时序因果发现

SCM：

We assume that the stochastic process $(\mathbf{X}_t)_{t\in\mathbb{Z}}$ admits a description by an SCM in which at most the past q values (for some q) of all variables occur:

$$X_t^j := f^j\left((\mathbf{PA}_q^j)_{t-q},\ldots,(\mathbf{PA}_1^j)_{t-1},(\mathbf{PA}_0^j)_t,N_t^j\right), \tag{10.1}$$

where

$$\ldots,N_{t-1}^1,\ldots,N_{t-1}^d,N_t^1,\ldots,N_t^d,N_{t+1}^1,\ldots,N_{t+1}^d,\ldots$$

are jointly independent noise terms. Here, for each $s\in\mathbb{Z}$, the symbol $(\mathbf{PA}_s^j)_{t-s}$ denotes the set of variables X_{t-s}^k, $k=1,\ldots,d$, that influence X_t^j. Note that $\mathbf{PA}_{t-s}^j$ may contain X_{t-s}^j for all $s>0$, but not for $s=0$. We assume the corresponding full time graph to be acyclic.

- **特例：VAR**

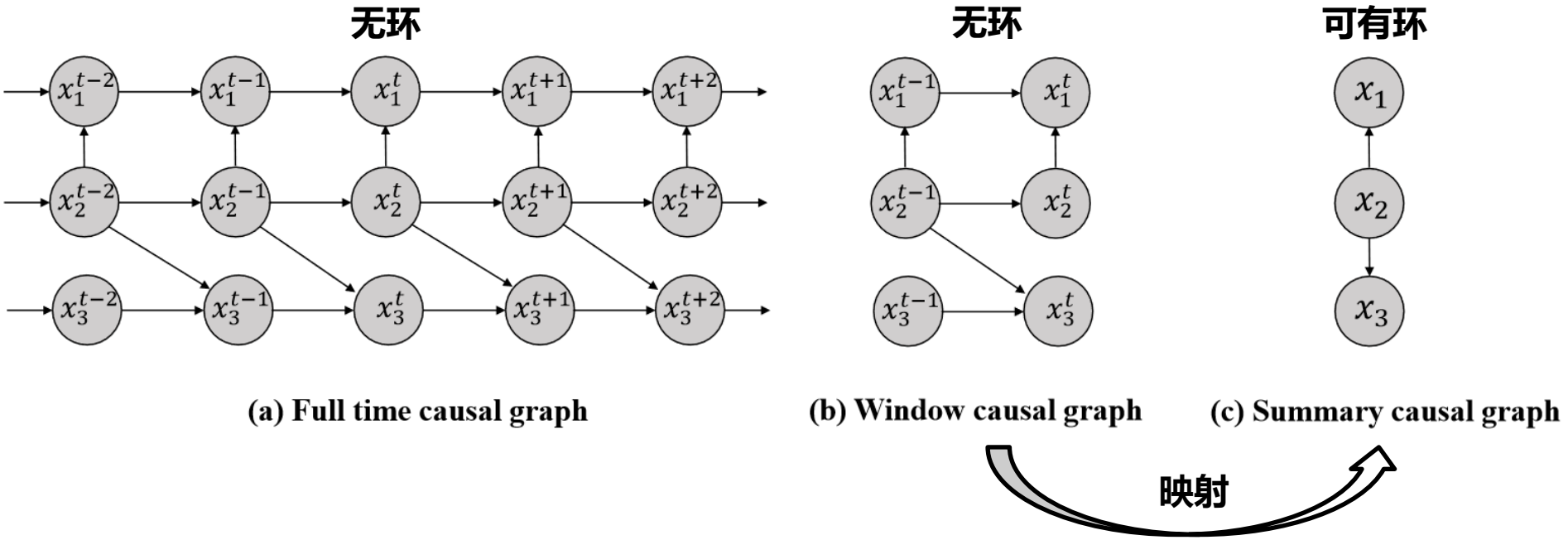
$$X_t^j := \sum_{i=1}^{q} A_i^j \mathbf{X}_{t-i} + N_t^j,$$

[Peters J, Janzing D, Schölkopf B. Elements of causal inference: foundations and learning algorithms[M]. The MIT Press, 2017.]

时序因果发现

□ 因果图：

- **Full time causal graph**：一次时序观测下，很难得到，**几乎没有**方法。
- **Window causal graph**：**少部分**方法能得到。
- **Summary causal graph**：**大部分**方法。



[Gong C, Yao D, Zhang C, et al. Causal Discovery from Temporal Data: An Overview and New Perspectives[J]. arXiv preprint arXiv:2303.10112, 2023.]

