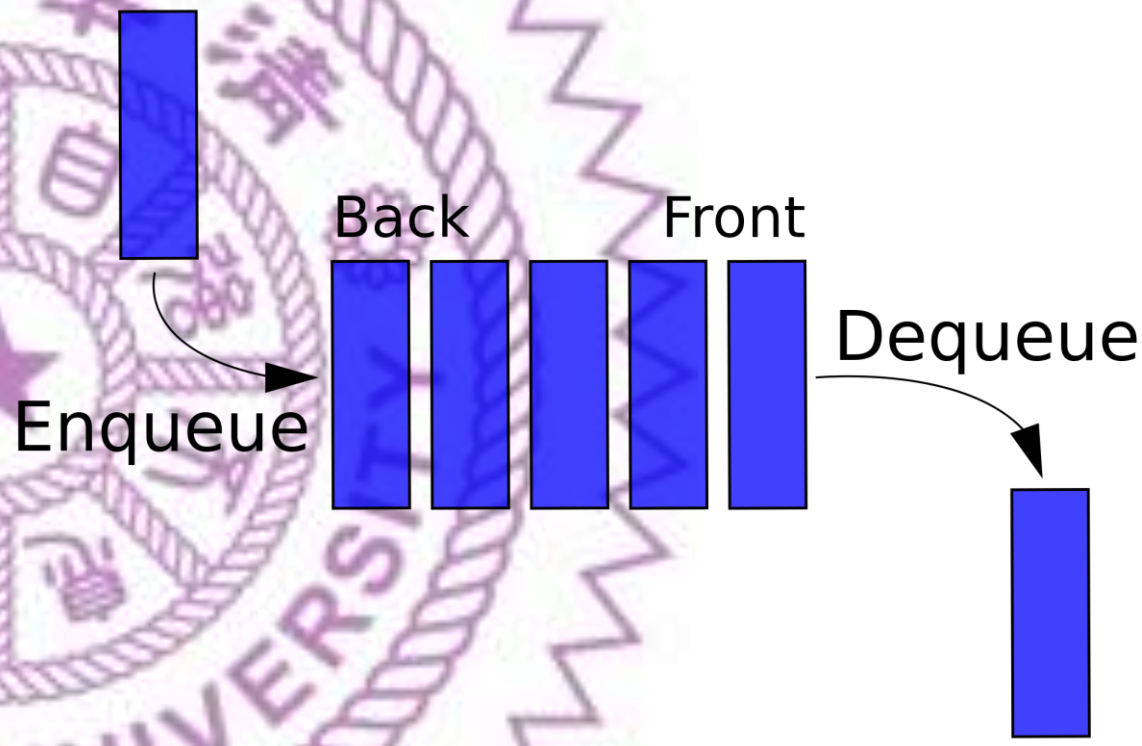
廣度優先搜尋

日月卦長

Queue

- push/ emplace(X) 加入資料
- front() 看最前面的資料
- pop() 刪掉最前面的資料
- First in first out



std::queue


```cpp
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#include <queue>
#include <vector>
using namespace std;
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  cout << Q.front() << ' ' << Q.back();
  return 0;
}
```

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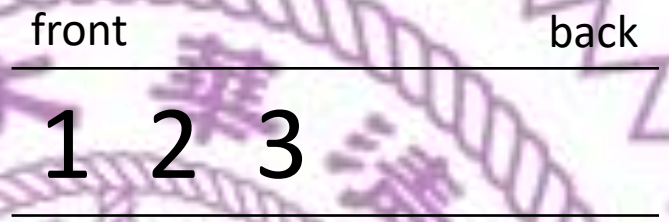
std::queue

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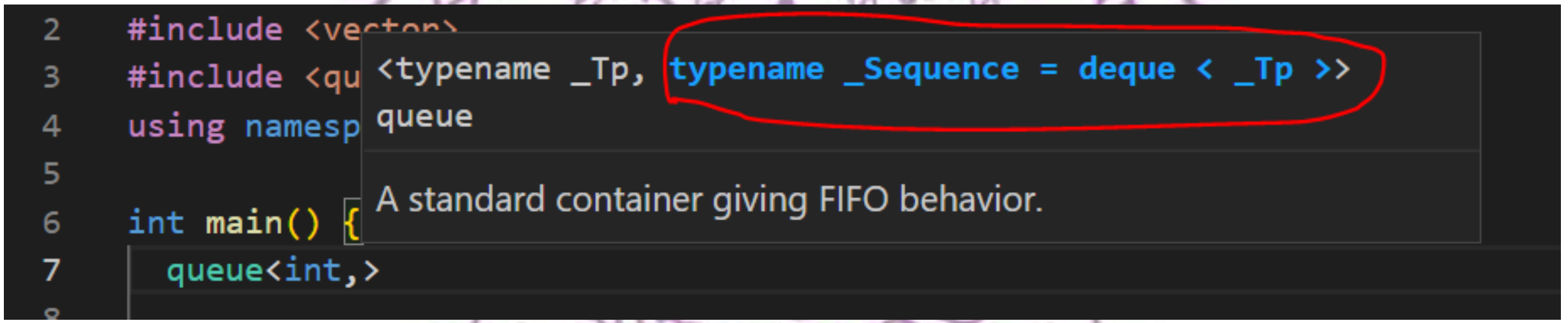
是用 deque 做出來的



元素存取

	queue
尾巴	back()
頭部	front()

數量資訊

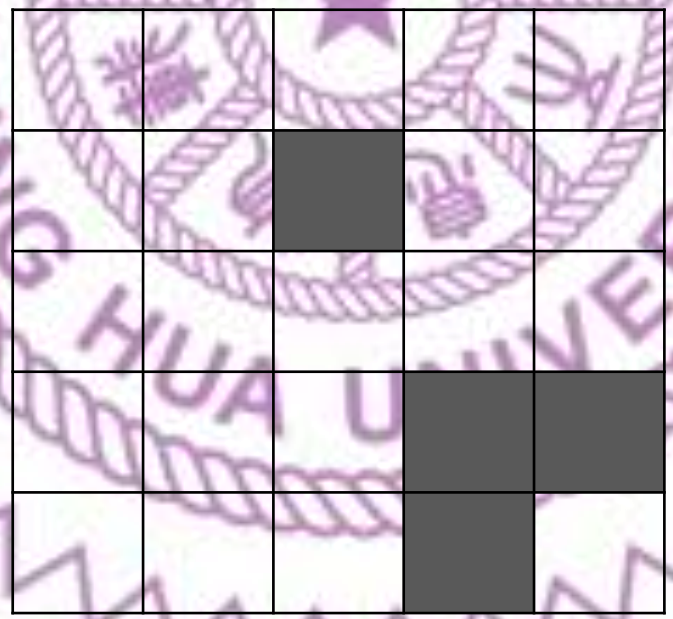
	queue
裡面是不是空的	empty()
裡面有多少東西	size()

