Random Contrastive Interaction for Particle Swarm Optimization in High-Dimensional Environment

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***Abstract*—In high-dimensional environment, the interaction among particles significantly affects their movements in searching the vast solution space and thus plays a vital role in assisting particle swarm optimization (PSO) to attain good performance. To this end, this article designs a random contrastive interaction (RCI) strategy for PSO, resulting in RCI-PSO, to tackle large-scale optimization problems (LSOPs) effectively and efficiently. Unlike existing interaction mechanisms for low-dimensional problems, RCI randomly chooses several different peers from the current swarm to construct a random interaction topology for each particle. Then, it lets the particle interact with the selected peers based on their current evolutionary information instead of their historical evolutionary information. Within the topology, RCI only propagates the evolutionary information of two contrastive dominators with the largest difference in fitness to direct the evolution of the particle. Therefore, particles with no more than two dominators in their topologies are not updated. Furthermore, a dynamic topology size adjustment scheme is devised to gradually enlarge the interaction topology. In this way, the swarm gradually switches from exploring the immense search space dispersedly to exploiting the found optimal regions intensively as the evolution continues. With these two strategies, RCI-PSO expectedly compromises search diversity and search convergence well at the swarm level and the particle level. Finally, extensive experiments executed on two public LSOP suites verify that RCI-PSO performs competitively with or even much better than totally 40 state-of-the-art large-scale approaches and preserves a good capability and scalability in tackling complex LSOPs.**

***Index Terms*—Adaptive topology, high-dimensional problems, large-scale optimization, particle swarm optimization (PSO), random contrastive interaction (RCI).**

I. Introduction

PARTICLE swarm optimization (PSO) has acquired remarkable success on low-dimensional optimization problems [1], [2]. Imitating the bird flocking behavior, PSO deploys a swarm of particles to traverse the solution space to collaboratively find the global optimum of the optimized problem [3], [4]. Specifically, each particle is denoted by two attributes: 1) the position and 2) the velocity, and is updated as

$$v_i^d \leftarrow wv_i^d + c_1r_1\left(p\text{best}_i^d - x_i^d\right) + c_2r_2\left(n\text{best}_i^d - x_i^d\right) \quad (1)$$

$$x_i^d \leftarrow x_i^d + v_i^d \quad (2)$$

where $X_i = [x_i^1, \ldots, x_i^d, \ldots, x_i^D]$ and $\mathrm{V}_i = [v_i^1, \ldots, v_i^d, \ldots, v_i^D]$ represent the position vector and the velocity vector of the ith particle, respectively. D denotes the dimensionality of the optimized problem. $p\text{best}_i = [p\text{best}_i^1, \ldots, p\text{best}_i^d, \ldots, p\text{best}_i^D]$ is the personal best position found by the ith particle so far, while $n\text{best}_i = [n\text{best}_i^1, \ldots, n\text{best}_i^d, \ldots, n\text{best}_i^D]$ is the best one among the personal best positions (***p*bests**) of its neighbors, which are connected with the ith particle by a specific topology [5], [6]. As for the parameters, w denotes the inertia weight, which controls the inheriting degree of the previous velocity [7]. c_1 and c_2 represent two acceleration coefficients [8]. r_1, as well as r_2, is a real value uniformly randomized within [0, 1].

In (1), each particle interacts with its neighbors via the best historical evolutionary information, namely, $nbest_i$ in its topology. In [1], [9], and [10], it has been experimentally demonstrated that the interaction mechanism plays a vital role in assisting PSO to achieve satisfactory performance. Thus, to improve the effectiveness of PSO, researchers have been dedicated to designing a variety of effective interaction schemes for particles [1], [9], [10], [11].

In the early research, a lot of static topology structures have been devised to realize the interaction among particles [10], such as ring topologies [12], cycle topologies [13], star topologies [14], Von Neumann topologies [5], and small-world topologies [6], [15]. However, these static topologies restrict the interaction ranges of particles, leading to very limited diversity of particle interaction. This is not beneficial for the promotion of the search diversity.

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To alleviate this issue, researchers have developed dynamic topologies [16], [17], [18], where the topology structures are dynamically adjusted for effective propagation of evolutionary information. Since different topologies have distinct strengths in helping PSO solve different types of optimization problems [1], [9], [10], some researchers have even attempted to hybridize multiple topologies for particles to interact with each other in each generation [19].

The above topology-based interaction mechanisms generally transmit the whole evolutionary information of particles, leading to that each particle learns from the historical experience of only one peer. To further enhance the interaction efficiency, some researchers have proposed dimension-level interaction schemes [20], [21], [22], [23]. In this kind of interaction mechanisms, for each dimension of a particle, a random topology is utilized to propagate the evolutionary information of its chosen neighbors. By virtue of this technique, each particle could learn from the historical experience of multiple peers and thus the learning effectiveness and efficiency of particles are expectedly improved to help PSO achieve promising performance [20], [21], [22], [23].

However, the above interaction mechanisms all transmit historical evolutionary information of particles. During the evolution, such information of particles probably remains unchanged in many generations, especially in the late evolution stage. Consequently, the interaction among particles may be insufficient. In high-dimensional environment, such insufficient interaction may occur more frequently. Consequently, the historical evolutionary information-based interaction mechanisms provide limited assistance for PSO in solving large-scale optimization problems (LSOPs).

In particular, different from low-dimensional problems, on the one hand, the solution space of an LSOP is exponentially enlarged. Therefore, seeking its global optimum in such vast space becomes much more difficult [24], [25], [26]. On the other hand, the correlation between variables likely becomes complicated in an LSOP, which leads to that the landscape of this LSOP is very complex along with abundant saddle areas or local regions [27], [28], [29]. In this case, optimization algorithms are at great risk of falling into local zones and premature convergence.

To handle the above challenges, effective and efficient particle interaction schemes are urgently demanded for PSO to guarantee high-search diversity for locating optimal regions fast whilst maintaining fast search convergence for finding high-quality solutions. To this end, many studies have been dedicated to designing novel particle interaction techniques [30], [31], [32], [33]. To the best of our knowledge, existing particle interaction in high-dimensional environment can be roughly partitioned into two main categories.

The first category of particle interaction mechanisms [25], [26], [30], [31], [34] still adopt some specific topologies to disseminate the whole evolutionary information of particles. Nevertheless, different from the ones in low-dimensional environment, these mechanisms transmit the current evolutionary information of particles instead of their historical information [25], [26], [35]. On account of the continuous update of particles, the transmitted information is different for different particles and also different for the same particle in different generations. As a result, the interaction diversity of particles probably remains high, leading to high-search diversity of the swarm [30], [31], [34], [36].

The second category of particle interaction techniques [37] aim to fully utilize those in low-dimensional environment. Therefore, they usually decompose an LSOP into several exclusive low-dimensional subproblems, and then separately solve these subproblems by PSO based on the particle interaction methods in low-dimensional environment [27], [28]. However, since the decomposed subproblems are individually solved, the decomposition accuracy plays a crucial role in this type of interaction methods. This is because the correlations between variables seriously affect the dissemination efficacy of the evolutionary information of particles. As a result, the research on this kind of particle interaction mainly concentrates on devising accurate decomposition methods [37], [38], [39], [40].

The above-mentioned interaction schemes have assisted PSO to effectively solve certain kinds of LSOPs. However, the dissemination of evolutionary information still encounters great challenges in complex high-dimensional environment with a lot of saddle areas and local regions. This results in that the learning effectiveness and efficiency of particles are not so high as one would have expected, such that the optimization performance of PSO in dealing with complicated LSOPs is not so satisfactory as one would have hoped.

To further boost the optimization effectiveness of PSO in solving LSOPs, this article designs a random contrastive interaction (RCI) mechanism to disseminate evolutionary information of particles with high efficacy. Embedding this interaction scheme into PSO, we develop a novel PSO variant, named RCI-PSO. To be specific, the main contributions of this article and the key components of RCI-PSO are summarized as follows.

1) An RCI scheme is devised for particles to interact with each other effectively and efficiently. Instead of disseminating historical evolutionary information of particles like traditional PSO [1], [9], [10], [41], RCI directly transmits the current evolutionary information of particles. To ensure high interaction diversity, RCI randomly forms a stochastic interaction topology for each particle. Then, it only updates those particles with at least two dominators in their interaction topologies via broadcasting the evolutionary information of two contrastive dominators with the largest discrepancy in fitness. By this means, on the one hand, some relatively poor particles have chances to directly enter the next iteration, which is conducive to diversity enhancement; on the other hand, each updated particle learns from two very different exemplars, which probably ensures a potential balance between search diversity and search convergence at the particle level.
2) RCI can actually be seen as a general interaction framework for PSO. With different settings of the topology size, it possesses different properties. A large topology size is conducive to fast convergence of the swarm to locate high-quality solutions with slight diversification. In contrast, a small topology size is beneficial for

enhancing diversity of the swarm to find optimal regions with slight intensification. Particularly, the large-scale PSO devised in [31] is a special case of RCI when the topology size is set as 2.

3) A dynamic topology size adjustment scheme is further devised to dynamically enlarge the interaction topology. In this way, the swarm gradually switches from exploring the search space with slight intensification to exploiting the found optimal zones with slight diversification. Therefore, a potential balance between search diversity and search convergence is sustained at the swarm level.

With the above designed techniques, RCI-PSO is anticipated to seek promising regions with high-search diversity whilst finding high-quality solutions in optimal areas with subtle exploitation in tackling LSOPs. To verify its effectiveness and efficiency, experiments are extensively done on the CEC2010 [42] and the CEC2013 [43] LSOP sets, by comparing RCI-PSO with 40 large-scale optimizers. Furthermore, experiments are also conducted to investigate the contributions of the devised two techniques to the effectiveness of RCI-PSO. Besides, experiments on the CEC2010 set with different dimension sizes are conducted to demonstrate the good scalability of RCI-PSO.

The structure of the remainder of this article is organized as follows. Section II reviews typical interaction schemes of PSO in both low-dimensional and high-dimensional environments. Then, in Section III, RCI-PSO is elaborated in detail. Following is the verification of the effectiveness of RCI-PSO by experiments in Section IV. Finally, Section V concludes this article.

II. Particle Interaction for PSO

It is widely recognized that particle interaction directly affects the evolutionary information transmission and hereafter influences the update of particles [1], [9], [10]. Due to this, many researchers have devised numerous effective particle interaction mechanisms to improve the learning efficacy of particles to boost the effectiveness of PSO [2], [18], [30], [31].

A. Particle Interaction in Low-Dimensional Environment

In the early research on PSO, researchers mainly concentrated on designing particle interaction schemes for PSO to effectively tackle low-dimensional optimization problems. In this direction, the most representative ones are topology-based interaction mechanisms [1], [9], [10], [41], [44].

First, researchers mainly focused on devising static particle interaction topologies for particles to communicate with each other [10]. In these interaction methods, the interaction topology structure and the topology size of particles keep fixed during the whole evolution. For instance, in the earliest standard PSO [45], a star topology connecting all particles was utilized for particles to exchange information. Such a topology leads to too greedy interaction among particles. As a result, the classical PSO probably falls into local regions and encounters premature convergence when dealing with multimodal problems [46], [47]. To alleviate this issue, many local topologies have been devised for particles to communicate locally. To name a few typical instances, in [12], a ring topology was proposed to arrange all particles into a ring. Then, each particle only interacts with its left and right neighbors to exchange historical evolutionary information. In [13], a cyclic-network topology was adopted to let each particle interact with successively numbered nearby neighbors for information exchange. In [5], the Von Neumann topology was tested on PSO by first arranging particles into a 2-D grid and then letting each particle communicate with its above, below, left, and right neighbors to exchange evolutionary information. In [6] and [15], the small-world topology was adopted to connect particles for information exchange. In particular, each particle is first connected with several successive peers and then each link is randomly selected with a given probability to reconnect the particle with a random peer chosen from the swarm.

However, particles in the above static topologies are restricted to interact with fixed objects. Though the positions of the fixed objects may be updated during the evolution, they cannot interact with other peers. As a consequence, the search diversity of the swarm is limited, which may result in entrapment in local minima or premature convergence. To alleviate the above predicament, many dynamic particle interaction methods have been proposed [16], [17], [18] to improve the interaction diversity of particles. For instance, an adaptive time-varying topology was proposed in [16] by dynamically adjusting the interaction topology of particles as the evolution continues. In this manner, a good balance between exploration and exploitation can be maintained. Wang et al. [17] devised a dynamic tournament topology scheme to improve the particle interaction efficiency. Specifically, each particle interacts with several better peers randomly selected from the swarm based on the tournament selection method. A distance-based dynamic neighborhood topology was designed in [18] to make full use of evolutionary information of particles. To be specific, for each particle, several nearest neighbors to the personal best position of this particle are obtained and several nearest neighbors to the global best position are attained as well. Then, the authors defined four interaction modes by setting different numbers of neighbors for particles to interact based on their search performance.

To combine the advantages of different interaction topologies, some researchers have attempted to utilize multiple interaction topologies to evolve the swarm. For instance, Liu et al. [19] utilized the regular graph to define the topology and then theoretically afforded an optimal setting of the topology size based on the given swarm size and the total computational budget. Wang et al. [48] first divided the swarm into a hybrid topology swarm and a scale-free topology swarm. Then, they mixed the star topology and the ring topology to evolve the former swarm, and adaptively generated the topology for the latter swarm based on a network growing model. Zheng et al. [4] proposed a fitness-distance balance scheme to construct an interaction topology for each particle to communicate their historical evolutionary information. They also designed an individual-exploitation method to build another topology for each particle to exchange their current evolutionary information.

To further enhance the interaction diversity and efficiency, some researchers have proposed dimension-level interaction schemes. Along this line, the most typical one is the comprehensive learning PSO (CLPSO) [21]. Specifically, for each dimension of each particle, the binary tournament topology was adopted to let the particle interact with two randomly selected peers. In this way, each particle exchanges information with multiple peers and learns from them simultaneously. Inspired by such thought of interaction schemes, many other PSO variants have been developed, such as orthogonal learning PSO [49], differential elite learning PSO [50], and genetic learning PSO [51].

The above interaction mechanisms have aided PSO to obtain good optimization performance in solving low-dimensional problems. However, their effectiveness rapidly deteriorates when dealing with LSOPs. This is because in high-dimensional environment, they cannot afford efficient and diverse interactions among particles [52], [53], [54], which results in that particles cannot search the vast space in diverse directions to locate promising regions.

B. Particle Interaction in High-Dimensional Environment

To improve the effectiveness of PSO in tackling LSOPs, researchers have been dedicated to designing novel particle interaction mechanisms for PSO in two major directions: 1) designing novel interaction schemes to increase the interaction diversity of particles so as to fully traverse the vast problem space [25], [26], [30], [34], [55] and 2) fully utilizing existing interaction schemes in low-dimensional environment by decomposing an LSOP into several exclusive low-dimensional subproblems [37], [38], [39], [56].

