

Game playing

CHAPTER 5, SECTIONS 1–5

Outline

- ◇ Perfect play
- ◇ Resource limits
- ◇ α - β pruning
- ◇ Games of chance

Games vs. search problems

“Unpredictable” opponent \Rightarrow solution is a contingency plan

Time limits \Rightarrow unlikely to find goal, must approximate

Plan of attack:

- algorithm for perfect play (Von Neumann, 1944)
- finite horizon, approximate evaluation (Zuse, 1945; Shannon, 1950; Samuel, 1952–57)
- pruning to reduce costs (McCarthy, 1956)

Types of games

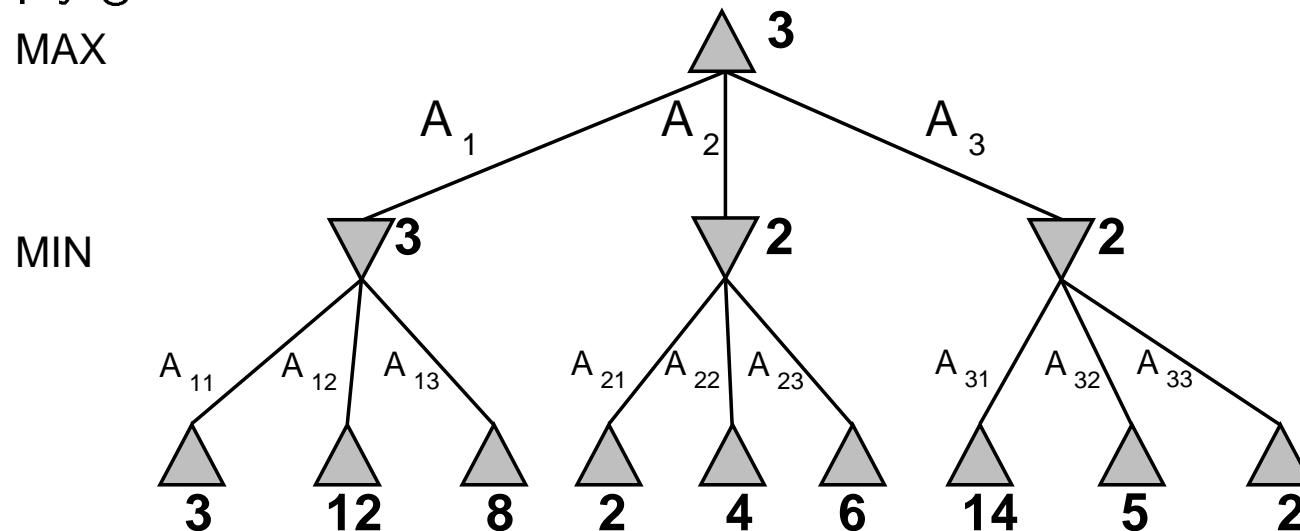
	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information		bridge, poker, scrabble nuclear war

Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest *minimax value*
= best achievable payoff against best play

E.g., 2-ply game:



Minimax algorithm

function MINIMAX-DECISION(*game*) *returns an operator*

for each *op* **in** OPERATORS[*game*] **do**

 VALUE[*op*] ← MINIMAX-VALUE(APPLY(*op*, *game*), *game*)

end

return the *op* with the highest VALUE[*op*]

function MINIMAX-VALUE(*state*, *game*) *returns a utility value*

if TERMINAL-TEST[*game*](*state*) **then**

return UTILITY[*game*](*state*)

else if MAX is to move in *state* **then**

return the highest MINIMAX-VALUE of SUCCESSORS(*state*)

else

return the lowest MINIMAX-VALUE of SUCCESSORS(*state*)

Properties of minimax

Complete??

Optimal??

Time complexity??