洪水算法

Flood fill algorithm

倒水問題

- 在一個 $N \times M$ 的格子圖中，選一個點倒水
- 溢出來水會往某些方向流(看題目規定水怎麼流)
- 有些時候會有障礙物無法淹沒



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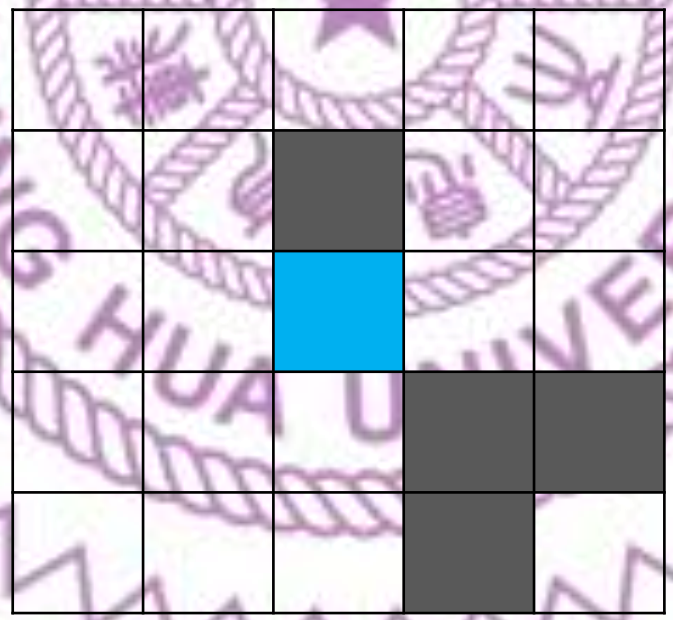

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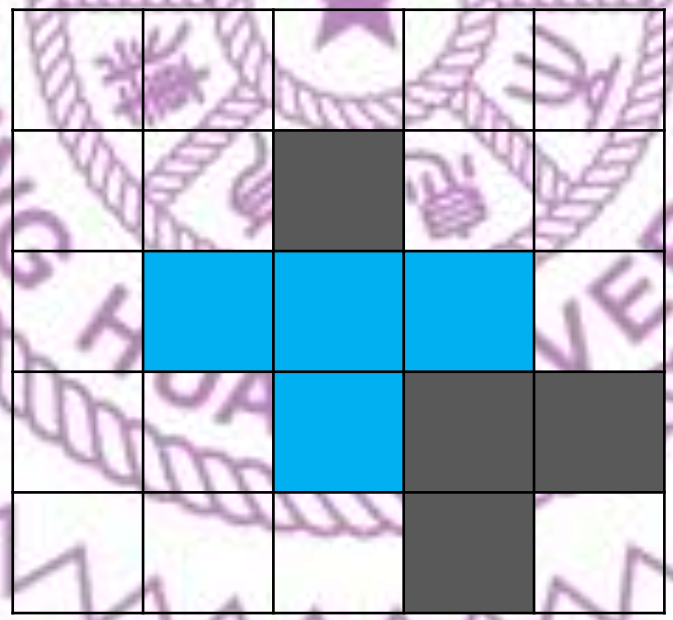

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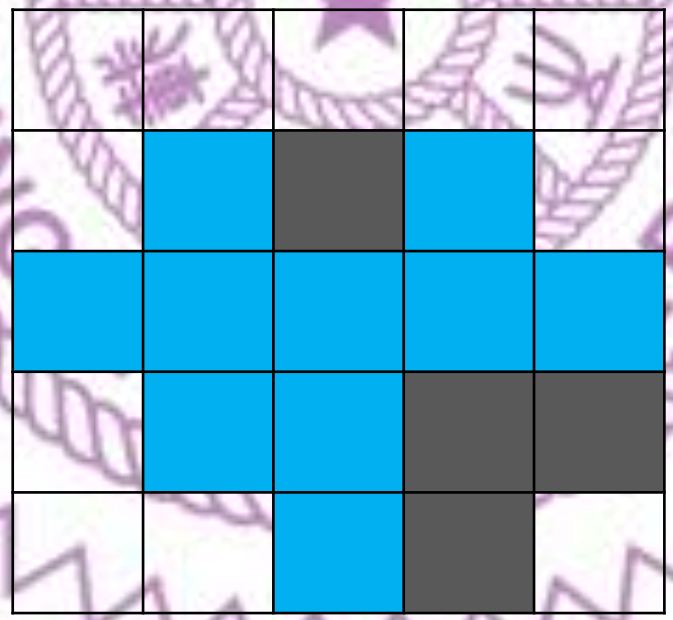

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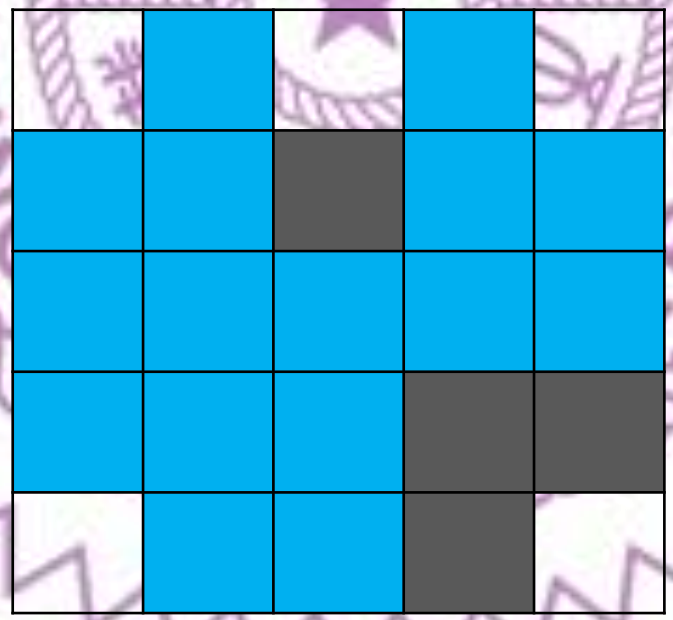

