Simulation

In this web appendix, we present more details about the simulations shown in the main manuscript. We also did more experiments to investigate our method in different scenarios. For each simulation setting, we use the six matrix models introduced in 3.1. In our main manuscript, the Frobenius error medians of different methods are presented in figures without exact numbers. In this section, all the error medians are displayed in tables with uncertainty, which is the difference of 75% quantile and 25% quantile of Frobenius errors in numbers of replicates.

Clustering-based exemplar algorithm

In this part, we show the exact Frobenius errors and uncertainties in 1. The result is as 1 shows.

[Table 1 about here.]

According to the result in 1, applying clustering does not have substantial effect on the accuracy of our MSG method but could dramatically reduce the computation cost as observed in 1. With larger number of clusters, our method generally has smaller error but longer running time. To balance the computation cost and accuracy, we take $K = p$ in the later simulations.

Comparison with CorShrink

In this part, we compared the behavior of different estimators on correlation and covariance estimation. We did this to make comparison with CorShrink Dey and Stephens (2018) which also adopts empirical Bayes method but aims to estimate correlation matrix. In each setting, we generated true covariance matrices $\mathbf{\Sigma}$ as in 3.1.

We first compared the estimation of covariance matrix $\mathbf{\Sigma}$. For CorShrink, covariance matrix is estimated as $\mathrm{diag}(\widehat{SD})\widehat{\boldsymbol{R}}\mathrm{diag}(\widehat{SD})$, where $\widehat{SD}$ are sample deviations for $\boldsymbol{X}_{\cdot j}, j = 1, \ldots, p$. The result is presented in 2.

[Table 2 about here.]

In the second setting, we compare the estimation of correlation matrix $\boldsymbol{R}$. For covariance matrix estimators, $\widehat{\boldsymbol{R}} = \mathrm{diag}(1/\widehat{SD})\widehat{\boldsymbol{\Sigma}}\mathrm{diag}(1/\widehat{SD})$, where $\widehat{SD}$ are standard deviations derived from $\widehat{\boldsymbol{\Sigma}}$. The result is presented in 3.

[Table 3 about here.]

The results in 2 and 3 show that CorShrink has outstanding performance in correlation estimation, it has the lowest error in most models, except for Block model when correlations and standard deviations are related and our MSG method can beat other methods. As for covariance estimation, when the empirical distribution of the parameter $(\sigma_j, \sigma_k, r_{jk})$ is simple and easy to approximate, like in Sparse, Block, Dense and Dense2, our method had better performance than CorShrink. In the models built on eigenvalue structure, like Orth and Spiked, CorShrink behaved slighter better. This phenomenon can also be observed in comparison between OracNonlin and OracMSG.

Misspecification

Our estimator is based on the assumption that data is generated from multivariate Gaussian distribution. In reality, data is not always normally distributed. So we are interested to investigate its performance when the model is misspecified. We generated $\boldsymbol{Y}$ from two non-normal distributions with variance of 1 and let $\boldsymbol{X} = \boldsymbol{L}\boldsymbol{Y}$ where $\boldsymbol{L}$ is Cholesky decomposition of covariance matrix $\boldsymbol{\Sigma}$. Then the observed data $\boldsymbol{X}$ has covariance matrix Σ. In first simulation, $\boldsymbol{Y} \sim U(0,1)/\sqrt{(}1/12)$. In the second simulation, $\boldsymbol{Y}$ was generated from negative binomial distributions with size of 10 and mean of 4, and then was normalized to have variance of 1.

[Table 4 about here.]

[Table 5 about here.]

The result shows that although MSG and CorShrink method assume normal distribution for data, it can adapt to misspecified data either for continuous distribution 4 and discrete distribution 5, especially in Sparse, Block Dense and Dense2 models.

Large dimension

In previous simulations, we made $p = 30, 100 and 200$. We are also interested in the case when p has large value. In this section, we let $p = 1000$. The median of errors is presented in 6.