In the first direction, to improve the interaction diversity, researchers mainly focused on designing novel interaction schemes based on random topologies to exchange the current evolutionary information of particles instead of their historical evolutionary information [24], [25], [26], [30], [34], [57], [58]. For instance, Cheng and Jin [24] proposed a random competitive particle interaction mechanism for PSO, leading to a competitive swarm optimizer (CSO), to effectively cope with LSOPs. During the evolution of the swarm, particles are randomly organized into pairs. Then, particles in each pair interact with each other with the loser following the guidance of the winner while the winner directly entering the next iteration. Similarly, in [59], a social interaction scheme was designed for PSO, resulting in a social learning PSO (SLPSO), to traverse the high-dimensional space efficiently. To be specific, each particle interacts with a random dominator with an interaction probability and follows the guidance of this dominator to move toward promising regions. Deng et al. [26] proposed a ranking-based biased learning swarm optimizer (RBLSO) along with two types of interaction strategies. The first is the ranking paired interaction scheme, which lets worse particles interact with better ones, while the second is the biased center interaction mechanism, which lets each particle interact with all particles by following the guidance of the fitness-weighted center of the entire swarm. In [30], an adaptive decoupled exploration and exploitation PSO (APSODEE) was designed along with two interaction schemes. First, a sparse interaction mechanism was developed to let each particle interact with those scattered in sparse areas. Second, a subswarm based interaction scheme was devised to first partition particles into subswarms and then let each particle interact with the best ones in all subswarms. In [60], a subswarm-based distributed interaction scheme was developed for PSO to tackle LSOPs. At each iteration, the whole swarm is separated into subswarms and then, a distributed model is used to separately evolve these subswarms by only letting the worst particle in each subswarm interact with the best peer of the subswarm and the best one of the entire swarm.

In the above methods, each updated particle interacts with only one dominator. As a result, slow convergence may occur. To alleviate this issue, Yang et al. [25] proposed a level-based interaction scheme for PSO, leading to a level-based learning swarm optimizer (LLSO). Particularly, in this interaction scheme, particles are partitioned into different levels of the same size according to their fitness. Afterward, each particle in lower levels interacts with two dominators randomly selected from two different higher levels. Lan et al. [34] separated the whole optimization process into two phases, and then devised two different interaction schemes for particles in the two phases, leading to a two-phase learning-based PSO (TPLSO). In the first phase, particles are first arranged into triads. Then, particles in each triad interact with each other by letting the loser follow the guidance of the winner. In contrast, in the second phase, a number of elite particles are first picked out. Then, each elite interacts with two random dominators by following their direction to move toward optimal areas. Wang et al. [61] also divided the whole evolution process into two stages and then designed different interaction schemes for particles in the two stages. In the first stage, each particle interacts with one of its dominators to find promising areas, while in the second stage, each particle interacts with two of its dominators to exploit the found optimal regions. Recently, Yang et al. [31] put forward a stochastic dominant interaction strategy for PSO, resulting in a stochastic dominant learning swarm optimizer (SDLSO). In this method, each particle interacts with two random peers stochastically selected from the swarm. Then, only when the particle is dominated by the two selected peers, it is updated by following their guidance; otherwise, it is not updated.

In the second direction, researchers mainly anticipated to make full use of existing effective interaction mechanisms in low-dimensional environment to cope with LSOPs based on the thought of the "divide-and-conquer" mechanism [37]. To this end, they first decomposed an LSOP into exclusive small-scale subproblems, and then adopted low-dimensional PSO variants to separately solve each subproblem [37].

Bergh and Engelbrecht [27] took the first attempt to randomly divide an LSOP with D variables into K subproblems with each containing D/K variables. Then, they optimized each subproblem using the classical PSO with the star interaction topology. Since the optimization of interacting variables usually intertwines together, they should be placed together to optimize [38], [39], [56]. However, without prior knowledge of the correlations between variables, it is hardly accurate to decompose an LSOP into low-dimensional subproblems. As a result, recent research in this direction mainly concentrated on devising accurate decomposition methods to separate an LSOP into subproblems as accurately as possible [37].

Along this road, the most remarkable work is the differential grouping (DG) strategy [29], which partitions an LSOP into subproblems by detecting variable correlations. Inspired by this work, many DG variants have been proposed to improve the decomposition accuracy and efficiency. To name a few, an extended DG (XDG) [32] and a global DG (GDG) [33] were proposed to detect both direct and indirect variable correlations. An improved version of DG, named DG2 [62], and a recursive DG (RDG) [38] were devised to improve the decomposition efficiency. Further, an improved RDG, called RDG2 [39], was developed to alleviate the sensitivity to decomposition parameters. Recently, a dual DG (DDG) [56] was designed to partition multiplicatively separable LSOPs into subproblems.

The above interaction schemes for high-dimensional environment have afforded useful support for PSO to effectively cope with certain kinds of LSOPs. However, the interaction among particles still encounters insufficiency in intricate high-dimensional environment with many saddle areas and local regions. This leads to low learning effectiveness and efficiency of particles, such that the performance of PSO in tackling complicated LSOPs is still not so satisfactory as one would have anticipated. To further promote the effectiveness of PSO in solving LSOPs, this article devises an RCI scheme for PSO to hopefully boost the interaction effectiveness and diversity of particles. As a result, a novel PSO variant named RCI-PSO is developed to let the swarm traverse the vast and complex solution space efficiently.

III. Random Contrastive Interaction-Based Particle Swarm Optimization

Standing up to the challenges brought by LSOPs, PSO desiderates effective and efficient particle interaction schemes to improve the search capabilities of particles to find high-quality solutions. To this end, this article devises an RCI mechanism for particles to exchange valuable evolutionary information effectively and efficiently in high-dimensional environment. Embedding RCI into PSO, we develop a novel large-scale PSO variant, which is named as RCI-PSO. The key components of RCI-PSO are elaborated in the following.

A. Random Contrastive Interaction

Fig. 1 shows the overall framework of RCI. Its core idea is to adopt a random topology to let each particle interact with others to transmit diverse evolutionary information. Instead of exchanging the historical evolutionary information of particles, in RCI, each particle directly communicates with others based on the current evolutionary information. Given that NP particles are maintained in the swarm and the topology size of the random topology is TS, RCI works principally as follows.

1) For each particle X_i (i= 1, 2, ..., NP), as shown in Fig. 1, TS different particles are randomly selected from the current swarm to form a stochastic topology. Subsequently, $\boldsymbol{X}_i$ only interacts with these chosen peers in the topology.
2) Subsequently, RCI finds all dominators of $\boldsymbol{X}_i$ in the topology. In particular, a particle $\boldsymbol{X}_j$ is defined as the dominator of $\boldsymbol{X}_i$ when $f(\boldsymbol{X}_j) \leq f(\boldsymbol{X}_i)$. Only when there exist at least two dominators in the topology, X_i is updated by interacting with these dominators; otherwise, it is not updated.
3) Once X_i is triggered to update, RCI lets it interact with the best dominator and the worst dominator and then follow their guidance to move. Concretely, X_i is updated as follows:

$$v_i{}^d \leftarrow R_1 v_i{}^d + R_2(x^d_{D\text{best}} - x^d_i) + \phi R_3(x^d_{D\text{worst}} - x^d_i) \quad (3)$$
$$x^d_i \leftarrow x^d_i + v_i{}^d \quad (4)$$

where $X_{D\text{best}} = [x^1_{D\text{best}}, \ldots, x^d_{D\text{best}}, \ldots, x^D_{D\text{best}}]$ is the position of the best dominator and $X_{D\text{worst}} = [x^1_{D\text{worst}}, \ldots, x^d_{D\text{worst}}, \ldots, x^D_{D\text{worst}}]$ is the position of the worst dominator in the topology. $X_i = [x^1_i, \ldots, x^d_i, \ldots, x^D_i]$ and $V_i = [v^1_i, \ldots, v^d_i, \ldots, v^D_i]$ represent the position and the velocity of the ith particle, respectively. R_1, R_2, and R_3 are three real values randomly generated from [0, 1] and ϕ is a control parameter in charge of the learning degree of X_i from $X_{D\text{worst}}$.

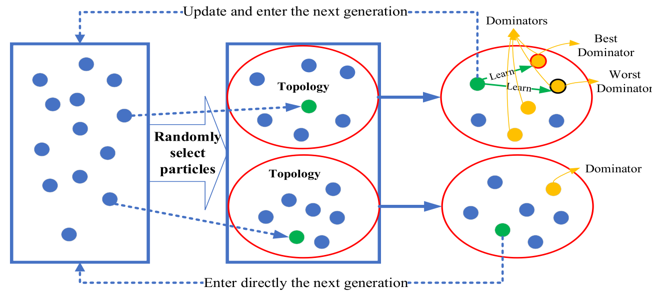

Fig. 1. Framework of RCI.

First, in step 1), though the topology size is the same, the topology structures of different particles are quite different due to the random selection of the components in the topologies. In this manner, different particles can interact with different peers and thus high interaction diversity could be maintained, which is conducive to diversity enhancement of the swarm.

Second, in step 2), the same as the classical PSOs [6], [10], [12], [13], [15], to ensure that each particle follows the guidance of better peers, RCI updates a particle only when there are at least two dominators in its topology; otherwise, RCI lets this particle directly enter the next iteration. By this means, each updated particle is expectedly directed to promising regions, and thus fast convergence of the swarm can be afforded.

Third, in step 3), RCI lets each updated particle communicate with the two most different dominators, namely, the best dominator and the worst dominator. With this interaction mechanism, each updated particle is anticipated to follow the direction of two very diverse leaders. Therefore, RCI-PSO hopefully compromises search diversity and search convergence appropriately at the particle level to traverse the vast solution space.

Remark: From Fig. 1 and (3) we discover that RCI implicitly allocates each particle an update probability, which is calculated as follows:

$$p_i = 1 - \frac{C^0_{\text{rank}(i)-1} \cdot C^{\text{TS}}_{\text{NP}-\text{rank}(i)} + C^1_{\text{rank}(i)-1} \cdot C^{\text{TS}-1}_{\text{NP}-\text{rank}(i)}}{C^{\text{TS}}_{\text{NP}-1}}$$
$$= 1 - \frac{C^{\text{TS}}_{\text{NP}-\text{rank}(i)}}{C^{\text{TS}}_{\text{NP}-1}} - \frac{C^1_{\text{rank}(i)-1} \cdot C^{\text{TS}-1}_{\text{NP}-\text{rank}(i)}}{C^{\text{TS}}_{\text{NP}-1}} \quad (5)$$

where rank(i) is the fitness ranking of the ith particle after all particles are sorted from the best to the worst in terms of fitness; p_i is its update probability; NP is the swarm size; TS is the topology size, and $\text{C}^{\text{TS}}_{\text{NP}-1}$ is the number of combinations when randomly selecting TS elements from (NP − 1) ones.

To visually observe the update probability of each particle, we plot the changing curve of the update probabilities of particles along with their fitness rankings in Fig. 2 under the same setting of NP = 900, but with different settings of TS. From (5) and Fig. 2, we attain the following findings.

1) The update probability is only dependent on the ranking of each particle. The better one particle is, the lower rank it has and thus the lower update probability it owns. Therefore, better particles have more chances to directly enter the next iteration to preserve valuable evolutionary information, while worse particles have higher update probabilities to move toward optimal regions. In this way, RCI implicitly assists PSO to balance exploration for seeking optimal zones and exploitation for finding high-quality solutions properly at the swarm level.
2) The update probabilities of the top two best particles are both 0, while the update probability of the worst particle is 1. This implies that the top two best particles in the swarm always become better and better as the evolution continues. As a result, these two particles eventually locate at or around the optimum in the vast solution space.
3) As the fitness ranking increases, the update probability grows quickly. Besides, as the topology size TS becomes large, the update probabilities of particles increase exponentially fast along with their fitness rankings. This indicates that a large TS leads to that more particles are updated in each generation. However, it should be noticed that a large TS also likely leads to a large difference between the best dominator and the worst dominator. As a result, each updated particle likely interacts with two more diverse exemplars and follows their diverse guidance to traverse the vast solution space. This is just a unique advantage of RCI. As a whole, RCI is capable of helping the swarm balance search diversity and search convergence well to traverse the immense solution space.

B. Adaptive Topology Size Adjustment

From Fig. 2, it is found that the topology size TS has great impact on the update probabilities of particles. Besides, from (3), TS also has great influence on the interaction between each updated particle and its dominators.

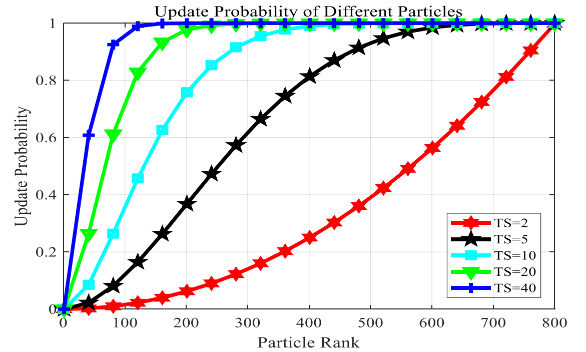

Fig. 2. Update probability of different particles.

Algorithm 1 RCI-PSO

Input: Swarm size *NP*, Control parameter ϕ, Maximum number of fitness evaluations $FES_{\max}$.

Initialize the swarm randomly and evaluate its fitness; $fes = NP$;
While ($fes \leqslant FES_{\max}$) **do**
  Calculate the topology size *TS* according to Eq. (6);
  For (each particle in the swarm) **do**
    Randomly choose *TS* different particles from the swarm to form an interaction topology;
    If (at least two dominators exist in the topology)
      Select the best dominator and the worst dominator in the topology as the leading exemplars;
      Update the particle according to Eq. (3) and Eq. (4);
      Evaluate the fitness of the updated particle and fes++;
    End If
  End For
End While
Get the best solution $\boldsymbol{x}$ in the swarm.

Output: The found best solution $\boldsymbol{x}$ and its fitness $f(\boldsymbol{x})$;

Specifically, a smaller TS affords more chances for particles to directly enter the next generation. This is very beneficial for high diversity maintenance. Furthermore, a smaller TS also leads to that each particle interacts with fewer peers and thus the difference between its best dominator and worst dominator is expectedly smaller. This may result in that each updated particle follows intensive guidance of two similar exemplars, which is profitable for fast convergence. On the contrary, a larger TS gives rise to exponentially increased update probabilities of particles. As a result, more particles are likely updated, which is conducive to the swarm to fast approach optimal regions. Besides, a larger TS also leads to that each particle interacts with more peers and thus the difference between its best dominator and worst dominator is expectedly larger, which affords benefit for diversity enhancement.