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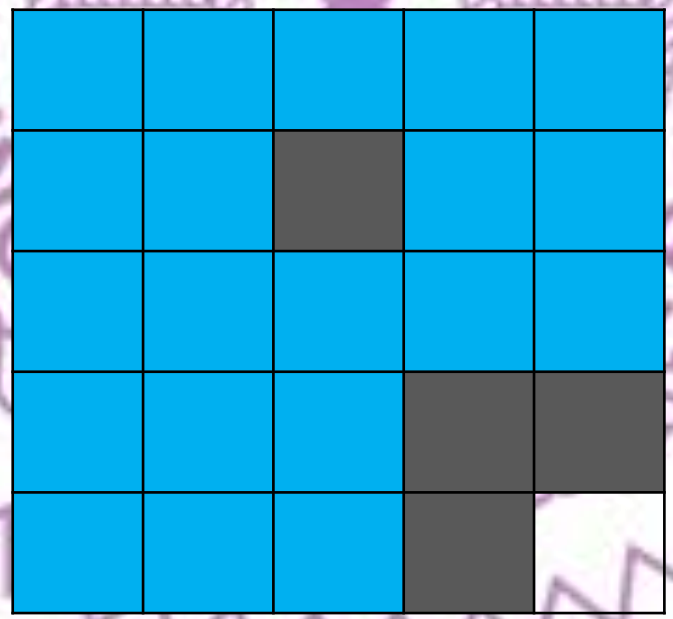


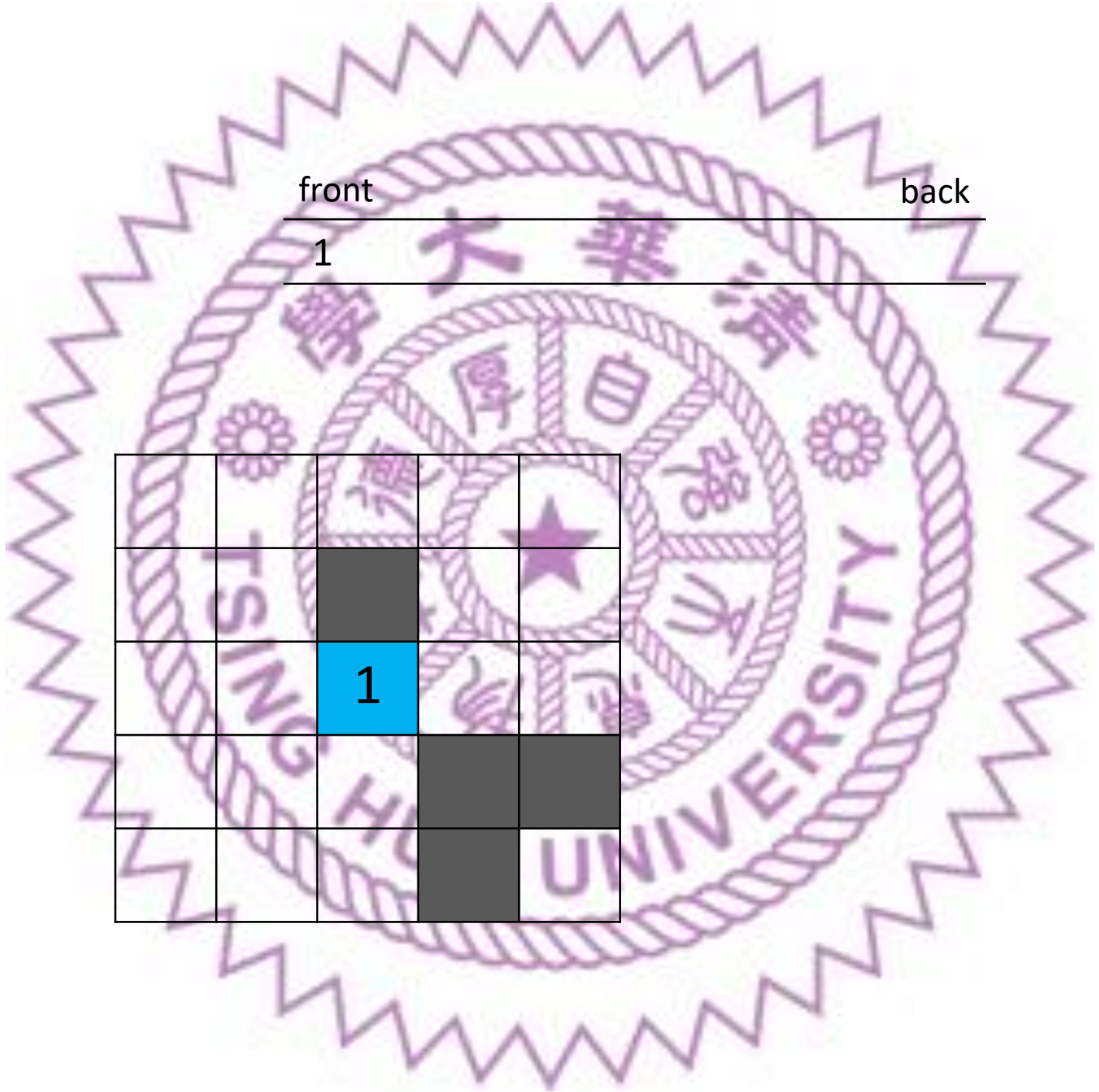
暴力法 $O(N^2M^2)$

- 設起始點的level為0
- L = 1
- While True
  - 掃描所有的格子 (x,y):
    - 如果(x,y) 非障礙物 且 沒設置level 且 上下左右的格子的 level 等於 L-1
      - 設(x,y)的level為 L
  - 如果都沒有格子的level為L-1
    - Break
  - L = L + 1

