CS 7280 - Spring 2015 Olga Vitek

Homework 2

Each part of the problems 5 points
Due on Blackboard before 11:45am on Thursday February 4.

Note: For all but the last problem it is best to perform the calculations by hand.

1. Often caffeine is used in order to replace the need for sleep. One recent study compares students' ability to recall memorized information after either the consumption of caffeine or a brief sleep. A random sample of 24 adults (between the ages of 18 and 39) were randomly divided into three groups and verbally given a list of words to memorize. During a break, one of the groups takes a nap for an hour and a half, another group is kept awake and then given a caffeine pill an hour prior to testing. The response variable of interest is the number of words participants are able to recall following the break.

   The summary statistics for the two groups are shown below. We are interested in testing whether there is evidence of difference in average recall ability.

Group	Sample Size	Mean	Standard Deviation
Sleep	15	15.25	3.3
Caffeine	12	12.25	3.6

   (a) Test the two groups for difference in recall ability, assuming an equal population variance between the two groups. State the null and the alternative hypothesis, the test statistic, and the conclusion at the 95% confidence level. According to the test results, would we be justified in concluding that caffeine impairs recall ability?

   (b) Repeat (a), without assuming an equal population variance between the two groups. Compare the results.

   (c) Repeat (b), while using the following confidence interval approach as opposed to testing approach.

      i. Produce two separate 95% confidence intervals for the mean recall in each population. Interpret the results.

      ii. Explain why the 95% confidence interval is not an interval that contains 95% of recalled words in each group, and explain what information the confidence interval DOES convey.

      iii. Produce a single 95% confidence interval for the difference between the means

   (d) What are the advantages/disadvantages of all the approaches above in answering the original question?

2. A 2015 Gallup U.S. Daily survey conducted telephone interviews of a random sample of 1,024 adults, aged 18 and older, living in all 50 U.S. states and the District of Columbia `http://www.gallup.com/poll/183272/record-high-americans-support-sex-marriage.aspx`. The survey found that 60% of responders support legalizing same-sex marriage.

(a) Calculate 90% and 95% confidence intervals for the proportion in the population who favor legalizing same-sex marriage. Which confidence interval is wider? Why?

(b) A similar poll conducted in 2014 found that 55% of respondents supported legalizing same-sex marriage in the previous year. What is the minimum number of 2015 interviews would have been required to establish that the proportion of those in favor of legalizing same-sex marriage increased since the previous year? (Note: since the number of respondents in 2014 is unknown, view 55% as a fixed value of interest).

3. What is wrong with each of these statements?

   (a) A researcher wants to test $H_0: \bar{x}_1 = \bar{x}_2$ against $H_a: \bar{x}_1 = \bar{x}_2$.

   (b) A study recorded the exam scores of 100 college students. The scores of 56 males in the study were compared with all 100 students in the study using the independent two-sample t-test.

   (c) A two-sample t statistic gave a p-value of 0.94. From this we can reject the null hypothesis with 90% confidence.

   (d) A researcher is interested in testing $H_0: \mu_1 - \mu_2 = 0$ against $H_a: \mu_1 - \mu_2 < 0$. The test gave t = 2.15. Since the p-value for the two-sided alternative ($H_a: \mu_1 - \mu_2 \neq 0$) gave p-value equal to 3.6%, the researcher concludes that the p-value for the one-sided test is 1.8%.

4. We will conduct a simulation that reproduces Figure 2 and Figure 4 `http://www.nature.com/nmeth/journal/v12/n3/abs/nmeth.3288.html`of the manuscript by Halsey et al, as follows.

   (a) Create a vector of differences between the population means in two groups, which contains values 0, 0.5, 1, 2, and 3. Create a vector of standard deviations, which contains values 0.01, 0.25, 0.5, 1, and 2. For each combination of means and standard deviations, simulate $n = 10$ observations from the two populations as in Figure 1, and record the p-value of a two-sample t-test. Repeat this a large number of times, say 500, and plot the histogram of the p-values.

   (b) Repeat (a) for $n = 30$ and $n = 300$.

   (c) Interpret the results. Comment of the interplay of the true difference between the population means, of the standard deviation in the population, and of the sample size on the ability to detect the differences.