Student's Name: Date

Introduction to Circuit Analysis Laboratory

Lab Experiment 6

Parallel Circuits, Kirchhoff's Current Law and Current Divider Rule

6.1. Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law, KCL, was introduced by German mathematician and physicist Gustav Kirchhoff. Gustav described that the sum of the currents leaving the node, junction point, was equal to the sum of the currents entering the same junction or node. A simple way to say this is that at any node, what goes in must come out.

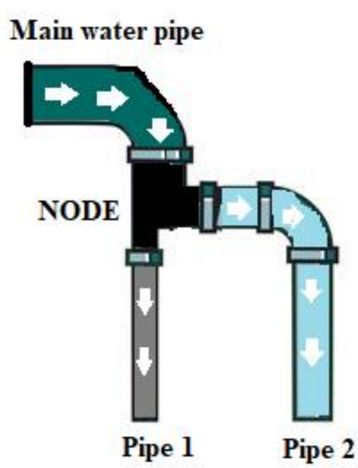


Figure 6.1 – Illustration of water distribution in water pipes

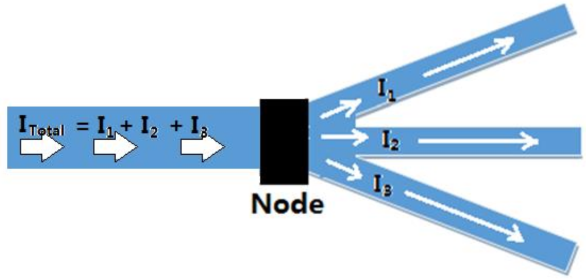


Figure 6.2 – Current Distribution

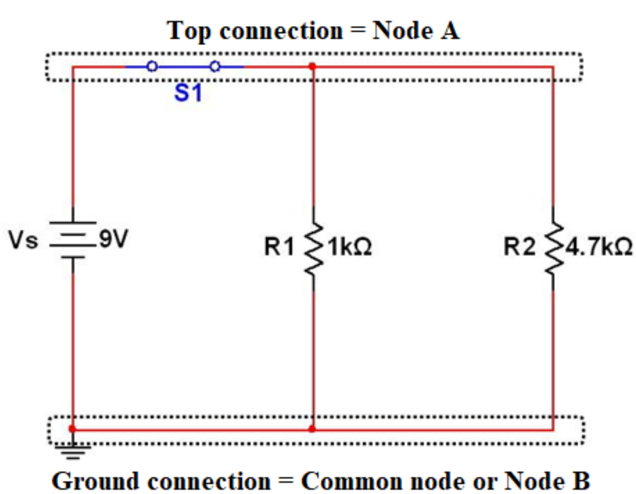
$$Sum\ of\ I_{in(node\ A)} = sum\ of\ I_{out(Node\ A)}$$

$$Sum\ of\ I_{in(Node\ A)} + sum\ of\ I_{out(Node\ B)} = 0\ A$$

Formula 6.1 – Kirchhoff's Current Law (KCL)

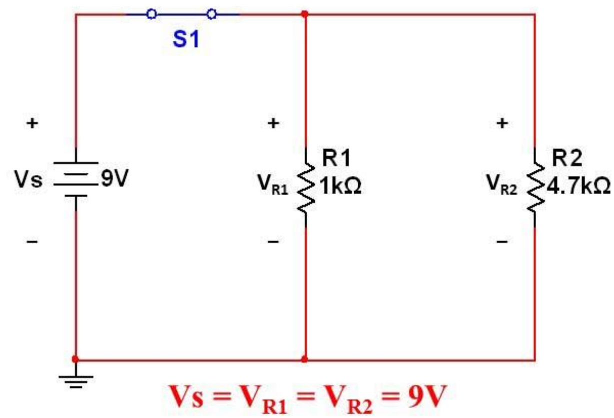
6.2. Elements Connected in Parallel

Components are connected in parallel if their component terminals are connected to the same node respectively, and have the same voltage drop. In other words, two or more components are in parallel if they are connected between the same two connection points or nodes. The shortcut notation for a parallel connection is two slashes "//" sometimes **"||"** is also used. If a 1kΩ resistor and a 4.7kΩ resistor are connected in parallel, one could write 1kΩ || 4.7kΩ. This is read as: 1kΩ in parallel with 4.7kΩ.



