Simulation

In this web appendix, we present more details about the simulations in the main manuscript as well as more simulations related to our method. For each simulation, we use models in 3.1.

Clustering-based exemplar algorithm

In our main manuscript, the Frobenius errors of different methods are shown in a figure without exact numbers. 1 contains the exact Frobenius error and uncertainty, which is the difference of 75% quantile and 25% quantile of 200 replicates.

[Table 1 about here.]

Comparison with corShrink

In this part, we did 4 simulations in order to compare the behavior of corShrink (**?**) with other covariance matrix estimators. The procedure is as the following 2 settings. In each setting, we generate true covariance matrices $\boldsymbol{\Sigma}$ as 6 models in 3.1. Sample size n is taken as 20 and 100.

In first setting, we compare the estimation of covariance matrix $\boldsymbol{\Sigma}$. For corShrink, covariance matrix is estimated as $\widehat{\boldsymbol{\Sigma}} = \mathrm{diag}(\widehat{SD})\widehat{\boldsymbol{R}}\mathrm{diag}(\widehat{SD})$ where $\widehat{SD}$ are sample standard deviations. The result is presented in .

[Table 2 about here.]

In second setting, we compare the estimation of correlation matrix $\boldsymbol{R}$. For other covariance matrix estimators, $\widehat{\boldsymbol{R}} = \mathrm{diag}(1/\widehat{SD})\widehat{\boldsymbol{\Sigma}}\mathrm{diag}(1/\widehat{SD})$, where $\widehat{SD}$ are standard deviations derived from $\widehat{\boldsymbol{\Sigma}}$.

Misspecification

Our method is based on the assumption that data is generated from multivariate Gaussian distribution. We are also interested on its performance when the data is misspecified. We generate data from uniform and negative binomial distributions and compare its performance with other methods.

Large dimension

We are also interested in the case when p is 1000.

Table 1: Simulations investigating Clustering-based exemplar algorithm, as described in Section 3.2.

Model	Method	p=30	p=100	p=200
Model1	no cluster	3.51 (0.95)	6.96 (1.04)	11.27 (0.81)
	ratio=2	3.68 (0.96)	7.21 (0.88)	11.54 (0.74)
	ratio=1	3.98 (0.9)	7.36 (0.99)	11.61 (0.77)
	ratio=0.5	4.26 (0.86)	7.84 (0.84)	11.9 (0.79)
	ratio=0.25	4.72 (0.96)	8.99 (1.22)	12.34 (0.97)
Model2	no cluster	5.19 (6.86)	15.75 (19.95)	32.23 (34.18)
	ratio=2	5.42 (7.2)	15.51 (20.15)	32.61 (33.87)
	ratio=1	5.34 (7.03)	15.25 (19.32)	32.06 (33.43)
	ratio=0.5	5.4 (6.75)	15.48 (18.94)	33.58 (34.96)
	ratio=0.25	5.22 (6.59)	16.31 (18.37)	33.03 (34.54)
Model3	no cluster	4.18(2.96)	13.79(8.17)	26.66(16.73)
	ratio=2	4.12(3.06)	13.81(7.99)	16.64(16.51)
	ratio=1	4.13(3.04)	13.83(8.13)	26.53(16.64)
	ratio=0.5	4.23(2.93)	13.95(8.44)	26.46(16.85)
	ratio=0.25	4.54(2.70)	13.74(8.35)	26.56(17.47)
Model4	no cluster	4.87(3.54)	17.52(11.62)	38.63(32.05)
	ratio=2	4.89(3.51)	17.57(11.54)	38.62(31.94)
	ratio=1	4.81(3.48)	17.40(11.66)	38.55(32.05)
	ratio=0.5	4.92(3.43)	17.19(11.99)	38.27(32.65)
	ratio=0.25	5.45(3.27)	16.90(12.01)	38.54(34.55)
Model5	no cluster	5.72 (0.27)	13.02 (0.16)	20.23 (0.25)
	ratio=2	5.71 (0.28)	13.01 (0.16)	20.22 (0.26)
	ratio=1	5.71 (0.25)	13.01 (0.18)	20.24 (0.26)
	ratio=0.5	5.78 (0.29)	13.01 (0.16)	20.25 (0.24)
	ratio=0.25	6.05 (0.39)	13.04 (0.16)	20.27 (0.26)
Model6	no cluster	2.62 (0.21)	3.71 (0.12)	4.76 (0.15)
	ratio=2	2.62 (0.2)	3.7 (0.12)	4.74 (0.16)
	ratio=1	2.63 (0.21)	3.71 (0.11)	4.75 (0.16)
	ratio=0.5	2.66 (0.2)	3.71 (0.11)	4.77 (0.16)
	ratio=0.25	2.77 (0.19)	3.77 (0.13)	4.77 (0.18)

Table 2: Simulations investigating behavior of corShrink.