Based on the above analysis, a fixed TS is hardly able to assist RCI-PSO to achieve satisfactory performance in solving LSOPs. In particular, in the early stage of the evolution, a small TS should be expectedly maintained, so that the swarm could explore the tremendous search space with slight intensification to seek promising areas fast. In contrast, in the late stage of the evolution, a large TS should be expectedly maintained such that the swarm could exploit the found optimal zones with slight diversification to find high-quality solutions.

Bearing the above consideration into mind, we design the following simple dynamic adjustment strategy for TS:

$$\mathrm{TS}_i = \mathrm{TS}^{\min} + \mathrm{round}\left(\left(\mathrm{TS}^{\max} - \mathrm{TS}^{\min}\right) \times \sqrt{\frac{\mathrm{FEs}}{\mathrm{MaxFEs}}}\right) \quad (6)$$

where FEs is the number of used fitness evaluations; MaxFEs is the given maximum number of fitness evaluations; $\mathrm{TS}^{\min}$ and $\mathrm{TS}^{\max}$ are the minimum and the maximum values of TS, respectively; and TS_i is the topology size in the ith generation. In this article, we set $\mathrm{TS}^{\min} = 2$ because in RCI, a particle is triggered to update only when at least two dominators exist in its interaction topology. As for $\mathrm{TS}^{\max}$, we set it as $\mathrm{TS}^{\max} = 25$ based on the investigation experiments conducted in Section IV-D.

With this dynamic strategy, the swarm gradually switches from exploring the immense space for seeking optimal zones to exploiting the found optimal regions for finding high-quality solutions as the evolution proceeds. However, such a switch is not at the serious sacrifice of the search diversity. On the one hand, the interaction topology of each particle is randomly constructed; on the other hand, for each updated particle, the difference between the best dominator and the worst dominator becomes hopefully larger as TS increases; this leads to that each updated particle follows diverse guidance of two very different exemplars. In this way, RCI-PSO expectedly keeps a good balance between exploration and exploitation at the swarm level. The effectiveness of this dynamic strategy is demonstrated by the experiments conducted in Section IV-D.

C. Overall Framework and Complexity Analysis

Combing the above two techniques together, we develop the complete RCI-PSO, which is outlined in Algorithm 1. To be specific, after the initialization of the swarm (line 1), the algorithm goes into the main iteration of the optimization (lines 2–12). In the main loop, the topology size for particles is first calculated (line 3). Subsequently, for each particle, a random interaction topology is constructed (line 5), and then the dominators of this particle in the interaction topology are identified. Only when at least two dominators exist, the particle is updated (lines 7 and 8). The above main iteration continues until the preset maximum number of fitness evaluations runs out. When the algorithm terminates, the found best solution is output.

From Algorithm 1, without consideration of the fitness evaluation time, in each generation, it takes $O(\mathrm{NP}\times\mathrm{TS})$ to construct the interaction topologies of all particles (line 5) and another $O(\mathrm{NP}\times\mathrm{TS})$ to find their dominators (line 7). Then, it takes $O(\mathrm{NP}\times D)$ to update particles (line 8). Since TS is generally much smaller than NP and D, the computing time of RCI-PSO is $O(\mathrm{NP}\times D)$ in each generation.

D. Difference Between RCI-PSO and Existing PSOs

First, the most distinguished difference between RCI-PSO and the classical PSOs [6], [7], [8], [13], [15] lies in the RCI interaction scheme. In most classical PSO methods [6], [7], [8], [13], [15], particles communicate with each other based on their historical evolutionary information. Nevertheless, in RCI-PSO, each particle interacts with others directly based on their current positions. The historical evolutionary information may remain unchanged for many iterations, especially in the late stage of the evolution, while the current positions of particles are generally updated iteration by iteration. Therefore, the particle interaction diversity in RCI-PSO is much higher than the classical PSO variants. Hence, RCI-PSO expectedly preserves higher search diversity and thus is suitable for solving LSOPs.

Second, the significant difference between RCI-PSO and existing large-scale PSO variants [24], [25], [26], [30], [31], [34], [59] is the contrastive interaction, which uses two dominators with the largest difference in terms of fitness to interact with each updated particle. A lot of existing studies on LSOPs, such as CSO [24], SLPSO [59], DLLSO [25], TPLSO [34], RBLSO [26], APSODEE [30], and SDLSO [31], also adopt the dominators in the associated topology to interact with each updated particle. However, they do not explicitly consider the difference among the dominators. As a consequence, the updated particle is likely guided by similar dominators, which may result in falling into local regions or premature convergence. Nevertheless, RCI-PSO uses the best dominator and the worst dominator to guide the interaction of each updated particle. Hence, each updated particle follows the diverse guidance of two very different dominators.

Third, RCI can be seen as a general particle interaction framework. With different settings of TS, RCI-PSO preserves different strengths in exploring and exploiting the immense solution space. In particular, we find that SDLSO [31] is a special case of RCI-PSO when TS is set as 2.

IV. Experiments

In this section, experiments are done to verify the effectiveness and efficiency of RCI-PSO on two LSOP benchmark suites, namely, the CEC2010 [42] and the CEC2013 [43] LSOP suites. These two suites have been widely used to validate the optimization effectiveness of large-scale optimizers in [25], [26], [30], [31], and [34]. The CEC2010 set contains 20 1000-D LSOPs with various properties, like separable, nonseparable, unimodal, and multimodal. The CEC2013 set consisting of 15 1000-D LSOPs is an extension of the CEC2010 set by introducing complicated features like overlapping and contribution imbalance. Thus, the latter set is more difficult to optimize than the former one. Tables SI and SII in the supplementary document show the main properties of these two suites, respectively.

In this section, this article first fine-tunes the swarm size NP and the control parameter ϕ in (3) for RCI-PSO in Section IV-A. Next, comparisons between RCI-PSO and several latest and well-performed large-scale methods are made in Section IV-B. Then, in Section IV-C1, RCI-PSO is compared with the winners of those competitions held in IEEE Congress on Evolutionary Computation (CEC) and other state-of-the-art large-scale methods on the two benchmark sets. In Section IV-D, deep investigations on the effectiveness of the key components in RCI-PSO are performed. Finally, Section IV-E conducts experiments to investigate the scalability of RCI-PSO by extending the dimensionality of the LSOPs in the CEC2010 suite from 200 to 2000.

Unless otherwise specified, the total number of fitness evaluations is set to $3000\times D$ (where D is the dimension size) for all optimizers in the experiments. For fair comparisons, we run each algorithm independently 30 times and then employ the median, the mean and the standard deviation (Std) over the 30 independent executions to evaluate its performance. Besides, to distinguish whether there is significant difference between RCI-PSO and each compared method on each benchmark LSOP, we conduct the Wilcoxon rank sum test at the significance level $\alpha = 0.05$, to compare the optimization result of RCI-PSO with that of each compared method on each LSOP. To observe the overall performance of each algorithm on one whole benchmark suite, we conduct the Friedman test at the significance level $\alpha = 0.05$ to get its average rank.

For fairness, all algorithms are executed on the same PC with eight Intel Core i7-10700 2.90-GHz CPUs, 8-GB memory, and Ubuntu 12.04 LTS 64-bit system. Due to the page limit, we have to attach some experimental results to the supplementary material. For better understanding, we present the abbreviations and the associated full names of all large-scale optimizers in Table SIII in the supplementary material.

A. Parameter Fine-Tuning of RCI-PSO

In RCI-PSO, two parameters, namely, the swarm size NP and the control parameter ϕ in (3), need fine-tuning. To investigate their optimal settings, this article conducts experiments on the 1000-D CEC2010 LSOPs by varying NP from 700 to 1200 and ranging ϕ from 0.1 to 0.5. The experimental results are displayed in Table SIV in the supplementary document. From this table, we acquire the following findings.

1) With different settings of NP, the optimal setting of ϕ for RCI-PSO is always 0.3. This indicates that the setting of ϕ is not related to the setting of NP. In particular, neither a too large ϕ (such as $\phi = 0.5$) nor a too small ϕ (such as $\phi<0.3$) is conducive to RCI-PSO to achieve good performance. This is because a too small ϕ reduces the influence of the worst dominator on each updated particle. In this case, each updated particle may move to the area where the best dominator locates too fast. As a result, premature convergence or falling into local regions may occur. In contrast, a too large ϕ enhances the effect of the worst dominator too much on each updated particle. This may do harm to the convergence of the swarm, and thus leads to unsatisfactory performance of RCI-PSO.
2) From the average rank, RCI-PSO with NP $= 900$ and $\phi = 0.3$ achieves the lowest rank and such a rank is much smaller than those of the other configurations. This indicates that NP $= 900$ and $\phi = 0.3$ is the optimal combination for RCI-PSO to achieve the best overall performance on the 1000-D CEC2010 suite.

In conclusion, the configuration of NP $= 900$ and $\phi = 0.3$ is adopted in the following experiments related to 1000-D LSOPs.

B. Comparison With State-of-the-Art Methods

This section mainly validates the effectiveness and efficiency of RCI-PSO by making comparisons with the latest and well-performed large-scale approaches. First, we choose 11 state-of-the-art evolutionary algorithms designed for LSOPs, namely, APSODEE [30], SDLSO [31], TPLSO [34], RBLSO [26], DLLSO [25], SLPSO [59], CSO [24], DECC-GDG [33], DECC-DG2 [62], DECC-RDG [38], and DECC-RDG2 [39]. The associated full names of these large-scale optimizers are shown in Table SIII in the supplementary material. The same with RCI-PSO, the former seven latest PSO variants consider optimizing all variables as a whole and adopt the dominators in the associated interaction topologies to direct the update of particles. Unlike RCI-PSO, the latter four latest approaches first decompose an LSOP into several low-dimensional subproblems and then utilize differential evolution (DE) designed for small-scale optimization problems to solve these subproblems separately. In these four decomposition-based methods [33], [38], [39], [62], DE was utilized instead of PSO, because in [29], [38], [39], and [56], DE has been demonstrated to be more effective than PSO to solve LSOPs under the CC framework. To ensure fairness, all parameters of the compared algorithms are set strictly according to the recommended settings in the original papers.

Tables II and III present the detailed optimization results of all algorithms and their comparisons on the 1000-D CEC2010 and the 1000-D CEC2013 LSOP suites, respectively. The bolded *p-values* signify that RCI-PSO performs significantly better than the corresponding compared methods on the associated LSOPs. Additionally, the signs “+,” “=,” and “−” in the right upper corner of *p-value* mean that RCI-PSO performs significantly better than, equivalently to, and significantly worse than the corresponding compared methods on the associated LSOPs. Furthermore, “*w/t/l*” in these two tables count the numbers of “+,” “=,” and “−” on the associated benchmark suite. For better observation, Table I summarizes the statistical comparison results with respect to “*w/t/l*” on different types of LSOPs and the average ranks on the two benchmark suites.

From Table II and the upper part of Table I, we discover the following findings on the 20 1000-D CEC2010 LSOPs.

1) Based on the Friedman test results, RCI-PSO gains the lowest average rank among all 12 large-scale optimizers. Besides, its rank value (1.75) is much smaller than those of the compared methods. These observations signify that on the one hand, RCI-PSO attains the best overall performance on the whole 1000-D CEC2010 suite; on the other hand, RCI-PSO presents significant superiority to the 11 compared approaches in solving the CEC2010 LSOPs.
2) Based on the Wilcoxon rank-sum test results, RCI-PSO significantly wins the 11 compared methods on at least 13 LSOPs and only shows failures on at most 5 LSOPs. This indicates that RCI-PSO is more effective in tackling the 1000-D CEC2010 LSOPs.
3) Comparison results on different types of LSOPs reveal that RCI-PSO achieves much better performance than

TABLE I
SUMMARIZED STATISTICAL COMPARISONS BETWEEN RCI-PSO AND THE COMPARED METHODS ON THE CEC2010 AND CEC2013 BENCHMARK SETS

Benchmark set	Problem Property	Index	RCI-PSO	APSODEE	SDLSO	TPLSO	RBLSO	DLLSO	SLPSO	CSO	DECC-	DECC-	DECC-	DECC-
CEC2010-1000	Fully Separable Unimodal	*w/t/l*	-	1/0/0	0/0/1	0/1/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Separable Multimodal	*w/t/l*	-	1/0/1	1/0/1	2/0/0	1/0/1	1/1/0	2/0/0	2/0/0	1/0/1	2/0/0	2/0/0	2/0/0
	Partially Separable Unimodal	*w/t/l*	-	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0
	Partially Separable Multimodal	*w/t/l*	-	6/0/3	4/2/3	9/0/0	9/0/0	6/2/1	9/0/0	8/0/1	9/0/0	9/0/0	9/0/0	9/0/0
	Fully Non-Separable Unimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Non-Separable Multimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Overall	*w/t/l*	-	16/0/4	13/2/5	19/1/0	19/0/1	16/3/1	20/0/0	19/0/1	19/0/1	20/0/0	20/0/0	20/0/0
	Overall	*rank*	**1.75**	3.80	3.20	6.40	7.65	6.00	8.40	8.20	9.30	9.40	7.00	6.90
CEC2013-1000	Fully Separable Unimodal	*w/t/l*	-	1/0/0	0/0/1	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Separable Multimodal	*w/t/l*	-	0/1/1	1/1/0	1/1/0	0/1/1	1/1/0	1/1/0	1/1/0	0/0/2	1/0/1	1/0/1	1/0/1
	Partially Separable Unimodal	*w/t/l*	-	3/0/0	3/0/0	3/0/0	3/0/0	3/0/0	3/0/0	3/0/0	3/0/0	3/0/0	3/0/0	3/0/0
	Partially Separable Multimodal	*w/t/l*	-	2/3/0	3/2/0	4/1/0	3/2/0	3/2/0	3/2/0	2/3/0	5/0/0	4/1/0	5/0/0	4/1/0
	Overlapping Unimodal	*w/t/l*	-	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0
	Overlapping Multimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Non-Separable Unimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Overall	*rank*	-	10/4/1	11/3/1	13/2/0	11/3/1	12/3/0	12/3/0	11/4/0	13/0/2	13/1/1	14/0/1	13/1/1
	Overall	*w/t/l*	**2.20**	3.47	5.33	4.80	5.73	5.47	7.87	6.80	9.53	9.80	8.87	8.13

the 11 compared methods on the complicated LSOPs. For instance, on the 15 partially separable LSOPs (including the six partially separable unimodal LSOPs and the nine partially separable multimodal LSOPs), RCI-PSO performs significantly better than the 11 compared methods on more than 10 LSOPs. Besides, on the two fully nonseparable LSOPs (including the one fully nonseparable unimodal LSOP and the one fully nonseparable multimodal LSOP), RCI-PSO significantly outperforms the 11 compared methods.