[Table 6 about here.]

The result shows that our estimator still has performance when p is extremely value.

References

Dey, K. K. and Stephens, M. (2018). Corshrink: Empirical bayes shrinkage estimation of correlations, with applications. *bioRxiv* page 368316.

Table 1: Simulations investigating Clustering-based exemplar algorithm, as described in Section 3.2.

Model	Method	p=30	p=100	p=200
Sparse	no cluster	3.02 (1.21)	7.35 (1.52)	11.4 (1.73)
	ratio=2	3.26 (1.25)	7.51 (1.47)	11.52 (1.78)
	ratio=1	3.38 (1.33)	7.86 (1.53)	11.93 (1.8)
	ratio=0.5	3.59 (1.34)	7.98 (1.69)	11.94 (1.86)
	ratio=0.25	3.95 (1.45)	8.4 (1.77)	11.97 (1.76)
Block	no cluster	5.19 (6.86)	15.75 (19.95)	32.23 (34.18)
	ratio=2	5.42 (7.2)	15.51 (20.15)	32.61 (33.87)
	ratio=1	5.34 (7.03)	15.25 (19.32)	32.06 (33.43)
	ratio=0.5	5.4 (6.75)	15.48 (18.94)	33.58 (34.96)
	ratio=0.25	5.22 (6.59)	16.31 (18.37)	33.03 (34.54)
Dense	no cluster	3.76 (3.81)	12.38 (13.37)	19.93 (21.49)
	ratio=2	3.59 (3.79)	12.45 (13.44)	20.07 (21.51)
	ratio=1	3.72 (3.6)	12.27 (12.87)	20.54 (21.65)
	ratio=0.5	3.68 (3.72)	12.57 (13.39)	20.08 (22.77)
	ratio=0.25	3.83 (3.81)	12.9 (12.75)	21.71 (22.82)
Dense2	no cluster	4.46 (5.23)	14.63 (21.61)	24.35 (37.1)
	ratio=2	4.48 (5.28)	14.83 (22.3)	24.58 (37.05)
	ratio=1	4.65 (5.21)	14.85 (21.99)	24.39 (36.78)
	ratio=0.5	4.48 (5.29)	14.43 (21.76)	24.24 (38.92)
	ratio=0.25	4.49 (5.2)	14.44 (21.0)	25.01 (39.0)
Orth	no cluster	5.72 (0.27)	13.02 (0.16)	20.23 (0.25)
	ratio=2	5.71 (0.28)	13.01 (0.16)	20.22 (0.26)
	ratio=1	5.71 (0.25)	13.01 (0.18)	20.24 (0.26)
	ratio=0.5	5.78 (0.29)	13.01 (0.16)	20.25 (0.24)
	ratio=0.25	6.05 (0.39)	13.04 (0.16)	20.27 (0.26)
Spiked	no cluster	2.62 (0.21)	3.71 (0.12)	4.76 (0.15)
	ratio=2	2.62 (0.2)	3.7 (0.12)	4.74 (0.16)
	ratio=1	2.63 (0.21)	3.71 (0.11)	4.75 (0.16)
	ratio=0.5	2.66 (0.2)	3.71 (0.11)	4.77 (0.16)
	ratio=0.25	2.77 (0.19)	3.77 (0.13)	4.77 (0.18)

Table 2: Simulations for covariance matrix estimation. $n = 100$.