能淹出水的格子

用 queue 記錄所有邊界格子

Demo



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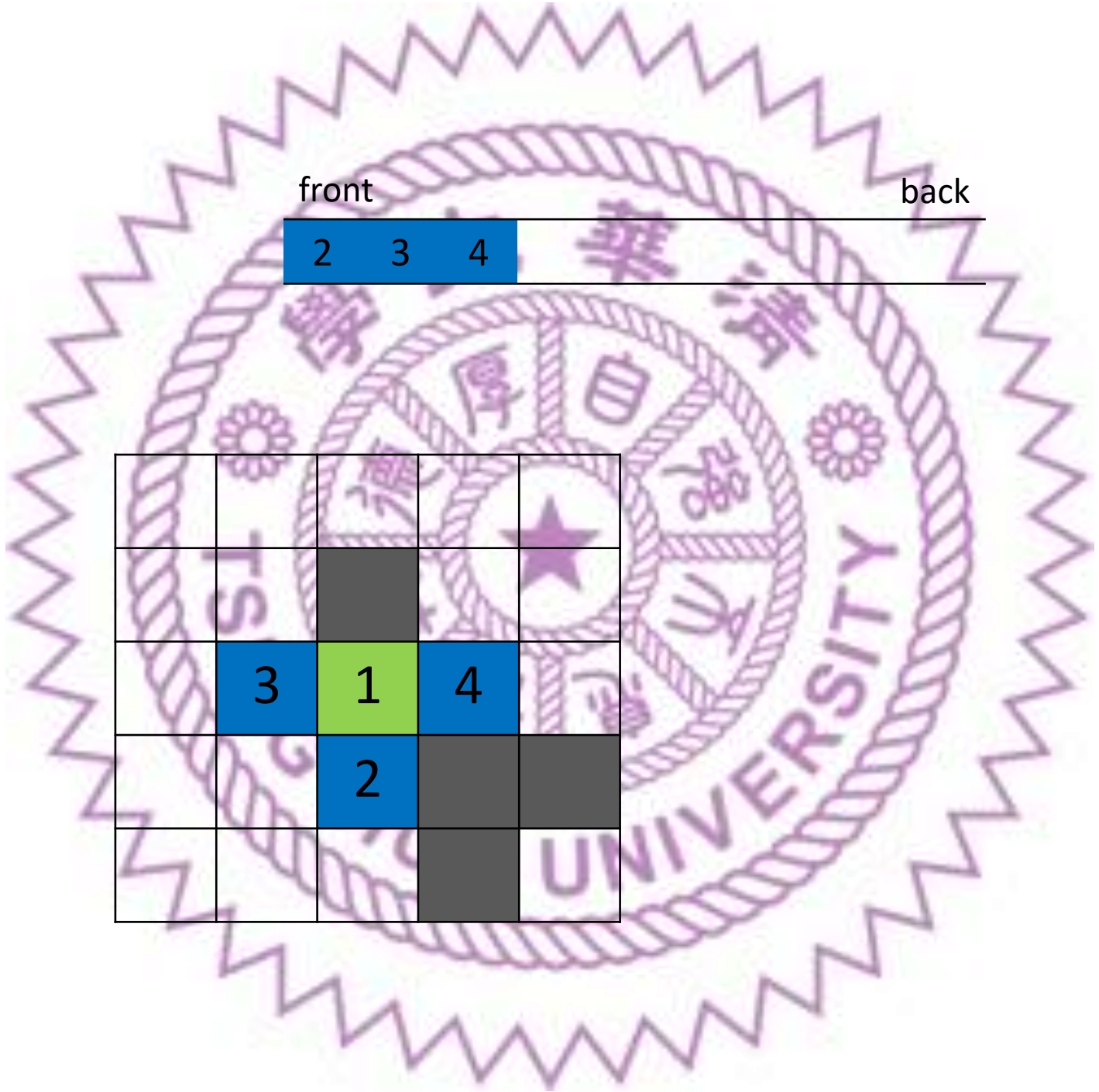

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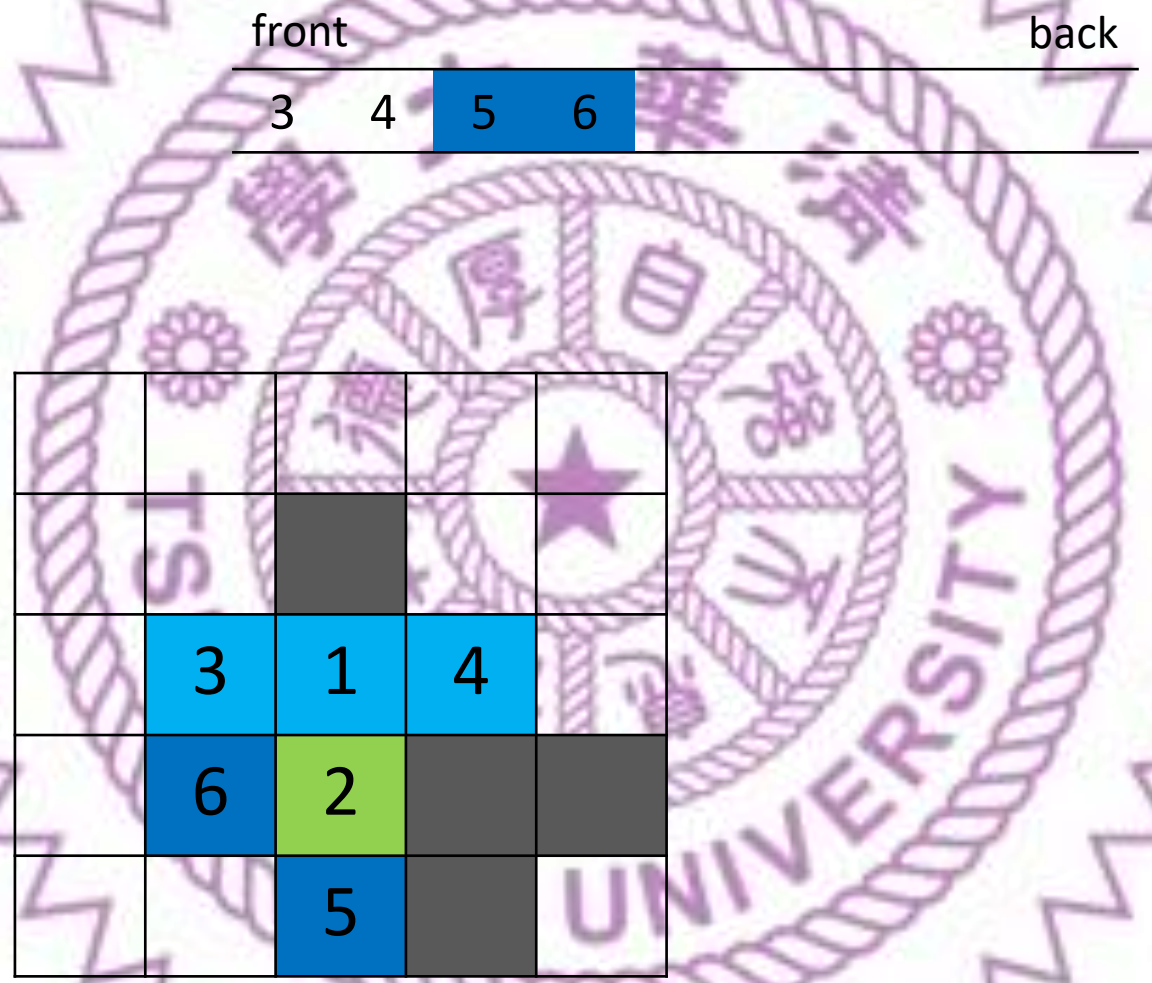