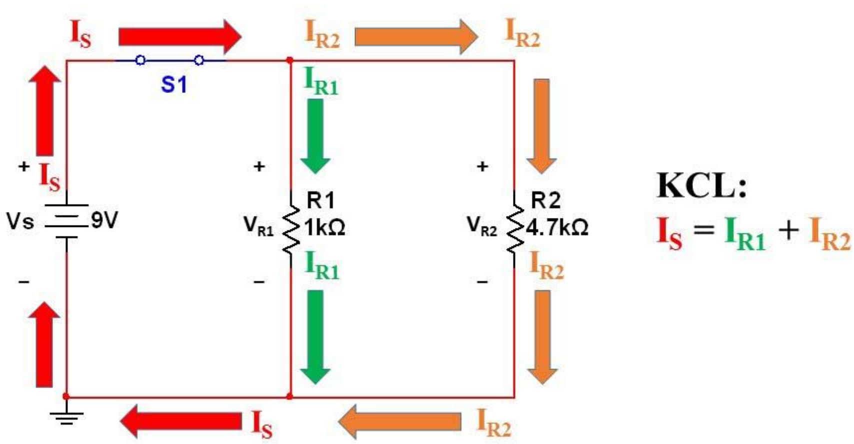
Circuit 6.1 – 1kΩ resistor in parallel with 4.7kΩ resistor

The voltage across parallel components is the same, because the voltage between two points is always the same.



Circuit 6.2 – Voltage across parallel components

The total current entering a junction with two parallel paths, however, divides between the two paths in such a way that the sum of the currents in the two paths is equal to the total current entering the parallel combination. As stated above, this is known as Kirchhoff's Current Law (KCL).



Circuit 6.3 – Current flow in a parallel circuit

6.3. Total Resistance and Conductance in a Parallel Circuit

Conductance is the reciprocal of resistance, is represented by the letter G and is measured in siemens [siemens=S].

(Conductance) $G = \frac{1}{R}$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_N}}$$ where N is the total number of resistor connected in parallel

Formula 6.2 – Total Resistance and Conductance formula

For example, to find the total resistance of the circuit Figure 6.1, the total resistance can then be obtained by taking the reciprocal of the total conductance.

$$G = \frac{1}{R_1} \qquad G_{1k\Omega} = \frac{1}{1k\Omega} = 1mS$$

$$G_{4.7k\Omega} = \frac{1}{R_2} \qquad G_{4.7k\Omega} = \frac{1}{4.7k\Omega} = 0.2128mS$$

$$G_T = G_{1k\Omega} + G_{4.7k\Omega} = 1mS + 0.2128mS = 1.2128mS = 1.21mS$$

$$R_T = \frac{1}{1.21mS} = 0.82645k\Omega = 826.45\Omega$$

In lab, the total resistance can me measure by placing the measuring leads of your DMM across the resistors connected in parallel as it is shown in Figure 6.3

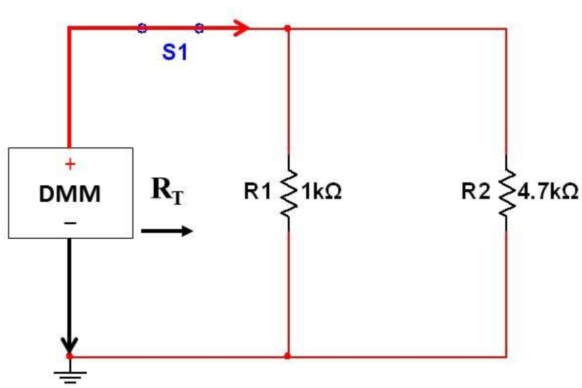


Figure 6.3 – Parallel Resistivity Circuit Measurement with a DMM

There is a special case for two resistor connected in parallel. The total resistance for two parallel resistors can also be calculated using the "product over sum" formula.