Space complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity?? $O(b^m)$

Space complexity?? $O(bm)$ (depth-first exploration)

For chess, $b \approx 35$, $m \approx 100$ for “reasonable” games
 \Rightarrow exact solution completely infeasible

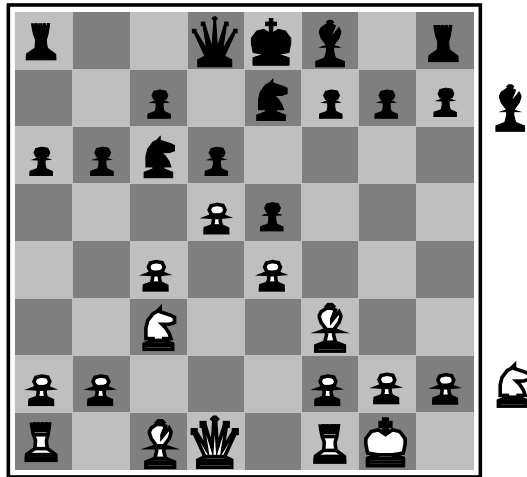
Resource limits

Suppose we have 100 seconds, explore 10^4 nodes/second
 \Rightarrow 10^6 nodes per move

Standard approach:

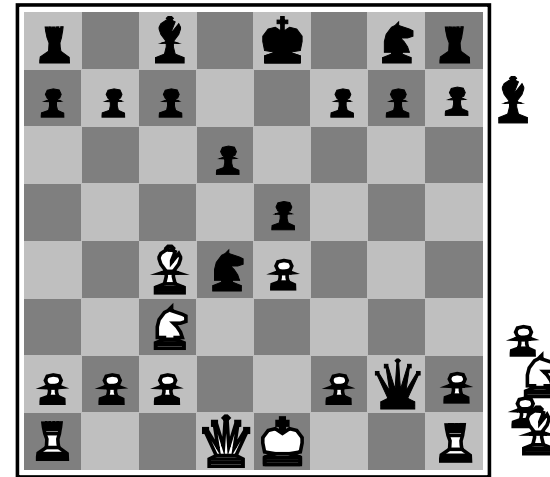
- *cutoff test*
e.g., depth limit (perhaps add *quiescence search*)
- *evaluation function*
= estimated desirability of position

Evaluation functions



Black to move

White slightly better



White to move

Black winning

For chess, typically *linear* weighted sum of features

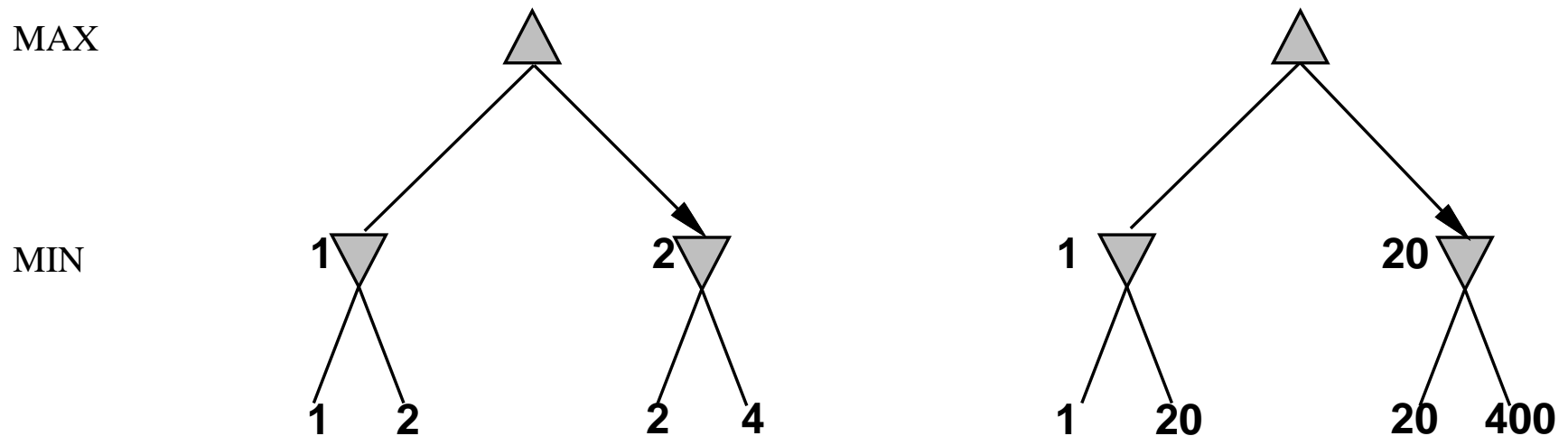
$$\text{EVAL}(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

e.g., $w_1 = 9$ with

$f_1(s) = (\text{number of white queens}) - (\text{number of black queens})$

etc.

