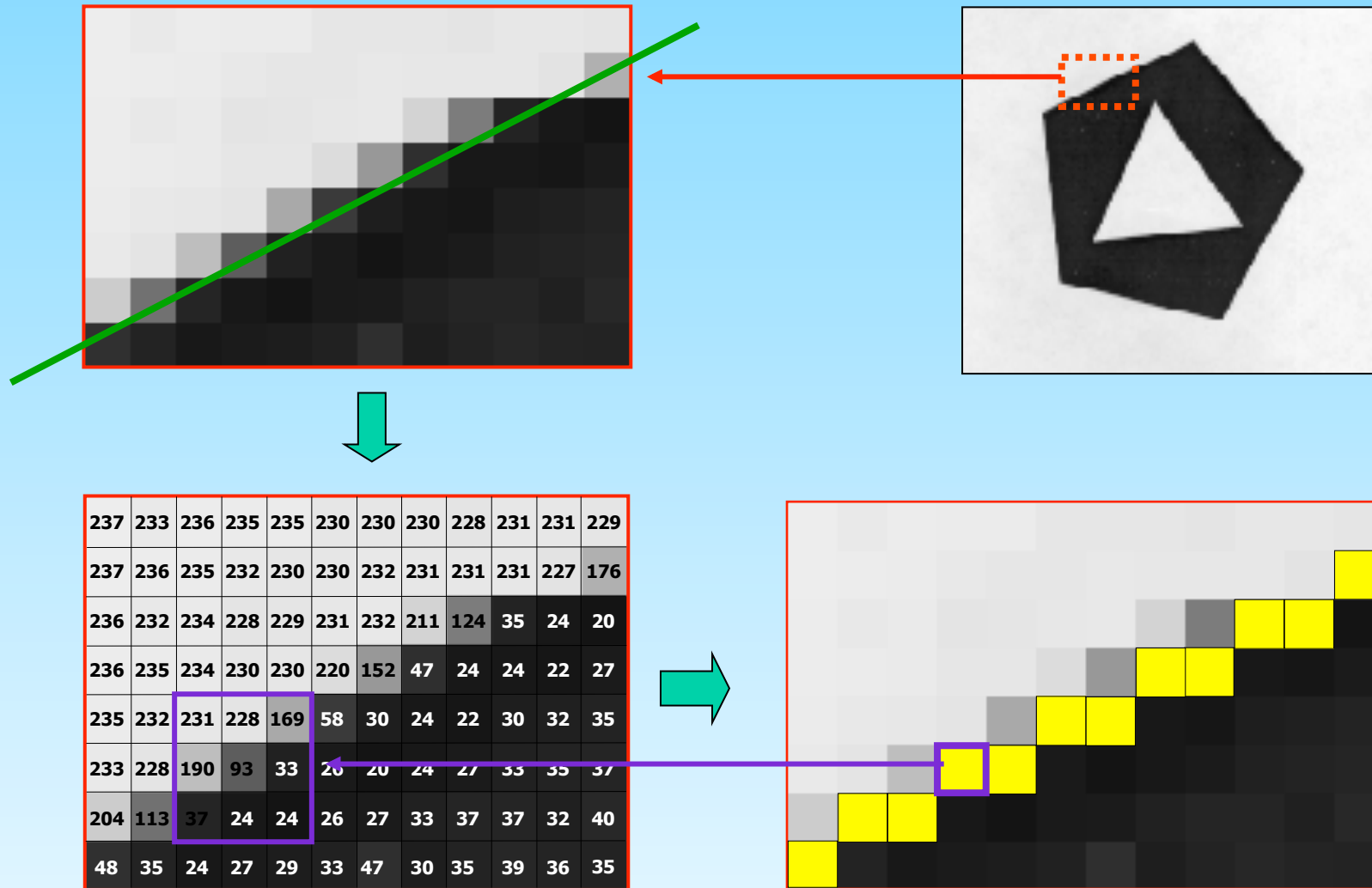




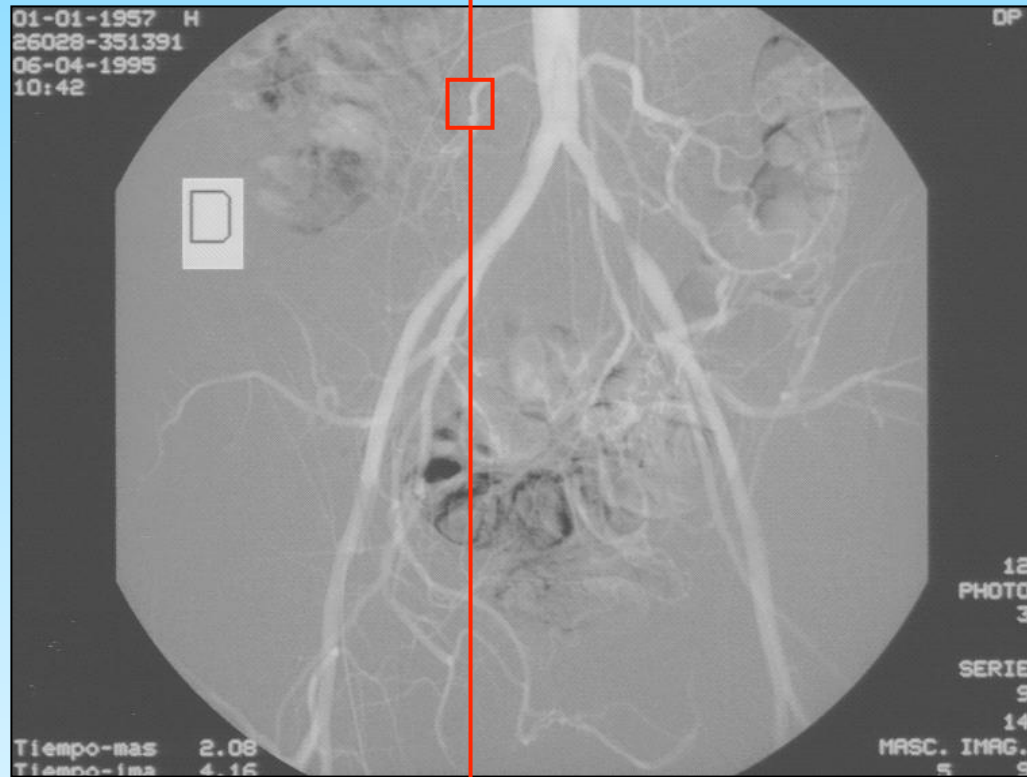
Accurate Subpixel Edge Location Based on Partial Area Effect

Agustín Trujillo-Pino
Karl Krissian
Miguel Alemán-Flores
Daniel Santana-Cedrés

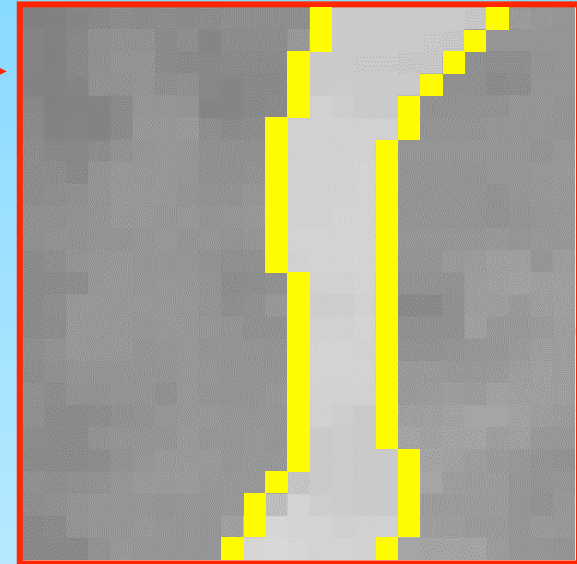
Edge Detection in the pixel level



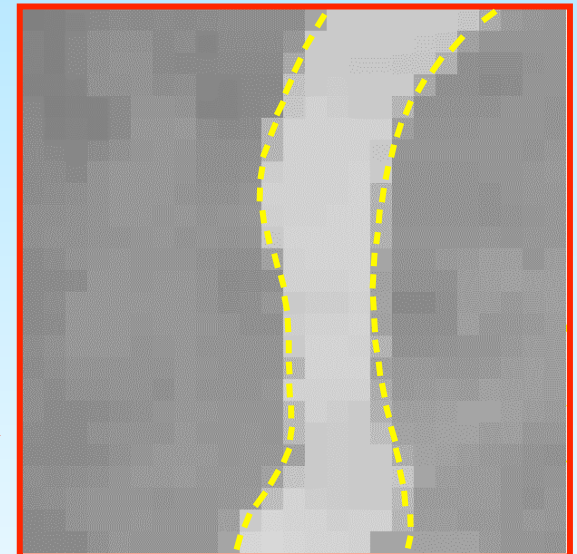
Subpixel Edge Detection



Edge detection in pixel level



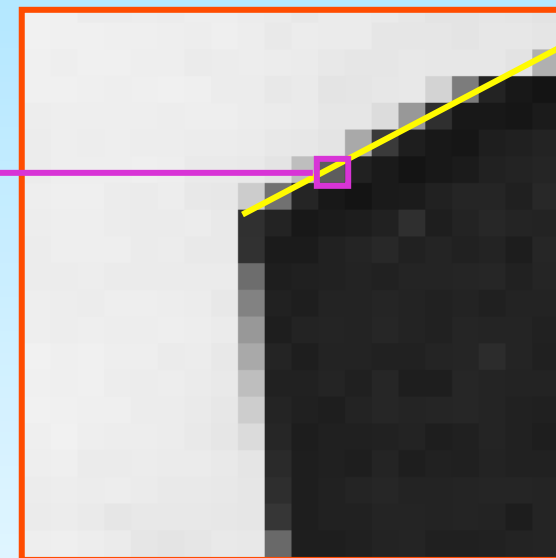
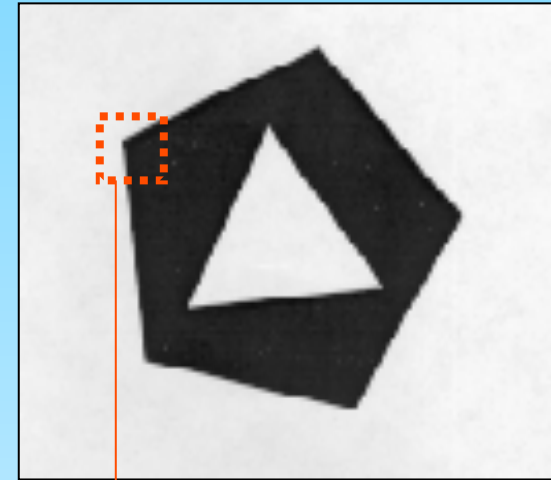
Subpixel edge detection