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

Formula 6.3 – Special case for two resistor connected in parallel

Once we have the total resistance, the total current can then be obtained by dividing the applied voltage by the total resistance.

$$I_T = \frac{9V}{824.56\Omega} = 0.0109149A = 10.91mA$$

6.4. The Current Divider Rule (CDR)

The current divider rule is a computational method that allows you to calculate how the current divides between two paths of known resistance. The current divider rule says that the current through one of two parallel paths is equal to the total current that comes into the junction multiplied by the ratio of the resistance of the other path divided by the sum of the resistance of the two paths. In symbolic form this is as follows:

$$I_X = I_T \frac{R_T}{R_X} \quad \text{Where } X \text{ is the unknown current of resistor X}$$

Formula 6.4 – Current Divider Rule

The advantage of using the Current Divider Rule (CDR) is that you obtain the percentage of the division of current between the paths. For this circuit, the current through the 1kΩ resistor will always be 0.82456 or 82.46% of the total. The current through the 4.7kΩ resistor will always be 0.17544 or 17.54% of the total. This current division ratio will always hold no matter what the total current is.

$$Ratio\ of\ current\ through\ R_1 = \frac{R_T}{R_1} = \frac{0.82456k\Omega}{1k\Omega} = 0.82456 = 82.46\%$$

$$Ratio\ of\ current\ through\ R_2 = \frac{R_T}{R_2} = \frac{0.82456k\Omega}{4.7k\Omega} = 0.17544 = 17.54\%$$

Appendix

$$\%\ of\ difference = \left(\frac{Calculated\ or\ actual\ value - measured\ value}{Calculated\ or\ actual\ value}\right) \times 100\%$$

Formula 6.5 – Percent of Difference between the Calculated and Measured Value Formula

6.5. Current Measurement

Measuring current is more complicated than measuring resistance or voltage. There are two main reasons for this:

1. *The connection of the DMM with the measure component. In* order for the DMM to measure the current through a component, the DMM has to be connected with the measure component in a way that the current can go through the DMM and the component. This means that the DMM must be made part of the current path of the circuit. In order to make the DMM part of the current path of the circuit, the original circuit must be "broken" and the meter connected across the two points of the open break. When the DMM is part of the open break, the DMM is connected in ***series*** with the measure component.

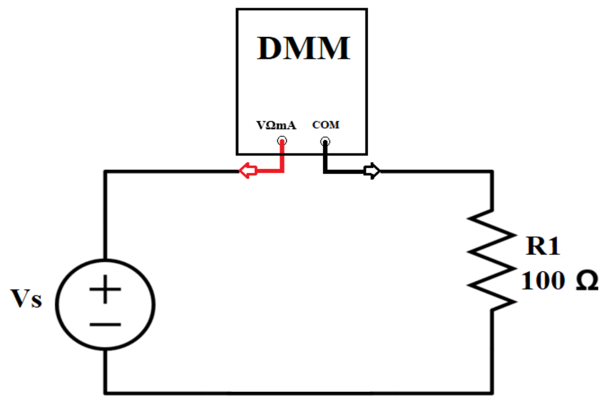
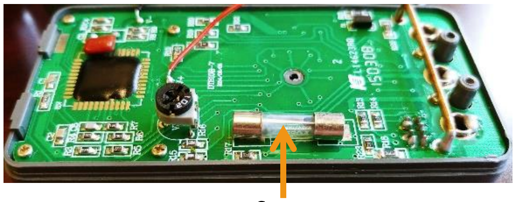


Figure 6.3 – Parallel Resistivity Circuit Measurement with a DMM

2. The fuse of the DMM. One of the most common mistakes with the use of the DMM to measure current is to connect the probes in parallel with the measure component. This will immediately short power to ground through the DMM causing the power supply current going through the DMM. As the current rushes through the DMM, the internal fuse will heat up and then burn out as 200 mA flows through it.[1]
   Remember that a fuse is a safety device consisting of a strip of wire that melts and breaks if the current exceeds a safe level. A fuse that is burned becomes an open circuit in an electric circuit.



fuse

   It is also important to set the DMM's probes and measure dial in a right position to measure current. Any mistake in the set up of the DMM and the circuit connection can burn the fuse of the DMM or damage your circuit.

6.6. Applications of Parallel Circuit

Every residence in the US has usually one or two electrical energy feeds. Each one of these feeds breaks out into several branch circuits. Each one of these circuits has many lighting loads and receptacles. All the electrical loads and receptacles connected to the same feed are in parallel. Therefore, all the electrical appliances in your house that are connected to the same feed are connected in parallel.

Each branch circuit has a fuse or a circuit breaker to protect the wiring against current overload in case you connect too many appliances in parallel, and therefore, exceed the current rating of the wires. Branch circuits in modern residences are wired with AWG # 12 wires which is capable of safely carrying 20 amperes. The circuit breakers used, therefore, are set to trip and interrupt the circuit if the current demand exceeds 20 amps.

