Lab Experiment 6
Parallel Circuits, Kirchhoff's Current Law and Current Divider Rule

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Introduction

Part 1. Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law, KCL, was introduced by German mathematician and physicist Gustav Kirchhoff. Gustav described that the sum of the currents leaving the node, junction point, was equal to the sum of the currents entering the same junction or node. A simple way to say this is that at any node, what goes in must come out.

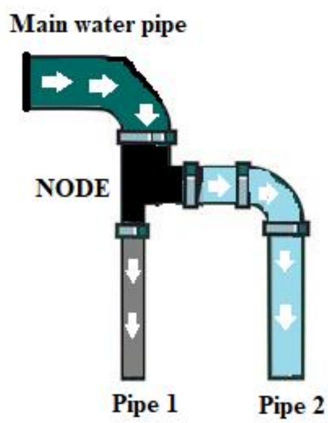


Figure 6.1 – Illustration of water distribution in water pipes

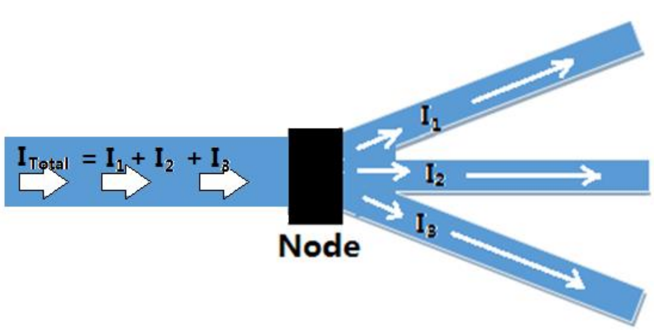


Figure 6.2 – Current Distribution

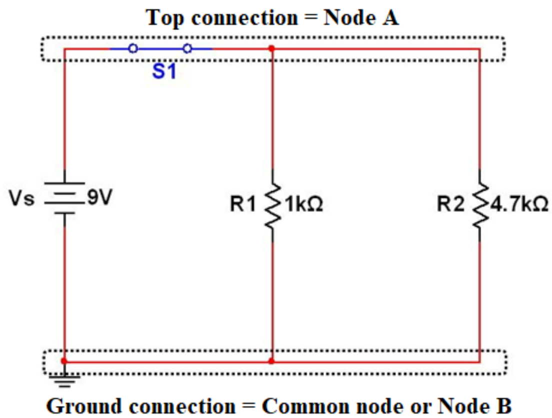
$$Sum\ of\ I_{in(node\ A)} = sum\ of\ I_{out(Node\ A)}$$

$$Sum\ of\ I_{in(Node\ A)} + sum\ of\ I_{out(Node\ B)} = 0\ A$$

Formula 6.1 – Kirchhoff's Current Law (KCL)

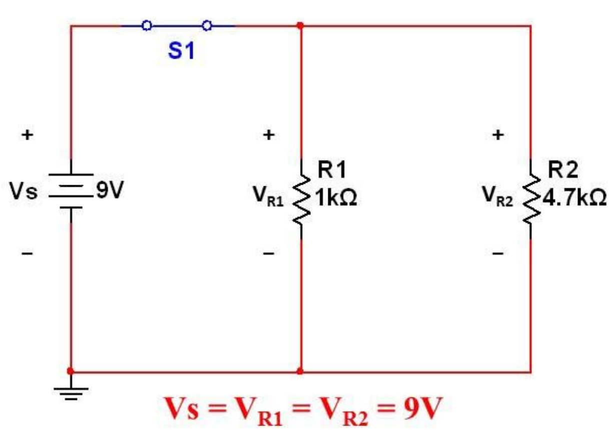
Part 2. Components Connected in Parallel

Components are connected in parallel if their component terminals are connected to the same common node respectively, and have the same voltage drop. In other words, two or more components are in parallel if they are connected between the same two connection points or nodes. The shortcut notation for a parallel connection is two slashes "//" sometimes **"||"** is also used. If a 1kΩ resistor and a 4.7kΩ resistor are connected in parallel, one could write 1kΩ || 4.7kΩ. This is read as: 1kΩ in parallel with 4.7kΩ.