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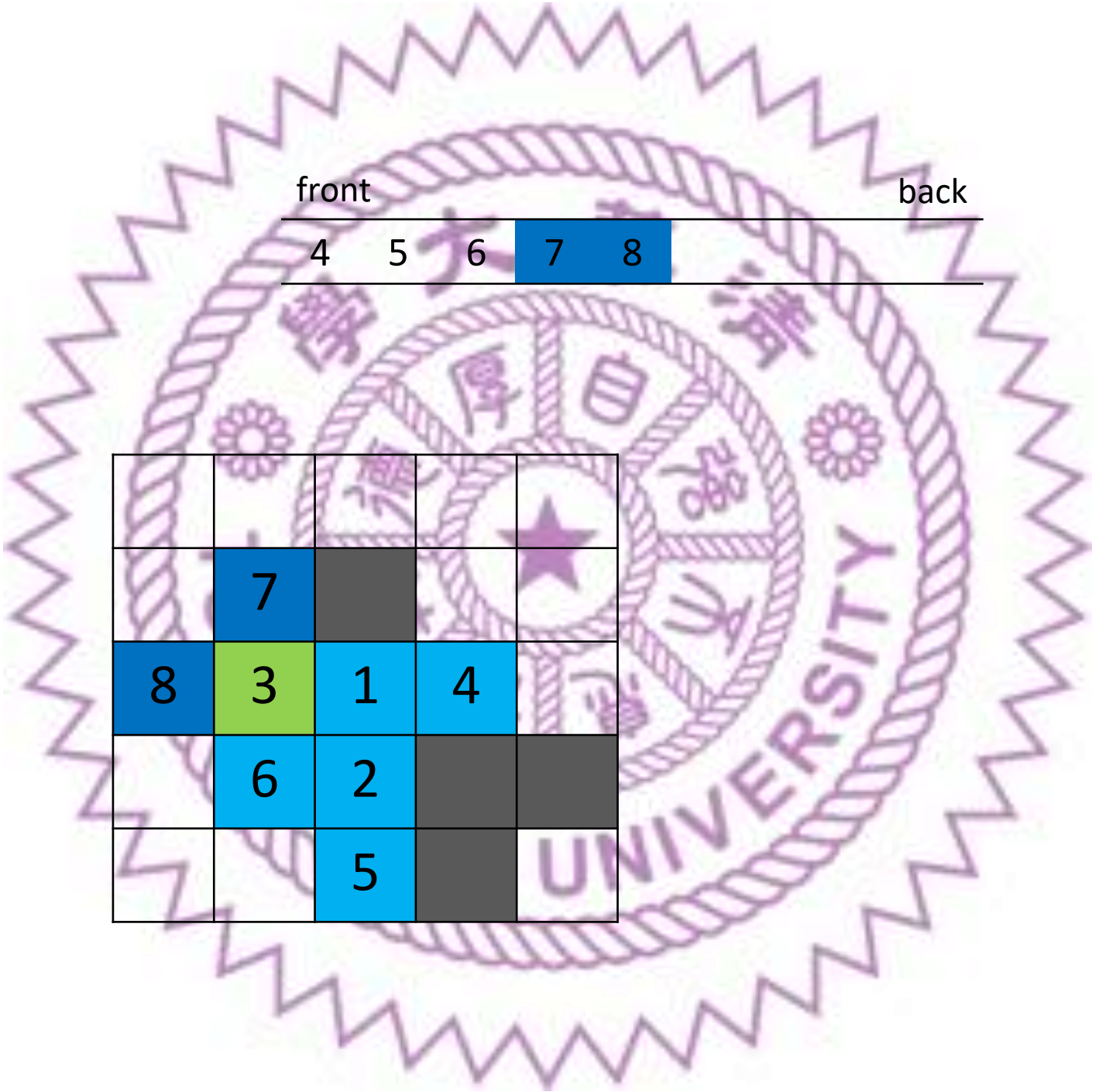

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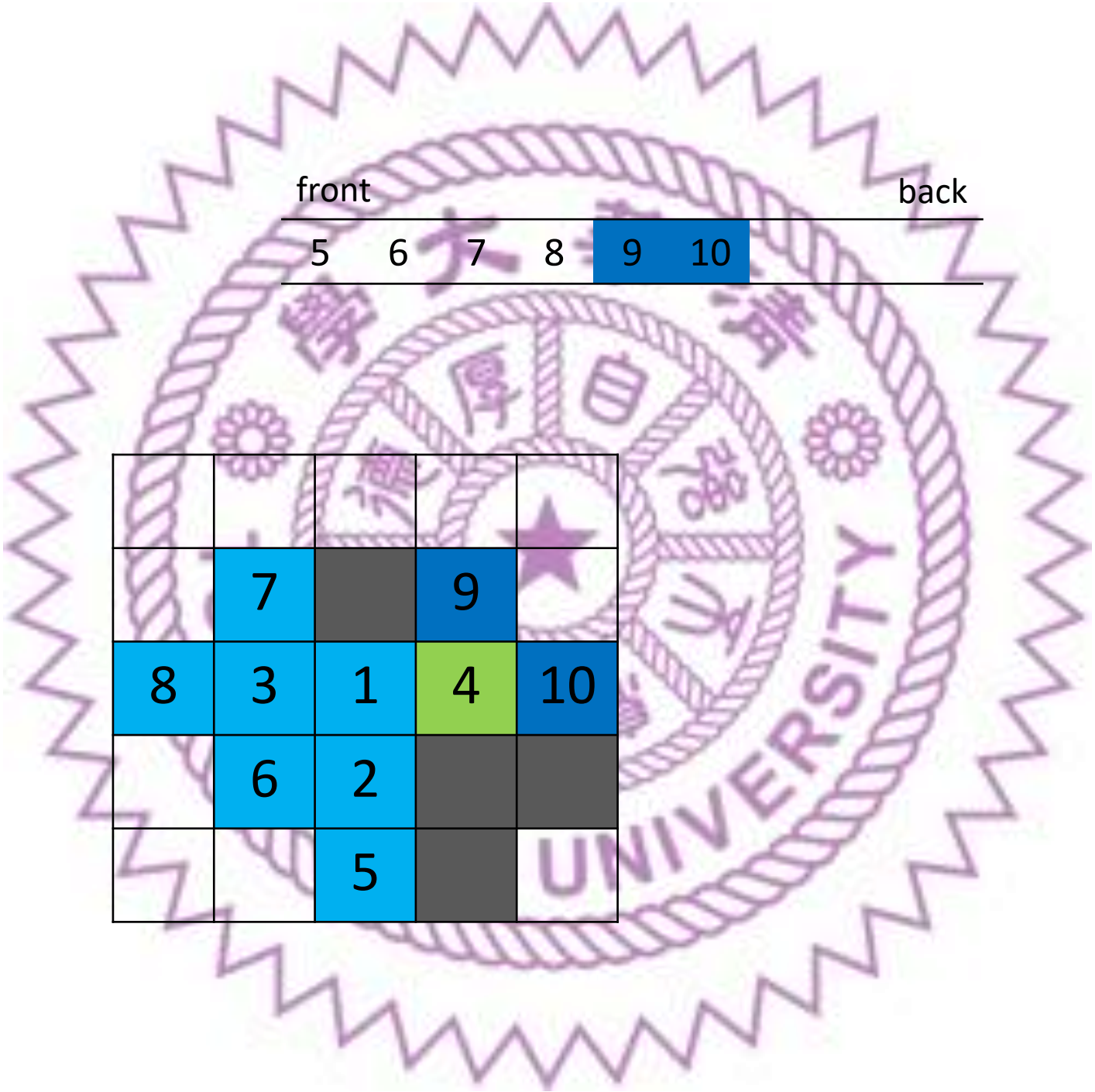