Digression: Exact values don't matter



Behaviour is preserved under any *monotonic* transformation of EVAL

Only the order matters:

payoff in deterministic games acts as an *ordinal utility* function

Cutting off search

MINIMAXCUTOFF is identical to MINIMAXVALUE except

1. TERMINAL? is replaced by CUTOFF?
2. UTILITY is replaced by EVAL

Does it work in practice?

$$b^m = 10^6, \quad b = 35 \quad \Rightarrow \quad m = 4$$

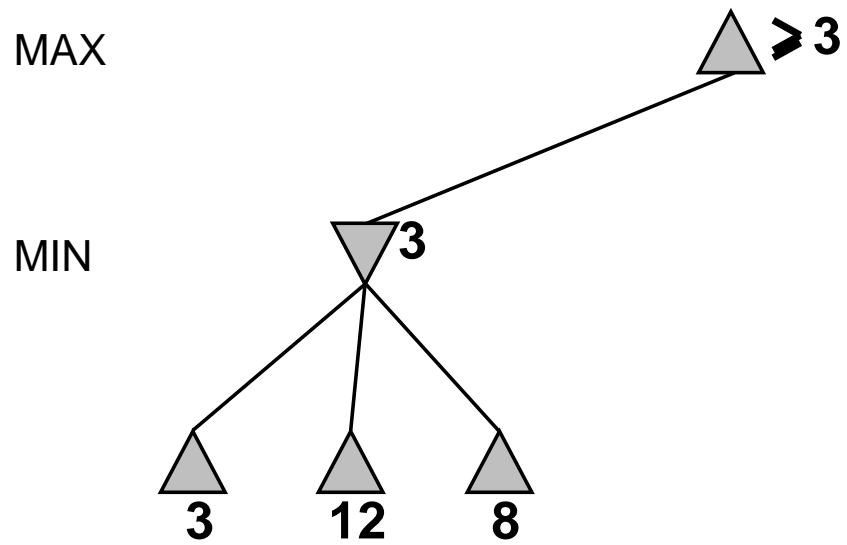
4-ply lookahead is a hopeless chess player!

