Chapter 2

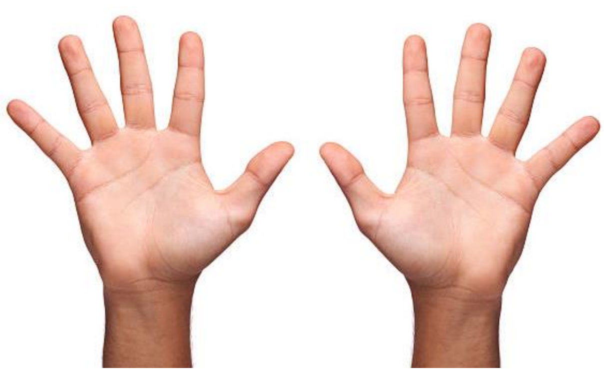
Number of Systems and Codes

Numerical system is a representative way to display digits, symbols, or letters. Binary number is the fundamental system use in computers and digital electronics because computers store information as 0 and 1, or “bits”, which is a short word for binary digit. On the other hand, outside the computer world, the universal number of system used to represent quantities is the decimal system. Decimal system is the number system used in every day to represent quantities from the beginning of many ancient cultures where they calculated with numerals based on ten, as the human hands have 10 digits.

In this chapter, students will learn the different number of system use between the world of computers and humans, and how to make the conversion between systems in order for the two different world to understand and communicate.

2.1. Decimal system

Decimal system is the quantity system uses by humans. The system started from ancient cultures when people used their 10 hands’ fingers to count. Thus, a decimal numbering system has 10 different digits to represent the quantities of the systems. As the number zero “0” is considered to be the starts points of the counting system, the digits for the decimal system are 0,1,2,3,4,5,6,7,8, and 9.



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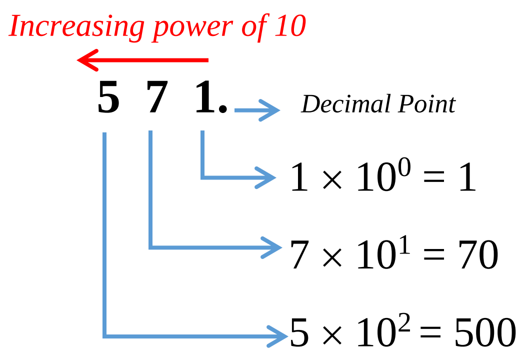
For instant, when digits are placed in a different position from right to left from the decimal point, the digits have a value ten times larger than the previous digits. Thus, a decimal systems is a base of 10 system, and digits have an increasing weight position of ten from the right to the left. Also, the first digit from the right side (rightmost) is the *Least Significant Digit*, LSD, since it is the digit at the first counting process, and it has a weighting factor of 10^0. On the other hand, the leftmost digit is the *Most Significant Digit*, MSD, since it is the last digit to count, and it has the greatest weighting factor.

Example 2.1. – Decimal number

Given a 3-digit decimal number 571, the lest significant digit is “1”, which has a weight position factor of 10^0, and the most significant digit is 5, which has a weigh position factor of 10^2. When the digit values are added up, the total is the decimal value, and a subscript is added next to LSD to identify the system that the digits represent. In this case *571*$_{10}$. Usually a subscript is not necessary for a decimal number but subscripts are recommended when numerical systems since digit might be repeated in different number of system, and a way to identify the different system is by subscript of their respective system base.



Total decimal quantity = 500 + 70 + 1 = 571_{10}

Exercise 2.1. – Decimal number

For the given decimal numbers, identify the LSD, MSD, and each digit weight position factor:

a. 3,708
b. 10,526
c. 490

2.2. Binary number

Digital devices use a different method called *binary system* to identify and represent digits. Binary system uses two digits, which is 0 and 1, to represent two distinct voltage level. The digit "0" is used to represent a voltage of or approximately 0 V, and the digit "1" is used to represent a voltage of or approximately 5 V.

Also, each binary digit is called a "bit", the most-right bit is known as *the least significant bit,* and the most-left bit is known as *the most significant bit.* Also, a subscript of 2 is added to the LSB to identify the number as a binary number or string.

Hence, the digits for the binary system is 0 and 1, and it is a base of 2 system.