Model	Method	p=30	p=100	p=200
Sparse	MSG	3.46 (1.39)	7.97 (1.67)	12.18 (1.87)
	MSGCor	3.42 (1.41)	7.75 (1.72)	11.79 (1.9)
	Adap	5.27 (1.53)	14.24 (1.61)	23.04 (1.67)
	Linear	4.91 (1.21)	15.28 (1.19)	28.06 (1.24)
	QIS	4.79 (1.26)	14.45 (1.28)	29.57 (1.48)
	NERCOME	4.94 (1.32)	14.54 (1.34)	26.61 (1.21)
	CorShrink	3.97 (1.49)	8.99 (1.53)	13.53 (1.74)
	SCM	4.98 (1.03)	16.56 (1.52)	33.21 (1.8)
	OracNonlin	3.99 (0.74)	13.5 (0.95)	25.65 (1.01)
	OracMSG	2.03 (0.76)	6.12 (1.35)	10.09 (1.49)
Block	MSG	5.26 (5.04)	17.71 (19.52)	29.12 (42.35)
	MSGCor	5.26 (5.04)	17.71 (19.52)	29.12 (42.35)
	Adap	13.59 (2.5)	55.12 (6.47)	114.28 (13.71)
	Linear	7.3 (3.24)	25.88 (13.03)	44.84 (28.3)
	QIS	7.08 (3.63)	24.74 (13.66)	65.1 (39.25)
	NERCOME	6.97 (4.13)	23.95 (15.33)	46.8 (31.58)
	CorShrink	6.73 (3.96)	23.0 (13.92)	42.03 (30.29)
	SCM	7.36 (3.66)	25.7 (13.44)	46.21 (28.97)
	OracNonlin	4.9 (1.15)	16.35 (2.03)	32.98 (4.15)
	OracMSG	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Dense	MSG	3.6 (3.96)	12.37 (13.53)	23.42 (28.93)
	MSGCor	3.6 (3.96)	12.37 (13.53)	23.42 (28.93)
	Adap	4.97 (2.99)	18.51 (10.55)	46.22 (32.62)
	Linear	5.09 (3.1)	16.57 (9.56)	33.6 (19.65)
	QIS	5.13 (3.23)	16.14 (10.2)	37.25 (30.02)
	NERCOME	5.01 (3.09)	16.84 (10.99)	30.67 (23.03)
	CorShrink	4.25 (3.16)	14.94 (11.29)	28.68 (24.66)
	SCM	5.03 (2.83)	17.18 (10.01)	32.92 (20.91)
	OracNonlin	3.27 (0.56)	10.83 (1.41)	21.3 (2.44)
	OracMSG	0.01 (0.08)	0.23 (0.61)	0.9 (1.52)
Dense2	MSG	4.64 (5.06)	14.23 (16.91)	30.05 (34.95)
	MSGCor	4.64 (5.06)	14.22 (16.91)	30.05 (34.95)
	Adap	4.77 (4.71)	16.17 (15.34)	33.03 (29.22)
	Linear	4.78 (4.77)	15.18 (15.51)	31.13 (29.03)
	QIS	4.8 (4.76)	16.14 (15.19)	34.61 (32.51)
	NERCOME	4.95 (4.85)	16.42 (16.58)	34.18 (32.12)
	CorShrink	4.72 (4.73)	15.83 (15.48)	33.28 (31.55)
	SCM	4.8 (4.74)	16.14 (14.96)	33.86 (30.36)
	OracNonlin	2.01 (0.43)	6.8 (0.81)	13.81 (1.6)
	OracMSG	0.0 (0.01)	0.0 (0.03)	0.01 (0.11)
Orth	MSG	4.24 (0.19)	8.89 (0.19)	13.85 (0.39)
	MSGCor	4.24 (0.19)	8.89 (0.19)	13.85 (0.39)
	Adap	5.0 (0.19)	9.54 (0.11)	12.93 (0.16)
	Linear	4.12 (0.12)	8.48 (0.09)	11.71 (0.1)
	QIS	4.3 (0.26)	10.19 (0.31)	37.19 (0.48)
	NERCOME	4.09 (0.15)	8.52 (0.06)	11.69 (0.05)
	CorShrink	4.52 (0.22)	9.07 (0.13)	12.71 (0.15)
	SCM	8.15 (0.42)	24.43 (0.68)	51.86 (0.81)
	OracNonlin	3.79 (0.12)	8.34 (0.06)	11.53 (0.03)
	OracMSG	4.13 (0.13)	8.47 (0.07)	11.64 (0.03)
Spiked	MSG	2.61 (0.2)	3.71 (0.09)	4.77 (0.2)
	MSGCor	2.61 (0.2)	3.71 (0.09)	4.77 (0.2)
	Adap	3.59 (0.07)	3.95 (0.06)	4.24 (0.08)
	Linear	2.58 (0.19)	3.52 (0.06)	3.71 (0.05)
	QIS	2.27 (0.26)	4.05 (0.2)	15.17 (0.21)
	NERCOME	2.24 (0.27)	3.25 (0.3)	3.66 (0.07)
	CorShrink	2.68 (0.2)	3.79 (0.09)	4.21 (0.08)
	SCM	3.67 (0.28)	10.72 (0.28)	20.73 (0.31)
	OracNonlin	2.05 (0.23)	2.99 (0.22)	3.4 (0.14)
	OracMSG	2.54 (0.22)	3.51 (0.08)	3.68 (0.04)