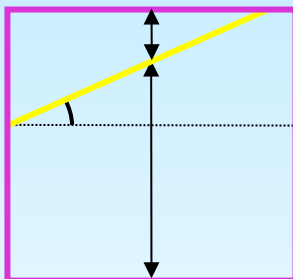
Main goal of this work

- Given an ideal image, locate accurately for every edge pixel the following features:

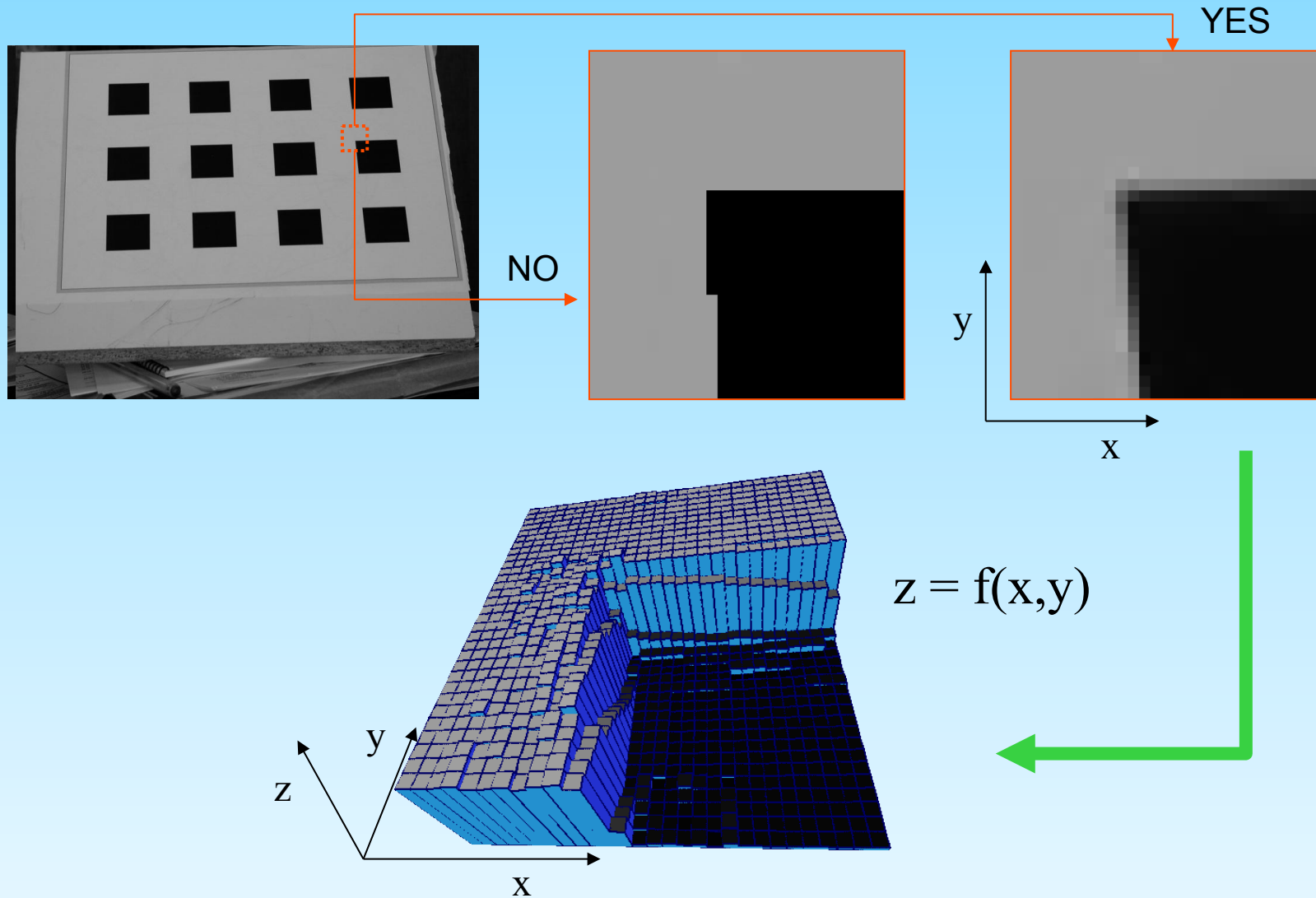
- orientation
- intensity difference at both sides
- subpixel position
- curvature



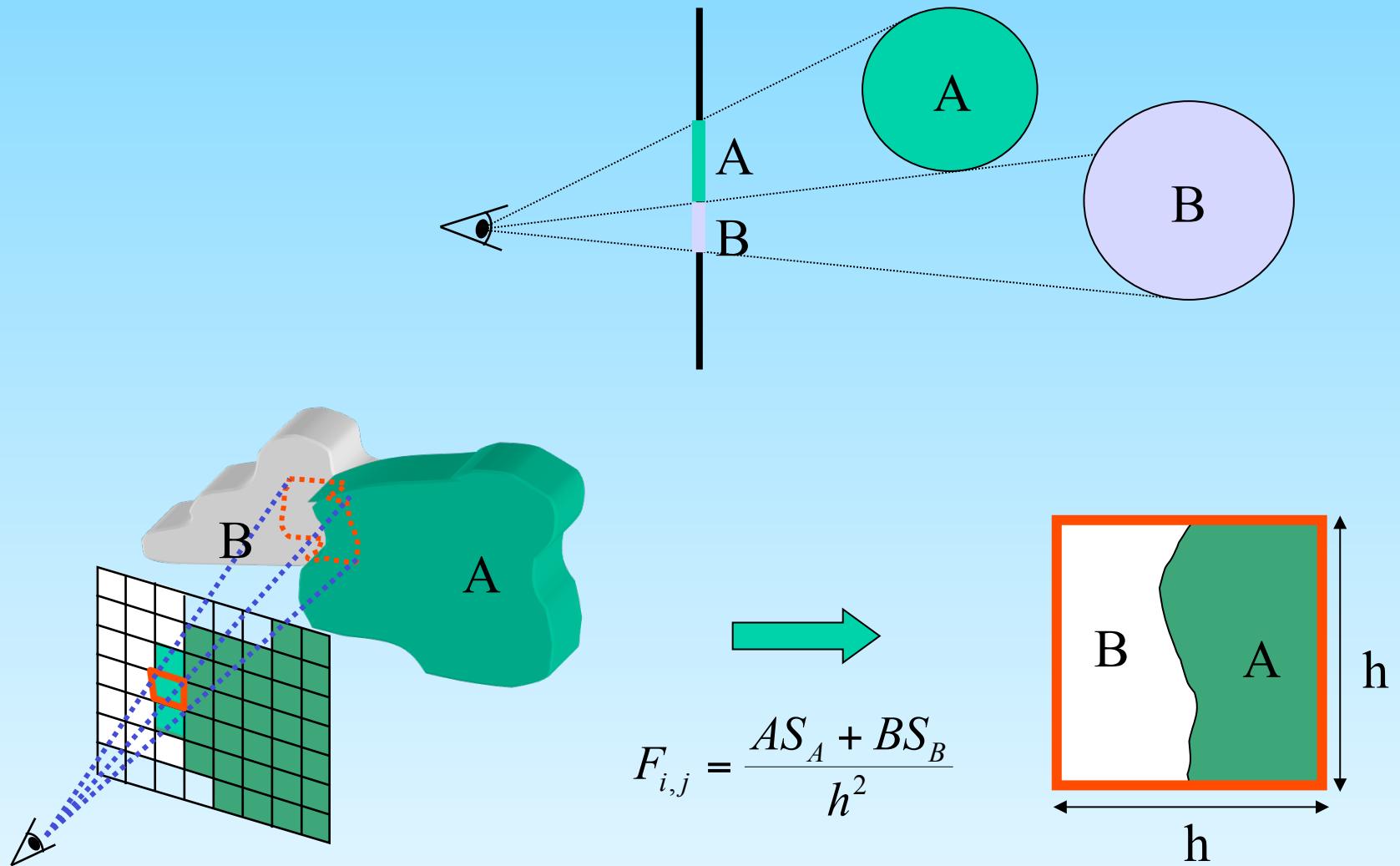
| | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 237 | 233 | 236 | 235 | 235 | 240 | 230 | 230 | 228 | 231 | 231 | 229 |
| 237 | 236 | 235 | 232 | 230 | 240 | 232 | 231 | 231 | 231 | 227 | 176 |
| 236 | 232 | 234 | 228 | 229 | 241 | 232 | 211 | 124 | 35 | 24 | 20 |
| 236 | 235 | 234 | 230 | 230 | 240 | 152 | 47 | 24 | 24 | 22 | 27 |
| 235 | 232 | 231 | 228 | 169 | 58 | 30 | 24 | 22 | 30 | 32 | 35 |
| 233 | 228 | 190 | 93 | 33 | 26 | 20 | 24 | 27 | 33 | 35 | 37 |
| 204 | 113 | 37 | 24 | 24 | 26 | 27 | 33 | 37 | 37 | 32 | 40 |
| 48 | 35 | 24 | 27 | 29 | 33 | 47 | 30 | 35 | 39 | 36 | 35 |



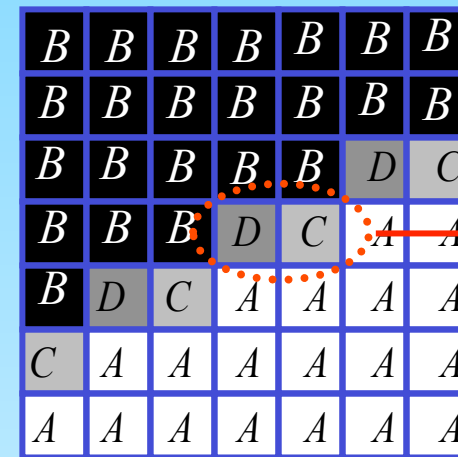
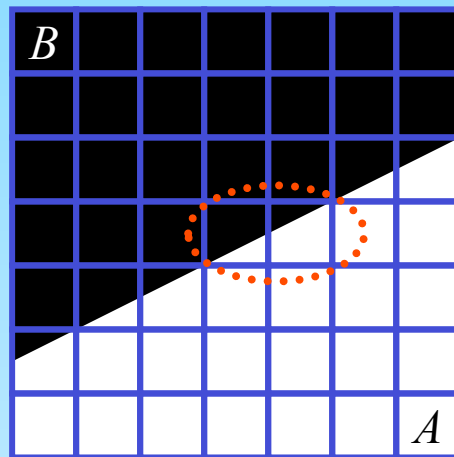
Acquired intensity in edge pixels



