Chapter 6

Karnaugh Map

The **Karnaugh Map (K-map)** is a graphical tool used to simplify a logic equation or to convert a truth table to its corresponding logic circuit in a simple, orderly process. Although a K-map can be used for problems involving any number of input variables, its practical usefulness is limited to five or six variables.

Karnaugh Map Format

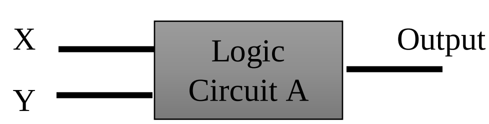
The K-map, like a truth table, is a means for showing the relationship between logic inputs and the desired output. Instead of being organized into columns and rows like a truth table, the Karnaugh map is an array of cells in which each cell represents a binary value of the input variables. The cells are arranged in a way so that simplification of a given expression is simply a matter of property grouping the cells. The results of grouping the cells will produce the simplest SOP or POS expression possible.

Karnaugh Map and Truth Table

2 variable Karnaugh Map

The 2 variable Karnaugh Map is an array of four cells, which is calculated with 2^N , where N is the number of inputs, $2^2 = 4$ possible combinations.

If we have a logic circuit with two inputs that confirms the following truth table:



Truth table of Logic Circuit A			
Decimal Value	Inputs		Output
	X (MSB)	Y (LSB)	
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	0

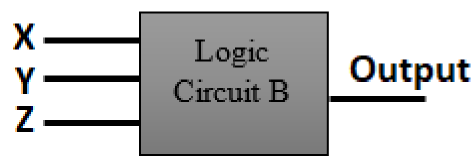
2-variable Karnaugh Map for Logic Circuit A

X \ Y	0	1
0	0	1
1	0	0

3 variable Karnaugh Map

The 3 variable Karnaugh Map is an array of eight cells, which is calculated with 2^N, where *N* is the number of inputs, $2^3 = 8$ possible combinations.

If we have a logic circuit with two inputs that confirms the following truth table:



Truth table of Logic Circuit B				
Decimal Value	Inputs			Output
	X (MSB)	Y	Z (LSB)	
0	0	0	0	**0**
1	0	0	1	**0**
2	0	1	0	**1**
3	0	1	1	**1**
4	1	0	0	**0**
5	1	0	1	**1**
6	1	1	0	**0**
7	1	1	1	**0**

3-variable Karnaugh Map for Logic Circuit B

X Y \ Z	0	1
0 0	0	0
0 1	1	1
1 1	0	0
1 0	0	1

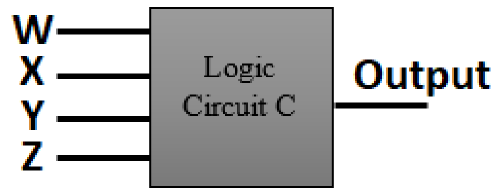
4 variable Karnaugh Map

The 4 variable Karnaugh Map is an array of 16 cells, which is calculated with 2^N , where N is the number of inputs, $2^4 = 16$ possible combinations.

If we have a logic circuit with two inputs that confirms the following truth table:

Truth table of Logic Circuit C

Decimal Value	Inputs				Output
	W (MSB)	X	Y	Z (LSB)	
0	0	0	0	0	**0**
1	0	0	0	1	**0**
2	0	0	1	0	**0**
3	0	0	1	1	**1**
4	0	1	0	0	**0**
5	0	1	0	1	**0**
6	0	1	1	0	**1**
7	0	1	1	1	**0**
8	1	0	0	0	**0**
9	1	0	0	1	**1**
10	1	0	1	0	**0**
11	1	0	1	1	**0**
12	1	1	0	0	**0**
13	1	1	0	1	**1**
14	1	1	1	0	**1**
15	1	1	1	1	**0**



4-variable Karnaugh Map for Logic Circuit C

YZ / WX	00	01	11	10
00	0	0	1	0
01	0	0	0	1
11	0	1	0	1
10	0	1	0	0

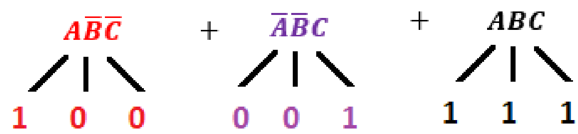
Karnaugh Map a Standard SOP expression

To place a standard SOP expression into a Karnaugh Map, a 1 is place for each product term. To do so, an input variable is considered to have a logic 1 and the inverse of it has the logic of 0. For example, variable A is logic 1 and $\bar{A}$ is logic 0.