Table 3: Simulations for correlation matrix estimation. $n = 100$.

Model	Method	p=30	p=100	p=200
Sparse	MSG	1.5 (0.44)	3.59 (0.55)	5.8 (0.66)
	MSGCor	1.49 (0.46)	3.54 (0.52)	5.68 (0.69)
	Adap	1.98 (0.58)	5.81 (0.61)	9.63 (0.78)
	Linear	2.58 (0.28)	8.21 (0.45)	14.85 (0.45)
	QIS	2.32 (0.34)	7.68 (0.4)	17.44 (0.46)
	NERCOME	2.35 (0.37)	7.76 (0.4)	14.12 (0.4)
	CorShrink	1.38 (0.4)	3.35 (0.55)	5.08 (0.62)
	SCM	2.78 (0.3)	9.75 (0.48)	19.83 (0.46)
	OracNonlin	2.19 (0.36)	7.58 (0.44)	14.29 (0.39)
	OracMSG	0.89 (0.39)	2.78 (0.64)	4.56 (0.68)
Block	MSG	1.0 (0.61)	2.28 (1.47)	4.34 (2.9)
	MSGCor	1.0 (0.61)	2.25 (1.43)	4.36 (2.92)
	Adap	6.51 (1.15)	26.75 (2.07)	56.71 (2.84)
	Linear	2.21 (0.43)	7.38 (1.09)	14.86 (2.2)
	QIS	1.73 (0.52)	5.66 (1.19)	23.33 (5.42)
	NERCOME	1.66 (0.45)	5.54 (1.09)	11.01 (2.02)
	CorShrink	2.01 (0.31)	6.53 (0.59)	12.91 (1.07)
	SCM	2.31 (0.35)	7.65 (0.75)	15.35 (1.25)
	OracNonlin	1.52 (0.37)	5.17 (0.77)	10.3 (1.44)
	OracMSG	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Dense	MSG	0.76 (0.74)	1.97 (2.2)	4.74 (4.55)
	MSGCor	0.76 (0.7)	2.01 (2.22)	4.64 (4.56)
	Adap	1.4 (0.48)	5.16 (2.51)	13.69 (10.33)
	Linear	1.5 (0.82)	4.94 (2.3)	9.88 (4.98)
	QIS	1.87 (1.02)	6.82 (2.99)	16.29 (5.92)
	NERCOME	1.74 (0.71)	7.04 (2.67)	14.37 (5.81)
	CorShrink	0.64 (0.83)	1.82 (2.37)	4.53 (4.44)
	SCM	1.39 (0.42)	4.55 (1.12)	9.41 (2.44)
	OracNonlin	1.5 (0.25)	6.41 (0.42)	13.66 (0.58)