Partial area effect hypothesis

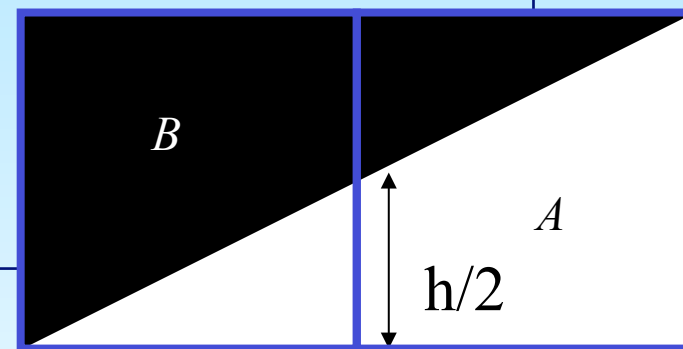


Ideal straight edge with slope 1/2



$$D = \frac{A + 3B}{4}$$

$$C = \frac{3A + B}{4}$$



Error when computing intensity change at both sides

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | B | B | B | B |
| B | B | B | B | B | B | B |
| B | B | B | B | B | D | C |
| B | B | B | D | C | A | A |
| B | D | C | A | A | A | A |
| C | A | A | A | A | A | A |
| A | A | A | A | A | A | A |

$$\begin{pmatrix} -1 & 0 & 1 \\ -\alpha & 0 & \alpha \\ -1 & 0 & 1 \end{pmatrix}$$

$$\rightarrow f_x = \frac{3\alpha+1}{8(\alpha+2)h} (A-B)$$

$$\begin{pmatrix} -1 & -\alpha & -1 \\ 0 & 0 & 0 \\ 1 & \alpha & 1 \end{pmatrix}$$

$$\rightarrow f_y = \frac{4\alpha+7}{8(\alpha+2)h} (A-B)$$

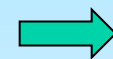
$$\|G\| = \frac{(A-B)}{8(\alpha+2)h} \sqrt{25\alpha^2 + 62\alpha + 50}$$

$$A=100$$

$$B=0$$

$$h=1$$

$$\alpha = \sqrt{2}$$



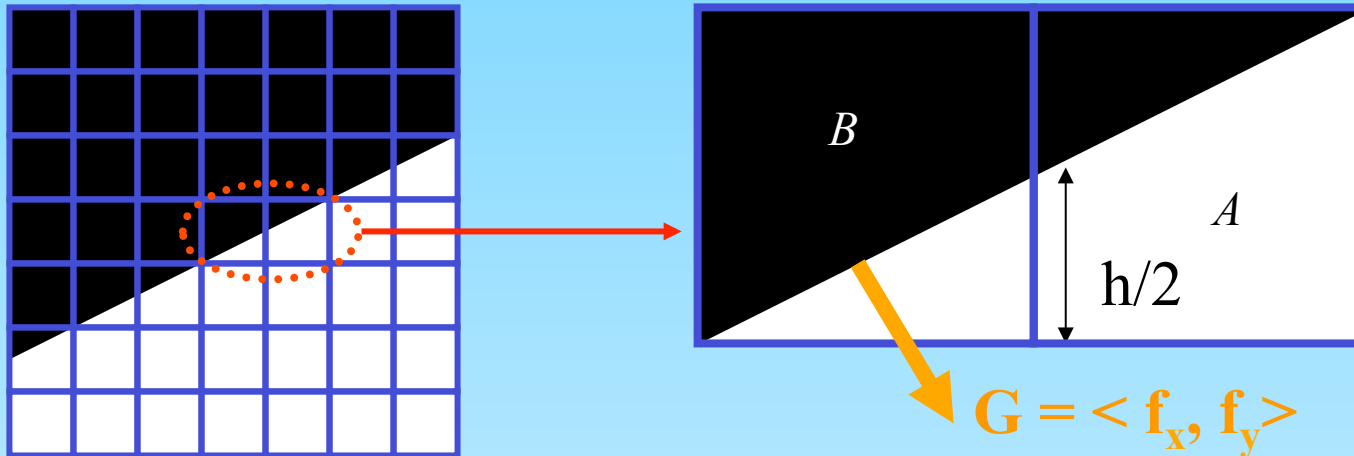
$$\|G\| =$$

| | | | | | | |
|----|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 0 | 0 | 5 |
| 0 | 0 | 0 | 0 | 5 | 19 | 37 |
| 0 | 0 | 5 | 19 | 37 | 51 | 51 |
| 5 | 19 | 37 | 51 | 51 | 37 | 19 |
| 37 | 51 | 51 | 37 | 19 | 5 | 0 |
| 51 | 37 | 19 | 5 | 0 | 0 | 0 |
| 19 | 5 | 0 | 0 | 0 | 0 | 0 |

WRONG INTENSITY DIFFERENCE

$$\|G\| \neq \frac{A-B}{2h}$$

Error when computing orientation



$f_x =$

| | | | | | | |
|----|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| 0 | 0 | 0 | 0 | 3 | 11 | 16 |
| 0 | 0 | 3 | 11 | 16 | 19 | 19 |
| 3 | 11 | 16 | 19 | 16 | 11 | |
| 16 | 19 | 19 | 16 | 11 | 3 | 0 |
| 19 | 16 | 11 | 3 | 0 | 0 | 0 |
| 11 | 3 | 0 | 0 | 0 | 0 | 0 |

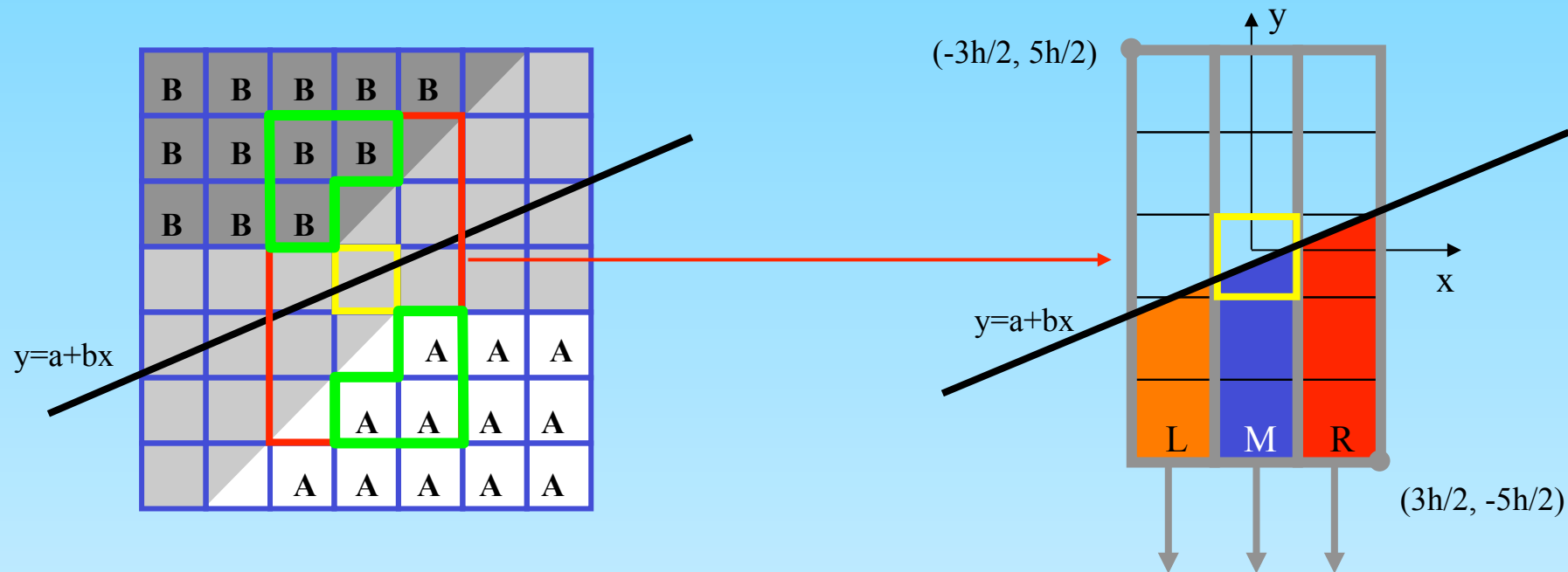
$f_y =$

| | | | | | | |
|----|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| 0 | 0 | 0 | 0 | 3 | 16 | 33 |
| 0 | 0 | 3 | 16 | 33 | 46 | 46 |
| 3 | 16 | 33 | 46 | 46 | 33 | 16 |
| 33 | 46 | 46 | 33 | 16 | 3 | 0 |
| 46 | 33 | 16 | 3 | 0 | 0 | 0 |
| 16 | 3 | 0 | 0 | 0 | 0 | 0 |