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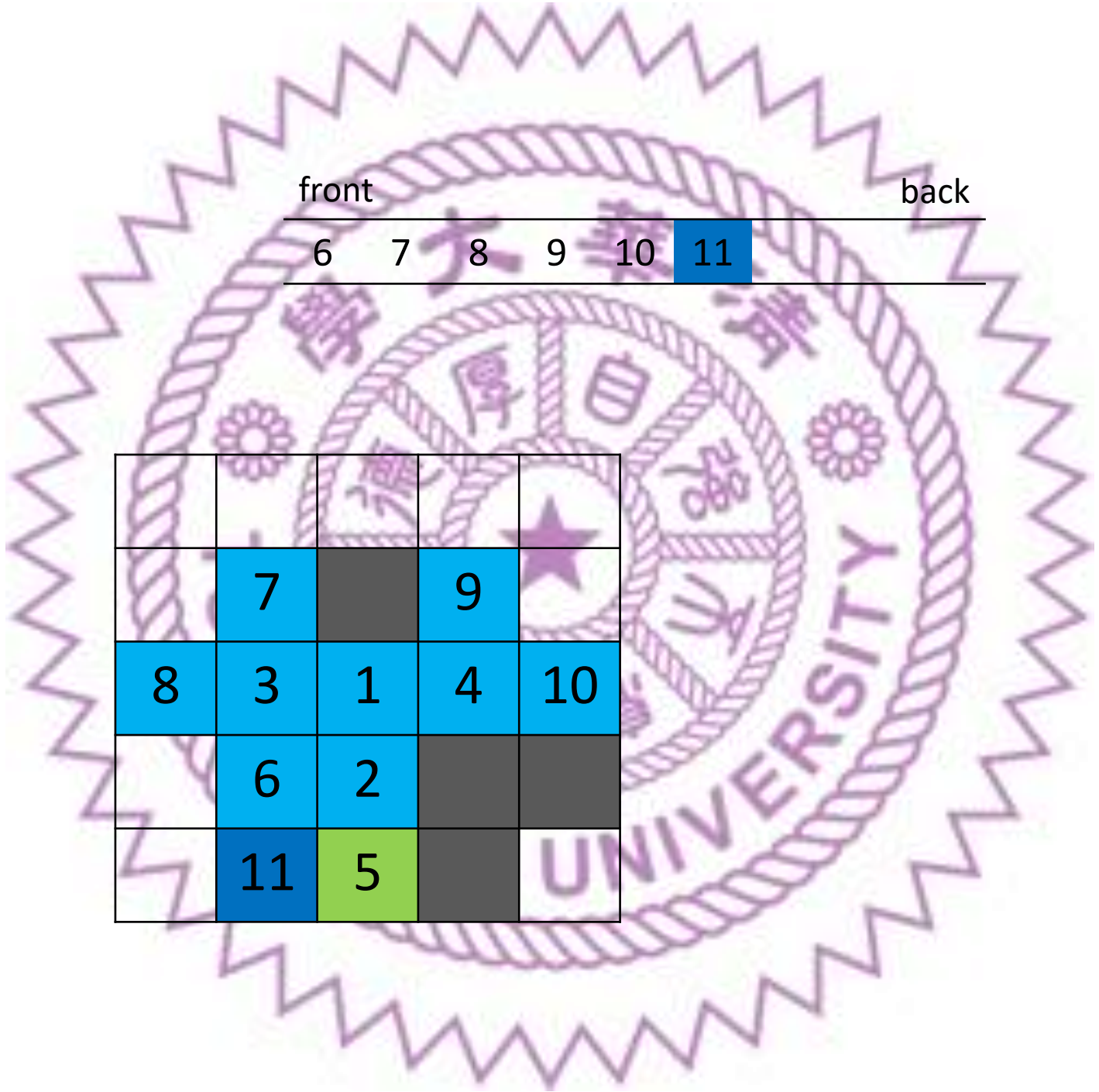

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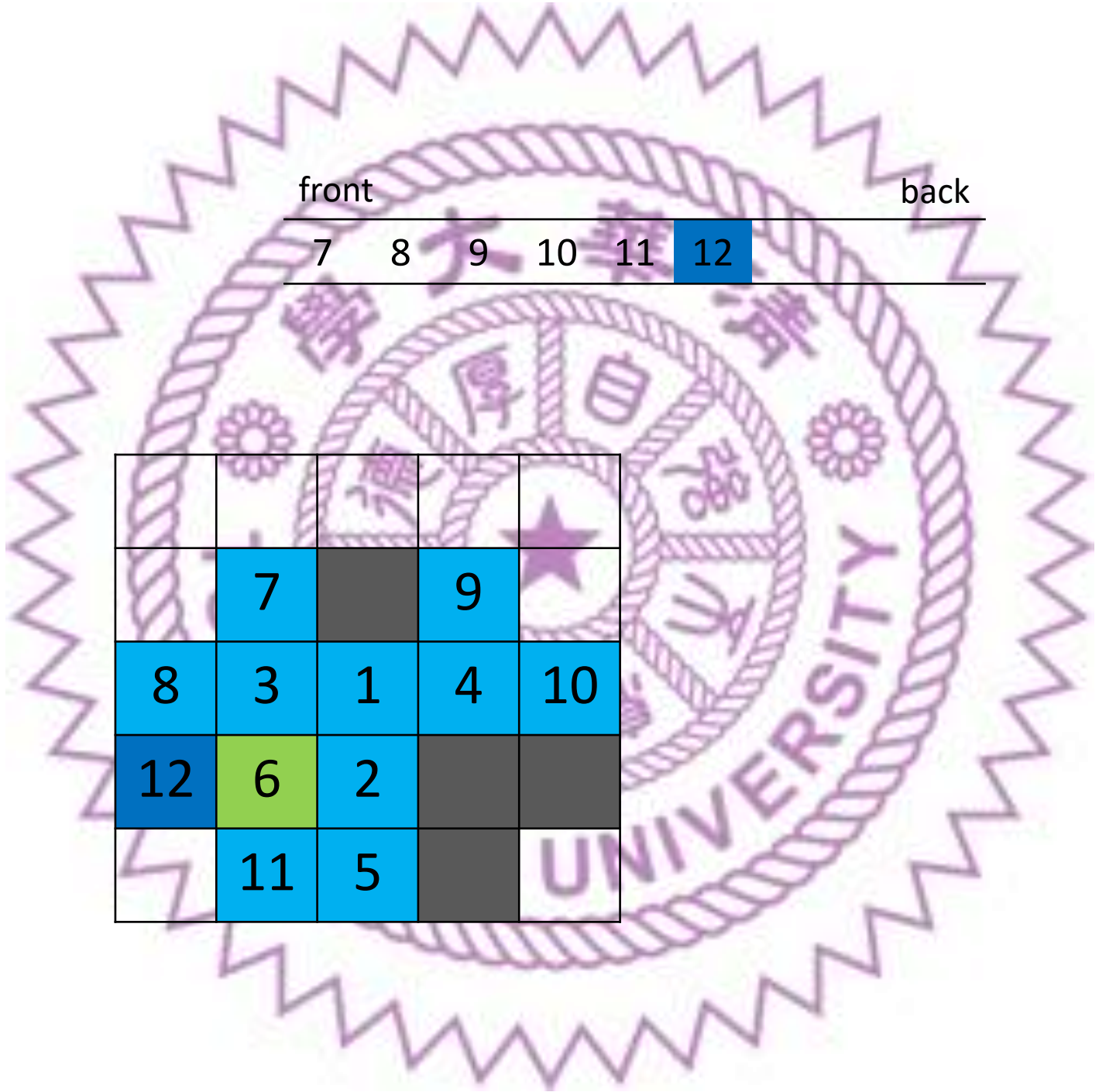