	OracMSG	0.01 (0.08)	0.21 (0.57)	0.73 (1.89)
Dense2	MSG	0.36 (0.27)	0.96 (0.86)	1.98 (2.25)
	MSGCor	0.32 (0.27)	0.94 (0.88)	1.88 (2.19)
	Adap	0.49 (0.17)	1.63 (0.56)	3.38 (1.56)
	Linear	0.76 (0.47)	2.59 (1.63)	5.49 (4.09)
	QIS	0.7 (0.45)	2.75 (1.33)	6.28 (2.55)
	NERCOME	0.72 (0.34)	2.95 (1.32)	6.51 (3.49)
	CorShrink	0.29 (0.32)	0.84 (1.04)	1.91 (2.17)
	SCM	0.49 (0.17)	1.63 (0.55)	3.36 (1.37)
	OracNonlin	0.6 (0.05)	2.78 (0.1)	6.01 (0.15)
	OracMSG	0.0 (0.02)	0.01 (0.11)	0.03 (0.35)
Orth	MSG	1.54 (0.07)	3.64 (0.09)	5.34 (0.14)
	MSGCor	1.54 (0.07)	3.64 (0.09)	5.34 (0.14)
	Adap	1.72 (0.03)	3.68 (0.0)	4.6 (0.0)
	Linear	1.5 (0.04)	3.47 (0.04)	4.52 (0.04)
	QIS	1.55 (0.09)	4.17 (0.14)	14.51 (0.1)
	NERCOME	1.48 (0.06)	3.48 (0.03)	4.51 (0.01)
	CorShrink	1.51 (0.05)	3.47 (0.03)	4.5 (0.02)
	SCM	2.95 (0.14)	9.98 (0.15)	20.03 (0.15)
	OracNonlin	1.39 (0.06)	3.41 (0.02)	4.46 (0.01)
	OracMSG	1.5 (0.05)	3.46 (0.03)	4.5 (0.01)
Spiked	MSG	1.99 (0.15)	3.35 (0.08)	4.53 (0.17)
	MSGCor	1.99 (0.15)	3.35 (0.08)	4.53 (0.17)
	Adap	2.69 (0.03)	3.31 (0.0)	3.49 (0.0)
	Linear	1.98 (0.14)	3.16 (0.05)	3.48 (0.05)
	QIS	1.72 (0.2)	3.7 (0.17)	14.78 (0.13)
	NERCOME	1.68 (0.2)	2.89 (0.22)	3.43 (0.06)
	CorShrink	2.02 (0.14)	3.16 (0.06)	3.46 (0.04)
	SCM	2.93 (0.16)	9.98 (0.14)	20.04 (0.14)
	OracNonlin	1.53 (0.22)	2.69 (0.18)	3.19 (0.13)
	OracMSG	1.93 (0.17)	3.14 (0.07)	3.46 (0.04)

Table 4: Simulations result for uniform distributed data.