Example 2.2. – Binary number

For a 4-bit binary number 1100_2, the MSB, LSB, and the weight position factor is identified as:

$1\ 1\ 0\ 0_2$ → *Base of 2 (binary system)*

→ 0×2^0 (LSB)

→ 0×2^1

→ 1×2^2

→ 1×2^3 (MSB)

Exercises 2.2 – binary number

For the given binary numbers, identify the LSB, MSB, and each digit weight position factor:

a. 101100_2
b. 11101_2
c. 101010_2

2.3. Hexadecimal system

$$16^n \ldots, 16^2, 16^1, 16^0$$

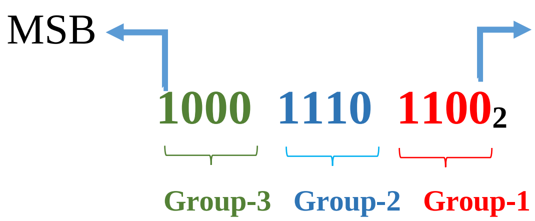
In digital, the term ***code*** refers to the sequence of symbols or characters, and a binary code is no more than a sequence of 1s and 0s. Since binary code are extremely long, one way to simplify a binary code is by grouping into a group of 4-bit or 3-bit. One group of 4-bit is called a ***nibbles*** and one group of 8-bit (2 nibbles) is called ***bytes***. A system to group a binary code into a group of 4-bit is called the ***hexadecimal number*** of system. It is a base 16 system because it has 16 digits, $2^4 = 16$. One group of 4-bit represents a single hexadecimal digit. The representation of each hexadecimal digit is by numbers from 0 to 9 and by letter in the alphabetic order from A to F to represent the values of 10, 11, 12, 13, 14, and 15, respectively.

Since hexadecimal uses 4-bit grouping, then instructions or data used in the computer systems is in two (1 byte), four (2-byte), or eight (3-byte) hexadecimal code, which is must easier and faster to read than a long string of binary digits.

Example 2.3. – Hexadecimal number

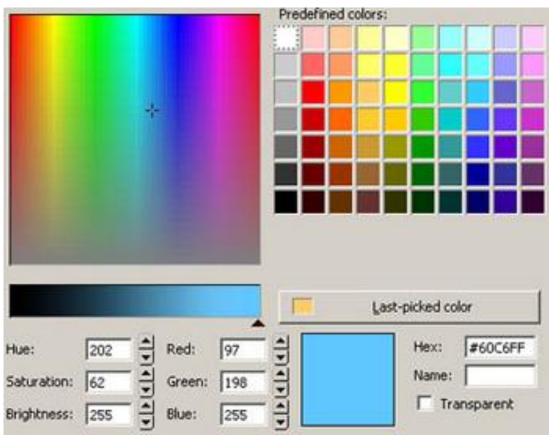
100011101100_2 (Binary code)

$1000\ 1110\ 1100_2$ (Hexadecimal system grouping)



Grouping of 4-bit stars from the LSB to the MSB direction. One group of 4-bit is equivalent to one

One example of hexadecimal application is the hexadecimal color palette (a hex triplet) of a graphic design application, where it uses two hexadecimal digit to represent one level of color variation among the color of Red, Green, and Blue (RGB).



2.4. Octal system

$$8^n ..., 8^2, 8^1, 8^0$$

Like the hexadecimal number system, the octal number system provides a convenient way to express binary number and codes. One octal digit is a decimal representation of a group of 3-bit. Therefore, the octal number system is a base of 8 ($2^3 = 8$) system composed of eight digits: 0,1,2,3,4,5,6,7.

Example 2.4. – Octal number

100011101100_2 (Binary code)

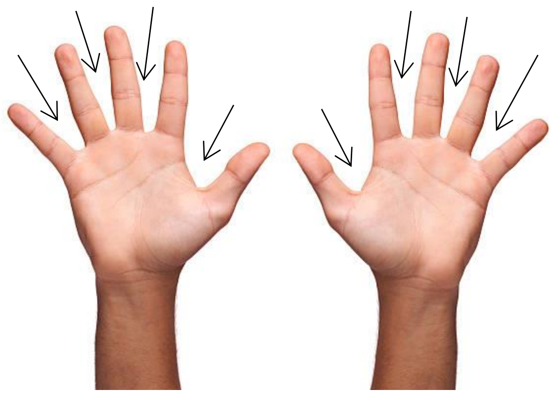
$100\ 011\ 101\ 100_2$ (Octal system grouping)

MSB LSB

$$\underbrace{100}_{\text{Group-4}}\quad \underbrace{011}_{\text{Group-3}}\quad \underbrace{101}_{\text{Group-2}}\quad \underbrace{100}_{\text{Group-1}}{}_2$$

Grouping of 3-bit stars from the LSB to the MSB direction. One group of 3-bit is equivalent to one octal digit

History has showed that some ancient culture such as the Native American Yuki and the Mexican Pamean cultures used octal system because the count using the spaces between their fingers instead of the fingers themselves.