[1] *How to Use a Multimeter*, https://learn.sparkfun.com/tutorials/how-to-use-a-multimeter/fuse, retrieve on 8/16/18

Laboratory Experiment

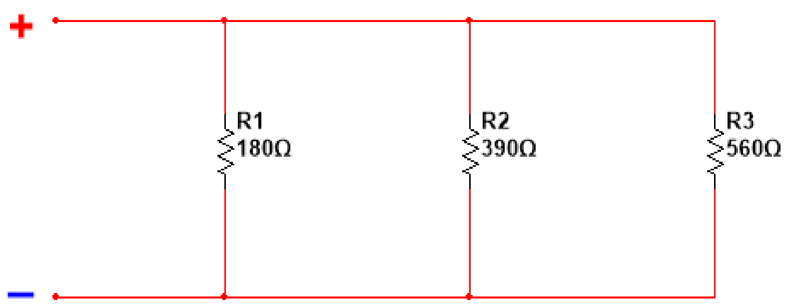
Part 1 – Building and measuring the current in a parallel resistivity circuit

1. Obtain 180 Ω, 390 Ω, 560 Ω, and 1.5 kΩ resistors from the lab kit, measure each resistor individually, and record the resistance in Table 6.1.

Actual resistance	Measured resistance	% of difference
R_1 = 390 Ω		
R_2 = 180 Ω		
R_3 = 560 Ω		
R_4 = 1.5 kΩ		

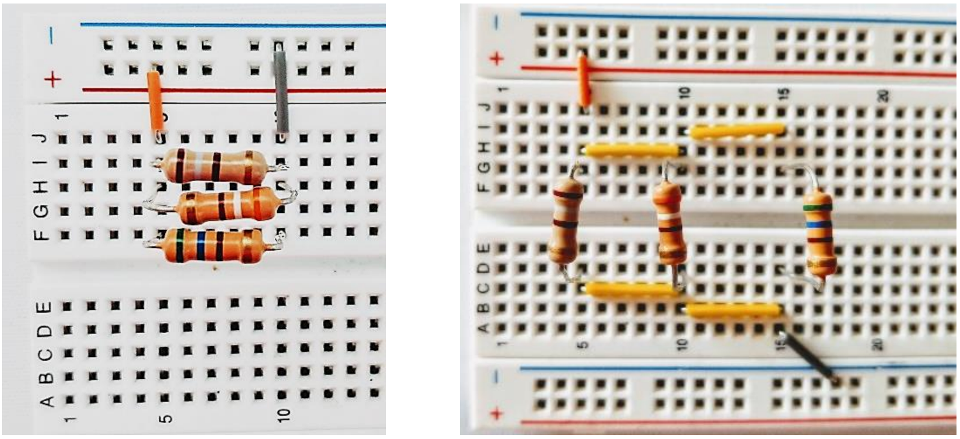
Table 6.1 – Individual resistance measurement

2. Using the resistors in Table 6.1, build Circuit 6.1 in the protoboard.



Circuit 6.1 – Parallel resistivity circuit

Alternatives to build Circuit 6.1 in the protoboard could be as:



Circuit 6.1a – Circuit 6.1 in a protoboard

3. Set the DMM to measure resistance. Measure the total resistance of Circuit 6.1 and record measurement in Table 6.2.
4. Calculate the total resistance and record calculation in Table 6.2.

Show calculations here:

Calculated total resistance of Circuit 6.1	Measured total resistance of Circuit 6.1	% of difference

Table 6.2 – Calculated and measured total resistance of Circuit 6.1

<u>Measuring current through each resistor</u>

Measuring the current through an element requires specific set up in the DMM and the circuit. It is recommended to set the DMM to the highest current range, which depending on the DMM could be 10 A or 20 A. This recommendation is to avoid to break open of the fuse of the DMM. The other important step to remember is to set up of the DMM in series with the element where the current is going to be measured. **Always remember:** to measure current of an element, one terminal of the element must be "**broken**" and the DMM must be placed in between the 'break'. In order words, the DMM is used as a bridge between the measured element and the other element on the circuit. Check Figure 6.3.