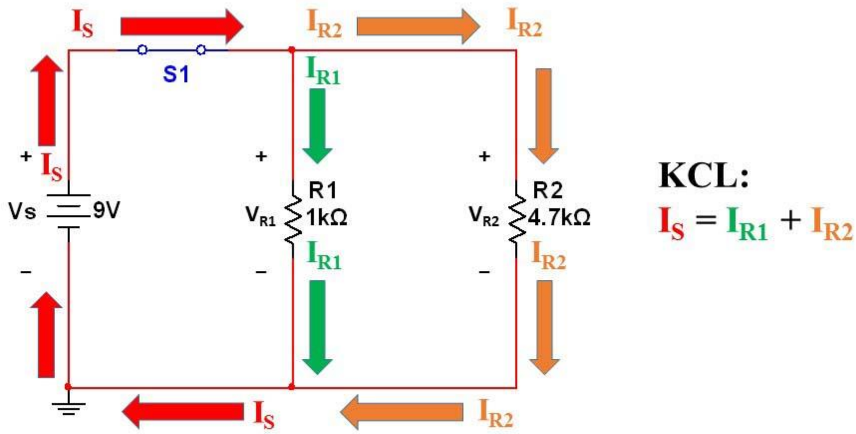
Circuit 6.1 – 1kΩ resistor in parallel with 4.7kΩ resistor

The voltage across parallel components is the same, because the voltage between two points is always the same.



Circuit 6.2 – Voltage across parallel components

The total current entering a junction with two parallel paths, however, divides between the two paths in such a way that the sum of the currents in the two paths is equal to the total current entering the parallel combination. As stated above, this is known as Kirchhoff's Current Law (KCL).



Circuit 6.3 – Current flow in a parallel circuit

Part 3. Total Resistance and Conductance in a Parallel Circuit

Conductance is the reciprocal of resistance, is represented by the letter G and is measured in siemens [siemens=S]. In parallel, the conductance value of the resistors adds.

$$\text{(Conductance)}\ G = \frac{1}{R}$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_N}} \qquad \text{where } N \text{ is the total number of resistor connected in parallel}$$

Formula 6.2 – Total Resistance and Conductance formula

For example, to find the total resistance of the circuit Figure 6.1, the total resistance can then be obtained by taking the reciprocal of the total conductance.

$$G = \frac{1}{R_1} \qquad G_{1k\Omega} = \frac{1}{1k\Omega} = 1mS$$

$$G_{4.7k\Omega} = \frac{1}{R_2} \qquad G_{4.7k\Omega} = \frac{1}{4.7k\Omega} = 0.2128mS$$

$$G_T = G_{1k\Omega} + G_{4.7k\Omega} = 1mS + 0.2128mS = 1.2128mS = 1.21mS$$

$$R_T = \frac{1}{1.21mS} = 0.82645k\Omega = 826.45\Omega$$

In lab, the total resistance can me measure by placing the measuring leads of your DMM across the resistors connected in parallel as it is shown in Figure 6.3

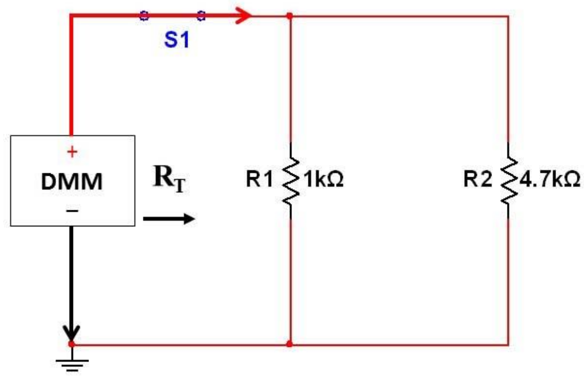


Figure 6.3 – Parallel Resistivity Circuit Measurement with a DMM

There is a special case for two resistor connected in parallel. The total resistance for two parallel resistors can also be calculated using the "product over sum" formula.