WRONG ORIENTATION

$$\frac{f_x}{f_y} = \frac{5\sqrt{2} - 4}{6 + \sqrt{2}} \neq \frac{1}{2}$$

Proposed method for isolated edges of first order



$$a = \frac{2S_M - 5(A + B)}{2(A - B)}$$

$$b = \frac{S_R - S_L}{2(A - B)}$$

$$S_M = 5B + \frac{A - B}{h^2} M$$

$$S_L = 5B + \frac{A - B}{h^2} L$$

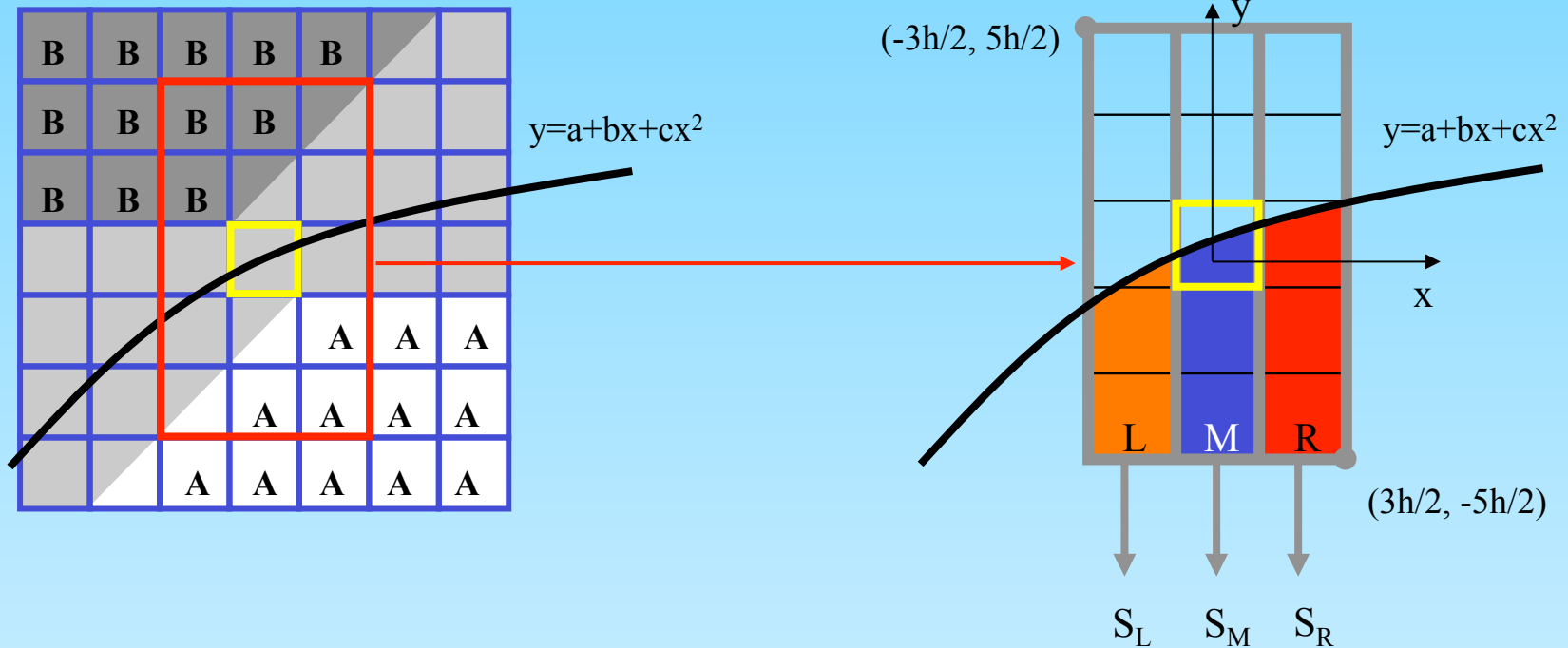
$$S_R = 5B + \frac{A - B}{h^2} R$$

$$M = \int_{-h/2}^{h/2} (a + bx + 5h/2) dx$$

$$L = \int_{-3h/2}^{-h/2} (a + bx + 5h/2) dx$$

$$R = \int_{h/2}^{3h/2} (a + bx + 5h/2) dx$$

Proposed method for isolated edges of second order



$$a = \frac{26S_M - S_L - S_R - 60(A + B)}{24(A - B)}$$

$$b = \frac{S_R - S_L}{2(A - B)}$$

$$c = \frac{S_L + S_R - 2S_M}{2(A - B)}$$

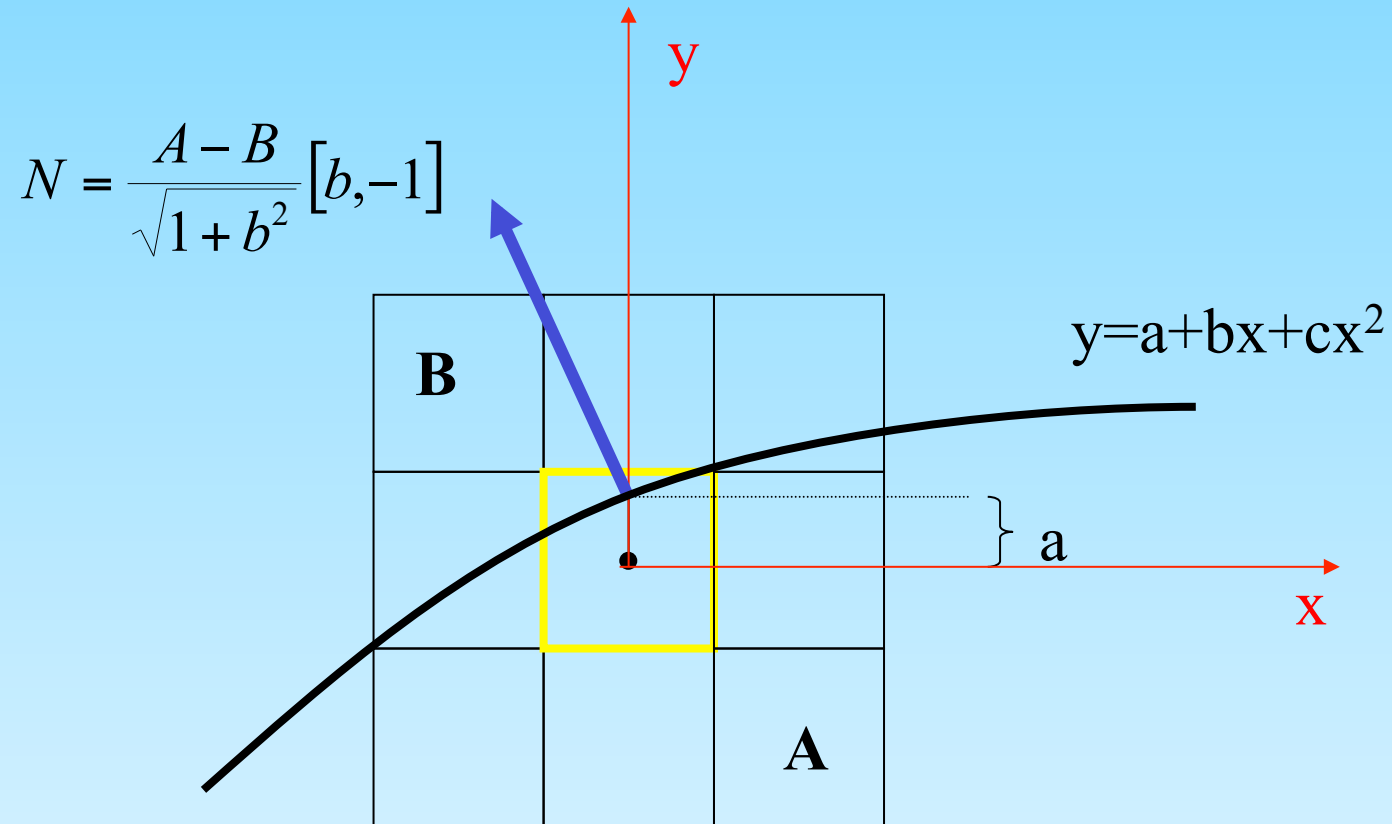
$$\left\{ \begin{array}{l} S_L = 5B + \frac{A - B}{h^2} L \\ S_M = 5B + \frac{A - B}{h^2} M \\ S_R = 5B + \frac{A - B}{h^2} R \end{array} \right.$$

$$L = \int_{-3h/2}^{-h/2} (a + bx + cx^2 + 5h/2) dx$$

$$M = \int_{-h/2}^{h/2} (a + bx + cx^2 + 5h/2) dx$$

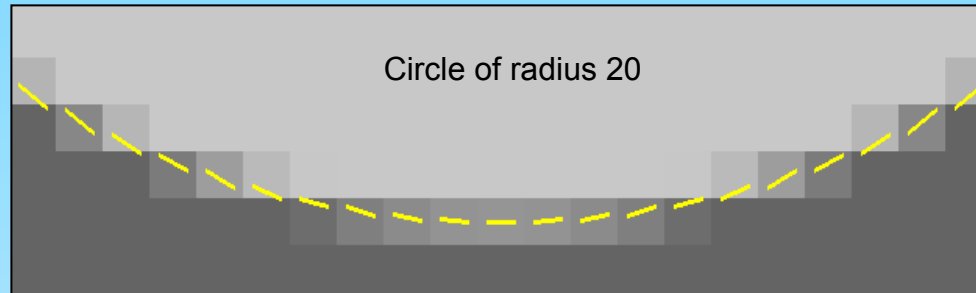
$$R = \int_{h/2}^{3h/2} (a + bx + cx^2 + 5h/2) dx$$

Estimating edge features

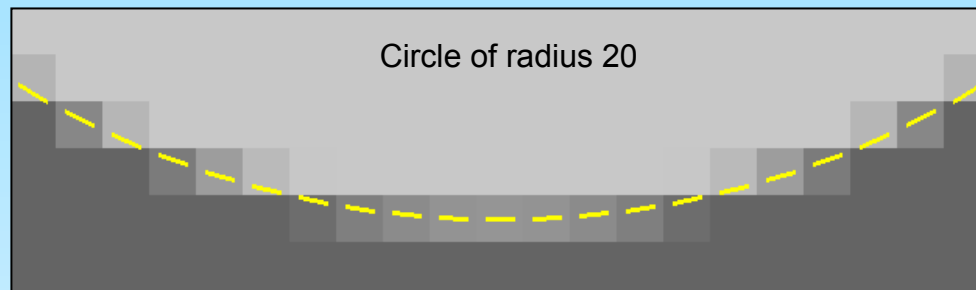


$$K = \frac{2c}{(1+b^2)^{3/2}}$$

Edge detection in an ideal circle

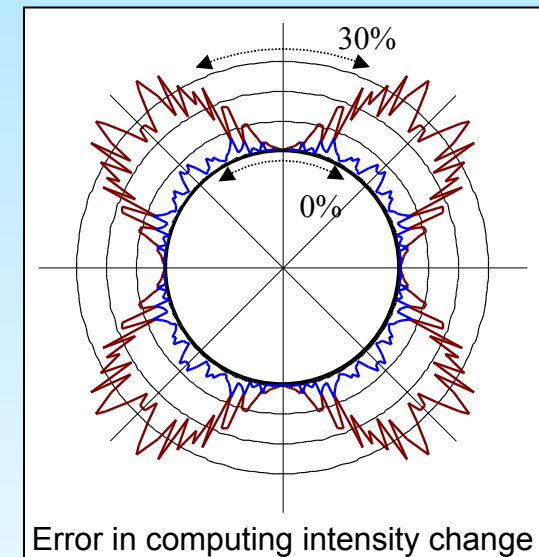
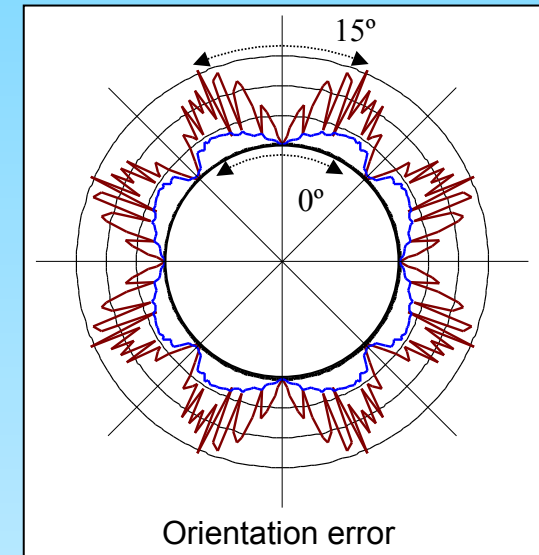