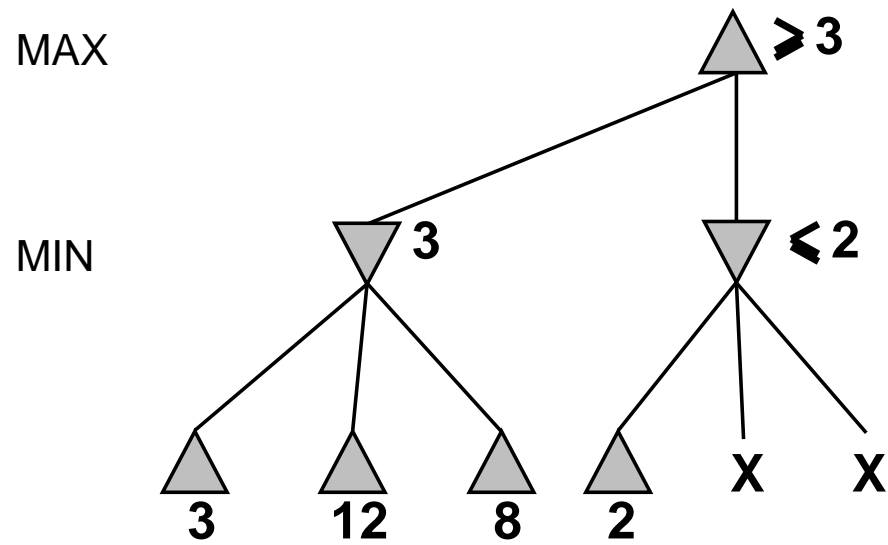
4-ply \approx human novice

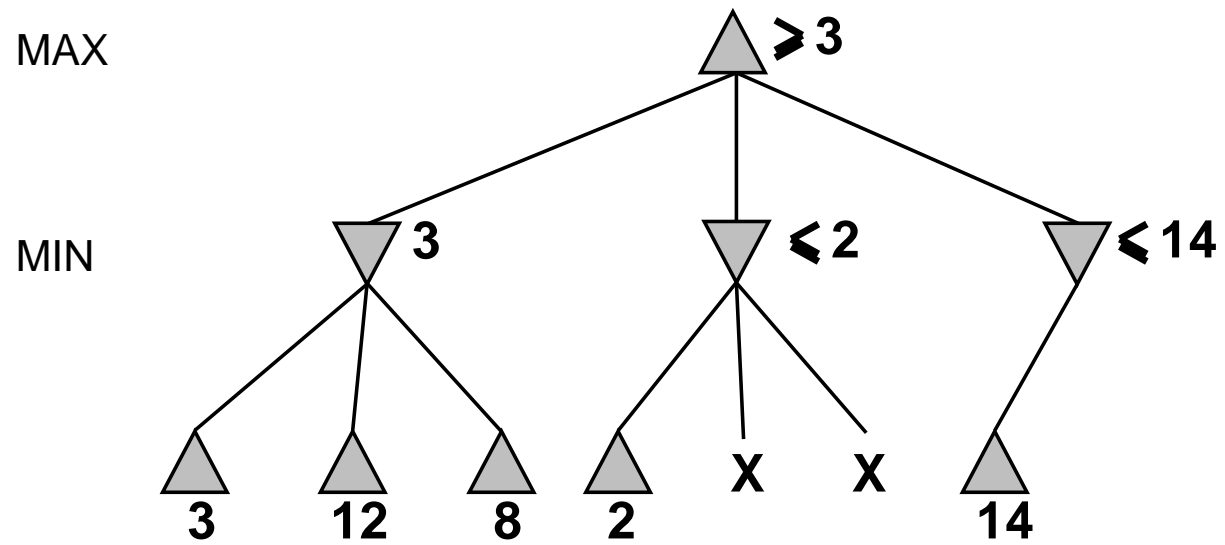
8-ply \approx typical PC, human master

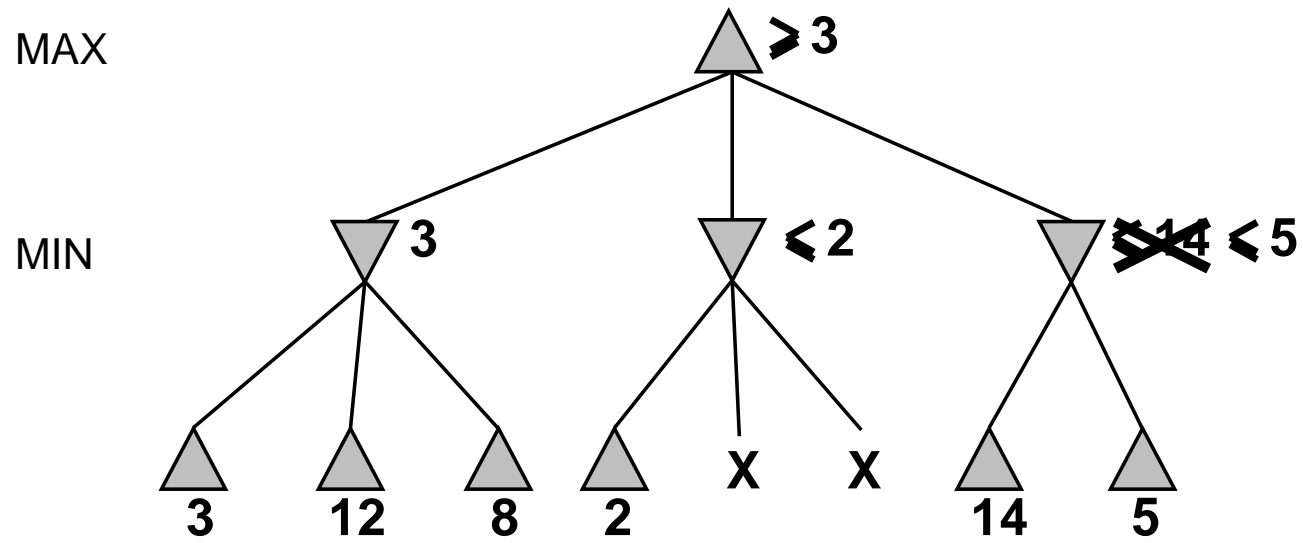
12-ply \approx Deep Blue, Kasparov

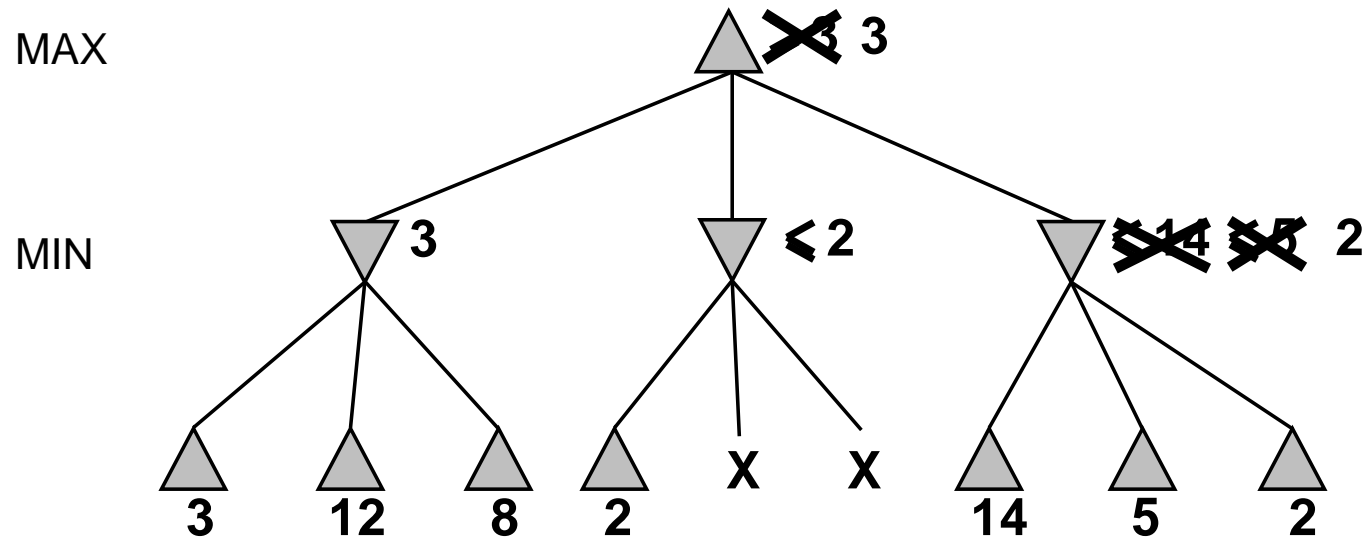