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Formula 6.3 – Special case for two resistor connected in parallel

Once we have the total resistance, the total current can then be obtained by dividing the applied voltage by the total resistance.

$$I_T = \frac{9V}{824.56\Omega} = 0.0109149A = 10.91mA$$

Part 4. The Current Divider Rule (CDR)

The current divider rule is a computational method that allows you to calculate how the current divides between two paths of known resistance. The current divider rule says that the current through one of two parallel paths is equal to the total current that comes into the junction multiplied by the ratio of the resistance of the other path divided by the sum of the resistance of the two paths. In symbolic form this is as follows:

$I_X = I_T \frac{R_T}{R_X}$	Where X is the unknown current of resistor X

Formula 6.4 – Current Divider Rule

The advantage of using the Current Divider Rule (CDR) is that you obtain the percentage of the division of current between the paths. For this circuit, the current through the 1kΩ resistor will always be 0.82456 or 82.46% of the total. The current through the 4.7kΩ resistor will always be 0.17544 or 17.54% of the total. This current division ratio will always hold no matter what the total current is.

$$Ratio\ of\ current\ through\ R_1 = \frac{R_T}{R_1} = \frac{0.82456k\Omega}{1k\Omega} = 0.82456 = 82.46\%$$

$$Ratio\ of\ current\ through\ R_2 = \frac{R_T}{R_2} = \frac{0.82456k\Omega}{4.7k\Omega} = 0.17544 = 17.54\%$$

$$\boxed{\%\_difference = \left(\frac{Measured\_Value - Calculated\_Value}{Calculated\_Value}\right) *100\%}$$

Formula 6.5 – Percent of Difference between the Calculated and Measured Value Formula

Part 5 - Applications of Parallel Circuit

Every residence in the US has usually one or two electrical energy feeds. Each one of these feeds breaks out into several branch circuits. Each one of these circuits has many lighting loads and receptacles. All the electrical loads and receptacles connected to the same feed are in parallel. Therefore, all the electrical appliances in your house that are connected to the same feed are connected in parallel.

Each branch circuit has a fuse or a circuit breaker to protect the wiring against current overload in case you connect too many appliances in parallel, and therefore, exceed the current rating of the wires. Branch circuits in modern residences are wired with AWG # 12 wires which is capable of safely carrying 20 amperes. The circuit breakers used, therefore, are set to trip and interrupt the circuit if the current demand exceeds 20 amps.

Laboratory Experiment

Part 1 – Two Resistors Connected In Parallel

Part 1 of your lab experiment is to measure the total resistance, and voltage and current through each element of a 2-resistors parallel circuit. For calculations, the current distribution in a parallel circuit can be found by applying Ohm's law, Kirchhoff's Current Law, and Current Divided Rule.

- Obtain a protoboard, jumper wires, and 1 kΩ and 4.7 kΩ resistors from your component kit.
- Build Circuit 6.1 in your protoboard
- Before powering your circuit, measure the total resistance (R_T) is indicated in Figure 6.3. Record this measurement in Table 6.1
- Set your circuit and DMM to measure current
- Power Circuit 6.1
- Measure the current through R_1, R_2, and voltage source, and record measurement in Table 6.1
- Set your circuit and DMM to measure voltage
- Measure the voltage across R_1, R_2, and voltage source, and record measurement in Table 6.1
- Calculate the total resistance using Formula 6.3. Record calculation in Table 6.1
- Calculate the current through R_1, R_2, and I_T using Ohm's law. Record calculation in Table 6.1
- Using the % of difference formula, Formula 6.5, calculate the percentage of difference between the measured and calculated value. Record calculation in Table 6.1

	Measured Value (Include Unit)	Calculated value (Include Unit)	% of Difference
Total Resistance, R_T			
Total Current, $I_S = I_T$			
Current through R1 (1 kΩ), I_{R1}			
Current through R2 (4.7 kΩ), I_{R2}			
Voltage source, V_S			
Voltage across R1 (1 kΩ), V_{R1}			
Voltage across R2 (4.7 kΩ), V_{R2}			