Traditional method

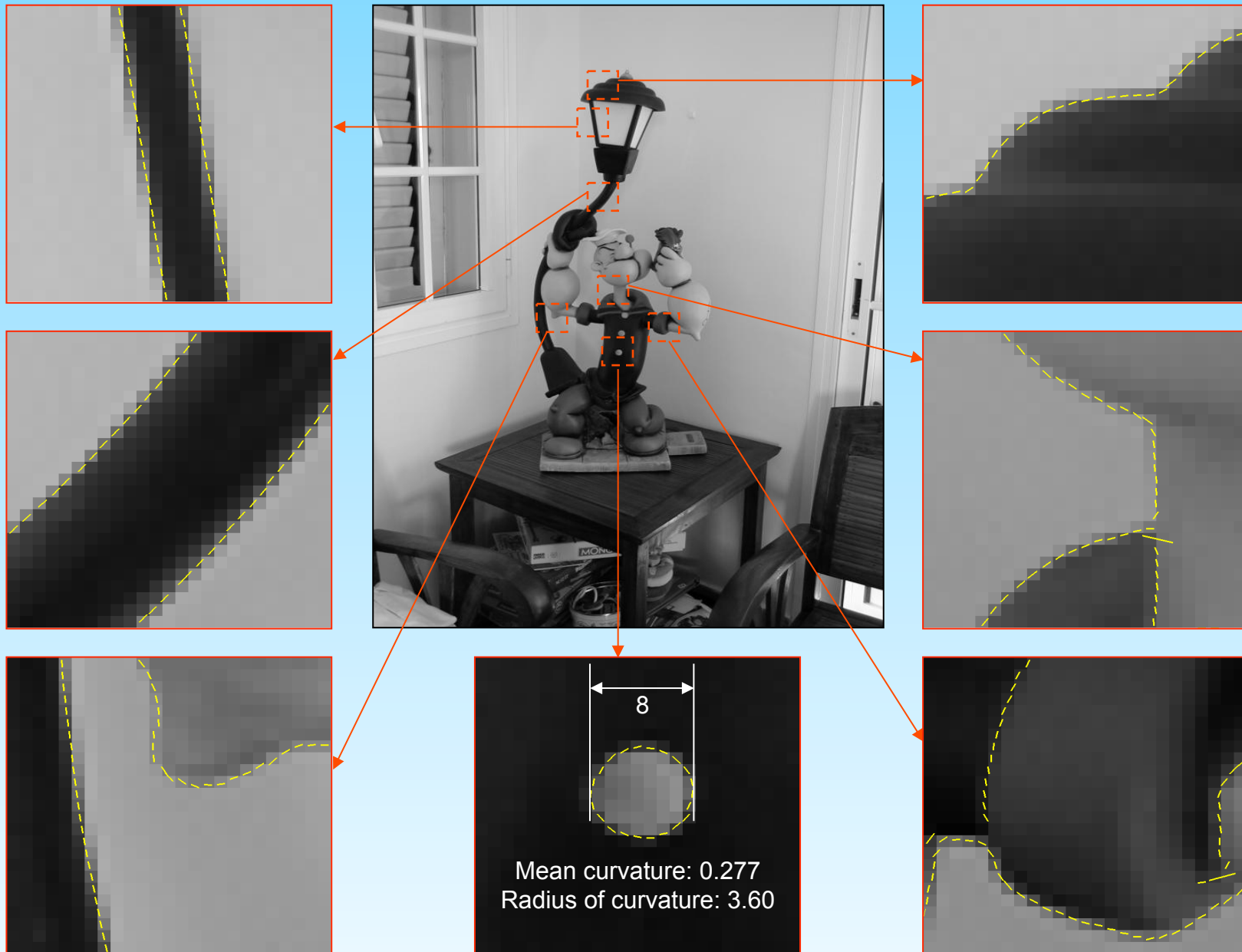


Proposed method

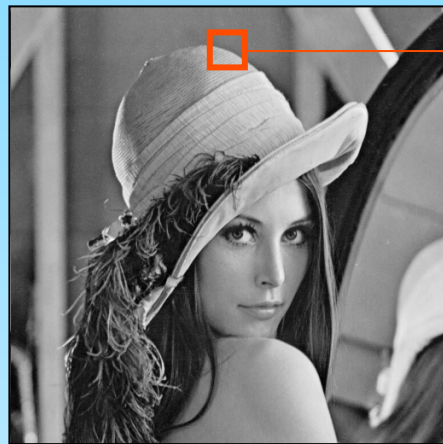
| Radius of curvature | Mean | Minimum | Maximum |
|---------------------|-------|---------|---------|
| Second derivatives | 28.32 | 12.49 | 32.45 |
| Analitic expression | 24.32 | 15.69 | 25.43 |
| Proposed method | 19.98 | 19.96 | 19.98 |



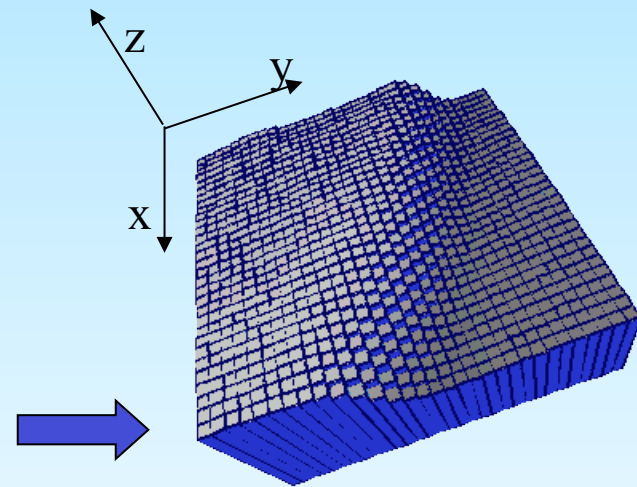
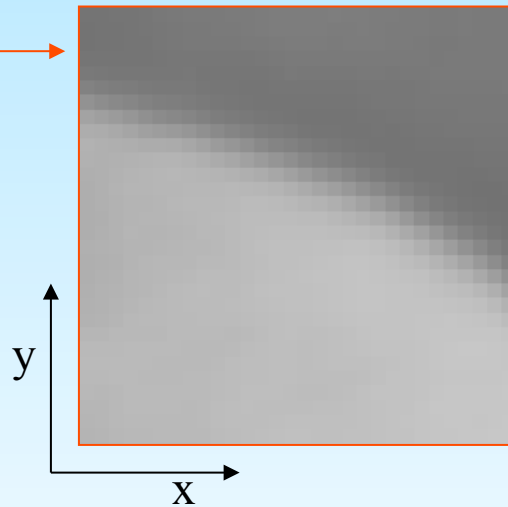
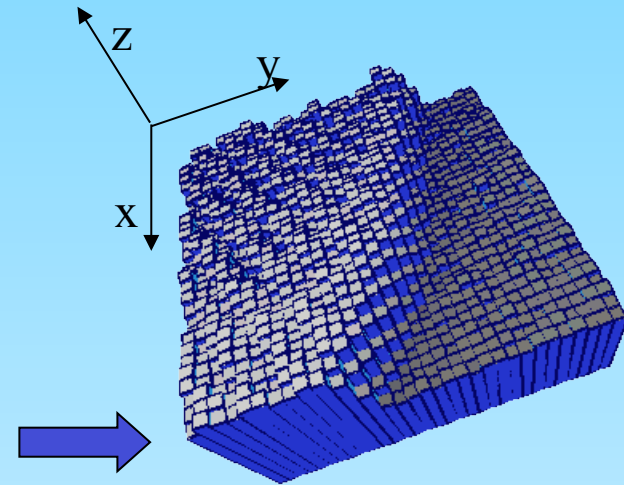
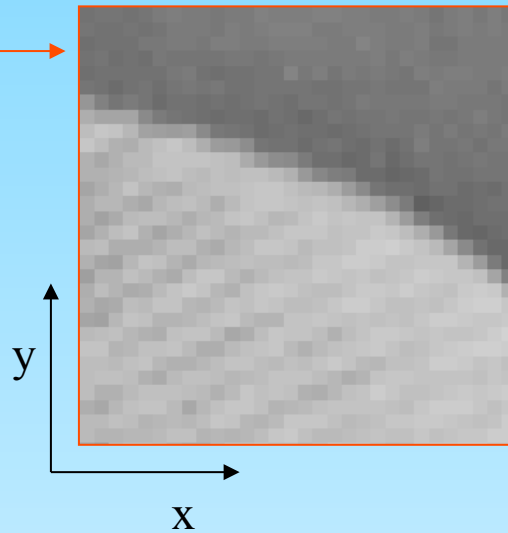
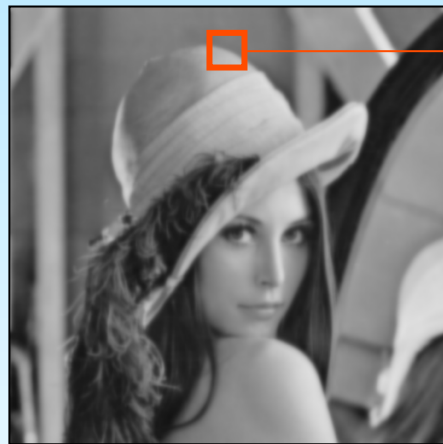
Experiment with real image