α - β pruning example











Properties of $\alpha-\beta$

Pruning *does not* affect final result

Good move ordering improves effectiveness of pruning

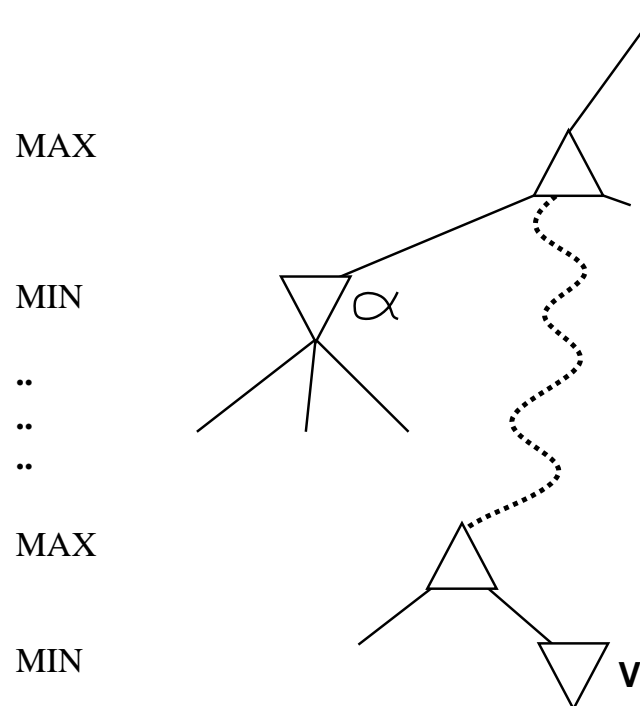
With “perfect ordering,” time complexity = $O(b^{m/2})$