Table 6.1 – 2 Resistors Circuit: Total resistance, Voltage, and Current Measurements

- Use the measured current I_T, I_{R1}, and I_{R2} from Table 6.1 and fill up the corresponding cell in Table 6.2
- Use the Current Divider Formula, Formula 6.4, and calculate I_{R1} and I_{R2}. Show calculation in Table 6.2
- Use KCL formula in Circuit 6.3 to find I_T. Show calculation in Table 6.2
- Use the current divider rule formula, Formula 6.3, to find the current through I_{R1} and I_{R2}. For this calculation, R_T and I_S values are the calculated value from Table 6.1.
- Using the % of difference formula, Formula 6.5, calculate the percentage of difference between the measured and calculated current. Show calculation in Table 6.2
- Ask lab instructor to check Table 6.1 and 6.2
- Once both tables are checked, disassembled circuit, organize your components in your components kit, and proceed to Part 2.

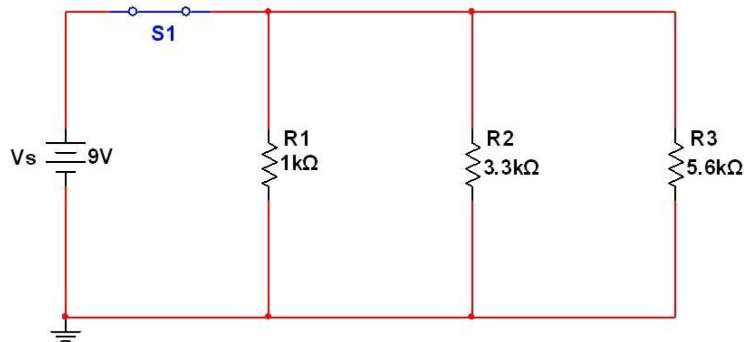
	I_T Total Current (Include Unit)	I_{R1} Current in R_1 (Include Unit)	I_{R2} Current in R_2 (Include Unit)
Measured value (Table 6.1)			
Calculated value			
% of Difference			

Table 6.2. Current divider rule in a 2 resistors circuit

Part 2 – 3 Resistors Connected in Parallel Configuration

Experiment Part 2 is to measure the total resistance of a 3 resistors parallel circuit, as well as the voltage and current distribution through each element of the parallel circuit. For the calculations part, the current distribution in a parallel circuit can be found by applying Ohm's law, Kirchhoff's Current Law, and Current Divided Rule.

— Obtain a 1 kΩ, 3.3 kΩ, and 5.6 kΩ resistors from your component list
— Build Circuit 6.4



Circuit 6.4 - Three Resistors Connected In Parallel Configuration

— Before powering your circuit, measure the total resistance and record this measurement in Table 6.3
— Set your circuit and DMM to measure current
— Power up circuit 6.4
— Measure the current through R_1, R_2, R_3, and voltage source, and record measurement in Table 6.3
— Set your circuit and DMM to measure voltage
— Measure the voltage across R_1, R_2, R_3, and voltage source, and record measurement in Table 6.3
— Calculate the total resistance using Formula 6.2. Record calculation in Table 6.3

Show Calculations of total resistance

— Calculate the current through R_1, R_2, R_3, and voltage source using Ohm's law. Record calculation in Table 6.3

Show Calculations of the current through R_1, R_2, and R_3

— Using the % of difference formula, Formula 6.5, calculate the percentage of difference between the measured and calculated value. Record calculation in Table 6.3

	Measured Value (Include Unit)	**Calculated value (Include Unit)**	**% of Difference**
$\mathbf{R_T}$			
$\mathbf{I_S = I_{Total}}$			
$\mathbf{I_{R1}}$			
$\mathbf{I_{R2}}$			
$\mathbf{I_{R3}}$			
$\mathbf{V_S}$			
$\mathbf{V_{R1}}$			
$\mathbf{V_{R2}}$			
$\mathbf{V_{R3}}$			

Table 6.3 – 3 Resistors Circuit: Total resistance, Voltage, and Current Measurements