From Table III and the lower part of Table I, we attain the following observations on the 15 1000-D CEC2013 LSOPs.

1) As revealed by the average rank, RCI-PSO still gains the lowest rank among all 12 methods. Furthermore, its rank value (2.20) is consistently much smaller than those of the 11 compared optimizers. These findings show that RCI-PSO not only attains the best overall performance on the whole CEC2013 set but also displays significant superiority to the 11 compared methods on such a difficult LSOP suite.
2) As disclosed by "*w/t/l*," RCI-PSO significantly dominates the 11 compared methods on at least 10 LSOPs and only shows inferiority to them on at most 2 LSOPs. This indicates that RCI-PSO is still more effective than the 11 compared methods in tackling such complex CEC2013 LSOPs.
3) Comparison results on different kinds of LSOPs disclose that RCI-PSO still obtains significantly better performance than the 11 compared methods on the complex LSOPs. For example, on the eight partially separable LSOPs (including the three partially separable unimodal LSOPs and the five partially separable multimodal LSOPs), RCI-PSO significantly beats the 11 compared methods down on more than 5 LSOPs and shows no worse performance than them on all 8 LSOPs. On the three overlapping LSOPs (including the two overlapping unimodal LSOPs and the one overlapping multimodal LSOP) and the one fully nonseparable LSOP, RCI-PSO presents significant superiority to all 11 compared methods on all 4 LSOPs. These observations further demonstrate that RCI-PSO is capable of solving complicated LSOPs.

The above comparison results have substantiated the effectiveness of RCI-PSO. To verify its efficiency in solving LSOPs, we conduct experiments to compare the convergence behavior of RCI-PSO with those of the 11 compared methods on the CEC2010 and the CEC2013 LSOP suites. The comparison results are shown in Figs. S2 and S3 in the supplementary material. In this series of experiments, the maximum number of fitness evaluations is set as $5000\times D$.

From Fig. S2 in the supplementary material, we get the following findings on the 1000-D CEC2010 LSOP suite: 1) at the first glimpse, RCI-PSO obtains the best solutions along with the fastest convergence on 11 LSOPs (F_4, F_7–F_9, F_{12}–F_{14}, F_{17}–F_{20}) and 2) RCI-PSO presents much superior performance to 10 compared algorithms on 4 LSOPs (F_1, F_{10}, F_{11}, and F_{16}), and performs much better than nine compared methods on the rest 5 LSOPs (F_2, F_3, F_5, F_6, and F_{15}).

From Fig. S3 in the supplementary material, we discover the following observations on the 1000-D CEC2013 LSOP suite: 1) at the first glance, RCI-PSO achieves significant superiority to the 11 compared methods in terms of convergence speed and solution quality on 10 LSOPs (F_4, F_5, F_7–F_9, and F_{11}–F_{15}) and 2) on the rest 5 LSOPs, RCI-PSO shows much better performance than ten compared methods on 2 LSOPs (F_1 and F_{10}).

In conclusion, the above comparison results between RCI-PSO and the 11 compared large-scale optimization methods have demonstrated that RCI-PSO is much more effective and efficient in coping with LSOPs. Such superiority of RCI-PSO mainly benefits from the devised RCI and the designed adaptive topology size adjustment strategy. In particular, in comparison with the seven holistic large-scale PSO variants (namely, APSODEE, SDLSO, TPLSO, RBLSO, DLLSO, SLPSO, and CSO), the better optimization performance of RCI-PSO demonstrates the great superiority of the RCI interaction scheme over the ones in these PSO variants. This is because RCI helps PSO maintain a better compromise between search diversity and search convergence at the swarm level and the particle level. Furthermore, the devised adaptive topology size adjustment strategy further helps RCI-PSO keep a good balance between exploration of the search space and exploitation of the found optimal regions by gradually switching from updating particles diversely to updating particles intensively. The collaboration between these two techniques makes great contributions to the good performance of RCI-PSO.

TABLE II
OPTIMIZATION PERFORMANCE COMPARISON BETWEEN RCI-PSO AND THE 11 COMPARED LARGE-SCALE ALGORITHMS ON THE 1000-D CEC2010 LSOPS

F	Quality	RCI-PSO	APSODEE	SDLSO	TPLSO	RBLSO	DLLSO	SLPSO	CSO	DECC-GDG	DECC-DG2	DECC-RDG	DECC-RDG2
F_1	Median	8.91E-23	1.54E-19	2.87E-23	1.97E-18	2.52E-20	2.93E-22	7.90E-18	4.40E-12	3.84E-10	3.17E+00	8.43E-02	1.16E-01
	Mean	8.78E-23	1.53E-19	2.88E-23	1.97E-18	2.51E-20	3.13E-22	8.73E-18	4.50E-12	3.94E-10	1.10E+02	7.35E+00	1.52E+01
	Std	1.86E-23	1.96E-20	7.46E-24	0.00E+00	1.23E-21	8.03E-23	3.30E-18	5.94E-13	1.18E-10	4.88E+02	3.29E+01	7.28E+01
	p-value	-	**3.02E-06**$^+$	3.02E-06$^-$	1.82E-01$^=$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_2	Median	8.47E+02	5.06E+02	9.16E+02	1.06E+03	7.29E+02	9.85E+02	1.93E+03	7.33E+03	7.65E+02	4.44E+03	4.41E+03	4.46E+03
	Mean	8.53E+02	5.07E+02	9.22E+02	1.05E+03	7.32E+02	9.82E+02	1.93E+03	7.42E+03	7.68E+02	4.43E+03	4.45E+03	4.47E+03
	Std	2.93E+01	1.29E+01	3.90E+01	6.48E+01	3.85E+01	4.39E+01	1.12E+02	2.86E+02	2.91E+01	1.45E+02	1.95E+02	1.57E+02
	p-value	-	2.98E-06$^-$	**5.22E-05**$^+$	**4.95E-07**$^+$	4.05E-06$^-$	**3.77E-06**$^+$	**3.01E-06**$^+$	**3.01E-06**$^+$	7.37E-06$^-$	**3.01E-06**$^+$	**3.01E-06**$^+$	**3.01E-06**$^+$
F_3	Median	2.89E-14	1.21E-13	2.53E-14	1.40E+00	1.21E-13	2.89E-14	1.85E+00	2.53E-09	1.66E+01	1.67E+01	1.68E+01	1.67E+01
	Mean	2.89E-14	1.25E-13	2.57E-14	1.41E+00	1.24E-13	2.76E-14	1.88E+00	2.60E-09	1.66E+01	1.66E+01	1.68E+01	1.67E+01
	Std	0.00E+00	7.78E-15	1.43E-15	1.25E-01	2.97E-15	2.38E-15	3.30E-01	2.62E-10	2.82E-01	3.26E-01	2.92E-01	3.46E-01
	p-value	-	**1.97E-06**$^+$	1.68E-06$^-$	**2.52E-06**$^+$	**9.86E-07**$^+$	7.67E-02$^=$	**2.52E-06**$^+$	**2.52E-06**$^+$	**2.52E-06**$^+$	**2.52E-06**$^+$	**2.52E-06**$^+$	**2.52E-06**$^+$
F_4	Median	4.51E+10	1.82E+11	1.33E+11	1.85E+11	7.68E+11	4.37E+11	3.04E+11	7.26E+11	1.79E+14	2.22E+12	6.05E+11	5.90E+11
	Mean	4.66E+10	1.80E+11	1.36E+11	1.96E+11	7.72E+11	4.40E+11	2.99E+11	7.25E+11	1.82E+14	2.27E+12	7.13E+11	6.76E+11
	Std	1.02E+10	2.59E+10	2.17E+10	7.54E+10	1.64E+11	1.10E+11	7.16E+10	1.23E+11	3.40E+13	7.14E+11	3.21E+11	3.03E+11
	p-value	-	**3.02E-06**$^+$	**3.02E-06**$^+$	**7.70E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_5	Median	7.46E+06	5.97E+06	6.97E+06	1.69E+07	2.74E+08	1.19E+07	3.29E+07	2.00E+06	3.57E+08	1.41E+08	1.36E+08	1.31E+08
	Mean	7.76E+06	5.67E+06	7.50E+06	1.64E+07	2.39E+08	1.22E+07	3.17E+07	2.86E+06	3.58E+08	1.42E+08	1.33E+08	1.32E+08
	Std	3.03E+06	1.55E+06	2.79E+06	3.26E+06	8.98E+07	3.43E+06	6.21E+06	1.79E+06	1.71E+07	2.21E+07	1.99E+07	1.87E+07
	p-value	-	2.35E-02$^-$	5.22E-01$^=$	**9.88E-07**$^+$	**3.02E-06**$^+$	**7.82E-04**$^+$	**3.02E-06**$^+$	2.14E-04$^-$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_6	Median	4.00E-09	4.05E-09	4.00E-09	2.27E+00	1.30E-08	4.00E-09	2.15E+01	8.23E-07	4.79E+03	1.54E+01	1.63E+01	1.63E+01
	Mean	4.00E-09	4.06E-09	4.00E-09	2.21E+00	1.29E-08	5.20E-01	2.08E+01	8.21E-07	4.78E+03	1.54E+01	1.63E+01	1.63E+01
	Std	4.83E-15	1.04E-11	8.41E-25	3.61E-01	1.03E-09	7.46E-01	2.63E+00	2.68E-08	1.21E+03	4.23E-01	3.42E-01	3.50E-01
	p-value	-	**2.83E-06**$^+$	2.32E-03$^-$	**4.89E-07**$^+$	**2.83E-06**$^+$	2.04E-01$^=$	**2.83E-06**$^+$	**2.83E-06**$^+$	**2.83E-06**$^+$	**2.83E-06**$^+$	**2.83E-06**$^+$	**2.83E-06**$^+$
F_7	Median	7.22E-13	9.77E-01	5.93E-02	1.09E+02	6.17E+03	1.22E+01	4.06E+04	2.04E+04	2.92E+10	4.09E+03	8.57E+00	5.92E+00
	Mean	8.11E-13	1.06E+00	5.54E-01	4.04E+03	6.73E+03	7.19E+02	6.49E+04	2.01E+04	3.00E+10	5.18E+03	1.13E+02	1.53E+02
	Std	4.73E-13	6.13E-01	2.13E+00	7.84E+03	2.85E+03	2.59E+03	5.60E+04	3.86E+03	5.49E+09	3.46E+03	2.78E+02	7.39E+02
	p-value	-	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_8	Median	5.44E+03	5.61E+06	2.38E+04	9.51E+04	3.58E+07	2.34E+07	7.43E+06	3.87E+07	1.20E+10	1.17E+07	2.77E+01	1.27E+01
	Mean	4.84E+03	5.79E+06	2.33E+04	2.59E+05	3.58E+07	2.34E+07	7.81E+06	3.87E+07	1.70E+10	1.48E+07	3.99E+05	6.65E+05
	Std	1.97E+03	8.45E+05	2.26E+03	2.55E+05	7.96E+04	2.46E+05	1.56E+06	6.81E+04	1.11E+10	1.62E+07	1.22E+06	1.51E+06
	p-value	-	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.96E-04**$^+$	**1.88E-03**$^+$
F_9	Median	7.45E+06	2.36E+07	2.22E+07	3.38E+07	4.96E+07	4.33E+07	3.22E+07	7.05E+07	3.90E+08	7.15E+07	4.67E+07	4.70E+07
	Mean	7.58E+06	2.35E+07	2.22E+07	3.48E+07	4.95E+07	4.36E+07	3.30E+07	7.03E+07	3.89E+08	7.22E+07	4.70E+07	4.69E+07
	Std	5.08E+05	1.40E+06	2.43E+06	8.54E+06	3.23E+06	4.28E+06	4.46E+06	5.73E+06	3.09E+07	7.05E+06	4.49E+06	4.88E+06
	p-value	-	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_{10}	Median	8.68E+02	5.07E+02	8.13E+02	9.92E+02	1.06E+03	8.89E+02	2.60E+03	9.59E+03	3.70E+03	5.34E+03	4.32E+03	4.31E+03
	Mean	8.77E+02	5.10E+02	8.14E+02	9.85E+02	1.06E+03	8.91E+02	2.56E+03	9.60E+03	3.69E+03	5.32E+03	4.30E+03	4.33E+03
	Std	3.39E+01	2.48E+01	3.57E+01	7.82E+01	5.02E+01	3.66E+01	2.17E+02	7.67E+01	7.03E+01	1.44E+02	1.57E+02	9.84E+01
	p-value	-	3.01E-06$^-$	7.27E-05$^-$	**5.80E-05**$^+$	**3.01E-06**$^+$	1.79E-01$^=$	**3.01E-06**$^+$	**3.01E-06**$^+$	**3.01E-06**$^+$	**3.01E-06**$^+$	**3.01E-06**$^+$	**3.01E-06**$^+$
F_{11}	Median	1.61E-13	3.43E-12	1.43E-13	3.79E+00	7.42E-13	2.75E+00	2.30E+01	3.80E-08	1.09E+01	1.08E+01	1.02E+01	1.02E+01
	Mean	1.64E-13	3.45E-12	1.44E-13	3.99E+00	7.44E-13	5.80E+00	2.32E+01	4.02E-08	1.08E+01	1.11E+01	1.02E+01	1.02E+01
	Std	8.00E-15	3.90E-13	4.11E-15	1.32E+00	1.75E-14	5.40E+00	2.10E+00	5.12E-09	7.93E-01	9.62E-01	9.15E-01	7.13E-01
	p-value	-	**3.00E-06**$^+$	1.84E-06$^-$	**4.95E-07**$^+$	**2.89E-06**$^+$	**3.00E-06**$^+$	**3.00E-06**$^+$	**3.00E-06**$^+$	**3.00E-06**$^+$	**3.00E-06**$^+$	**3.00E-06**$^+$	**3.00E-06**$^+$
F_{12}	Median	7.19E+01	1.94E+04	5.80E+03	5.66E+03	3.16E+04	1.24E+04	1.31E+04	4.23E+05	4.04E+05	1.21E+05	1.67E+03	1.49E+03
	Mean	7.46E+01	1.96E+04	5.91E+03	6.46E+03	3.15E+04	1.25E+04	1.75E+04	4.37E+05	4.02E+05	1.19E+05	1.99E+03	1.64E+03
	Std	1.72E+01	1.32E+03	4.09E+02	4.94E+03	2.18E+03	1.46E+03	9.07E+03	6.22E+04	1.66E+04	8.84E+03	9.32E+02	3.90E+02
	p-value	-	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_{13}	Median	2.87E+02	4.49E+02	4.26E+02	6.55E+02	7.34E+02	7.28E+02	8.48E+02	5.47E+02	8.75E+02	1.09E+03	6.92E+02	6.51E+02
	Mean	2.97E+02	4.72E+02	4.87E+02	6.67E+02	8.53E+02	7.35E+02	9.59E+02	6.29E+02	9.23E+02	1.34E+03	8.27E+02	8.06E+02
	Std	6.87E+01	7.30E+01	1.17E+02	1.44E+02	3.87E+02	1.93E+02	3.74E+02	2.32E+02	1.78E+02	7.64E+02	5.02E+02	4.69E+02
	p-value	-	**2.66E-05**$^+$	**3.05E-05**$^+$	**6.48E-07**$^+$	**3.02E-06**$^+$	**4.08E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.74E-06**$^+$	**4.08E-06**$^+$
F_{14}	Median	2.46E+07	7.14E+07	6.61E+07	9.67E+07	1.34E+08	1.25E+08	8.45E+07	2.52E+08	5.95E+08	5.99E+08	3.48E+08	3.51E+08
	Mean	2.48E+07	7.16E+07	6.61E+07	9.93E+07	1.34E+08	1.24E+08	8.41E+07	2.49E+08	5.95E+08	6.06E+08	3.48E+08	3.52E+08
	Std	1.24E+06	2.95E+06	3.26E+06	2.38E+07	4.61E+06	7.38E+06	6.31E+06	1.53E+07	2.98E+07	4.00E+07	2.42E+07	2.04E+07
	p-value	-	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_{15}	Median	8.88E+02	5.50E+02	6.11E+02	1.01E+04	1.04E+04	8.40E+02	1.12E+04	1.01E+04	6.70E+03	6.72E+03	5.84E+03	5.90E+03
	Mean	8.90E+02	5.58E+02	1.57E+03	7.76E+03	1.04E+04	8.33E+02	1.12E+04	1.01E+04	6.70E+03	6.73E+03	5.84E+03	5.89E+03
	Std	2.79E+01	2.70E+01	2.70E+03	3.93E+03	8.46E+01	4.31E+01	8.65E+01	5.23E+01	9.50E+01	9.57E+01	7.93E+01	9.23E+01
	p-value	-	3.02E-06$^-$	**6.25E-04**$^+$	**4.97E-07**$^+$	**3.02E-06**$^+$	5.57E-04$^-$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_{16}	Median	1.90E-13	2.99E-12	1.90E-13	1.70E+01	1.04E-12	3.70E+00	2.21E+01	5.75E-08	1.11E-02	1.09E-02	4.17E-13	4.05E-13
	Mean	1.90E-13	3.04E-12	1.92E-13	1.74E+01	2.93E-02	4.25E+00	2.51E+01	5.89E-08	1.13E-02	1.08E-02	4.17E-13	4.08E-13
	Std	5.67E-15	3.14E-13	6.78E-15	6.20E+00	1.60E-01	2.41E+00	1.16E+01	5.61E-09	8.31E-04	8.26E-04	2.70E-14	2.43E-14
	p-value	-	**2.98E-06**$^+$	3.45E-01$^=$	**4.94E-07**$^+$	**2.84E-06**$^+$	**2.98E-06**$^+$	**2.98E-06**$^+$	**2.98E-06**$^+$	**2.98E-06**$^+$	**2.98E-06**$^+$	**2.92E-06**$^+$	**2.84E-06**$^+$
F_{17}	Median	4.42E+03	1.80E+05	9.37E+04	6.53E+04	1.92E+05	9.02E+04	8.92E+04	2.22E+06	7.17E+04	7.28E+04	3.95E+04	4.09E+04
	Mean	4.42E+03	1.80E+05	9.32E+04	6.48E+04	1.93E+05	9.05E+04	9.00E+04	2.20E+06	7.26E+04	7.29E+04	4.00E+04	4.11E+04
	Std	7.77E+02	6.91E+03	4.87E+03	3.69E+04	6.77E+03	3.53E+03	1.58E+04	1.55E+05	4.43E+03	3.89E+03	2.55E+03	2.25E+03
	p-value	-	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_{18}	Median	8.51E+02	1.05E+03	1.27E+03	2.15E+03	1.93E+03	2.29E+03	2.66E+03	1.76E+03	1.36E+03	1.41E+03	1.15E+03	1.21E+03
	Mean	8.32E+02	1.06E+03	1.34E+03	2.18E+03	2.06E+03	2.55E+03	2.77E+03	1.73E+03	1.35E+03	1.41E+03	1.15E+03	1.21E+03
	Std	8.50E+01	1.28E+02	3.16E+02	4.73E+02	5.10E+02	8.32E+02	8.33E+02	5.22E+02	1.46E+02	1.68E+02	1.37E+02	1.52E+02
	p-value	-	**1.75E-05**$^+$	**3.02E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.74E-06**$^+$	**7.39E-06**$^+$
F_{19}	Median	8.82E+05	2.54E+06	4.92E+06	3.40E+06	7.77E+06	1.79E+06	5.11E+06	1.00E+07	2.33E+06	2.35E+06	1.76E+06	1.69E+06
	Mean	8.84E+05	2.54E+06	4.90E+06	3.51E+06	7.61E+06	1.80E+06	5.10E+06	1.01E+07	2.32E+06	2.35E+06	1.77E+06	1.72E+06
	Std	2.07E+04	1.18E+05	2.92E+05	5.08E+05	3.96E+05	9.96E+04	7.05E+05	5.64E+05	1.35E+05	1.07E+05	7.35E+04	1.11E+05
	p-value	-	**3.02E-06**$^+$	**3.02E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
F_{20}	Median	8.90E+02	9.92E+02	1.19E+03	1.85E+03	1.60E+03	1.85E+03	1.79E+03	9.85E+02	3.88E+07	5.34E+07	4.31E+03	4.18E+03
	Mean	9.15E+02	1.01E+03	1.22E+03	1.90E+03	1.64E+03	1.88E+03	1.85E+03	1.05E+03	1.96E+08	1.94E+08	6.58E+04	5.05E+03
	Std	5.77E+01	5.04E+01	1.26E+02	2.03E+02	3.19E+02	1.90E+02	2.59E+02	1.49E+02	3.50E+08	3.09E+08	2.62E+05	2.75E+03
	p-value	-	**1.01E-04**$^+$	**4.74E-06**$^+$	**4.99E-07**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**1.01E-04**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$	**3.02E-06**$^+$
w/t/l		-	16/0/4	13/2/5	19/1/0	19/0/1	16/3/1	20/0/0	19/0/1	19/0/1	20/0/0	20/0/0	20/0/0
rank		**1.75**	3.80	3.20	6.40	7.65	6.00	8.40	8.20	9.30	9.40	7.00	6.90