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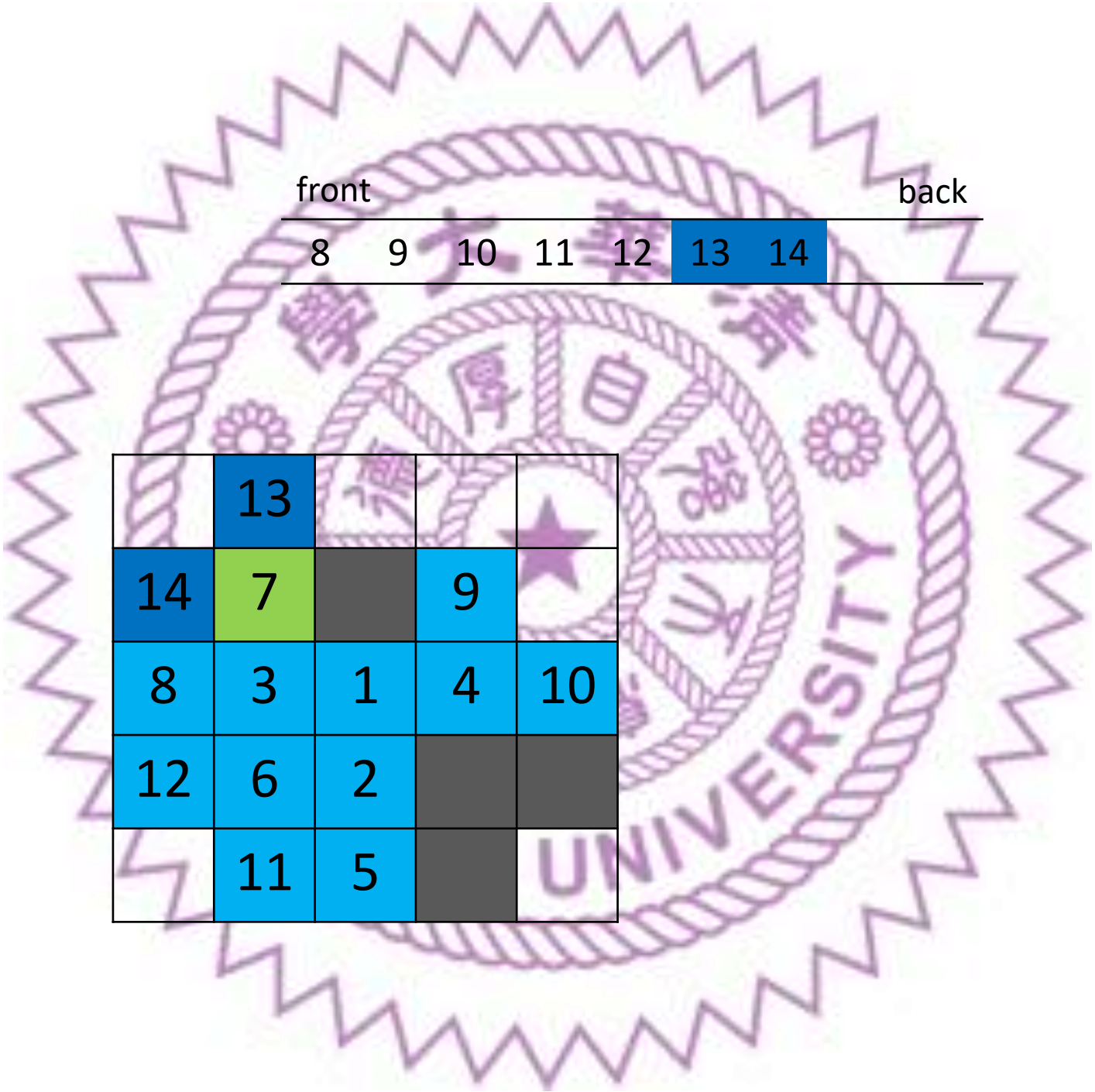