Traditional image smoothing

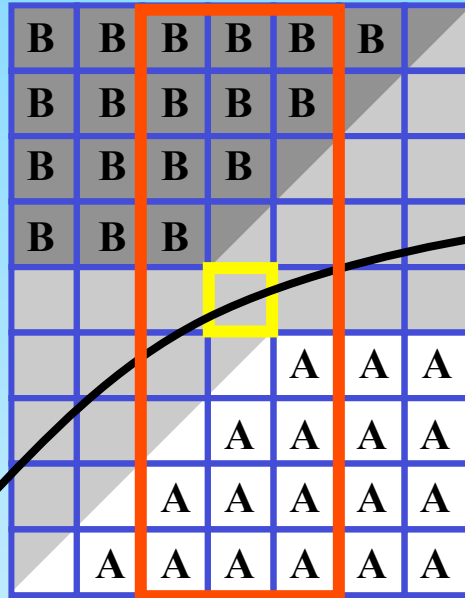


↓ Smoothing



Edge detection in smooth images

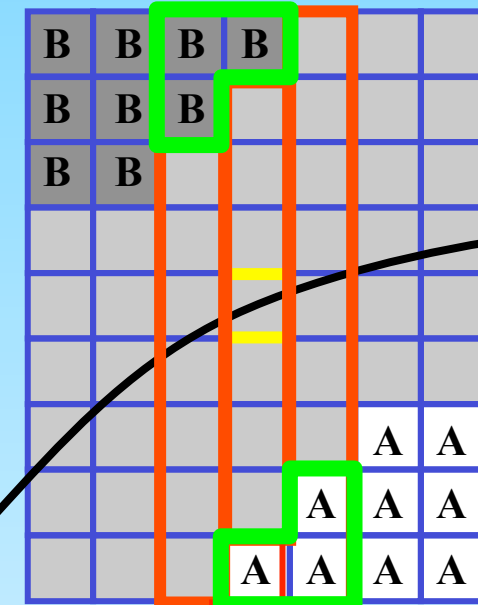
Original image F



$$* \begin{pmatrix} a_{11} & a_{01} & a_{11} \\ a_{01} & a_{00} & a_{01} \\ a_{11} & a_{01} & a_{11} \end{pmatrix} =$$

$$G_{x,y} = \sum_{i,j} a_{i,j} F_{x+i,y+j}$$

Smooth image G



$$a = \frac{2S_M - 7(A+B)}{2(A-B)} - \frac{1 + 24a_{01} + 48a_{11}}{12} c$$

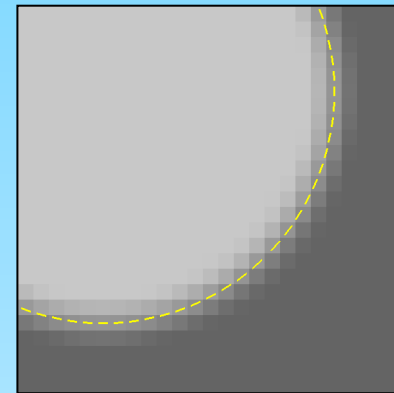
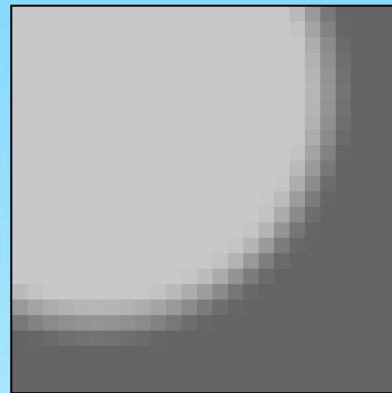
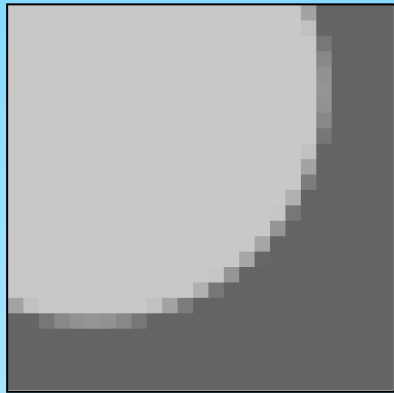
$$b = 1 + \frac{S_R - S_L}{2(A-B)}$$

$$c = \frac{S_L + S_R - 2S_M}{2(A-B)}$$

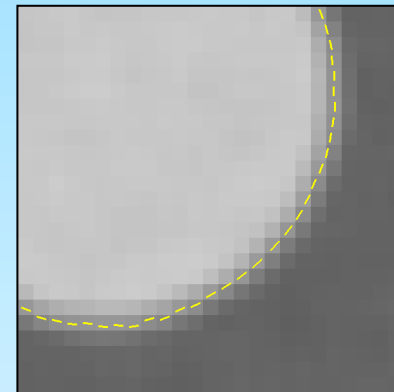
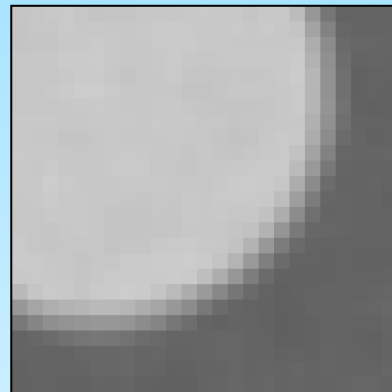
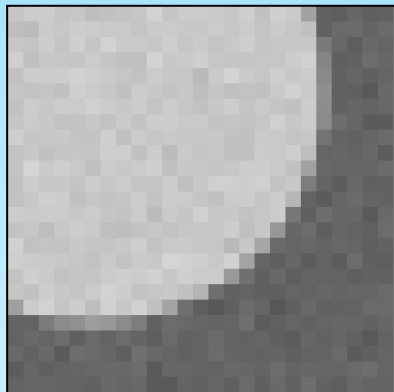


Experiment with noisy synthetic images

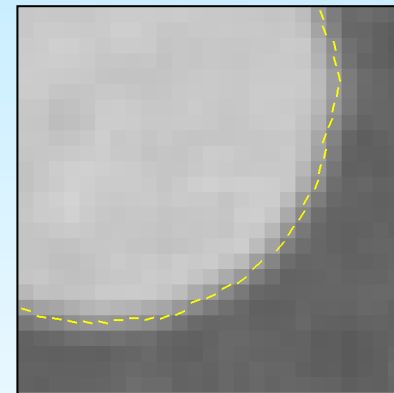
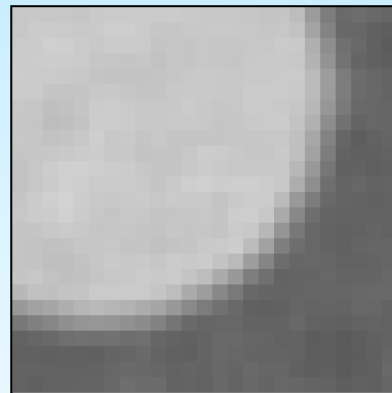
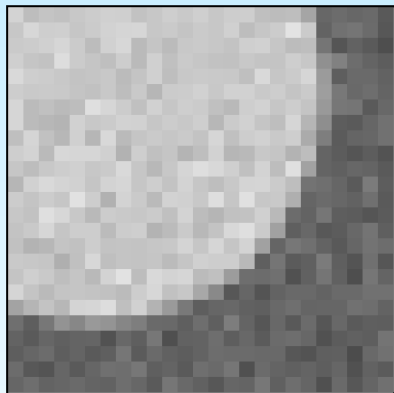
Noise 0