5. Set the DMM to measure current.
6. Break open one terminal of R1: to measure the current through R1, we can break open one terminal of R1 from the protoboard. For example, if you have R1 connected in between node 5 and 10, then, you can break open the terminal connected in node 5 or node 10.
7. Connect the red probe of the DMM to the node where R1 was connected before the break and connect the black probe of the DMM to open terminal of R1.
8. Connect the power supply to Circuit 6.1 and turn ON the power.
9. Read the current from the DMM and record the measurement in Table 6.3.
10. Turn OFF the power supply, repeat the previous steps and measure the current through each resistor. Record the measurements in Table 6.3

Current through elements	Measured current	Calculated current	% of difference
I_{R1}			
I_{R2}			
I_{R3}			
I_S = Current through the battery			

Table 6.3 – Measured current through each resistor in Circuit 6.1

11. Disconnect the power supply from the circuit and dissemble the resistors from the protoboard.
12. Calculate the current through each element and record calculation in table 6.3.

Show calculations here:

Note: The current distribution and flow for Circuit 3.1 is showed in below, Figure 3.4, which is also known as Kirchhoff's Current Law. You can use the measured current value in Table 6.3 and compare them with Figure 3.4.

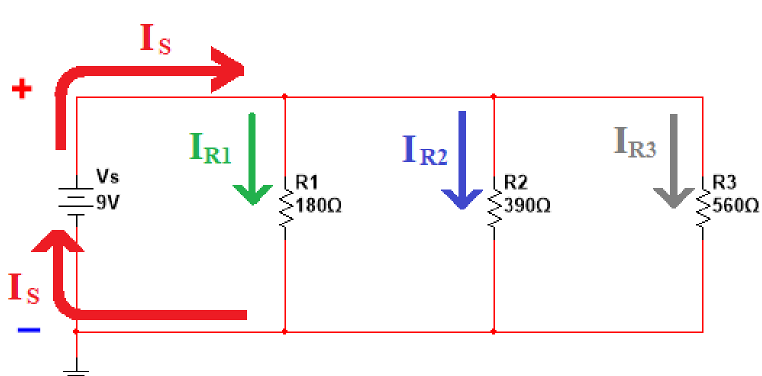
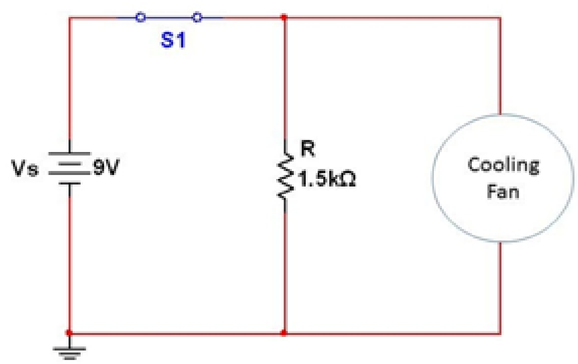


Figure 6.4 – Current flow within Circuit 6.1

Part 2 – Building and measuring the current in a parallel non-resistivity circuit

Circuit 6.2 shows a 1.5kΩ resistor connected in parallel with a computer chip cooling fan. The parallel combination is powered by a 9V supply. According to the fan's specifications, the fan current should be less than 50 mA for small fan and less 120 mA for bigger fan. Here, however, we are energizing the fan with 9V, therefore the fan current will be less.



Circuit 6.2 A Typical Heater and a fan in Parallel Circuit

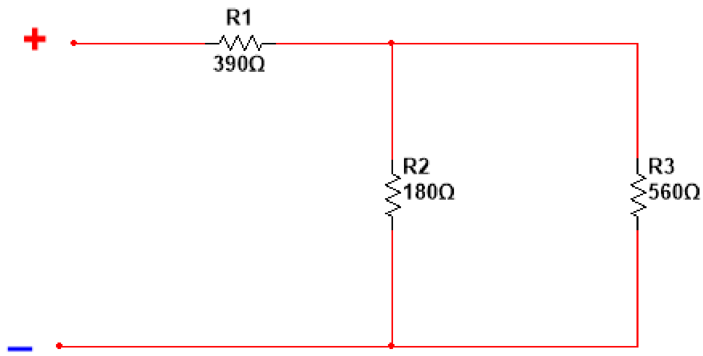
13. Obtain a cooling fan from lab technician.
14. Using resistor R4, 1.5 kΩ, and the cooling fan, build Circui 6.2.
15. Set your circuit and DMM to measure current.
16. Break open one terminal of the voltage source, and measure the current throgut voltage source (total current). Record measurement in Table 6.3.
17. Repeat the previous step and measure the current throguh 1.5 kΩ resistor, and the cooling fan. Record measurement in Table 6.3. Remember that you MUST turn OFF or disconnected the power supply first before making changes to the circuit.