Example 6.1) Place the following SOP equation into a 3-variable Karnaugh Map

$$A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$$

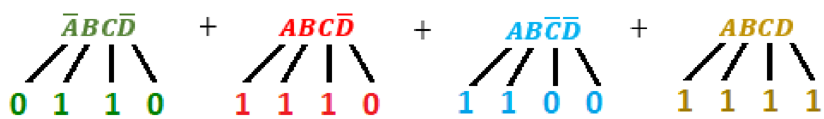
AB \ C	0	1
00		1
01		
11		1
10	1	



Example) Place the following SOP equation into a 4-variable Karnaugh Map

$\bar{A}BC\bar{D} + ABC\bar{D} + AB\bar{C}\bar{D} + ABCD$

AB \ CD	00	01	11	10
00				
01				1
11	1		1	1
10				



Rules to Simplify a Karnaugh Map

When an SOP expression is completely mapped in a Karnaugh Map, there will be a number of 1s on the K-map equal to the number of product terms in the standard SOP equation. The cells that do not have a 1 are the cells for which the expression is 0. To simplify the Karnaugh Map, we need to consider the following minimization rules:

1. Examine the map for adjacent 1s and check if a group of adjacent 1s can be made. Groups may be horizontal or vertical, but not diagonal.

WRONG

AB \ CD	00	01	11	10
00	0	0	0	1
01	0	0	1	0
11	0	1	0	0
10	1	0	0	0

RIGHT

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

WRONG

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	1	0	0
10	0	0	0	0

2. The number of adjacent 1s in a group MUST be according to the base of 2 such as $16 = 2^4$, $8 = 2^3$, $4 = 2^2$, $2 = 2^1$, $1 = 2^0$.

WRONG

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	1	0	0
10	0	0	0	0

CANNOT group of 6 adjacent 1s because 6 is not part of the base of 2

RIGHT

AB \ C	0	1
00	1	1
01	1	1
11	0	0
10	0	0

Group of 4 adjacent 1s is valid because 4 is part of the base of 2. $2^2 = 4$

WRONG

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

CANNOT group of 12 adjacent 1s because 12 is not part of the base of 2

3. Try to group the greatest number of adjacent 1s in a group. For example, always check if a group of 16 adjacent 1s can be grouped first. If there is not a group of 16 adjacent 1s, then check if it is possible to group 8 adjacent 1s, and so on.

WRONG

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

RIGHT

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

Although the group of 4 is valid, in this K-Map grouping four 1s will result in two groups. However, if we group eight 1s the outcome will be one group. Since the objective of applying K-Map is to look for the most simplified logic expression, therefore, in this case is considered invalid grouping four 1s when a group of eight 1s is possible.

4. Also check if it is possible to group adjacent-corner of 1s. This means that adjacent groups may wrap around the table, the leftmost cell in a row may be grouped with the rightmost cell, the top cell in a column may be grouped with the bottom cell, and the edge cell may be grouped as well.

Top and bottom adjacent-corner group of eight 1s.

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

Edge adjacent-corner group of four 1s.

Leftmost and rightmost adjacent-corner group of four 1s.

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

Leftmost and rightmost adjacent-corner group of four 1s.

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

5. Groups may overlap: reusing adjacent 1s is allowed if by reusing the 1s can form a new group of adjacent 1s.

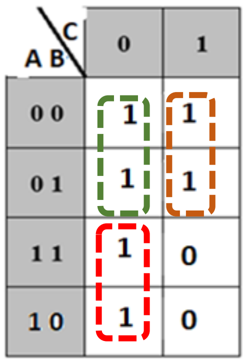
AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	0	0
11	0	0	0	0
10	0	0	0	0

AB \ CD	00	01	11	10
00	1	0	0	0
01	1	0	0	1
11	1	0	0	0
10	1	0	0	0

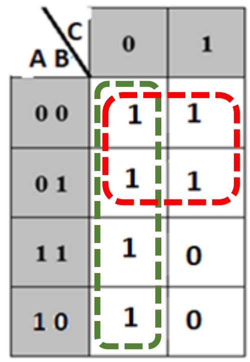
AB \ C	0	1
00	1	1
01	1	1
11	0	0
10	1	0

6. Always keep in mind that the objective of applying K-Map is to look for the most simplified logic expression. Therefore when grouping adjacent 1s should have as few groups as possible, and that those groups do not contradict any of the previous rules.