*The p-value of the Friedman test is 1.27E-16.

C. Comparison With CEC Winners and Other State-of-the-Art Large-Scale Methods

1) Comparison With CEC Winners and State-of-the-Art Large-Scale Methods on the CEC2010 Benchmark Suite: This section further validates the effectiveness of RCI-PSO by comparing it with the winners in the competitions on large-scale optimization held in IEEE CEC and the state-of-the-art large-scale methods in [63], which were all tested on the CEC2010 benchmark set. Especially, totally 19 state-of-the-art large-scale methods are utilized here to compare with RCI-PSO. They are BICA [64], MLSHADE-SPA [63], SEE [65], MMO-CC [66], EADE [67], ANDE [68],

TABLE III
OPTIMIZATION PERFORMANCE COMPARISON BETWEEN RCI-PSO AND THE 11 COMPARED LARGE-SCALE ALGORITHMS ON THE 1000-D CEC2013 LSOPS

F	Quality	RCI-PSO	APSODEE	SDLSO	TPLSO	RBLSO	DLLSO	SLPSO	CSO	DECC-GDG	DECC-DG2	DECC-RDG	DECC-RDG2
F_1	Median	1.07E-22	2.42E-19	4.71E-23	2.93E-18	2.88E-20	3.85E-22	1.04E-17	7.91E-12	5.04E-10	1.65E+01	1.97E-01	5.72E-01
	Mean	1.03E-22	2.55E-19	4.69E-23	3.08E-18	2.88E-20	3.92E-22	5.80E+02	7.88E-12	5.12E-10	9.61E+04	5.17E+00	5.50E+01
	Std	2.08E-23	6.26E-20	1.44E-23	1.26E-18	1.54E-21	9.07E-23	1.83E+03	1.21E-12	1.12E-10	5.25E+05	2.13E+01	1.96E+02
	p-value	-	**3.02E-11**$^+$	1.33E-10$^-$	**3.01E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**2.99E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_2	Median	8.03E+02	5.62E+02	9.95E+02	1.18E+03	7.48E+02	1.16E+03	2.09E+03	8.57E+03	4.56E+02	1.28E+04	1.26E+04	1.26E+04
	Mean	8.04E+02	5.68E+02	9.94E+02	1.20E+03	7.49E+02	1.15E+03	2.10E+03	8.58E+03	4.60E+02	1.28E+04	1.27E+04	1.26E+04
	Std	3.58E+01	2.31E+01	4.05E+01	1.26E+02	2.51E+01	7.27E+01	2.74E+01	1.79E+02	2.13E+01	5.99E+02	6.37E+02	5.42E+02
	p-value	-	3.00E-11$^-$	**3.01E-11**$^+$	**2.98E-09**$^+$	1.15E-07$^-$	**3.02E-11**$^+$	**2.65E-06**$^+$	**3.02E-11**$^+$	3.02E-11$^-$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_3	Median	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.14E+01	2.14E+01	2.14E+01	2.14E+01
	Mean	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.16E+01	2.14E+01	2.14E+01	2.14E+01	2.14E+01
	Std	7.87E-03	6.73E-03	4.87E-03	6.36E-03	5.86E-03	7.20E-03	1.58E-02	5.99E-03	1.51E-02	1.63E-02	1.77E-02	1.75E-02
	p-value	-	6.41E-01$^=$	7.23E-01$^=$	1.84E-01$^=$	6.79E-01$^=$	7.17E-01$^=$	2.06E-01$^=$	6.31E-01$^=$	3.02E-11$^-$	3.02E-11$^-$	3.02E-11$^-$	3.02E-11$^-$
F_4	Median	4.29E+08	3.20E+09	3.02E+09	3.23E+09	1.06E+10	6.05E+09	4.53E+09	1.22E+10	2.47E+11	5.53E+10	5.08E+10	4.70E+10
	Mean	4.35E+08	3.11E+09	3.02E+09	3.45E+09	1.06E+10	5.94E+09	6.28E+09	1.35E+10	2.50E+11	6.06E+10	4.95E+10	4.80E+10
	Std	6.63E+07	4.26E+08	6.28E+08	1.23E+09	1.58E+09	1.47E+09	5.08E+09	3.17E+09	6.89E+10	2.31E+10	1.57E+10	1.34E+10
	p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.01E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_5	Median	5.66E+05	6.31E+05	6.50E+05	6.39E+05	8.88E+05	6.54E+05	1.09E+06	5.90E+05	7.83E+06	5.19E+06	5.12E+06	5.19E+06
	Mean	5.70E+05	6.37E+05	6.71E+05	6.42E+05	8.87E+05	6.61E+05	1.09E+06	5.97E+05	7.83E+06	5.24E+06	5.13E+06	5.21E+06
	Std	8.39E+04	1.02E+05	1.28E+05	8.65E+04	1.48E+05	1.11E+05	4.01E+02	1.04E+05	4.31E+05	4.21E+05	4.50E+05	4.34E+05
	p-value	-	**1.63E-02**$^+$	**1.11E-03**$^+$	**7.28E-03**$^+$	**3.16E-10**$^+$	**2.50E-03**$^+$	**2.76E-06**$^+$	1.58E-01$^=$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_6	Median	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06
	Mean	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06	1.06E+06
	Std	9.59E+02	1.08E+03	1.23E+03	1.17E+03	9.52E+02	9.38E+02	5.24E+02	1.20E+03	1.50E+03	1.50E+03	1.20E+03	1.61E+03
	p-value	-	1.69E-01$^=$	4.51E-01$^=$	5.59E-01$^=$	1.67E-01$^=$	7.36E-02$^=$	2.05E-01$^=$	1.01E-01$^=$	**1.56E-08**$^+$	7.98E-02$^=$	**1.09E-05**$^+$	1.54E-01$^=$
F_7	Median	2.37E+04	5.36E+05	2.70E+05	7.50E+05	7.24E+06	1.54E+06	2.07E+06	5.45E+06	3.93E+08	4.43E+07	8.89E+07	5.40E+07
	Mean	2.59E+04	5.46E+05	3.47E+05	8.66E+05	8.02E+06	2.45E+06	1.32E+07	5.81E+06	4.03E+08	4.42E+07	9.82E+07	5.69E+07
	Std	1.46E+04	1.66E+05	3.05E+05	5.02E+05	3.17E+06	3.62E+06	2.56E+07	3.09E+06	9.76E+07	1.54E+07	4.17E+07	2.05E+07
	p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.01E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_8	Median	1.73E+13	5.33E+13	4.58E+13	5.14E+13	2.84E+14	1.16E+14	8.80E+13	2.42E+14	9.13E+15	5.73E+15	4.96E+15	3.98E+15
	Mean	1.91E+13	5.39E+13	4.91E+13	5.69E+13	2.64E+14	1.28E+14	1.83E+14	2.46E+14	9.51E+15	5.68E+15	4.83E+15	4.31E+15
	Std	6.16E+12	1.16E+13	1.57E+13	2.30E+13	6.54E+13	4.70E+13	2.09E+14	8.86E+13	3.21E+15	1.99E+15	1.71E+15	1.61E+15
	p-value	-	**3.02E-11**$^+$	**3.82E-10**$^+$	**1.76E-08**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_9	Median	3.17E+07	3.37E+07	1.07E+08	4.51E+07	4.70E+07	1.05E+08	6.57E+07	5.94E+07	5.15E+08	5.21E+08	4.87E+08	4.93E+08
	Mean	3.18E+07	3.41E+07	1.11E+08	4.29E+07	4.73E+07	1.17E+08	6.76E+07	6.08E+07	5.09E+08	5.16E+08	4.88E+08	4.87E+08
	Std	4.34E+06	6.45E+06	3.08E+07	8.32E+06	8.15E+06	4.22E+07	5.56E+06	1.31E+07	3.16E+07	2.97E+07	2.45E+07	3.28E+07
	p-value	-	2.34E-01$^=$	**3.02E-11**$^+$	**2.80E-05**$^+$	**1.17E-09**$^+$	**3.02E-11**$^+$	**3.02E-06**$^+$	**3.69E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_{10}	Median	9.40E+07	9.40E+07	9.40E+07	9.42E+07	9.40E+07	9.41E+07	9.40E+07	9.41E+07	9.46E+07	9.46E+07	9.46E+07	9.45E+07
	Mean	9.39E+07	9.40E+07	9.40E+07	9.42E+07	9.40E+07	9.40E+07	9.40E+07	9.40E+07	9.46E+07	9.45E+07	9.45E+07	9.45E+07
	Std	2.62E+05	2.15E+05	1.89E+05	2.52E+05	2.39E+05	2.28E+05	1.03E+05	2.25E+05	2.16E+05	1.95E+05	2.89E+05	2.45E+05
	p-value	-	7.06E-01$^=$	1.49E-01$^=$	**4.48E-03**$^+$	4.38E-01$^=$	3.63E-01$^=$	6.49E-01$^=$	8.50E-02$^=$	**1.61E-10**$^+$	**4.20E-10**$^+$	**9.26E-09**$^+$	**8.89E-10**$^+$
F_{11}	Median	5.45E+06	1.02E+08	9.22E+11	1.02E+08	3.38E+08	9.25E+11	9.20E+11	9.25E+11	6.50E+08	3.35E+09	5.53E+08	1.78E+09
	Mean	6.51E+06	1.02E+08	9.28E+11	1.17E+08	3.47E+08	9.29E+11	9.20E+11	9.29E+11	6.68E+08	4.95E+09	6.67E+08	4.17E+09
	Std	3.14E+06	3.00E+07	1.59E+10	7.52E+07	9.35E+07	9.30E+09	5.79E+07	9.80E+09	1.63E+08	4.82E+09	5.36E+08	6.33E+09
	p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.01E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**2.95E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_{12}	Median	9.51E+02	1.05E+03	1.25E+03	1.96E+03	1.50E+03	1.78E+03	2.00E+03	1.04E+03	6.22E+03	6.24E+03	3.97E+03	4.05E+03
	Mean	9.49E+02	1.06E+03	1.27E+03	1.96E+03	1.48E+03	1.79E+03	2.26E+03	1.08E+03	2.73E+04	7.92E+03	4.83E+03	7.62E+03
	Std	4.55E+01	4.55E+01	1.15E+02	1.80E+02	1.34E+02	1.18E+02	7.14E+02	7.51E+01	8.84E+04	4.20E+03	3.30E+03	1.86E+04
	p-value	-	**2.44E-09**$^+$	**3.34E-11**$^+$	**3.01E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-06**$^+$	**1.69E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_{13}	Median	2.88E+06	1.08E+08	2.55E+08	6.69E+07	6.80E+08	3.66E+08	2.89E+09	7.08E+08	1.50E+09	1.27E+09	2.92E+09	6.78E+08
	Mean	2.86E+06	1.12E+08	2.65E+08	1.09E+08	6.53E+08	3.80E+08	3.43E+09	7.48E+08	1.58E+09	1.30E+09	3.13E+09	6.93E+08
	Std	6.85E+05	3.33E+07	1.21E+08	1.21E+08	1.29E+08	1.59E+08	3.97E+09	2.89E+08	4.48E+08	3.16E+08	8.61E+08	2.00E+08
	p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.01E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_{14}	Median	1.14E+07	5.18E+07	5.20E+07	3.77E+07	1.51E+09	8.99E+07	1.43E+08	2.90E+09	5.82E+09	6.02E+09	2.74E+09	3.05E+09
	Mean	1.16E+07	5.63E+07	6.91E+07	4.43E+07	2.19E+09	1.21E+08	1.39E+09	3.67E+09	6.59E+09	5.72E+09	3.40E+09	3.06E+09
	Std	1.93E+06	2.57E+07	4.84E+07	1.79E+07	1.68E+09	8.75E+07	2.61E+09	3.38E+09	4.05E+09	2.74E+09	2.23E+09	1.51E+09
	p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.01E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_{15}	Median	2.25E+06	6.12E+06	1.21E+07	1.03E+07	2.49E+07	4.40E+06	6.30E+07	7.60E+07	1.13E+07	1.11E+07	9.48E+06	9.32E+06
	Mean	2.27E+06	6.20E+06	1.20E+07	1.06E+07	2.49E+07	4.47E+06	6.70E+07	7.61E+07	1.19E+07	1.14E+07	1.03E+07	9.59E+06
	Std	1.86E+05	5.21E+05	8.00E+05	2.18E+06	2.57E+06	4.12E+05	1.09E+07	6.24E+06	3.01E+06	2.29E+06	2.18E+06	1.66E+06
	p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.01E-09**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.01E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
w/t/l		-	10/4/1	11/3/1	13/2/0	11/3/1	12/3/0	12/3/0	11/4/0	13/0/2	13/1/1	14/0/1	13/1/1
rank		**2.20**	3.47	5.33	4.80	5.73	5.47	7.87	6.80	9.53	9.80	8.87	8.13