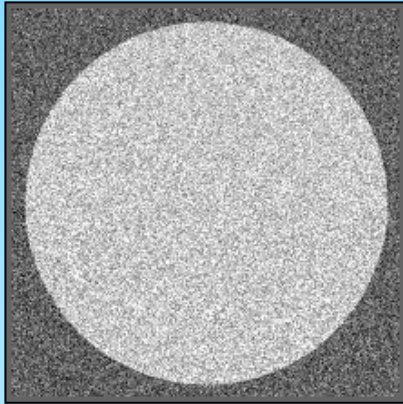
Noise 5



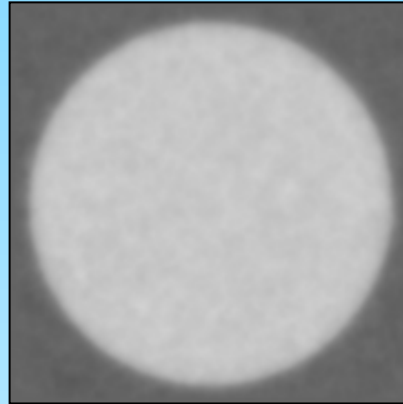
Noise 10



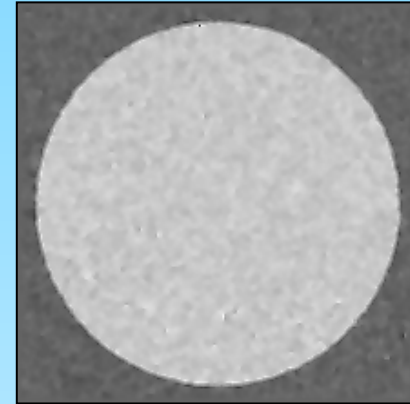
Tradition image restoration



Ideal image with noise added



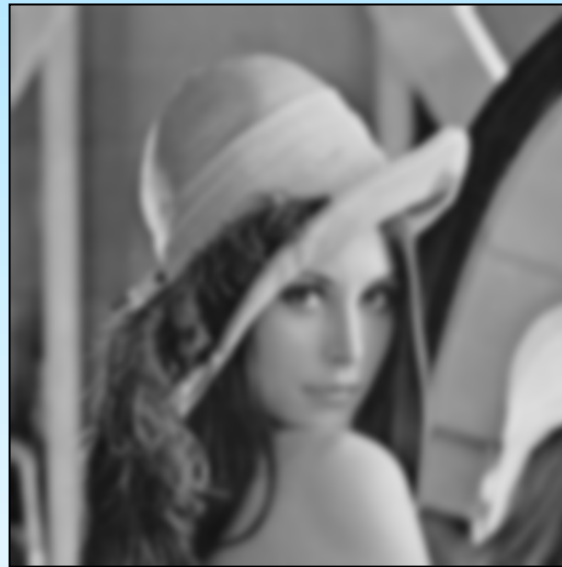
Gaussian smoothing



Anisotropic diffusion



Real image

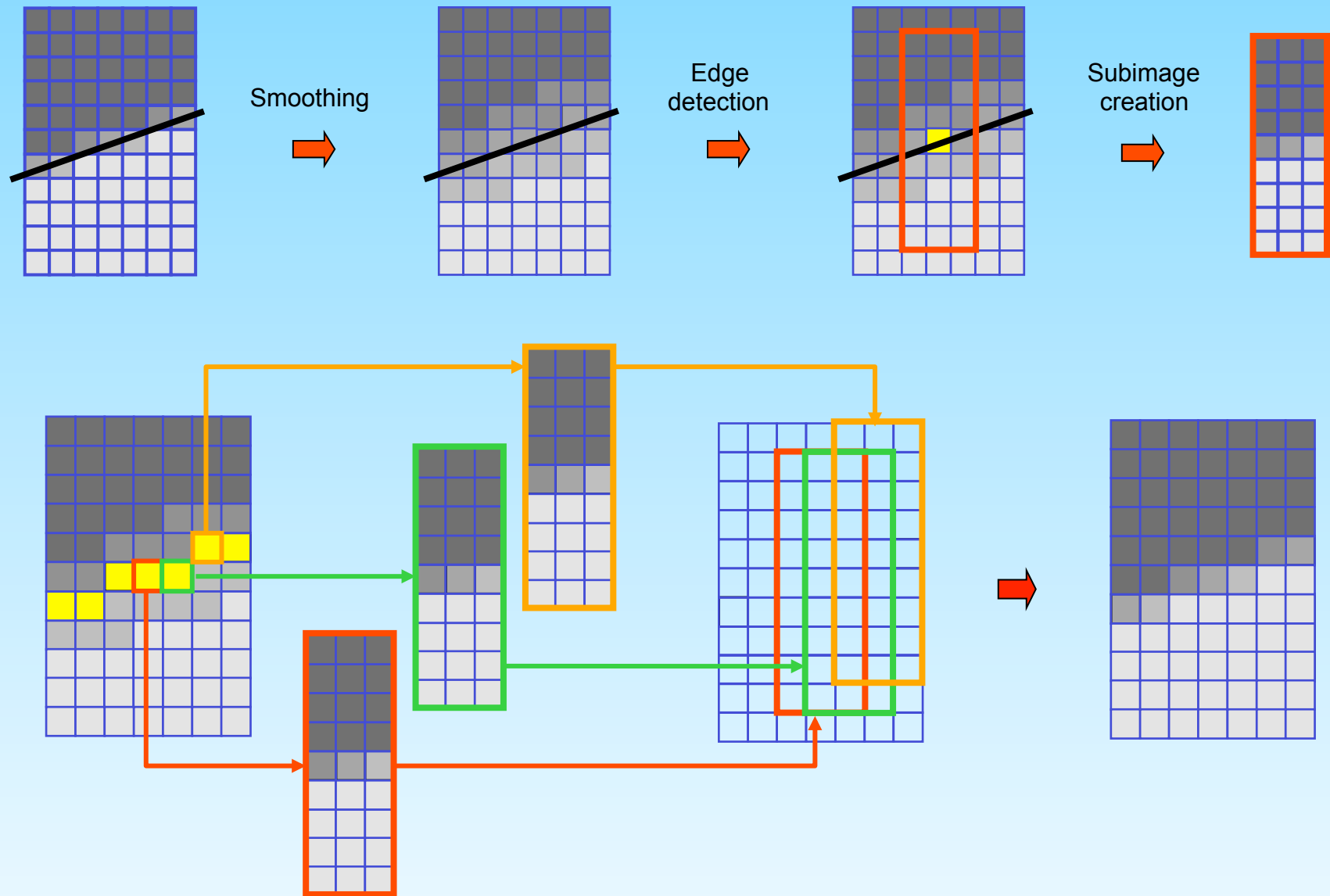


Gaussian smoothing



Anisotropic diffusion

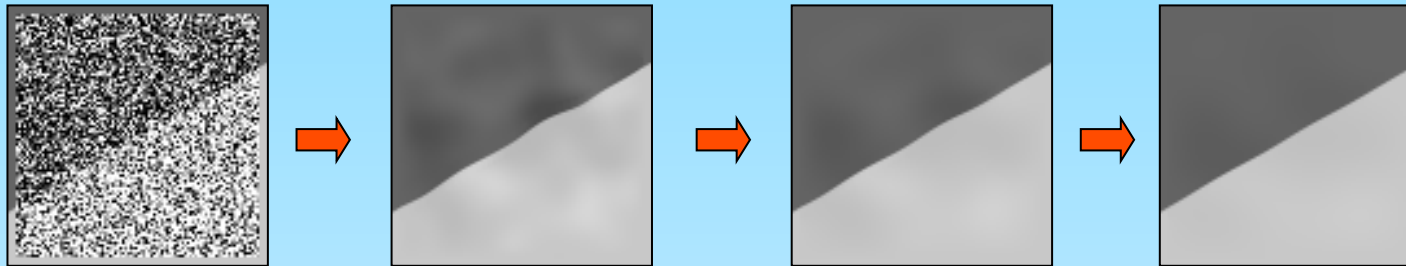
Restoration proposed method



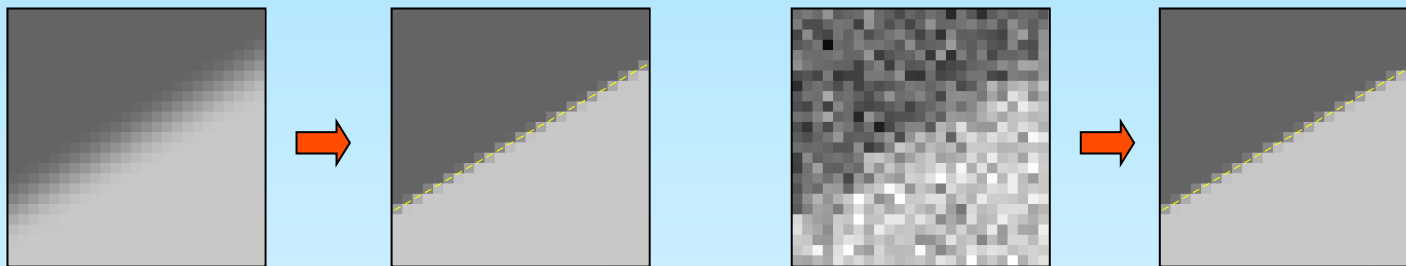
Features of the restoration proposed method