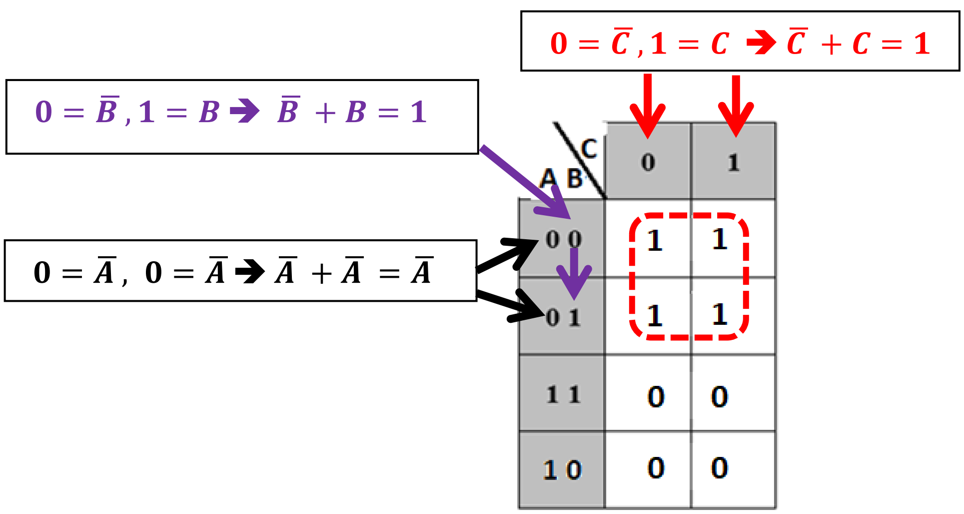
WRONG



RIGHT



After forming a group, to find the simplify product term, OR each like variable in the group and then AND the variables in the group. For this, let The final simplified SOP equation is the sum of all group product terms. Remember, an input variable is considered to have a logic 1 and the inverse of it has the logic of 0, for example, variable A is logic 1 and $\bar{A}$ is logic 0.



If we AND each variable involve in the group, $\bar{A} \cdot 1 \cdot 1 = \bar{A}$, the simplified SOP equation is $\bar{A}$

Example 6.2) Use a Karnaugh Map to simplify the following standard SOP equation:

$$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

First, place each product term into a Karnaugh map:

AB \ C	0	1
00	1	1
01	0	1
11	0	0
10	1	1

Second, applying the six Karnaugh mapping rules to loop all adjacent 1s.