Octal or base eight system is used in older computer as their base system especially in system that employed 12-bit, 24-bit, or 36-bit words. Since each octal digit represents three binary digit, computer with octal base system has their word size divisible by three-bit.

However, most modern computing platforms use 16-bit, 32-bit, or 64-bit words, which the word size is divisible by eight-bit, a byte.

2.5. Conversion between systems

The original computers were used primarily as calculators, but later they were used to manipulate other forms of information, such as words, symbols, and pictures. In each case, engineers and programmers sat down and decided how they were going to represent a new type of information in binary form. Although it is pretty complicated to do so, sounds and pictures can also be converted into binary numbers, too. They have to be divided up into small elements ("samples" in audio or "pixels" for pictures), and then every element has to be assigned a number. The result is a huge array of binary numbers, and the volume of all this data is one reason why image files on a computer are so large, and why it is relatively slow to view video or download audio over an internet connection.

One of the practices by which it is important to understand the conversion between numerical systems is the representation of a binary code in words. For example, with 8 bit binary, $0101\ 1100_2$, many logic and protocol analyzers and Binary Editors will show these sequences with a hex value as **$5C_{Hex}$**. A familiar binary code sequence $010\ 0011_2$, represents as **23_{Hex}** also having the ASCII representation the symbol hash #

2.5.1. Conversion from binary, hexadecimal, or octal system to decimal system

The conversion from binary, hexadecimal, or octal system to decimal number is done by using the weight position values and the base of the given number. The basic steps for the conversion is:

1. Identify and label the LSD and the MSD of the given number
2. Identity the appropriate base weigh position factor for each given digit from the LSD to the MSD
3. Multiply each given digit with their respective base weight
4. Sum all the product from step 3.
5. Label the result in step 4 with subscript 10 to identify that the number is the converted decimal number.

Example 2.5. – convert from binary to decimal

Given 100101_2, what would be its decimal value?

Step 1: Identify and label the LSD and the MSD of the given number

$$\mathbf{1\ 0\ 0\ 1\ 0\ 1_2}$$

(LSB)

(MSB)

Step 2: Identity the appropriate base weigh position factor for each given digit from the LSD to the MSD

$$\mathbf{1\ 0\ 0\ 1\ 0\ 1_2}$$

2^0 (LSB)

2^1

2^2

2^3

2^4

2^5 (MSB)

Step 3: Multiply each given digit with their respective base from step 2.

$$\mathbf{1\ 0\ 0\ 1\ 0\ 1_2}$$

$1\times2^0 = 1$(LSB)

$0\times2^1 = 0$

$1\times2^2 = 4$

$0\times2^3 = 0$

$0\times2^4 = 0$

$1\times2^5 = 32$(MSB)

Step 4. Sum all the product from step 3.

$$\mathbf{32+0+0+4+0+1 = 37}$$

Step 5. Label the result in step 4 with subscript 10 to identify that the number is the converted decimal number.

Answer = $\mathbf{37_{10}}$

Example 2.6. – convert from hexadecimal to decimal

Given $C27F_{16}$, what would be its decimal value?

Step 1: Identify and label the LSD and the MSD of the given number

$$\mathbf{C\ 2\ 7\ F_{16}}$$

(LSB)

(MSB)

Step 2: Identity the appropriate base weigh position factor for each given digit from the LSD to the MSD

$$\mathbf{C\ 2\ 7\ F_{16}}$$

16^0 (LSB)

16^1

16^2

16^3 (MSB)

Step 3: Multiply each given digit with their respective base from step 2.

C 2 7 F_{16}

F = $15 \times 16^0 = 15$ (LSB)

$7 \times 16^1 = 112$

$2 \times 16^2 = 512$

C = $12 \times 16^3 = 49152$ (MSB)

Step 4. Sum all the product from step 3.