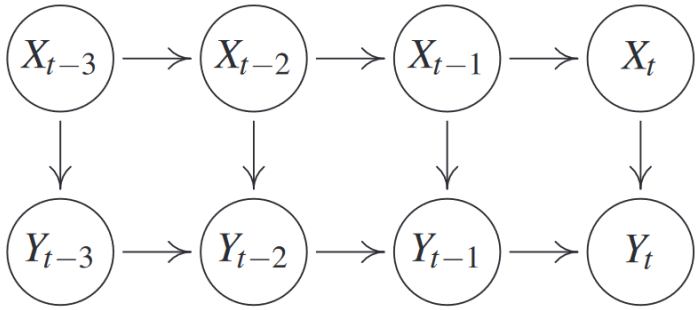
时序因果发现

□ 时序因果可识别性：从观测数据中唯一确定因果方向。

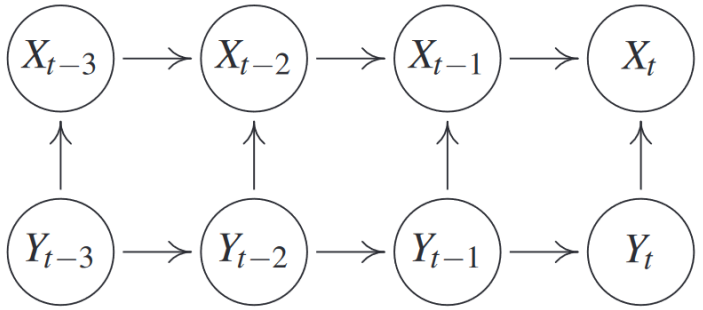
➢ 马尔科夫等价类的充要条件：

Lemma 6.25 (Graphical criteria for Markov equivalence) *Two DAGs $\mathcal{G}_1$ and $\mathcal{G}_2$ are Markov equivalent if and only if they have the same skeleton and the same immoralities.*

➢ 时间优先性：原因必先于结果发生，只考虑时滞因果效应。



(a) There are v-structures at all nodes of $(Y_t)_{t\in\mathbb{Z}}$.



(b) There are v-structures at all nodes of $(X_t)_{t\in\mathbb{Z}}$.

Figure 10.5: Two DAGs that are not Markov equivalent although they coincide up to instantaneous effects.

[Peters J, Janzing D, Schölkopf B. Elements of causal inference: foundations and learning algorithms[M]. The MIT Press, 2017.]

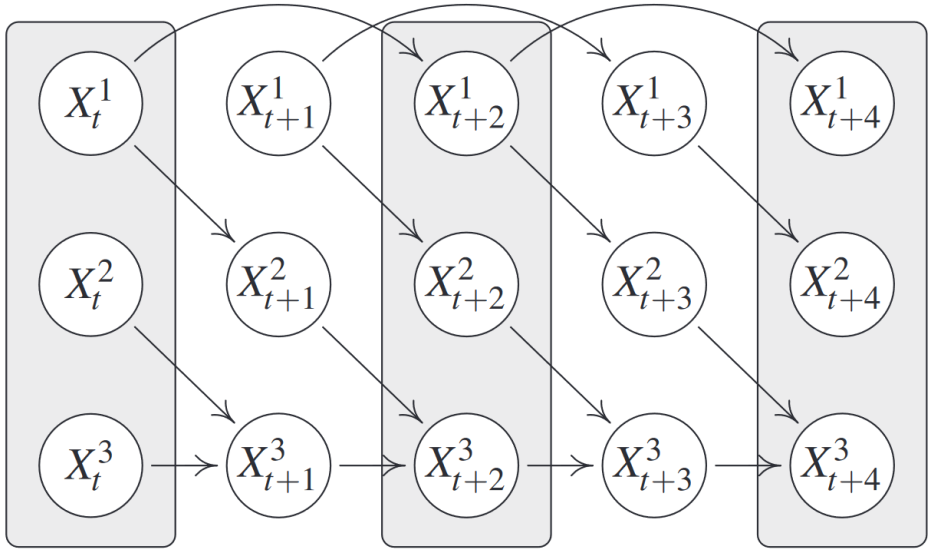
时序因果发现

- **时间优先性假设不成立的情况：**
  - 公鸡先于太阳升起打鸣，但太阳升起是公鸡打鸣的因：

[Some pictures from Google.]

时序因果发现&UCN

- **时间优先性假设不成立的情况：**
  - **欠采样/低时间分辨率数据：**



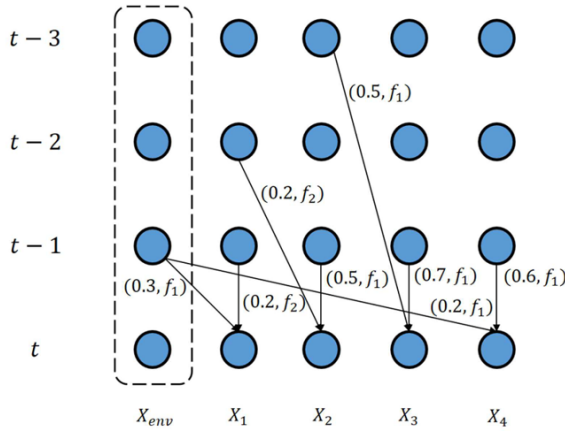


Figure 10.4: Example of a subsampled time series: only the variables in the shaded areas are observed.

在Real-World Data中，考虑瞬时+时滞因果效应更合理

[Peters J, et al. Elements of causal inference: foundations and learning algorithms[M]. The MIT Press, 2017.]
[Kang M, et al. Identifying Unique Causal Network from Nonstationary Time Series[J]. arXiv preprint arXiv:2211.10085, 2022.]