*The p-value of the Friedman test is 9.30E-23.

DISCC [69], DM-HDMR [70], HACC-D [71], MOS2012 [72], LMDEa [73], jDEsps [74], CCGS [75], SDENS [76], DECC-DML [77], MA-SW-chains [78], DASA [79], EOEA [80], and jDElsgo [81]. Among these methods, MA-SW-chains is the first winner in IEEE CEC2010; MOS2012, jDElsgo, and CCGS are the first, the second, and the third winners in IEEE CEC2012; MLSHADE-SPA and BICA are the second and the fourth winners in IEEE CEC2018. The full names of these large-scale optimizers are shown in Table SIII in the supplementary material for saving space.

For fair comparisons, we directly borrow the optimization results of the 19 compared methods from [63]. Table SV in the supplementary material shows the comparison results. From this table, we attain the following observations on the 20 1000-D CEC2010 LSOPs.

1) In view of the Friedman test results, RCI-PSO achieves the lowest rank among the 20 methods. Furthermore, the rank value of RCI-PSO is much smaller than those of the 19 compared methods. These findings verify that RCI-PSO attains the best overall performance and also shows significant superiority to the 19 compared methods on the whole CEC2010 benchmark suite.
2) In comparison with the six winners (namely, MA-SW-chains, MOS2012, jDElsgo, CCGS, MLSHADE-SPA, and BICA), RCI-PSO performs much better. In addition, RCI-PSO achieves the best results on 4 LSOPs, while MOS2012, MLSHADE-SPA and BICA attain the best results on 4, 3, and 2 LSOPs, respectively. As for MA-SW-chains, jDElsgo, and CCGS, they achieve the best results on no problems.

To summarize, the above comprehensive comparison results further demonstrate that RCI-PSO is particularly effective in solving the CEC2010 LSOPs.

2) Comparison With CEC Winners and State-of-the-Art Large-Scale Methods on the CEC2013 Benchmark Suite: In this section, we compare RCI-PSO with the winners in the competitions on large-scale optimization held in IEEE CEC and the state-of-the-art large-scale methods in [63], which were all tested on the CEC2013 benchmark set. In particular, totally 12 state-of-the-art large-scale methods are adopted. They are BICA [64], MLSHADE-SPA [63], SHADE-ILS [82], CCFR-IDG2 [83], CCFR-I [83], CRO [84], BCC3-DG2 [85], IHDELS [86], MOS2013 [87], VGDE [88], DECC-CG [89], and SACC [90]. Among these methods, SHADE-ILS, MLSHADE-SPA, MOS2013, and BICA are the first, the second, the third, and the fourth winners in IEEE CEC2018; besides, MOS2013 is also the first winner in both IEEE CEC2013 and IEEE CEC2015. IHDELS is the second winner in IEEE CEC2015. The full names of these large-scale optimizers are shown in Table SIII in the supplementary material.

Likewise, for fairness, we directly borrow the optimization results of the 12 compared methods from [63]. Table SVI in the supplementary material shows the detailed comparison results. From this table, we attain the following observations on the fifteen1000-D CEC2013 LSOPs.

1) From the perspective of the Friedman test results, the rank value of RCI-PSO is much larger than those of SHADE-ILS, MOS2013, and MLSHADE-SPA, slightly smaller than BICA, but is much lower than those of the other eight compared methods. This indicates that RCI-PSO achieves competitive performance with BICA, worse performance than SHADE-ILS, MOS2013, and MLSHADE-SPA, but significantly better performance than the other eight compared methods.
2) In comparison with the winners, RCI-PSO significantly outperforms the second winner in IEEE CEC2015, namely, IHDELS, performs competitively with the fourth winner in CEC2018, namely, BICA, but attains much worse performance than the top three winners in IEEE CEC2018, namely SHADE-ILS, MLSHADE-SPA, and MOS2013. However, compared with the three winners, RCI-PSO has a better usability because on the one hand, fewer parameters exist in RCI-PSO; on the other hand, there are no any trivial techniques in RCI-PSO. Therefore, compared with the three winners, RCI-PSO is easier to implement and thus is more convenient to be applied to solve real-world problems.

In summary, the above comprehensive comparison results on the CEC2013 suite further demonstrate that RCI-PSO is particularly promising for solving LSOPs.

D. Ablation Study

This section mainly conducts experiments to verify the contributions of the devised RCI and the adaptive topology size adjustment strategy to the good performance of RCI-PSO.

1) Effectiveness of the Devised RCI Scheme: First, we conduct experiments to testify the usefulness of the devised RCI. To this end, we first develop several different versions of RCI-PSO. In particular, in RCI, each particle is triggered to update only when there are at least two dominators in the associated topology. Besides, for each updated particle, RCI utilizes the best dominator and the worst dominator to direct its evolution. To verify the effectiveness of the above working principles, we first develop a new version of RCI-PSO, where RCI utilizes two random particles selected from the associated topology to direct the evolution of each particle. We name it as "RCI-PSO-2R." Then, we develop another new version of RCI-PSO, where RCI employs two random dominators in the associated topology to direct the evolution of each particle. We name it as "RCI-PSO-2RD."

In addition, in RCI, particles interact with each other based on their current evolutionary information. To verify this, we let particles in RCI-PSO interact with each other based on their historically personal best positions (*p*best*s*) and then develop three other versions of RCI-PSO, namely, "RCI-PSO-2CP," "RCI-PSO-2RP," and "RCI-PSO-2RDP." Specifically, in "RCI-PSO-2CP," in the random topology of each particle ($\boldsymbol{x}_i$), only when at least two dominant *p*best*s* to $p\text{best}_i$ exist, this particle is updated; otherwise, it is not updated. Besides, each updated particle is directed by the best and the worst *p*best*s* in its topology. In "RCI-PSO-2RP," we randomly select two *p*best*s* in the associated topology to direct the evolution of each particle. Instead of using the best and the worst *p*best*s* in "RCI-PSO-2CP," we randomly select two dominant *p*best*s* to $p\text{best}_i$ to update each particle $\boldsymbol{x}_i$ in "RCI-PSO-2RDP."

After the above preparation, we conduct experiments on the 1000-D CEC2010 LSOPs to compare RCI-PSO with the five developed versions of RCI-PSO. Table SVII in the supplementary document shows the comparison results. From this table, we discover the following findings.

1) No matter from the perspective of the average rank or from the view of the number of LSOPs where an algorithm achieves the best performance, RCI-PSO obtains the best results. This substantiates the great effectiveness of RCI.
2) Comparison results between RCI-PSO and RCI-PSO-2RD and RCI-PSO-2R show that RCI-PSO performs much better. This verifies the effectiveness of the two working principles in RCI, namely, one particle is updated only when at least two dominators exist in its topology and each updated particle follows the guidance of the best and the worst dominators.
3) Comparison results between RCI-PSO and "RCI-PSO-2CP," "RCI-PSO-2RP," and "RCI-PSO-2RDP" demonstrate that RCI-PSO is much better. This validates the superiority of exchanging the current evolutionary information of particles over exchanging their historical evolutionary information.

In conclusion, by means of the above experiments, the effectiveness of the devised RCI scheme is well verified.

2) Effectiveness of the Adaptive Topology Adjustment: This section conducts experiments to testify the influence of the designed adaptive topology size adjustment strategy on the 1000-D CEC2010 LSOP set. To make comparisons, we first develop different RCI-PSO with eight different fixed settings

TABLE IV
SETTINGS OF THE POPULATION SIZE FOR ALL ALGORITHMS TO TACKLE THE CEC2010 LSOPS WITH DIFFERENT DIMENSION SIZES

Alg. / *D*	*NP*											
	RCI-PSO	APSODEE	SDLSO	TPLSO	RBLSO	DLLSO	SLPSO	CSO	DECC-GDG	DECC-DG2	DECC-RDG	DECC-RDG2
D=200	400	400	300	300	200	300	100	250	50	50	50	50
D=500	600	600	300	400	300	300	100	250	50	50	50	50
D=800	800	800	400	500	500	500	100	500	50	50	50	50
D=1000	900	1000	400	600	500	500	100	500	50	50	50	50
D=2000	1100	1400	600	800	800	1000	100	1000	50	50	50	50

of TS, namely, TS = 5, TS = 10, TS = 15, TS = 20, TS = 25, TS = 30, TS = 35, and TS = 40. Then, we compare the RCI-PSO with the adaptive strategy and those with the fixed settings. Table SVIII in the supplementary document lists the comparison results among these versions of RCI-PSO. From this table, we dig out the following findings.

1) RCI-PSO with the adaptive strategy achieves the lowest average rank among all versions. Besides, its rank value is much smaller than the ones of RCI-PSO with different fixed settings of TS. This demonstrates that the adaptive scheme assists RCI-PSO to obtain the best overall optimization performance on the CEC2010 suite, and it also presents great superiority to the fixed settings of TS.
2) RCI-PSO with the adaptive scheme performs the best on 11 LSOPs. On the other 9 LSOPs, the optimization results of RCI-PSO with the adaptive TS scheme are very close to those of RCI-PSO with the associated optimal settings of TS. This further demonstrates that the adaptive method is very effective for RCI-PSO to achieve satisfactory performance.

All in all, by virtue of the above comparisons, it is basically confirmed that the devised adaptive TS adjustment strategy is very helpful for RCI-PSO to effectively cope with LSOPs.

3) Sensitivity Analysis of RCI-PSO to TS^{max}: This section conducts investigation experiments to observe the influence of TS^{max} in (6) on the optimization performance of RCI-PSO. To this end, we first set TS^{max} with different values, namely, 5, 10, 15, 20, 25, 30, 35, and 40. Then, we compare RCI-PSO with the above settings of TS^{max} on the 1000-D CEC2010 benchmark set. Table SIX in the supplementary material shows the comparison results.

From Table SIX in the supplementary material, we find that $TS^{max} = 25$ assists RCI-PSO to achieve the lowest rank among all TS^{max} settings. This indicates that $TS^{max} = 25$ is the most suitable setting for RCI-PSO to solve LSOPs. In addition, we also find that neither a too small TS^{max} nor a too large TS^{max} is suitable for RCI-PSO. This is because a too small TS^{max} leads to that TS starting from two changes to a small maximum value, which limits the interaction range of particles and thus may slow down the converge of the swarm. In contrast, a too large TS^{max} results in that TS changes to a too large value in the late stage, which may lead to serious loss of diversity and thus premature convergence of the swarm.