49152+512+112+15 = 49791

Step 5. Label the result in step 4 with subscript 10 to identify that the number is the converted decimal number.

49791_{10}

Example 2.7. – convert from octal to decimal

Given 35_8, what would be its decimal value?

Step 1: Identify and label the LSD and the MSD of the given number

3 5_8

(LSB)

(MSB)

Step 2: Identity the appropriate base weigh position factor for each given digit from the LSD to the MSD

3 5_8

8^0 (LSB)

8^1 (MSB)

Step 3: Multiply each given digit with their respective base from step 2.

3 5_8

$5 \times 8^0 = 5$(LSB)

$3 \times 8^1 = 24$(MSB)

Step 4. Sum all the product from step 3.

24+5=29

Step 5. Label the result in step 4 with subscript 10 to identify that the number is the converted decimal number.

29_{10}

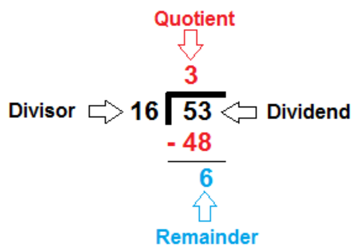
Exercise 2.3 – conversion to decimal system

Convert the following numbers into a decimal number

a. 1001010_2
b. $A07_{16}$
c. 721_8

2.5.2. Conversion from decimal system to binary, hexadecimal, or octal system

Recalling the part of a division



The conversion decimal system to binary, hexadecimal, or octal system is done by using repeating division with the base value of the given number, and the remainder of each division is the converted number. The basic steps for the conversion is:

1. Divide the given decimal number (divident) with the base value of the system (divisor) that we are converting to.

2. The remainder of step 1 is the converted digit. The first remainder digit is the LSD

3. Take the quotient from the previous division and divide it again with the base value of the system (divisor) that we are converting to. The remainder is the next converted digit.

4. Repeat step 3 until the quotient of the division becomes zero "0". The last remainder digit is the MSD

5. Write all the converted digit, which are all the remainders digit, from the LSD to the MSD (from rightmost digit to the leftmost digit). Do not forget to include the base subscript

Example 2.8. – Conversion from decimal system to binary system

What is the binary code of 37_{10}?

Step 1: Divide the given decimal number (dividend) with the base value of the system (divisor) that we are converting to.

37 is the dividend, and 2 is the divisor since the conversion is to binary code, which is base of 2

$$\frac{37}{2}$$

Step 2: The remainder of step 1 is the converted digit. The first remainder digit is the LSB

$$\frac{37}{2} = 18\frac{1}{2} \rightarrow Remainder = 1\ (LSB)$$

Step 3. Take the quotient from the previous division and divide it again with the base value of the system (divisor) that we are converting to. The remainder is the next converted digit.

$$\frac{18}{2} = 9.0\ \rightarrow Remainder = 0$$

Step 4. Repeat Step 3 until the quotient of the division becomes zero "0". The last remainder is the MSB

$$\frac{9}{2} = 4\frac{1}{2}\ \rightarrow Remainder = 1$$

$$\frac{4}{2} = 2.0\ \rightarrow Remainder = 0$$

$$\frac{2}{2} = 1.0\ \rightarrow Remainder = 0$$

$$\frac{1}{2} = 0\frac{1}{2}\ \rightarrow Remainder = 1 \ldots .Quotient = 0\ \rightarrow STOP\ division\ \ (MSB)$$

Step 5. Write all the converted digit, which are all the remainders digit, from the LSB to the MSB (from rightmost digit to the leftmost digit). Do not forget to include the base subscript

$$\mathbf{1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1_2}$$

(LSB)

(MSB)

Example 2.9. – Conversion from decimal system to hexadecimal system

What is the hexadecimal code of 215_{10}?

Step 1: Divide the given decimal number (dividend) with the base value of the system (divisor) that we are converting to.

215 is the dividend, and 16 is the divisor since the conversion is to hexadecimal code, which is base of 16

$$\frac{215}{16}$$

Step 2: The remainder of step 1 is the converted digit. The first remainder digit is the LSD

$$\frac{215}{16} = 13\frac{7}{16} \rightarrow Remainder = 7 \quad (LSD)$$

Step 3. Take the quotient from the previous division and divide it again with the base value of the system (divisor) that we are converting to. The remainder is the next converted digit.