Model	Method	p=30	p=100	p=200
Sparse	MSG	3.39 (1.58)	7.64 (1.64)	11.92 (1.81)
	MSGCor	3.36 (1.58)	7.48 (1.79)	11.58 (1.81)
	Adap	5.17 (1.4)	13.71 (1.43)	22.21 (1.54)
	Linear	4.84 (1.22)	15.15 (1.23)	28.15 (1.04)
	QIS	4.73 (1.32)	14.34 (1.29)	29.63 (1.47)
	NERCOME	4.82 (1.44)	14.49 (1.19)	26.71 (1.15)
	CorShrink	3.89 (1.52)	8.91 (1.42)	13.51 (1.78)
	SCM	4.97 (1.25)	16.46 (1.56)	33.05 (1.79)
	OracNonlin	4.01 (0.81)	13.42 (1.01)	25.66 (0.93)
	OracMSG	1.9 (0.91)	6.01 (1.16)	10.07 (1.64)
Block	MSG	4.18 (3.87)	14.2 (16.03)	28.91 (31.48)
	MSGCor	4.18 (3.87)	14.2 (16.03)	28.91 (31.48)
	Adap	12.14 (2.75)	50.89 (5.48)	110.43 (9.1)
	Linear	6.54 (2.74)	22.22 (9.85)	47.03 (20.36)
	QIS	6.21 (2.86)	21.13 (10.67)	60.5 (42.01)
	NERCOME	6.3 (2.66)	21.98 (10.21)	43.9 (21.92)
	CorShrink	5.83 (2.71)	20.68 (10.95)	42.38 (21.83)
	SCM	6.64 (2.4)	23.08 (10.13)	47.14 (20.15)
	OracNonlin	4.8 (0.85)	16.2 (2.04)	33.0 (3.52)
	OracMSG	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Dense	MSG	3.4 (3.29)	10.07 (11.07)	21.37 (25.63)
	MSGCor	3.4 (3.29)	10.05 (11.07)	21.37 (25.63)
	Adap	4.75 (2.23)	15.17 (6.74)	31.85 (16.09)
	Linear	4.68 (2.49)	15.11 (7.61)	30.87 (15.31)
	QIS	4.53 (2.48)	14.63 (7.85)	37.31 (30.31)
	NERCOME	4.54 (2.39)	14.44 (6.97)	30.14 (18.25)
	CorShrink	3.96 (2.54)	12.19 (8.2)	26.07 (19.72)
	SCM	4.73 (2.09)	15.08 (6.66)	30.69 (16.01)
	OracNonlin	3.17 (0.66)	10.79 (1.4)	21.13 (2.43)
	OracMSG	0.0 (0.02)	0.07 (0.31)	0.34 (0.83)
Dense2	MSG	3.23 (3.58)	11.32 (11.95)	20.44 (24.74)
	MSGCor	3.23 (3.59)	11.32 (11.95)	20.44 (24.74)
	Adap	3.72 (3.08)	13.4 (10.75)	24.37 (19.79)
	Linear	3.7 (3.15)	13.61 (11.23)	24.28 (20.68)
	QIS	3.72 (3.13)	13.42 (10.95)	24.46 (18.67)
	NERCOME	4.0 (2.99)	13.49 (10.66)	24.32 (19.74)
	CorShrink	3.75 (2.9)	13.0 (10.73)	24.31 (20.56)
	SCM	3.86 (2.79)	13.4 (10.41)	25.16 (19.93)
	OracNonlin	2.03 (0.39)	6.89 (0.94)	13.97 (1.4)
	OracMSG	0.0 (0.0)	0.0 (0.01)	0.0 (0.01)
Orth	MSG	4.14 (0.14)	8.68 (0.13)	13.11 (0.23)
	MSGCor	4.14 (0.14)	8.68 (0.13)	13.11 (0.23)
	Adap	4.79 (0.13)	9.23 (0.09)	12.37 (0.1)
	Linear	4.07 (0.12)	8.46 (0.08)	11.7 (0.09)
	QIS	4.15 (0.24)	10.09 (0.28)	37.24 (0.41)
	NERCOME	4.04 (0.17)	8.51 (0.07)	11.68 (0.05)
	CorShrink	4.23 (0.15)	8.72 (0.11)	12.13 (0.1)
	SCM	7.92 (0.43)	24.31 (0.49)	51.82 (0.7)
	OracNonlin	3.74 (0.15)	8.34 (0.05)	11.53 (0.02)
	OracMSG	4.09 (0.13)	8.49 (0.07)	11.64 (0.03)
Spiked	MSG	2.6 (0.21)	3.61 (0.08)	4.44 (0.1)
	MSGCor	2.6 (0.21)	3.61 (0.08)	4.44 (0.1)
	Adap	3.55 (0.06)	3.8 (0.05)	3.94 (0.04)
	Linear	2.59 (0.22)	3.51 (0.06)	3.71 (0.05)
	QIS	2.26 (0.28)	4.0 (0.18)	15.18 (0.2)
	NERCOME	2.25 (0.28)	3.22 (0.27)	3.65 (0.09)
	CorShrink	2.64 (0.22)	3.63 (0.07)	3.9 (0.04)
	SCM	3.63 (0.23)	10.65 (0.22)	20.7 (0.28)
	OracNonlin	2.02 (0.26)	2.98 (0.2)	3.4 (0.17)
	OracMSG	2.53 (0.23)	3.48 (0.09)	3.66 (0.03)