In conclusion, based on the above experiment, we utilize $TS^{max} = 25$ as the default setting for RCI-PSO to solve optimization problems.

E. Scalability Investigation

In this section, this article executes experiments to demonstrate the scalability of RCI-PSO in solving LSOPs by expanding the dimension size of the CEC2010 LSOPs from 200 to 2000. To make comparisons, the selected 11 compared large-scale methods are still used here. However, for fairness, we fine-tune the population sizes for all algorithms on the CEC2010 set with different dimension sizes. After preliminary experiments, the settings of the population size for all algorithms are listed in Table IV. In this table, it deserves notice that the population sizes of the four CC based large-scale methods (DECC-GDG, DECC-DG2, DECC-RDG, and DECC-RDG2) are the same for the CEC2010 LSOPs with different dimension sizes. This is because the scales of the decomposed subproblems of each CEC2010 LSOP remain the same as those of the associated 1000-D CEC2010 LSOP since its correlation structure of variables is not changed.

Tables SX–SXIII in the supplementary material present the detailed optimization results of the 12 algorithms and their comparisons on the CEC2010 suite with the dimensionality set as 200-D, 500-D, 800-D, and 2000-D, respectively. Table V summarizes the statistical comparisons among the 12 methods. From these tables, we discover the following conclusions.

1) In view of the average rank, RCI-PSO consistently achieves the lowest rank among the 12 algorithms and its average rank value is particularly smaller than those of the 11 compared methods. These reveal that RCI-PSO consistently performs the best and shows significant superiority to the 11 compared large-scale methods on the whole 200-D, 500-D, 800-D, and 2000-D CEC2010 test suites.
2) From the perspective of "*w/t/l*," RCI-PSO significantly outperforms the 11 compared algorithms on at least 15, 13, 11, and 12 LSOPs on the 200-D, 500-D, 800-D, and 2000-D CEC2010 LSOP sets, respectively. Besides, it only shows significant inferiority to them on at most 5, 4, 4, and 7 LSOPs on the four kinds of the CEC2010 sets, respectively. Except for SDLSO, which is a special case of RCI-PSO, the superiority of RCI-PSO to the other ten compared methods becomes more and more significant as the dimensionality increases. These observations demonstrate that RCI-PSO is more promising than the 11 compared approaches in coping with the 200-D, 500-D, 800-D, and 2000-D CEC2010 LSOPs.
3) Deep observations show that RCI-PSO is very promising in solving complex LSOPs. Particularly, on the six partially separable unimodal problems, as the dimensionality increases, RCI-PSO significantly outperforms the 11 compared approaches. On the nine partially separable multimodal problems, with the growth of the dimension size, except for SDLSO, RCI-PSO obtains increasingly better performance than the 11 compared methods. On the two fully nonseparable problems (one unimodal problem and one multimodal

TABLE V
SUMMARIZED COMPARISON RESULTS BETWEEN RCI-PSO AND THE COMPARED METHODS ON CEC2010 SET WITH DIFFERENT DIMENSION SIZES

Benchmark set	Problem Property	Index	RCI-PSO	APSODEE	SDLSO	TPLSO	RBLSO	DLLSO	SLPSO	CSO	DECC-GDG	DECC-DG2	DECC-RDG	DECC-RDG2
CEC2010-200	Fully Separable Unimodal	*w/t/l*	-	0/1/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Separable Multimodal	*w/t/l*	-	1/0/1	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0
	Partially Separable Unimodal	*w/t/l*	-	6/0/0	4/0/2	6/0/0	6/0/0	5/1/0	5/1/0	6/0/0	6/0/0	6/0/0	5/1/0	5/1/0
	Partially Separable Multimodal	*w/t/l*	-	5/0/4	6/0/3	9/0/0	8/0/1	7/0/2	9/0/0	7/0/2	9/0/0	8/0/1	8/0/1	8/0/1
	Fully Non-Separable Unimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Non-Separable Multimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Overall	*w/t/l*	-	14/1/5	15/0/5	20/0/0	19/0/1	17/1/2	19/1/0	18/0/2	20/0/0	19/0/1	18/1/1	18/1/1
	Overall	*rank*	**2.10**	2.90	4.60	5.45	5.15	6.70	8.50	8.20	9.90	9.60	7.50	7.40
CEC2010-500	Fully Separable Unimodal	*w/t/l*	-	1/0/0	0/10	1/0/0	1/0/0	0/10/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Separable Multimodal	*w/t/l*	-	1/0/1	1/0/1	2/0/0	2/0/0	1/1/0	2/0/0	2/0/0	1/0/1	2/0/0	2/0/0	2/0/0
	Partially Separable Unimodal	*w/t/l*	-	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0
	Partially Separable Multimodal	*w/t/l*	-	4/2/3	4/2/3	8/1/0	7/1/1	7/2/0	9/0/0	7/1/1	9/0/0	8/1/0	8/1/0	8/1/0
	Fully Non-Separable Unimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Non-Separable Multimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Overall	*w/t/l*	-	14/2/4	13/3/4	19/1/0	18/1/1	16/4/0	20/0/0	18/1/1	19/0/1	19/1/0	19/1/0	19/1/0
	Overall	*rank*	**1.95**	3.15	2.75	7.70	7.00	5.60	9.10	8.65	9.45	8.75	6.95	6.95
CEC2010-800	Fully Separable Unimodal	*w/t/l*	-	1/0/0	0/0/1	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Separable Multimodal	*w/t/l*	-	1/0/1	0/2/0	2/0/0	1/0/1	1/1/0	2/0/0	2/0/0	1/0/1	2/0/0	2/0/0	2/0/0
	Partially Separable Unimodal	*w/t/l*	-	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0
	Partially Separable Multimodal	*w/t/l*	-	6/0/3	3/3/3	9/0/0	9/0/0	6/2/1	9/0/0	8/0/1	9/0/0	9/0/0	9/0/0	8/1/0
	Fully Non-Separable Unimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Non-Separable Multimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Overall	*w/t/l*	-	16/0/4	11/5/4	20/0/0	19/0/1	16/3/1	20/0/0	19/0/1	19/0/1	20/0/0	20/0/0	19/1/0
	Overall	*rank*	**1.85**	3.05	3.70	7.55	7.90	5.95	8.70	8.30	9.50	8.45	6.40	6.65
CEC2010-2000	Fully Separable Unimodal	*w/t/l*	-	1/0/0	0/0/1	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Separable Multimodal	*w/t/l*	-	1/0/1	0/0/2	2/0/0	2/0/0	1/0/1	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0
	Partially Separable Unimodal	*w/t/l*	-	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0
	Partially Separable Multimodal	*w/t/l*	-	6/0/3	4/1/4	9/0/0	9/0/0	7/0/2	9/0/0	8/0/1	9/0/0	9/0/0	9/0/0	9/0/0
	Fully Non-Separable Unimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0
	Fully Non-Separable Multimodal	*w/t/l*	-	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	1/0/0	0/1/0	1/0/0	1/0/0	1/0/0	1/0/0
	Overall	*w/t/l*	-	16/0/4	12/1/7	20/0/0	20/0/0	17/0/3	20/0/0	18/1/1	20/0/0	20/0/0	20/0/0	20/0/0
	Overall	*rank*	**1.80**	2.95	3.50	6.50	6.95	5.20	9.50	7.15	10.15	10.35	6.95	7.00

problem), RCI-PSO consistently shows significant superiority to the 11 compared algorithms.

By the above experiments, the good scalability of RCI-PSO is verified. In particular, RCI-PSO is much more effective than the 11 compared large-scale approaches in tackling complex LSOPs, like partially separable and fully nonseparable ones. Such advantages of RCI-PSO mainly benefit from the RCI scheme and the adaptive scheme for adjusting the topology size. The two strategies cooperatively let particles interact with each other effectively and efficiently, and thus endow RCI-PSO a powerful optimization ability to solve LSOPs.

V. CONCLUSION

This article has designed an RCI strategy for PSO to let particles interact with each other effectively and efficiently in high-dimensional environment. As a result, a novel PSO variant, named RCI-PSO, was developed to cope with LSOPs. To traverse the vast search space effectively and efficiently, RCI builds a random interaction topology for each particle and only allows the particle to interact with the dominators in its topology. Then, RCI lets each updated particle follow the guidance of two contrastive dominators, namely, the best dominator and the worst dominator in its topology, to move toward promising regions. In this way, a good compromise between approaching optimal areas fast and avoiding falling into local regions can be potentially maintained at the particle level. Furthermore, a dynamic topology size adjustment strategy was designed to help the swarm gradually switch from exploring the search space to exploiting the found optimal areas. In this manner, a promising balance between search diversity and search convergence can be ensured at the swarm level. With these two kinds of promising balances, RCI-PSO is anticipated to effectively and efficiently solve LSOPs.

A large series of experiments have been conducted on two LSOP benchmark suites to verify the effectiveness and efficiency of RCI-PSO. In comparisons with totally 40 latest and well-performed large-scale optimizers, RCI-PSO has been experimentally substantiated to be more effective and efficient in solving LSOPs, especially the complicated ones. In addition, the good scalability of RCI-PSO has also been verified and the usefulness of its two key components has been investigated too.

In the future, to further improve the usability of RCI-PSO, we aim to devise adaptive parameter strategies for the swarm size NP and the control parameter ϕ, so that users can employ it to tackle practical optimization problems easily. Another direction is to deploy RCI-PSO to solve real-world optimization problems in engineering and academics, like neural architecture search [91], information spreading in social networks [92], and expensive optimization problems [93].

REFERENCES

[1] J. Peng, Y. Li, H. Kang, Y. Shen, X. Sun, and Q. Chen, "Impact of population topology on particle swarm optimization and its variants: An information propagation perspective," *Swarm. Evol. Comput.*, vol. 69, Mar. 2022, Art. no. 100990.

[2] S. Zhao and D. Wang, "Elite-ordinary synergistic particle swarm optimization," *Inf. Sci.*, vol. 609, pp. 1567–1587, Sep. 2022.

[3] H. Gui et al., "Boundary element method with particle swarm optimization for solving potential problems," *Adv. Eng. Softw.*, vol. 172, Oct. 2022, Art. no. 103191.

[4] K. Zheng et al., "Hybrid particle swarm optimizer with fitness-distance balance and individual self-exploitation strategies for numerical optimization problems," *Inf. Sci.*, vol. 608, pp. 424–452, Aug. 2022.

[5] J. Kennedy and R. Mendes, "Population structure and particle swarm performance," in *Proc. Congr. Evol. Comput.*, vol. 2, 2002, pp. 1671–1676.

[6] J. Kennedy, "Small worlds and mega-minds: Effects of neighborhood topology on particle swarm performance," in *Proc. Congr. Evol. Comput.*, vol. 3, 1999, pp. 1931–1938.

[7] Y. Shi and R. Eberhart, "A modified particle swarm optimizer," in *Proc. IEEE Congr. Evol. Comput.*, 1998, pp. 69–73.

[8] R. C. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," in *Proc. Int. Symp. MHS*, vol. 1, 1995, pp. 39–43.

[9] N. Lynn, M. Z. Ali, and P. N. Suganthan, "Population topologies for particle swarm optimization and differential evolution," *Swarm. Evol. Comput.*, vol. 39, pp. 24–35, Apr. 2018.

[10] T. Blackwell and J. Kennedy, "Impact of communication topology in particle swarm optimization," *IEEE Trans. Evol. Comput.*, vol. 23, no. 4, pp. 689–702, Aug. 2019.

[11] Q. Yang et al., "Stochastic cognitive dominance leading particle swarm optimization for multimodal problems," *Mathematics*, vol. 10, no. 5, p. 761, 2022.

[12] X. Li, "Niching without niching parameters: Particle swarm optimization using a ring topology," *IEEE Trans. Evol. Comput.*, vol. 14, no. 1, pp. 150–169, Feb. 2010.

[13] I. Maruta, T.-H. Kim, D. Song, and T. Sugie, "Synthesis of fixed-structure robust controllers using a constrained particle swarm optimizer with cyclic neighborhood topology," *Exp. Syst. Appl.*, vol. 40, no. 9, pp. 3595–3605, 2013.

[14] Q. Jiang and J. Li, "A novel method for finding global best guide for multiobjective particle swarm optimization," in *Proc. Int. Symp. Intell. Inf. Technol. Appl.*, vol. 3, 2009, pp. 146–150.

[15] Y.-J. Gong and J. Zhang, "Small-world particle swarm optimization with topology adaptation," in *Proc. Conf. Genet. Evol. Comput.*, 2013, pp. 25–32.

[16] W. H. Lim and N. A. M. Isa, "Particle swarm optimization with adaptive time-varying topology connectivity," *Appl. Soft Comput.*, vol. 24, pp. 623–642, Nov. 2014.

[17] L. Wang et al., "Particle swarm optimization using dynamic tournament topology," *Appl. Soft Comput.*, vol. 48, pp. 584–596, Nov. 2016.

[18] N. Zeng, Z. Wang, W. Liu, H. Zhang, K. Hone, and X. Liu, "A dynamic neighborhood-based switching particle swarm optimization algorithm," *IEEE Trans. Cybern.*, vol. 52, no. 9, pp. 9290–9301, Sep. 2022.

[19] Q. Liu, W. Wei, H. Yuan, Z.-H. Zhan, and Y. Li, "Topology selection for particle swarm optimization," *Inf. Sci.*, vol. 363, pp. 154–173, Oct. 2016.

[20] Y. Cao et al., "Comprehensive learning particle swarm optimization algorithm with local search for multimodal functions," *IEEE Trans. Evol. Comput.*, vol. 23, no. 4, pp. 718–731, Aug. 2019.

[21] J.J. Liang, A. K. Qin, P. N. Suganthan, and S. Baskar, "Comprehensive learning particle swarm optimizer for global optimization of multimodal functions," *IEEE Trans. Evol. Comput.*, vol. 10, no. 3, pp. 281–295, Jun. 2006.

[22] A. Lin, W. Sun, H. Yu, G. Wu, and H. Tang, "Adaptive comprehensive learning particle swarm optimization with cooperative archive," *Appl. Soft Comput.*, vol. 77, pp. 533–546, Apr. 2019.

[23] K. Zhang, Q. Huang, and Y. Zhang, "Enhancing comprehensive learning particle swarm optimization with local optima topology," *Inf. Sci.*, vol. 471, pp. 1–18, Jan. 2019.

[24] R. Cheng and Y. C. Jin, "A competitive swarm optimizer for large scale optimization," *IEEE Trans. Cybern.*, vol. 45, no. 2, pp. 191–204, Feb. 2015.

[25] Q. Yang, W. N. Chen, J. D. Deng, Y. Li, T. Gu, and J. Zhang, "A level-based learning swarm optimizer for large-scale optimization," *IEEE Trans. Evol. Comput.*, vol. 22, no. 4, pp. 578–594, Aug. 2018.