Step 4. Repeat step 3 until the quotient of the division becomes zero "0". The last remainder is the MSD

$$\frac{13}{16} = 0\frac{13}{16} \rightarrow Remainder = 13 \ \ldots Quotient = 0 \ \rightarrow STOP\ division\ (MSD)$$

13 is equivalent to ***D*** in hexadecimal

Step 5. Write all the converted digit, which are all the remainders digit, from the LSD to the MSD (from rightmost digit to the leftmost digit). Do not forget to include the base subscript

D $\mathbf{7_{16}}$

(LSD)

(MSD)

Example 2.10. – Conversion from decimal system to octal system

What is the octal code of 96_8?

Step 1: Divide the given decimal number (dividend) with the base value of the system (divisor) that we are converting to.

96 is the dividend, and 8 is the divisor since the conversion is to octal code, which is base of 8

$$\frac{96}{8}$$

Step 2: The remainder of step 1 is the converted digit. The first remainder digit is the LSD

$$\frac{96}{8} = 12.0 \rightarrow Remainder = 0 \ \ (LSD)$$

Step 3. Take the quotient from the previous division and divide it again with the base value of the system (divisor) that we are converting to. The remainder is the next converted digit.

$$\frac{12}{8} = 1\frac{4}{8} \rightarrow Remainder = 4$$

Step 4. Repeat step 3 until the quotient of the division becomes zero "0". The last remainder is the MSD

$$\frac{1}{8} = 0\frac{1}{8} \rightarrow Remainder = 1 \ldots . Quotient = 0 \ \rightarrow STOP\ division\ (MSD)$$

Step 5. Write all the converted digit, which are all the remainders digit, from the LSD to the MSD (from rightmost digit to the leftmost digit). Do not forget to include the base subscript

1 4 $\mathbf{0_8}$

(LSD)

(MSD)

Exercise 2.4 – conversion from decimal to binary, hexadecimal, and octal system

a. What is the binary code of 70_{10}? 1000110
b. What is the hexadecimal code of 526_{10}? 20E
c. What is the octal code of 278_8? 426

2.5.3. Conversion from binary to hexadecimal system

Converting a binary number to hexadecimal is a straightforward procedure. Simply break the binary code into 4-bit groups from the LSB to the MSB order, and replaces each 4-bit group with the equivalent hexadecimal symbol. One way to do the replacement is to convert the 4-bit group into its equivalent decimal value and convert the decimal value into a hexadecimal digit.

Binary code	**Decimal equivalent**	**Hexadecimal digit**
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

1. Break the binary code into 4-bit groups from the LSB to the MSB
2. If the last group does not have 4 bit in the group, fill the bit place with “0” up to 4-bit
3. Convert each group of 4-bit into its decimal value
4. Convert each decimal value to its respective hexadecimal digit

Example 2.11. – Conversion from binary to hexadecimal system

Convert the following binary code into a hexadecimal code 10111010100100_2

Step 1: Break the binary code into 4-bit groups from the LSB to the MSB

$$\mathbf{10\ \ 1110\ \ 1010\ \ 0100_2}$$

(LSB)

(MSB)

Step 2: If the last group does not have 4 bit in the group, fill the bit place with "0" up to 4-bit

$$\mathbf{0010\ \ 1110\ \ 1010\ \ 0100_2}$$

(LSB)

(MSB)

Step 3: convert each group of 4-bit into its decimal value

$$\underbrace{\mathbf{0010}}_{\mathbf{2}}\quad \underbrace{\mathbf{1110}}_{\mathbf{14}}\quad \underbrace{\mathbf{1010}}_{\mathbf{10}}\quad \underbrace{\mathbf{0100_2}}_{\mathbf{3}}$$

Step 4: convert each decimal value to its respective hexadecimal digit

$$\underbrace{\mathbf{0010}}_{\mathbf{2}}\quad \underbrace{\mathbf{1110}}_{\mathbf{14=E}}\quad \underbrace{\mathbf{1010}}_{\mathbf{10=A}}\quad \underbrace{\mathbf{0100_2}}_{\mathbf{3}}$$

Answer: $2EA3_{16}$

Exercise 2.5 – conversion from binary system to hexadecimal system

Convert the following binary code into hexadecimal code

a. 1011101_2
b. 11110001001_2
c. 101010101_2

2.5.4. Conversion from hexadecimal system to binary system

Conversion from hexadecimal number to binary number is a straightforward method. It replaces each hexadecimal value with its respective 4-bit equivalent.

