulations result for negative binomial distributed data.

Model	Method	p=30	p=100	p=200
Model1	msg_sgrid_km	4.26 (3.26)	15.5 (12.09)	29.23 (22.1)
	msg_km_cor	4.26 (3.26)	15.49 (12.11)	29.14 (22.19)
	adap.thrsd	5.03 (3.7)	22.77 (17.21)	55.57 (40.25)
	linear	4.8 (2.72)	17.16 (10.75)	32.32 (18.78)
	QIS	4.71 (2.97)	16.68 (11.61)	39.52 (33.2)
	NERCOME	4.56 (3.2)	16.5 (11.43)	32.19 (21.33)
	corShrink	4.0 (3.55)	14.66 (12.46)	28.66 (22.84)
	SCM	4.74 (3.15)	16.52 (10.73)	32.64 (20.51)
	oracle_nonlin	3.28 (0.69)	10.46 (1.46)	20.64 (2.49)
	oracle_gmleb	3.12 (0.99)	10.04 (3.31)	19.0 (5.2)
Model2	msg_sgrid_km	5.7 (5.13)	17.72 (14.5)	38.45 (32.1)
	msg_km_cor	5.63 (5.18)	17.53 (14.69)	38.12 (32.58)
	adap.thrsd	5.36 (5.74)	18.16 (18.15)	39.9 (40.12)
	linear	5.29 (5.37)	15.88 (14.64)	34.97 (38.41)
	QIS	5.25 (5.67)	15.93 (16.04)	32.59 (37.67)
	NERCOME	5.02 (6.18)	16.19 (16.14)	35.33 (37.56)
	corShrink	5.23 (5.86)	15.73 (16.92)	34.87 (36.67)
	SCM	5.33 (5.66)	15.88 (16.58)	35.03 (36.6)
	oracle_nonlin	2.09 (0.38)	6.57 (0.96)	13.59 (1.76)
	oracle_gmleb	4.06 (1.59)	11.84 (4.36)	24.66 (12.36)
Model3	msg_sgrid_km	4.16 (0.78)	7.75 (1.01)	12.4 (0.8)
	msg_km_cor	4.0 (0.83)	7.46 (1.09)	11.89 (0.91)
	adap.thrsd	4.36 (1.13)	10.58 (1.27)	17.68 (1.6)
	linear	5.02 (0.98)	13.75 (0.98)	24.68 (0.74)
	QIS	4.69 (1.02)	13.05 (0.95)	26.73 (0.86)
	NERCOME	4.8 (0.92)	13.06 (1.0)	23.98 (0.82)
	corShrink	3.52 (1.08)	7.17 (1.18)	11.1 (0.93)
	SCM	5.34 (0.77)	16.11 (0.96)	32.37 (1.18)
	oracle_nonlin	4.12 (0.7)	12.42 (0.79)	23.36 (0.72)
	oracle_gmleb	3.76 (0.73)	6.84 (0.85)	10.39 (0.7)
Model4	msg_sgrid_km	5.47 (5.91)	18.3 (23.07)	37.5 (46.38)
	msg_km_cor	5.47 (5.91)	18.3 (23.07)	37.5 (46.38)
	adap.thrsd	14.43 (3.16)	56.78 (11.33)	122.54 (26.37)
	linear	7.65 (4.76)	26.2 (15.39)	51.14 (32.25)
	QIS	7.56 (5.19)	25.59 (16.34)	67.24 (51.77)
	NERCOME	7.18 (5.4)	25.74 (17.11)	49.84 (38.45)
	corShrink	6.91 (4.94)	23.99 (17.23)	48.59 (35.77)
	SCM	7.45 (4.3)	25.64 (15.36)	52.07 (33.74)
	oracle_nonlin	4.97 (1.08)	16.63 (2.3)	32.93 (3.86)
	oracle_gmleb	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Model5	msg_sgrid_km	5.77 (0.34)	13.11 (0.2)	20.5 (0.28)
	msg_km_cor	5.77 (0.34)	13.11 (0.2)	20.5 (0.28)
	adap.thrsd	8.25 (0.14)	15.64 (0.3)	22.54 (0.45)
	linear	5.68 (0.27)	12.94 (0.11)	19.61 (0.09)
	QIS	4.95 (0.47)	13.46 (0.31)	35.4 (0.68)
	NERCOME	4.91 (0.39)	12.65 (0.18)	19.54 (0.08)
	corShrink	5.9 (0.32)	13.44 (0.16)	20.27 (0.1)
	SCM	7.74 (0.62)	25.41 (0.72)	50.53 (1.04)
	oracle_nonlin	4.45 (0.39)	12.28 (0.13)	19.21 (0.06)
	oracle_gmleb	5.65 (0.28)	12.95 (0.11)	19.64 (0.08)
Model6	msg_sgrid_km	2.7 (0.23)	3.82 (0.12)	5.16 (0.21)
	msg_km_cor	2.7 (0.23)	3.82 (0.12)	5.16 (0.21)
	adap.thrsd	3.65 (0.1)	4.29 (0.29)	5.3 (0.34)
	linear	2.64 (0.23)	3.53 (0.06)	3.72 (0.05)
	QIS	2.36 (0.29)	4.08 (0.22)	15.19 (0.27)
	NERCOME	2.31 (0.28)	3.25 (0.24)	3.66 (0.08)
	corShrink	2.78 (0.23)	3.91 (0.1)	4.4 (0.12)
	SCM	3.72 (0.3)	10.75 (0.31)	20.81 (0.36)
	oracle_nonlin	2.08 (0.26)	3.01 (0.23)	3.4 (0.16)
	oracle_gmleb	2.64 (0.28)	3.53 (0.09)	3.71 (0.05)

Table 8: Simulations investigating behavior when $p = 1000$.

Method	Model3	Model4	Model1	Model2	Model5	Model6
msg_sgrid_km	144.07(116.84)	159.32(123.43)	43.81(1.35)	158.29(195.58)	60.34(0.87)	16.82(0.42)
msg_km_cor	142.34(118.24)	156.24(126.42)	42.60 (1.46)	158.27(195.62)	60.34(0.87)	16.82(0.42)
adap.thrsd	421.48(308.18)	159.04(152.63)	46.68 (1.53)	637.30(103.61)	49.05(0.19)	6.35(0.26)
linear	148.95(104.53)	151.72(145.75)	75.12 (0.32)	225.10(118.93)	47.19(0.15)	4.78(0.17)
QIS	381.59(230.76)	220.16(183.11)	161.69 (5.49)	731.37(291.45)	239.71(2.94)	97.78(0.76)
NERCOME	147.99(107.34)	156.33(136.08)	75.34 (0.40)	228.94(121.97)	46.81(0.20)	3.83(0.01)
SCM	167.26(103.16)	140.31(136.63)	159.23 (2.23)	242.71(118.40)	251.22(1.80)	101.24(0.42)
oracle_nonlin	101.36(10.01)	66.23(6.27)	74.62 (0.41)	164.04(18.25)	46.54(0.01)	3.73(0.00)
oracle_gmleb	90.11(32.66)	111.13(42.20)	30.07(0.79)	0.0(0.0)	46.65(4.59)	3.74(0.00)