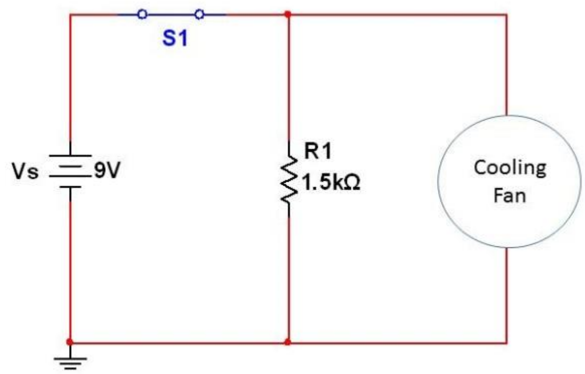
— Use the measured current I_T, I_{R1}, I_{R2}, and I_{R3} from Table 6.3 and fill up the corresponding cell in Table 6.4
— Use the Current Divider Formula, Formula 6.4, and calculate I_{R1}, I_{R2}, and I_{R3}. Show calculation in Table 6.4. For this calculation, R_T and IS values are the calculated value from Table 6.3.
— Use KCL formula in Circuit 6.3 to find I_T. Show calculation in Table 6.4
— Use the current divider rule formula, Formula 6.3, to find the current through I_{R1} and I_{R2}
— Using the % of difference formula, Formula 6.5, calculate the percentage of difference between the measured and calculated current. Show calculation in Table 6.4
— Ask lab instructor to check Table 6.3 and 6.4
— Once both tables are checked, disassembled circuit, organize your components in your components kit, and proceed to Part 3.

	$\mathbf{I_{Total}}$ **(Include Unit)**	$\mathbf{I_{R1}}$ **(Include Unit)**	$\mathbf{I_{R2}}$ **(Include Unit)**	$\mathbf{I_{R3}}$ **(Include Unit)**
Measured value (Table 6.3)				
Calculated value (Show calculations)				
% of Difference				

Table 6.4. Current divider rule in a 3 resistors circuit

Part 3. Non-Resistive Components in Parallel

Circuit 6.5 shows a 1.5kΩ resistor connected in parallel with a computer chip cooling fan. The parallel combination is powered by a 9V supply. According to the fan's specifications, the fan current should be less than 50 mA for small fan and less 120 mA for bigger fan. Here, however, we are energizing the fan with 9V, therefore the fan current will be less.



Circuit 6.5 A Typical Heater and a fan in Parallel Circuit

- Obtain a 1.5 kΩ resistor from your component kit
- Obtain a cooling fan from lab technician
- Build Circui 6.5
- Set your circuit and DMM to measure current
- Measure the current throgut voltage source (total current), 1.5 kΩ resistor, and the cooling fan. Record measurement in Table 6.5
- Ask lab instructor to check Table 6.5
- Once both tables are checked, disassembled circuit, organize your components in your components kit, and proceed with lab report.

	I_{TOTAL} (Include Unit)	$I_{R=1.5\,k\Omega}$ (Include Unit)	$I_{Cooling\ Fan}$ (Include Unit)	Does KCL Hold? (Yes/No) Explain
Measured				

Table 6.5 - Parallel Components and KCL

Question

1. Three resistors, 5.6 kΩ, 8.2 kΩ, and 2.7 kΩ, are connected in parallel. When a student measured the total resistance, the DMM read **6.027536 kΩ.** Without calculations, do you think this measurement may be correct? Justify your answer

2. A global outlet power strip has a maximum current load of 15A. If a 10A air conditioner and a 1A desk lamp is already connected in the power strip. What do you think it would happen if you connect a 12 A hair dryer to the same power strip? Explain your answer.
3. A student built a circuit with two resistors, 2 kΩ and 3 kΩ, connected in parallel. The student measured the current through 3 kΩ and the DMM displayed 4 mA. Using this measurement, how can the student predict the total voltage of the parallel circuit? Justify your answer.

Student's Name: ____________________________ Lab Instructor's Signature____________________

-- LAB EXPERIMENT ENDS HERE --