1. convert each hexadecimal digit into its decimal value

2. convert each decimal digit into a set of 4-bit code

Example 2.12. – Conversion from hexadecimal to binary system

Represent the hexadecimal number $3B_{16}$ into a binary code.

Step 1: convert each hexadecimal digit into its decimal value

$$3\ B_{16}$$

$$3 \quad 11$$

Step 2: convert each decimal digit into a set of 4-bit code

$$3\ B_{16}$$

$$\underbrace{3}_{0011} \quad \underbrace{11}_{1011}$$

Answer: 00111011_2 or 111011_2

Exercise 2.6 – conversion from hexadecimal system to binary system

Convert the following hexadecimal numbers into its respective binary code

a. $1C8_{16}$
b. 230_{16}
c. $D5E_{16}$

2.5.5. Conversion from binary to octal system

Since each digit of the octal system present a 3-bit group, then the conversion from binary to octal system is done by replacing each octal digit with its respective 3-bit equivalent.

1. Identify the LSB and MSB, and break the binary code into 3-bit groups from the LSB to the MSB

↓

2. If the last group does not have 3 bit in the group, fill the bit place with "0" up to 3-bit

↓

3. Convert each group of 3-bit into its decimal value

Example 2.13. – Conversion from binary system to octal system

Convert the following binary code into octal code

Step 1: Identify the LSB and MSB, and break the binary code into 3-bit groups from the LSB to the MSB

$$\mathbf{11\ \ 010\ \ 101_2}$$

(LSB)

(MSB)

Step 2: If the last group does not have 3 bit in the group, fill the bit place with "0" up to 3-bit

$$\mathbf{011\ \ 010\ \ 101_2}$$

(LSB)

(MSB)

Step 3: convert each group of 3-bit into its decimal value

$$\underbrace{\mathbf{011}}_{\mathbf{3}}\ \ \underbrace{\mathbf{010}}_{\mathbf{2}}\ \ \underbrace{\mathbf{101_2}}_{\mathbf{5}}$$

Answer: 325_8

Exercise 2.7 – conversion from binary system to octal system

Convert the following binary code into octal code

a. 1000111_2
b. 1011000101_2
c. 11110100_2

2.5.4. Conversion from octal system to binary system

Conversion from octal number to binary number is also a straightforward method since each octal digit is also an equivalent decimal value. It replaces each octal value with its respective 3-bit equivalent.

Example 2.14. – Conversion from octal system to binary system

Represent the octal number 25_8 into a binary code.

Step 1: convert each octal digit into a set of 3-bit code

$$2\ 5_8$$

$$\underbrace{2}_{010}\quad\underbrace{5}_{101}$$

Answer: 010101_2 or 10101_2

Exercise 2.8 – conversion from octal system to binary system

Convert the following octal system into a binary code

a. 35_8
b. 106_8
c. 427_8

2.6. Binary-Coded-Decimal (BCD)

Binary coded decimal (BCD) is a way to express each of the decimal digits with a binary code. Since a decimal digit can be as large as 9, four bits are required to code each digit (the binary code for 9 is 1001).

Decimal value	4-bit BCD code
0	0000
1	0001
2	0010
3	0110
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

There are only ten code groups in the BCD system, so it is very easy to convert between decimal and BCD. Since we like to read and write in decimal, the BCD code provides an excellent interface to binary systems. Examples of such interfaces are keypad inputs and digital readouts.

To illustrate the BCD code, take a decimal number such as **874_{10}**. Each digit is changed to its binary equivalent as follows:

$$\underbrace{\mathbf{8}}\quad\underbrace{\mathbf{7}}\quad\underbrace{\mathbf{4}}$$

1000 0111 0100 (BCD)

The BCD system offers relative ease of conversion between machine-readable and human-readable numerals. As compared to the simple binary system, however, BCD increases the circuit complexity. The BCD system is not as widely used today as it was a few decades ago, although some systems still employ BCD in financial applications.

Exercise 2.9 – Binary-Coded-Decimal (BCD)

Convert the following decimal number into its BCD code:

a. 703_{10}
b. 95_{10}
c. 2460_{10}