Current through elements	Measured current
I_{R1}	
I_{R2}	
I_{R3}	
I_S = Current through the battery	

Table 6.3 – Measured current through each resistor in Circuit 6.1

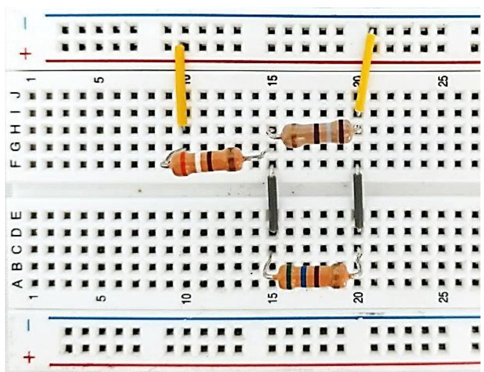
Part 3 – Building and measuring the current in a series-parallel resistivity circuit

18. Using the resistors R1, R2, and R3, build Circuit 6.3 in the protoboard.



Circuit 6.3 – Series-parallel resistivity circuit

One alternative of building Circuit 6.3 in the protoboard could be as:



Circuit 6.3a – Circuit 6.3 in a protoboard

19. Set the DMM to measure resistance. Measure the total resistance of Circuit 6.3 and record measurement in Table 6.4.

20. Calculate the total resistance and record calculation in Table 6.4.

Show calculations here:

Calculated total resistance of Circuit 6.3	Measured total resistance of Circuit 6.3	% of difference

Table 6.4 – Calculated and measured total resistance of Circuit 6.1

21. Set the DMM to measure current.
22. Break open one terminal of R1 open in Circuit 6.3.
23. Connect the red probe of the DMM to the node where R1 was connected before the break and connect the black probe of the DMM to open terminal of R1.
24. Connect the power supply to Circuit 6.3 and turn ON the power.
25. Read the current from the DMM and record the measurement in Table 6.5.
26. Turn OFF the power supply, repeat the previous steps and measure the current through each resistor. Record the measurements in Table 6.5

Current through elements	Measured current	Calculated current	% of difference
I_{R1}			
I_{R2}			
I_{R3}			
I_S = Current through the battery			

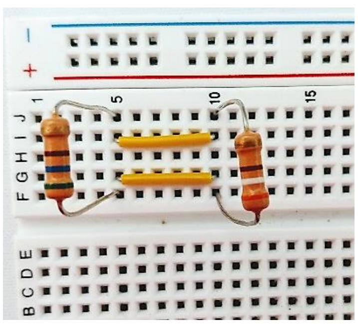
Table 6.5 – Measured current through each resistor in Circuit 6.3

27. Disconnect the power supply from the circuit and dissemble the resistors from the protoboard.
28. Calculate the current through each element and record calculation in table 6.5.

Show calculations here:

Question

1. Three resistors, 5.6 kΩ, 8.2 kΩ, and 2.7 kΩ, are connected in parallel. When a student measured the total resistance, the DMM read **6.027536 kΩ.** Without calculations, do you think this measurement may be correct? Justify your answer
2. A global outlet power strip has a maximum current load of 15A. If a 10A air conditioner and a 1A desk lamp is already connected in the power strip. What do you think it would happen if you connect a 12 A hair dryer to the same power strip? Explain your answer.
3. A student built a circuit with two resistors, 2 kΩ and 3 kΩ, connected in parallel. The student measured the current through 3 kΩ and the DMM displayed 4 mA. Using this measurement, how can the student predict the total voltage of the parallel circuit? Justify your answer.
4. A student is asked to build a parallel circuit with two resistors in a protoboard. The student built the circuit as shown in the picture below and show the circuit the lab instructor.



Would the lab instructor tell the student that the circuit connection is correct or incorrect? Justify your answer.

Answers here:

Student's Name: ________________ Lab Instructor's Signature______________ Date: _______

------------------------------------- **LAB EXPERIMENT ENDS HERE** ------------------------------------