Table 5: Simulations result for negative binomial distributed data.

Model	Method	p=30	p=100	p=200
Sparse	MSG	3.56 (1.39)	8.37 (2.14)	12.75 (1.86)
	MSGCor	3.5 (1.41)	8.2 (2.25)	12.45 (1.98)
	Adap	5.49 (1.62)	14.8 (1.69)	24.13 (1.98)
	Linear	5.03 (1.28)	15.44 (1.36)	28.33 (1.12)
	QIS	4.84 (1.29)	14.65 (1.29)	29.6 (1.65)
	NERCOME	4.98 (1.37)	14.85 (1.36)	26.82 (1.18)
	CorShrink	3.93 (1.47)	9.37 (1.65)	14.08 (1.73)
	SCM	5.04 (1.36)	16.65 (1.59)	33.02 (2.07)
	OracNonlin	3.96 (0.78)	13.68 (0.89)	25.83 (1.12)
	OracMSG	2.07 (0.89)	6.41 (1.46)	10.29 (1.55)
Block	MSG	5.13 (5.59)	15.17 (21.1)	38.68 (45.87)
	MSGCor	5.13 (5.59)	15.16 (21.13)	38.68 (45.87)
	Adap	14.02 (3.18)	56.02 (9.97)	121.63 (26.82)
	Linear	7.44 (3.8)	23.47 (13.98)	52.9 (33.53)
	QIS	7.13 (4.24)	22.34 (16.16)	60.51 (52.2)
	NERCOME	6.94 (4.38)	22.88 (14.15)	50.61 (36.64)
	CorShrink	6.83 (4.17)	21.38 (15.8)	49.5 (35.57)
	SCM	7.36 (4.08)	23.61 (14.1)	52.89 (34.0)
	OracNonlin	4.83 (0.99)	16.34 (2.41)	32.87 (4.19)
	OracMSG	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Dense	MSG	3.68 (4.3)	12.52 (11.77)	22.57 (27.91)
	MSGCor	3.68 (4.3)	12.52 (11.8)	22.56 (27.91)
	Adap	5.42 (3.46)	21.91 (15.03)	58.59 (41.83)
	Linear	4.83 (2.82)	17.7 (9.35)	32.7 (18.85)
	QIS	4.85 (2.96)	16.81 (9.72)	36.33 (31.04)
	NERCOME	4.95 (3.53)	15.86 (9.61)	31.35 (21.77)
	CorShrink	4.53 (3.3)	14.91 (10.05)	28.64 (22.39)
	SCM	5.07 (2.94)	17.19 (8.85)	32.61 (19.27)
	OracNonlin	3.23 (0.64)	10.66 (1.6)	20.95 (2.89)
	OracMSG	0.02 (0.13)	0.52 (1.08)	1.54 (2.48)
Dense2	MSG	4.66 (6.0)	17.77 (20.8)	34.8 (37.69)
	MSGCor	4.65 (6.0)	17.77 (20.81)	34.79 (37.69)
	Adap	5.09 (5.39)	21.68 (21.41)	41.3 (36.24)
	Linear	5.01 (5.01)	19.26 (18.01)	36.06 (35.59)
	QIS	5.07 (5.2)	19.89 (18.7)	35.65 (34.73)
	NERCOME	5.46 (5.44)	19.21 (19.57)	35.68 (37.16)
	CorShrink	4.87 (5.24)	18.97 (19.15)	36.71 (35.06)
	SCM	4.93 (5.19)	19.25 (19.16)	37.16 (34.44)
	OracNonlin	2.02 (0.49)	6.93 (1.21)	14.01 (1.99)
	OracMSG	0.0 (0.02)	0.01 (0.2)	0.05 (0.5)
Orth	MSG	4.34 (0.3)	9.13 (0.28)	14.66 (0.48)
	MSGCor	4.34 (0.3)	9.13 (0.28)	14.66 (0.48)
	Adap	5.18 (0.32)	10.25 (0.51)	14.93 (0.74)
	Linear	4.13 (0.13)	8.49 (0.08)	11.7 (0.09)
	QIS	4.31 (0.27)	10.2 (0.28)	37.35 (0.61)
	NERCOME	4.09 (0.18)	8.53 (0.07)	11.7 (0.05)
	CorShrink	4.66 (0.27)	9.29 (0.19)	13.04 (0.19)
	SCM	8.25 (0.49)	24.45 (0.75)	52.11 (0.99)
	OracNonlin	3.79 (0.13)	8.35 (0.06)	11.54 (0.03)
	OracMSG	4.13 (0.13)	8.49 (0.07)	11.64 (0.03)
Spiked	MSG	2.66 (0.23)	3.82 (0.11)	5.13 (0.23)
	MSGCor	2.66 (0.23)	3.82 (0.11)	5.13 (0.23)
	Adap	3.64 (0.1)	4.33 (0.24)	5.22 (0.38)
	Linear	2.6 (0.23)	3.52 (0.06)	3.71 (0.06)
	QIS	2.31 (0.26)	4.09 (0.21)	15.18 (0.24)
	NERCOME	2.28 (0.27)	3.28 (0.24)	3.66 (0.07)
	CorShrink	2.75 (0.24)	3.91 (0.1)	4.39 (0.11)
	SCM	3.71 (0.28)	10.73 (0.33)	20.75 (0.39)
	OracNonlin	2.05 (0.27)	3.01 (0.2)	3.4 (0.13)
	OracMSG	2.6 (0.27)	3.53 (0.08)	3.7 (0.05)