Model	Method	p=30	p=100	p=200
Model1	msg_sgrid_km	4.13 (0.77)	7.64 (0.96)	12.19 (0.93)
	msg_km_cor	3.91 (0.91)	7.26 (1.06)	11.76 (0.94)
	adap.thrsd	4.12 (1.09)	10.07 (1.16)	16.64 (1.19)
	linear	4.88 (0.92)	13.57 (0.81)	24.67 (0.67)
	QIS	4.56 (0.94)	12.84 (0.83)	26.65 (0.86)
	NERCOME	4.67 (0.98)	13.01 (0.8)	23.97 (0.71)
	corShrink	3.38 (1.07)	7.0 (1.25)	10.84 (1.31)
	SCM	5.22 (0.92)	16.05 (0.78)	32.14 (1.13)
	oracle_nonlin	3.95 (0.71)	12.33 (0.7)	23.34 (0.81)
	oracle_gmleb	3.63 (0.76)	6.74 (0.78)	10.3 (0.85)
Model2	msg_sgrid_km	5.52 (5.47)	16.59 (16.08)	34.58 (40.06)
	msg_km_cor	5.52 (5.47)	16.59 (16.08)	34.58 (40.05)
	adap.thrsd	13.59 (2.89)	54.68 (5.96)	116.37 (15.91)
	linear	7.45 (3.89)	23.91 (11.67)	50.65 (26.53)
	QIS	7.19 (4.36)	23.1 (12.04)	69.21 (54.8)
	NERCOME	7.29 (4.28)	23.01 (13.47)	48.57 (30.61)
	corShrink	6.75 (4.39)	22.26 (11.45)	46.82 (28.74)
	SCM	7.45 (3.93)	24.36 (11.11)	50.47 (28.47)
	oracle_nonlin	4.91 (0.99)	16.44 (2.06)	32.86 (3.94)
	oracle_gmleb	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Model3	msg_sgrid_km	4.31 (3.41)	13.83 (10.19)	25.41 (17.42)
	msg_km_cor	4.31 (3.41)	13.79 (10.24)	25.3 (17.41)
	adap.thrsd	4.81 (3.38)	17.81 (10.68)	39.11 (21.77)
	linear	4.78 (3.05)	15.8 (8.85)	29.33 (15.16)
	QIS	4.65 (3.23)	15.33 (9.41)	40.79 (28.32)
	NERCOME	4.74 (3.47)	14.92 (10.12)	30.14 (16.05)
	corShrink	4.21 (3.54)	13.58 (11.03)	26.01 (19.31)
	SCM	4.76 (3.12)	15.61 (9.22)	30.68 (16.19)
	oracle_nonlin	3.3 (0.74)	10.37 (1.37)	20.45 (2.22)
	oracle_gmleb	3.09 (1.04)	9.32 (2.68)	17.89 (4.98)
Model4	msg_sgrid_km	5.9 (4.82)	16.68 (13.9)	32.96 (22.78)
	msg_km_cor	5.84 (4.86)	16.37 (14.09)	32.48 (23.17)
	adap.thrsd	5.46 (5.23)	15.19 (15.64)	29.23 (27.79)
	linear	5.56 (4.9)	15.35 (15.25)	28.44 (26.14)
	QIS	5.44 (5.29)	15.06 (15.71)	27.28 (24.79)
	NERCOME	5.31 (5.56)	15.71 (13.22)	27.71 (24.15)
	corShrink	5.26 (5.36)	14.62 (16.06)	27.72 (25.8)
	SCM	5.32 (5.26)	14.97 (15.82)	28.48 (25.29)
	oracle_nonlin	2.03 (0.41)	6.78 (0.83)	13.65 (1.68)
	oracle_gmleb	3.95 (1.58)	11.43 (4.27)	22.62 (7.98)
Model5	msg_sgrid_km	5.72 (0.28)	13.03 (0.18)	20.24 (0.24)
	msg_km_cor	5.72 (0.28)	13.03 (0.18)	20.24 (0.24)
	adap.thrsd	8.2 (0.13)	15.31 (0.09)	21.7 (0.1)
	linear	5.63 (0.23)	12.93 (0.12)	19.61 (0.1)
	QIS	4.86 (0.36)	13.39 (0.29)	35.44 (0.51)
	NERCOME	4.84 (0.36)	12.65 (0.17)	19.54 (0.07)
	corShrink	5.82 (0.26)	13.31 (0.14)	20.12 (0.1)
	SCM	7.64 (0.56)	25.29 (0.7)	50.46 (0.88)
	oracle_nonlin	4.37 (0.34)	12.27 (0.14)	19.21 (0.07)
	oracle_gmleb	5.64 (0.19)	12.94 (0.12)	19.66 (0.07)