[26] H. Deng, L. Peng, H. Zhang, B. Yang, and Z. Chen, "Ranking-based biased learning swarm optimizer for large-scale optimization," *Inf. Sci.*, vol. 493, pp. 120–137, Aug. 2019.

[27] F. V. D. Bergh and A. P. Engelbrecht, "A cooperative approach to particle swarm optimization," *IEEE Trans. Evol. Comput.*, vol. 8, no. 3, pp. 225–239, Jun. 2004.

[28] X. Li and X. Yao, "Cooperatively coevolving particle swarms for large scale optimization," *IEEE Trans. Evol. Comput.*, vol. 16, no. 2, pp. 210–224, Apr. 2012.

[29] M. N. Omidvar, X. D. Li, Y. Mei, and X. Yao, "Cooperative co-evolution with differential grouping for large scale optimization," *IEEE Trans. Evol. Comput.*, vol. 18, no. 3, pp. 378–393, Jun. 2014.

[30] D. Li, W. Guo, A. Lerch, Y. Li, L. Wang, and Q. Wu, "An adaptive particle swarm optimizer with decoupled exploration and exploitation for large scale optimization," *Swarm. Evol. Comput.*, vol. 60, Feb. 2021, Art. no. 100789.

[31] Q. Yang, W.-N. Chen, T. Gu, H. Jin, W. Mao, and J. Zhang, "An adaptive stochastic dominant learning swarm optimizer for high-dimensional optimization," *IEEE Trans. Cybern.*, vol. 52, no. 3, pp. 1960–1976, Mar. 2022.

[32] Y. Sun, M. Kirley, and S. K. Halgamuge, "Extended differential grouping for large scale global optimization with direct and indirect variable interactions," in *Proc. Conf. Genet. Evol. Comput.*, 2015, pp. 313–320.

[33] Y. Mei, M. N. Omidvar, X. Li, and X. Yao, "A competitive divide-and-conquer algorithm for unconstrained large-scale black-box optimization," *ACM Trans. Math. Softw.*, vol. 42, no. 2, pp. 1–24, 2016.

[34] R. Lan, Y. Zhu, H. Lu, Z. Liu, and X. Luo, "A two-phase learning-based swarm optimizer for large-scale optimization," *IEEE Trans. Cybern.*, vol. 51, no. 12, pp. 6284–6293, Dec. 2021.

[35] F. Wang, X. Wang, and S. Sun, "A reinforcement learning level-based particle swarm optimization algorithm for large-scale optimization," *Inf. Sci.*, vol. 602, pp. 298–312, Jul. 2022.

[36] Q. Yang et al., "Segment-based predominant learning swarm optimizer for large-scale optimization," *IEEE Trans. Cybern.*, vol. 47, no. 9, pp. 2896–2910, Sep. 2017.

[37] X. Ma et al., "A survey on cooperative co-evolutionary algorithms," *IEEE Trans. Evol. Comput.*, vol. 23, no. 3, pp. 421–441, Jun. 2019.

[38] Y. Sun, M. Kirley, and S. K. Halgamuge, "A recursive decomposition method for large scale continuous optimization," *IEEE Trans. Evol. Comput.*, vol. 22, no. 5, pp. 647–661, Oct. 2018.

[39] Y. Sun, M. N. Omidvar, M. Kirley, and X. Li, "Adaptive threshold parameter estimation with recursive differential grouping for problem decomposition," in *Proc. Genet. Evol. Comput. Conf.*, 2018, pp. 889–896.

[40] Q. Yang, W.-N. Chen, and J. Zhang, "Evolution consistency based decomposition for cooperative coevolution," *IEEE Access*, vol. 6, pp. 51084–51097, 2018.

[41] Q. Yang et al., "Predominant cognitive learning particle swarm optimization for global numerical optimization," *Mathematics*, vol. 10, no. 10, p. 1620, 2022.

[42] K. Tang, X. Li, P. N. Suganthan, Z. Yang, and T. Weise, "Benchmark functions for the CEC2010 special session and competition on large-scale global optimization," Nat. Inspired Comput. Appl. Lab., Univ. Sci. Technol. China, Anhui, China, Rep. TR-2010, 2010.

[43] X. Li et al., "Benchmark functions for the CEC2013 special session and competition on large-scale global optimization," Evol. Comput. Mach. Learn. Group, RMIT Univ., Melbourne, VIC, Australia, Rep. TR-2013, 2013.

[44] Q. Yang et al., "Stochastic triad topology based particle swarm optimization for global numerical optimization," *Mathematics*, vol. 10, no. 7, p. 1032, 2022.

[45] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proc. IEEE Int. Conf. Neural Netw.*, vol. 4, 1995, pp. 1942–1948.

[46] Z. Ren, A. Zhang, C. Wen, and Z. Feng, "A scatter learning particle swarm optimization algorithm for multimodal problems," *IEEE Trans. Cybern.*, vol. 44, no. 7, pp. 1127–1140, Jul. 2014.

[47] S. Jang-Ho et al., "Multimodal function optimization based on particle swarm optimization," *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 1095–1098, Apr. 2006.

[48] C. Wang et al., "A hybrid topology scale-free gaussian-dynamic particle swarm optimization algorithm applied to real power loss minimization," *Eng. Appl. Artif. Intell.*, vol. 32, pp. 63–75, Jun. 2014.

[49] Z. Zhan et al., "Orthogonal learning particle swarm optimization," *IEEE Trans. Evol. Comput.*, vol. 15, no. 6, pp. 832–847, Dec. 2011.

[50] Q. Yang, X. Guo, X.-D. Gao, D.-D. Xu, and Z.-Y. Lu, "Differential elite learning particle swarm optimization for global numerical optimization," *Mathematics*, vol. 10, no. 8, p. 1261, 2022.

[51] Y.-J. Gong et al., "Genetic learning particle swarm optimization," *IEEE Trans. Cybern.*, vol. 46, no. 10, pp. 2277–2290, Oct. 2016.

[52] S. Mahdavi, M. E. Shiri, and S. Rahnamayan, "Metaheuristics in large-scale global continues optimization: A survey," *Inf. Sci.*, vol. 295, pp. 407–428, Feb. 2015.

[53] A. LaTorre, S. Muelas, and J.-M. Peña, "A comprehensive comparison of large scale global optimizers," *Inf. Sci.*, vol. 316, pp. 517–549, Sep. 2015.

[54] R. Lan, L. Zhang, Z. Tang, Z. Liu, and X. Luo, "A hierarchical sorting swarm optimizer for large-scale optimization," *IEEE Access*, vol. 7, pp. 40625–40635, 2019.

[55] Q. Yang et al., "A distributed swarm optimizer with adaptive communication for large-scale optimization," *IEEE Trans. Cybern.*, vol. 50, no. 7, pp. 3393–3408, Jul. 2020.

[56] J.-Y. Li, Z.-H. Zhan, K. C. Tan, and J. Zhang, "Dual differential grouping: A more general decomposition method for large-scale optimization," *IEEE Trans. Cybern.*, vol. 53, no. 6, pp. 3624–3638, Jun. 2023.

[57] Q. Yang, Y. Zhu, X. Gao, D. Xu, and Z. Lu, "Elite directed particle swarm optimization with historical information for high-dimensional problems," *Mathematics*, vol. 10, no. 9, p. 1384, 2022.

[58] Q. Yang et al., "A dimension group-based comprehensive elite learning swarm optimizer for large-scale optimization," *Mathematics*, vol. 10, no. 7, p. 1072, 2022.

[59] R. Cheng and Y. C. Jin, "A social learning particle swarm optimization algorithm for scalable optimization," *Inf. Sci.*, vol. 291, pp. 43–60, Jan. 2015.

[60] Z.-J. Wang, Z.-H. Zhan, S. Kwong, H. Jin, and J. Zhang, "Adaptive granularity learning distributed particle swarm optimization for large-scale optimization," *IEEE Trans. Cybern.*, vol. 51, no. 3, pp. 1175–1188, Mar. 2021.

[61] H. Wang, M. Liang, C. Sun, G. Zhang, and L. Xie, "Multiple-strategy learning particle swarm optimization for large-scale optimization problems," *Complex Intell. Syst.*, vol. 7, no. 1, pp. 1–16, 2021.

[62] M. N. Omidvar, M. Yang, Y. Mei, X. Li, and X. Yao, "DG2: A faster and more accurate differential grouping for large-scale black-box optimization," *IEEE Trans. Evol. Comput.*, vol. 21, no. 6, pp. 929–942, Dec. 2017.

[63] A. A. Hadi, A. W. Mohamed, and K. M. Jambi, "LSHADE-SPA memetic framework for solving large-scale optimization problems," *Complex Intell. Syst.*, vol. 5, no. 1, pp. 25–40, 2019.

[64] H. Ge et al., "Bi-space interactive cooperative coevolutionary algorithm for large scale black-box optimization," *Appl. Soft Comput.*, vol. 97, Dec. 2020, Art. no. 106798.

[65] P. Yang, K. Tang, and X. Yao, "Turning high-dimensional optimization into computationally expensive optimization," *IEEE Trans. Evol. Comput.*, vol. 22, no. 1, pp. 143–156, Feb. 2018.

[66] X. Peng, Y. Jin, and H. Wang, "Multimodal optimization enhanced cooperative coevolution for large-scale optimization," *IEEE Trans. Cybern.*, vol. 49, no. 9, pp. 3507–3520, Sep. 2019.

[67] A. W. Mohamed, "Solving large-scale global optimization problems using enhanced adaptive differential evolution algorithm," *Complex Intell. Syst.*, vol. 3, no. 4, pp. 205–231, 2017.

[68] A. W. Mohamed and A. S. Almazyad, "Differential evolution with novel mutation and adaptive crossover strategies for solving large scale global optimization problems," *Appl. Comput. Intell. Soft Comput.*, vol. 2017, pp. 1–18, Mar. 2017.

[69] G. Dai, X. Chen, L. Chen, M. Wang, and L. Peng, "Cooperative coevolution with dependency identification grouping for large scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2016, pp. 5201–5208.

[70] S. Mahdavi, M. E. Shiri, and S. Rahnamayan, "Cooperative co-evolution with a new decomposition method for large-scale optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2014, pp. 1285–1292.

[71] S. Ye, G. Dai, L. Peng, and M. Wang, "A hybrid adaptive coevolutionary differential evolution algorithm for large-scale optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2014, pp. 1277–1284.

[72] A. LaTorre et al., "Multiple offspring sampling in large scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2012, pp. 1–8.

[73] T. Takahama and S. Sakai, "Large scale optimization by differential evolution with landscape modality detection and a diversity archive," in *Proc. IEEE Congr. Evol. Comput.*, 2012, pp. 1–8.

[74] J. Brest, B. Bošković, A. Zamuda, I. Fister, and M. S. Maučec, "Self-adaptive differential evolution algorithm with a small and varying population size," in *Proc. IEEE Congr. Evol. Comput.*, 2012, pp. 1–8.

[75] K. Zhang and B. Li, "Cooperative coevolution with global search for large scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2012, pp. 1–7.

[76] H. Wang, Z. Wu, S. Rahnamayan, and D. Jiang, "Sequential de enhanced by neighborhood search for large scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2010, pp. 1–7.

[77] M. N. Omidvar, X. Li, and X. Yao, "Cooperative co-evolution with delta grouping for large scale non-separable function optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2010, pp. 1–8.

[78] D. Molina, M. Lozano, and F. Herrera, "MA-SW-chains: Memetic algorithm based on local search chains for large scale continuous global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2010, pp. 1–8.

[79] P. Korošec et al., "The differential ant-stigmergy algorithm for large-scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2010, pp. 1–8.

[80] Y. Wang and B. Li, "Two-stage based ensemble optimization for large-scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2010, pp. 1–8.

[81] J. Brest, A. Zamuda, I. Fister, and M. S. Maučec, "Large scale global optimization using self-adaptive differential evolution algorithm," in *Proc. IEEE Congr. Evol. Comput.*, 2010, pp. 1–8.

[82] D. Molina, A. LaTorre, and F. Herrera, "SHADE with iterative local search for large-scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2018, pp. 1–8.

[83] M. Yang et al., "Efficient resource allocation in cooperative co-evolution for large-scale global optimization," *IEEE Trans. Evol. Comput.*, vol. 21, no. 4, pp. 493–505, Aug. 2017.

[84] S. Salcedo-Sanz, C. Camacho-Gómez, D. Molina, and F. Herrera, "A coral reefs optimization algorithm with substrate layers and local search for large scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2016, pp. 3574–3581.

[85] M. N. Omidvar, B. Kazimipour, X. Li, and X. Yao, "CBCC3—A contribution-based cooperative co-evolutionary algorithm with improved exploration/exploitation balance," in *Proc. IEEE Congr. Evol. Comput.*, 2016, pp. 3541–3548.

[86] D. Molina and F. Herrera, "Iterative hybridization of de with local search for the CEC'2015 special session on large scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2015, pp. 1974–1978.

[87] A. LaTorre, S. Muelas, and J. M. Peña, "Large scale global optimization: Experimental results with MOS-based hybrid algorithms," in *Proc. IEEE Congr. Evol. Comput.*, 2013, pp. 2742–2749.

[88] F. Wei, Y. Wang, and T. Zong, "Variable grouping based differential evolution using an auxiliary function for large scale global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2014, pp. 1293–1298.

[89] Z. Yang, K. Tang, and X. Yao, "Large scale evolutionary optimization using cooperative coevolution," *Inf. Sci.*, vol. 178, no. 15, pp. 2985–2999, 2008.

[90] F. Wei, Y. Wang, and Y. Huo, "Smoothing and auxiliary functions based cooperative coevolution for global optimization," in *Proc. IEEE Congr. Evol. Comput.*, 2013, pp. 2736–2741.

[91] Z. Lu, S. Liang, Q. Yang, and B. Du, "Evolving block-based convolutional neural network for hyperspectral image classification," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, pp. 1–21, 2022.

[92] W. N. Chen, D. Z. Tan, Q. Yang, T. Gu, and J. Zhang, "Ant colony optimization for the control of pollutant spreading on social networks," *IEEE Trans. Cybern.*, vol. 50, no. 9, pp. 4053–4065, Sep. 2020.

[93] F.-F. Wei et al., "A classifier-assisted level-based learning swarm optimizer for expensive optimization," *IEEE Trans. Evol. Comput.*, vol. 25, no. 2, pp. 219–233, Apr. 2021.

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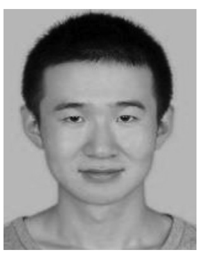

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