AB \ C	0	1
00	1	1
01	0	1
11	0	0
10	1	1

Identify the product term of each group

$0 = \bar{A}, 1 = A \rightarrow \bar{A} + A = 1$

$0 = \bar{B}, 0 = \bar{B} \rightarrow \bar{B} + \bar{B} = \bar{B}$

$0 = \bar{C}, 1 = C \rightarrow \bar{C} + C = 1$

$1 \cdot \bar{B} \cdot 1 = \bar{B}$

AB \ C	0	1
00	1	1
01	0	1
11	0	0
10	1	1

$0 = \bar{A}, 0 = \bar{A} \rightarrow \bar{A} + \bar{A} = \bar{A}$

$0 = \bar{B}, 1 = B \rightarrow \bar{B} + B = 1$

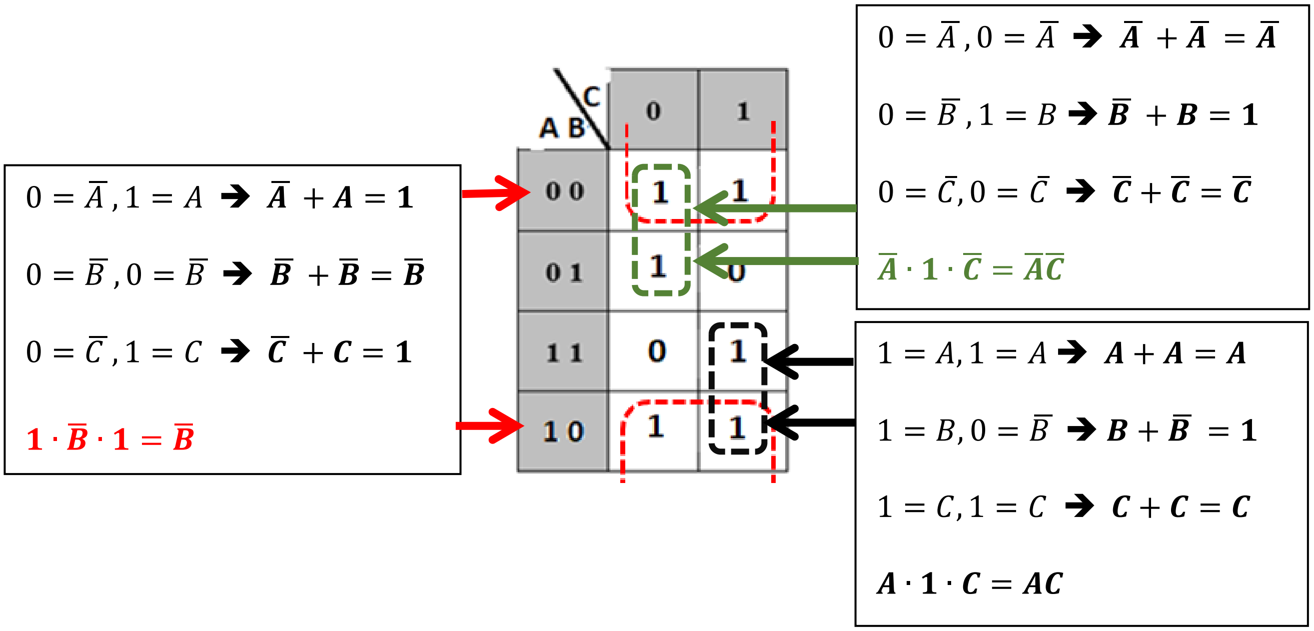
$1 = C, 1 = C \rightarrow C + C = C$

$\bar{A} \cdot 1 \cdot C = \bar{A}C$

After finding each product term, write the simplified SOP equation: $\bar{B} + \bar{A}C$

Example 6.3) Use a Karnaugh Map to simplify the following standard SOP equation:

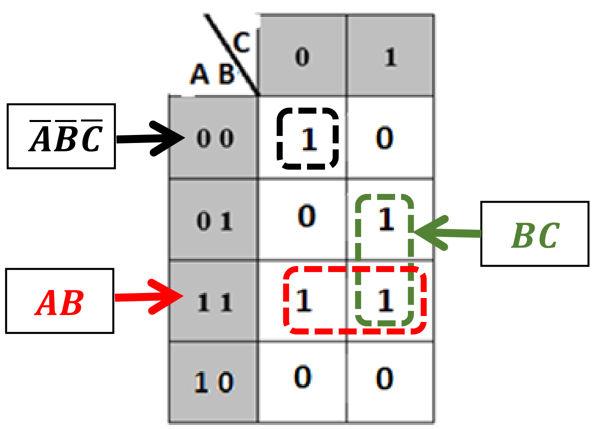
$$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC + A\bar{B}\bar{C} + A\bar{B}C$$



The simplified SOP equation: $\bar{B} + \bar{A}\,\bar{C} + AC$

Example 6.4) Use a Karnaugh Map to simplify the following standard SOP equation:

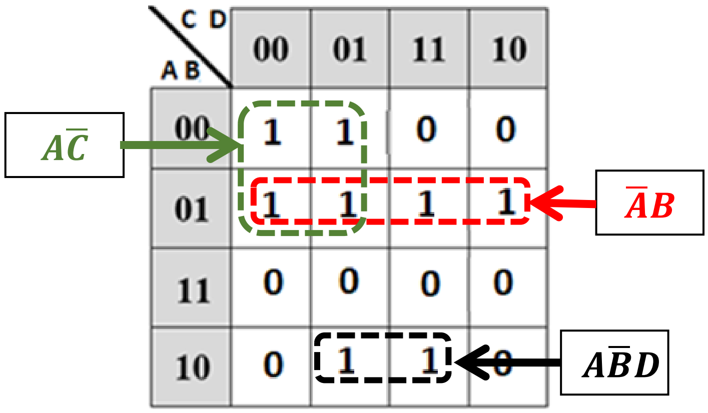
$$\bar{A}\bar{B}\bar{C} + \bar{A}BC + AB\bar{C} + ABC$$



The simplified SOP equation: $AB + BC + \bar{A}\bar{B}\bar{C}$

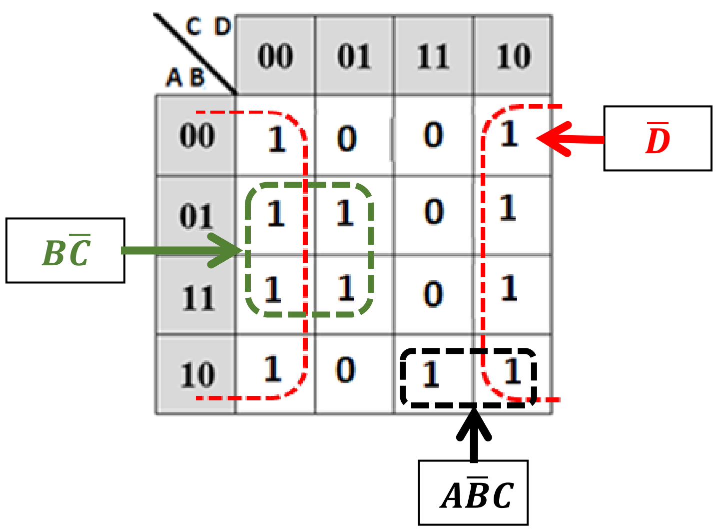
Example 6.5) Use a Karnaugh Map to simplify the following standard SOP equation:

a) $\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}CD$



The simplified SOP equation: $\bar{A}B + A\bar{C} + A\bar{B}D$

b) $\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$



The simplified SOP equation: $\bar{D} + B\bar{C} + A\bar{B}C$