⇒ *doubles* depth of search

⇒ can easily reach depth 8 and play good chess

A simple example of the value of reasoning about which computations are relevant (a form of *metareasoning*)

Why is it called α - β ?



α is the best value (to MAX) found so far off the current path

If V is worse than α , MAX will avoid it \Rightarrow prune that branch

Define β similarly for MIN

The α - β algorithm

Basically MINIMAX + keep track of α , β + prune

function MAX-VALUE(*state*, *game*, α , β) **returns** the minimax value of *state*

inputs: *state*, current state in game

game, game description

α , the best score for MAX along the path to *state*

β , the best score for MIN along the path to *state*

if CUTOFF-TEST(*state*) **then return** EVAL(*state*)

for each *s* **in** SUCCESSORS(*state*) **do**

$\alpha \leftarrow \text{MAX}(\alpha, \text{MIN-VALUE}(s, \text{game}, \alpha, \beta))$

if $\alpha \geq \beta$ **then return** β

end

return α

function MIN-VALUE(*state*, *game*, α , β) **returns** the minimax value of *state*

if CUTOFF-TEST(*state*) **then return** EVAL(*state*)

for each *s* **in** SUCCESSORS(*state*) **do**

$\beta \leftarrow \text{MIN}(\beta, \text{MAX-VALUE}(s, \text{game}, \alpha, \beta))$

if $\beta \leq \alpha$ **then return** α

end

return β

Deterministic games in practice

Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

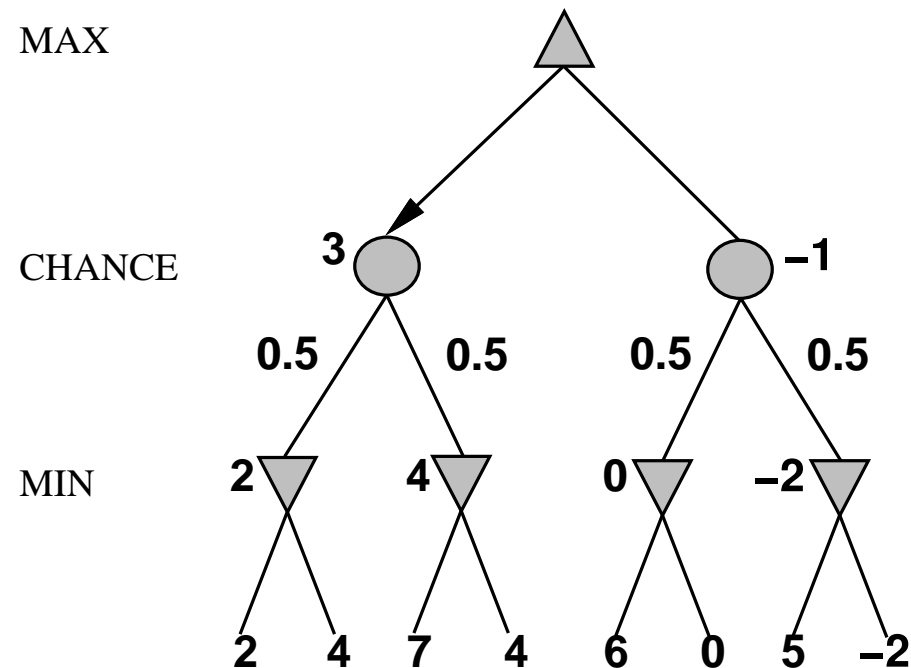
Othello: human champions refuse to compete against computers, who are too good.

Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

Nondeterministic games

E..g, in backgammon, the dice rolls determine the legal moves

Simplified example with coin-flipping instead of dice-rolling:



Algorithm for nondeterministic games

EXPECTIMINIMAX gives perfect play

Just like MINIMAX, except we must also handle chance nodes:

...

if *state* is a chance node **then**

return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

...

A version of α - β pruning is possible

but only if the leaf values are bounded. Why??

Nondeterministic games in practice

Dice rolls increase b : 21 possible rolls with 2 dice

Backgammon \approx 20 legal moves (can be 6,000 with 1-1 roll)

$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

As depth increases, probability of reaching a given node shrinks

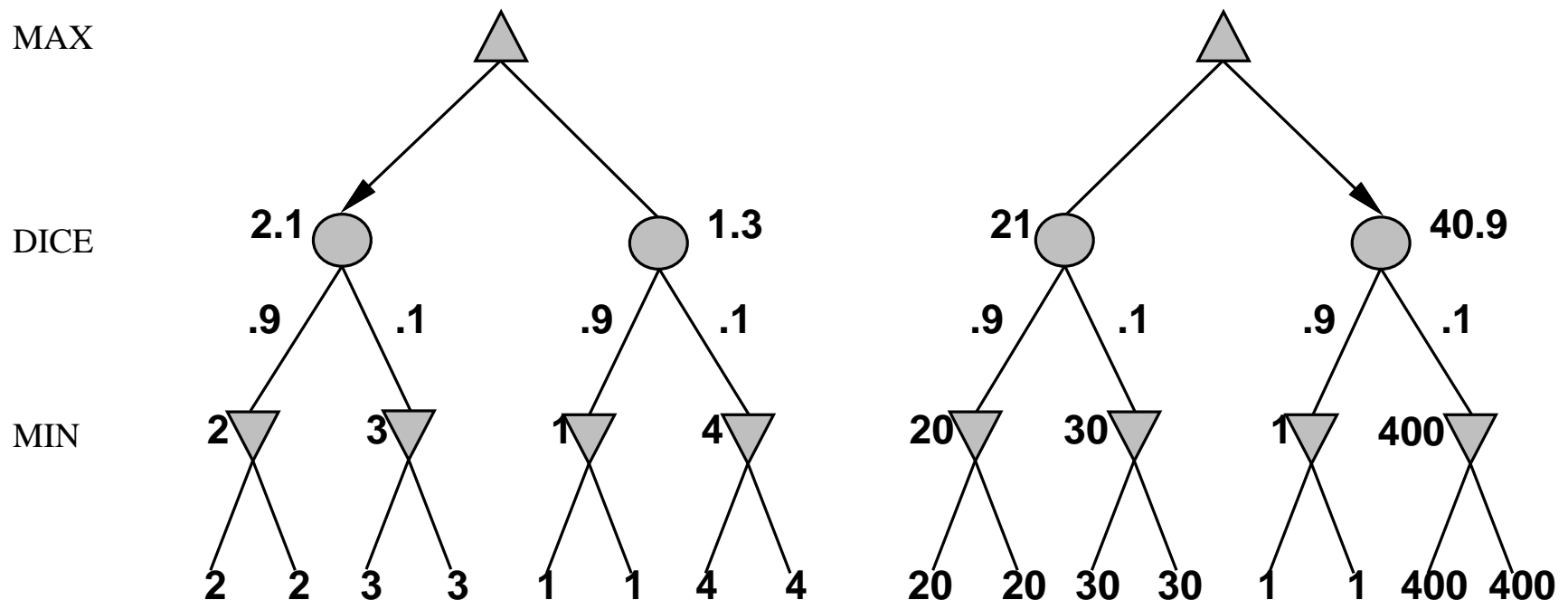
\Rightarrow value of lookahead is diminished

α - β pruning is much less effective

TDGAMMON uses depth-2 search + very good EVAL

\approx world-champion level

Digression: Exact values DO matter



Behaviour is preserved only by *positive linear* transformation of $EVAL$

Hence $EVAL$ should be proportional to the expected payoff

Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI

- ◇ perfection is unattainable \Rightarrow must approximate
- ◇ good idea to think about what to think about
- ◇ uncertainty constrains the assignment of values to states

Games are to AI as grand prix racing is to automobile design