Table 6: Simulations investigating behavior when $p = 1000$.

Method	Sparse	Block	Dense	Dense2	Orth	Spiked
MSG	43.79(1.91)	17.23(21.37)	118.17(141.00)	182.41(203.76)	60.57(1.23)	16.82(0.42)
MSGCor	42.38(1.86)	17.26(21.37)	116.58(141.91)	179.05(207.58)	60.57(1.23)	16.82(0.42)
Adap	46.96(1.49)	64.48(11.57)	322.20 (262.64)	164.11(233.82)	49.06(0.29)	6.35(0.26)
Linear	74.99(0.35)	23.44(13.66)	140.98 (110.96)	150.00(201.23)	47.17(0.16)	4.78(0.17)
QIS	161.49(3.03)	68.03(28.94)	433.54 (151.17)	205.06(175.77)	240.09(2.53)	97.78(0.76)
NERCOME	75.29(0.58)	22.93(18.16)	141.50 (113.74)	174.27(174.87)	46.81(0.20)	3.83(0.01)
CorShrink	26.63(0.93)	22.75(14.11)	118.93 (147.66)	168.85(218.66)	47.96(0.11)	101.24(0.42)
Sample	159.30(2.62)	24.80(14.45)	143.48 (129.75)	170.29(217.02)	251.14(1.80)	101.24(0.42)
OracNonlin	74.45(0.32)	16.29(1.27)	99.63(6.74)	66.90(7.45)	46.54(0.01)	3.73(0.00)
OracMSG	29.97(0.83)	0.0(0.0)	84.75(32.78)	115.78(72.52)	46.65(0.01)	3.74(0.00)