- Ideal images remain unchanged

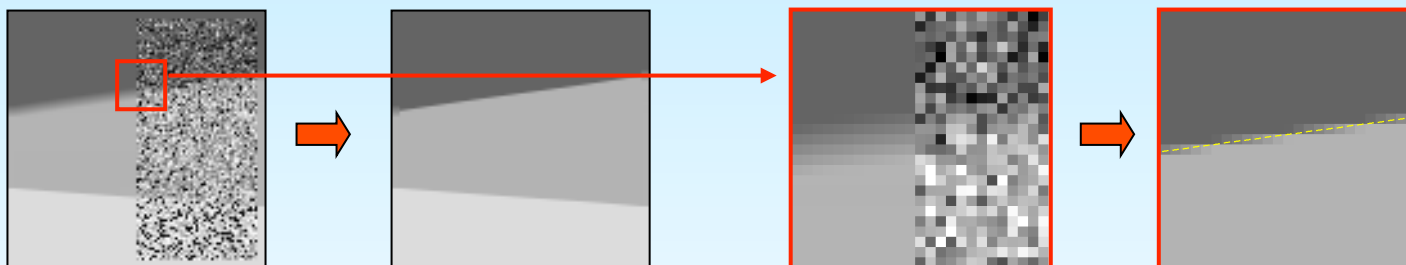
- Effective noise removal



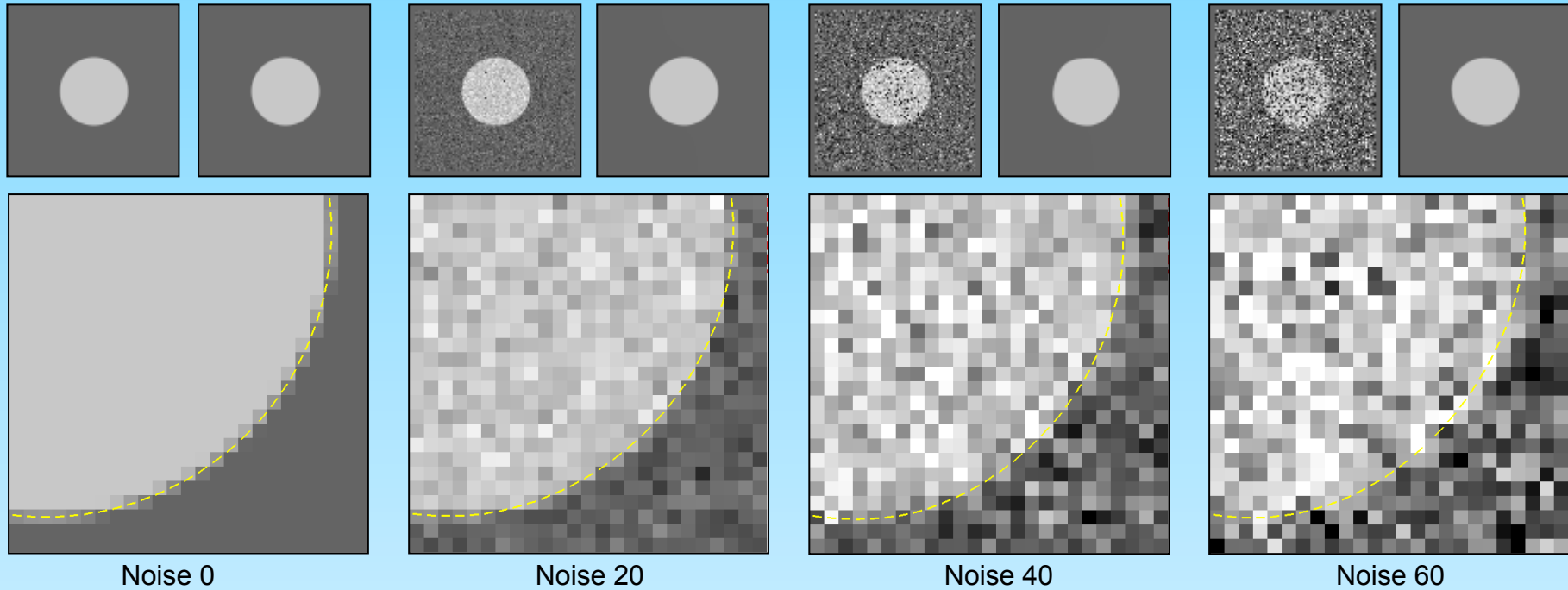
- Autofocus



- Robustness to different noise and intensity levels

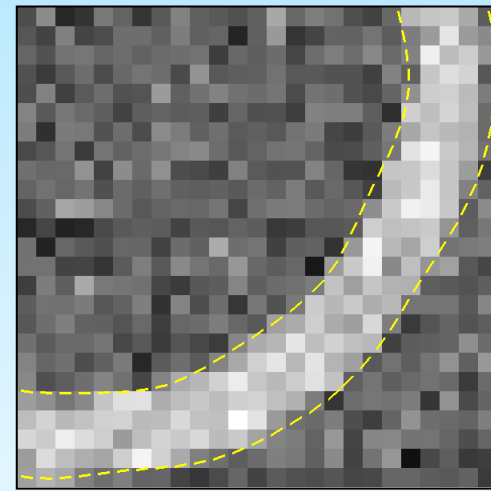
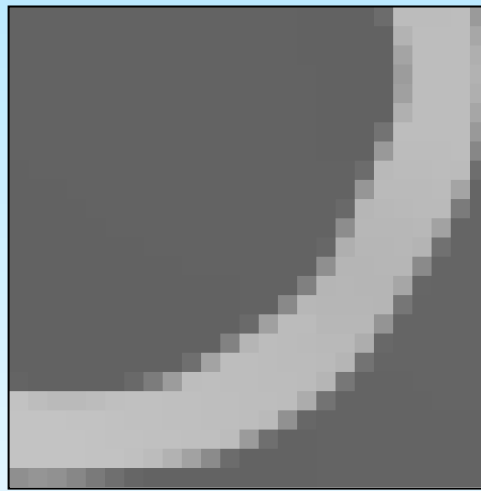
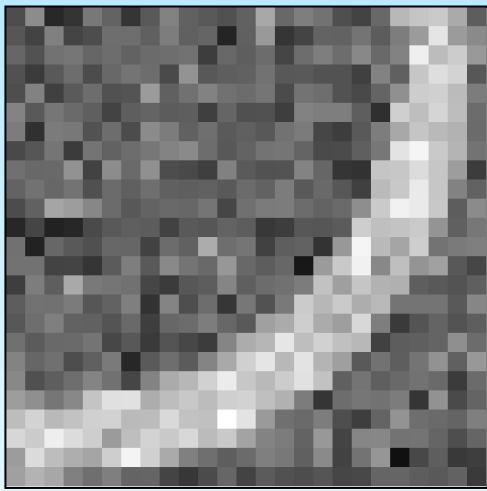
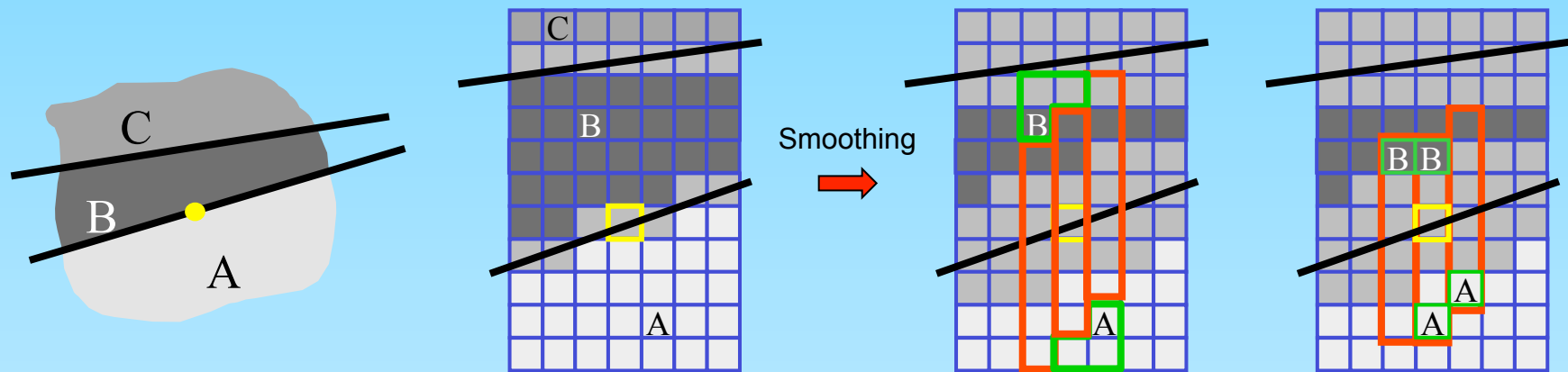


Experiments with synthetic circle of radius 20

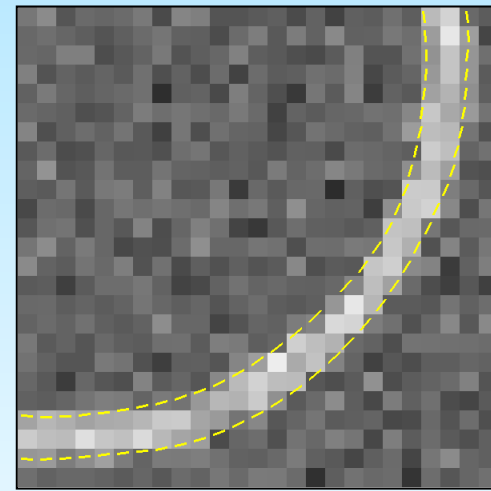
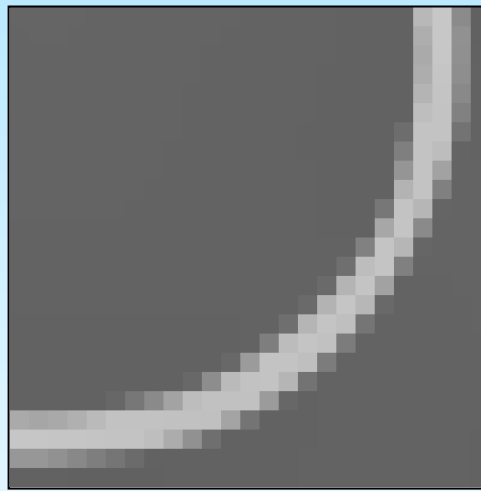
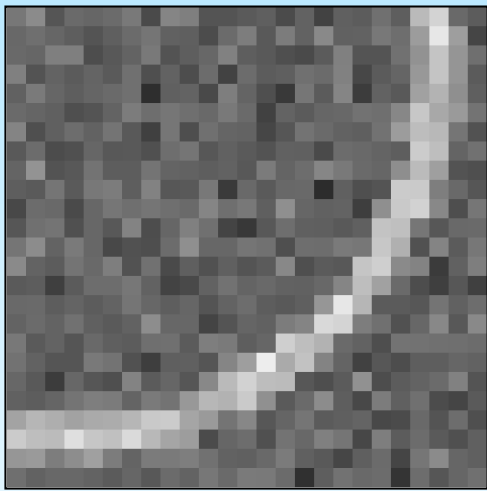
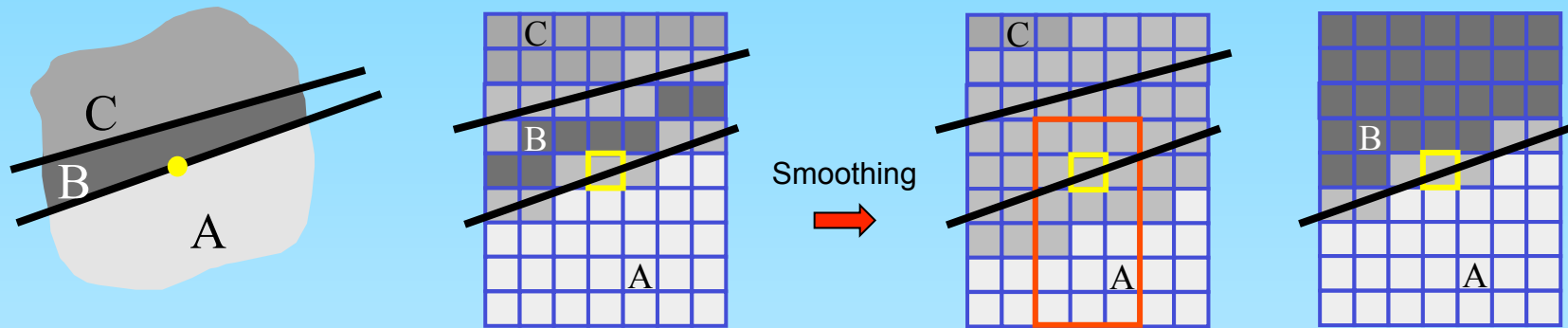


| Noise | Inten. chan. | | Orientation | | Position | | Radius of curvat. | | |
|-------|--------------|------|-------------|------|----------|------|-------------------|------|------|
| | Mean | Max | Mean | Max | Mean | Max | Mean | Min | Max |
| 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 20.0 | 20.0 | 20.0 |
| 20 | 0.48 | 0.66 | 0.76 | 2.00 | 10.8 | 25.2 | 20.0 | 17.2 | 22.9 |
| 40 | 0.74 | 0.94 | 1.54 | 4.25 | 26.8 | 67.8 | 19.9 | 15.1 | 30.5 |
| 60 | 1.06 | 1.30 | 1.92 | 5.31 | 30.4 | 85.2 | 19.8 | 15.2 | 35.4 |

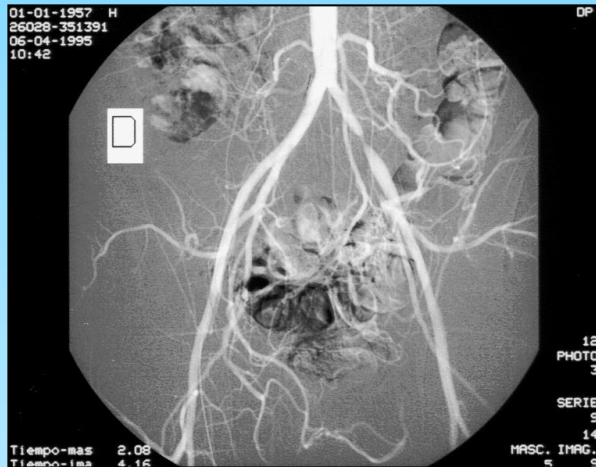
Nearby edge location



Tackling very close edges



Experiments with real angiographic image



Restoration