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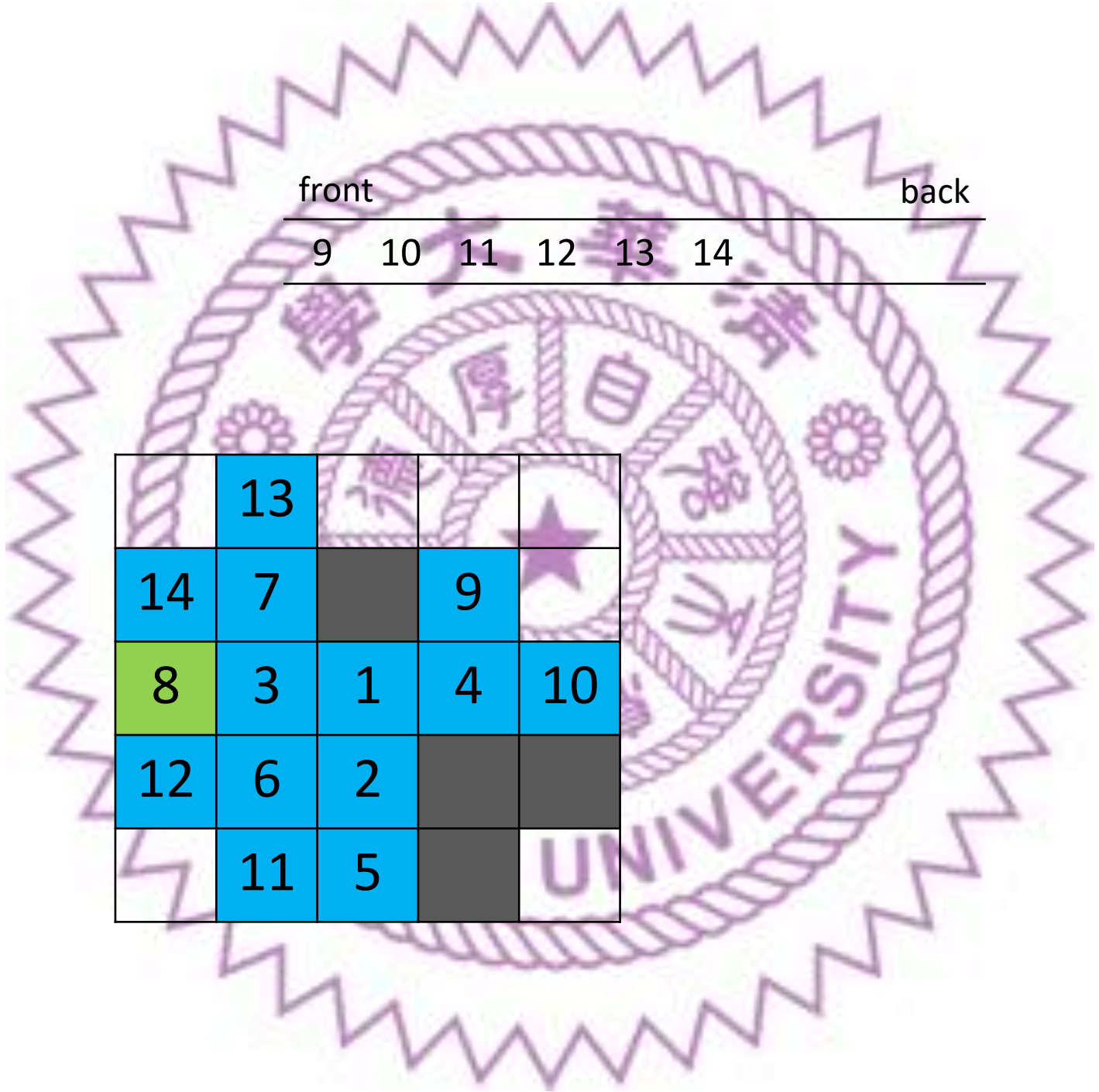

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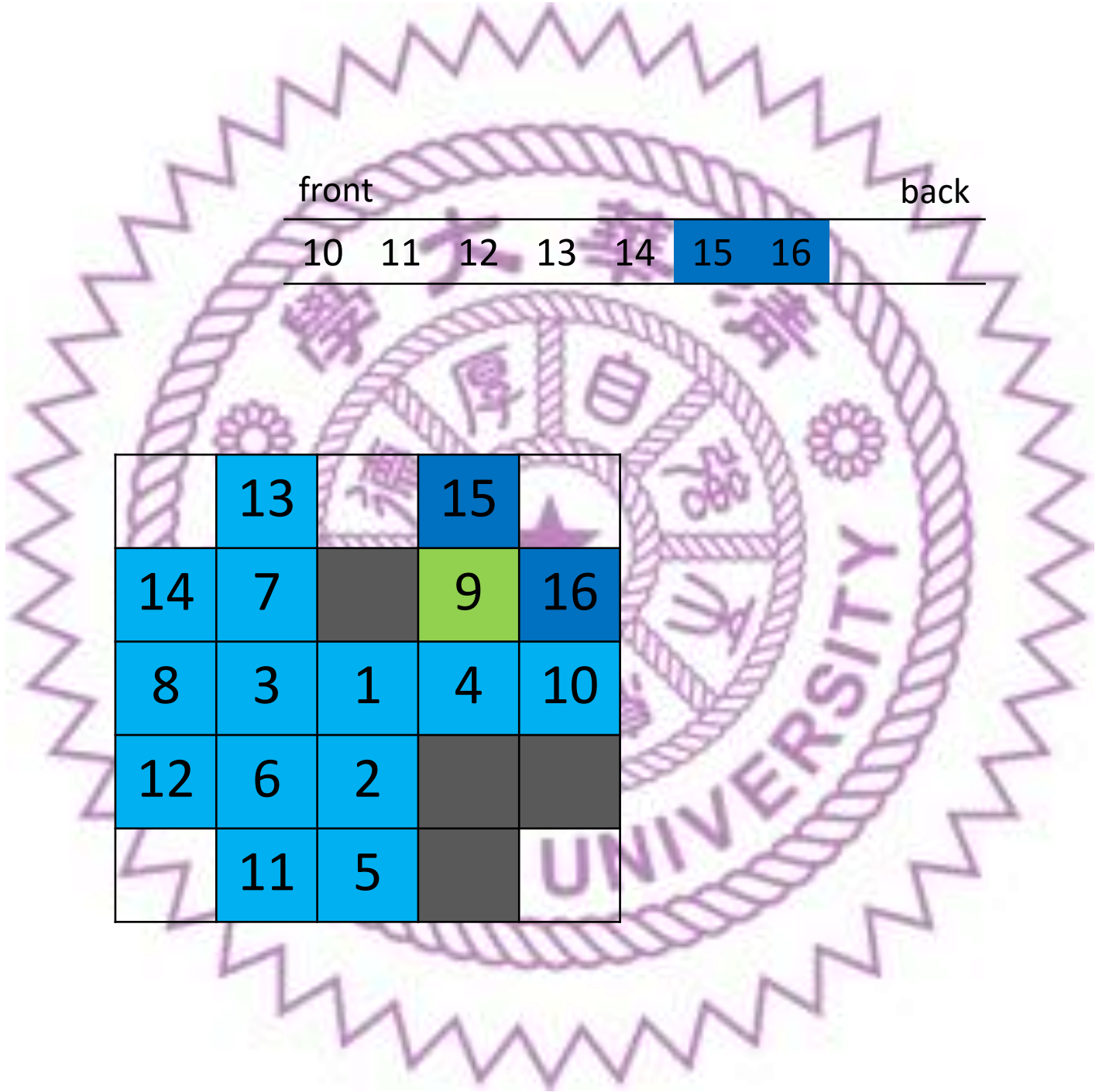


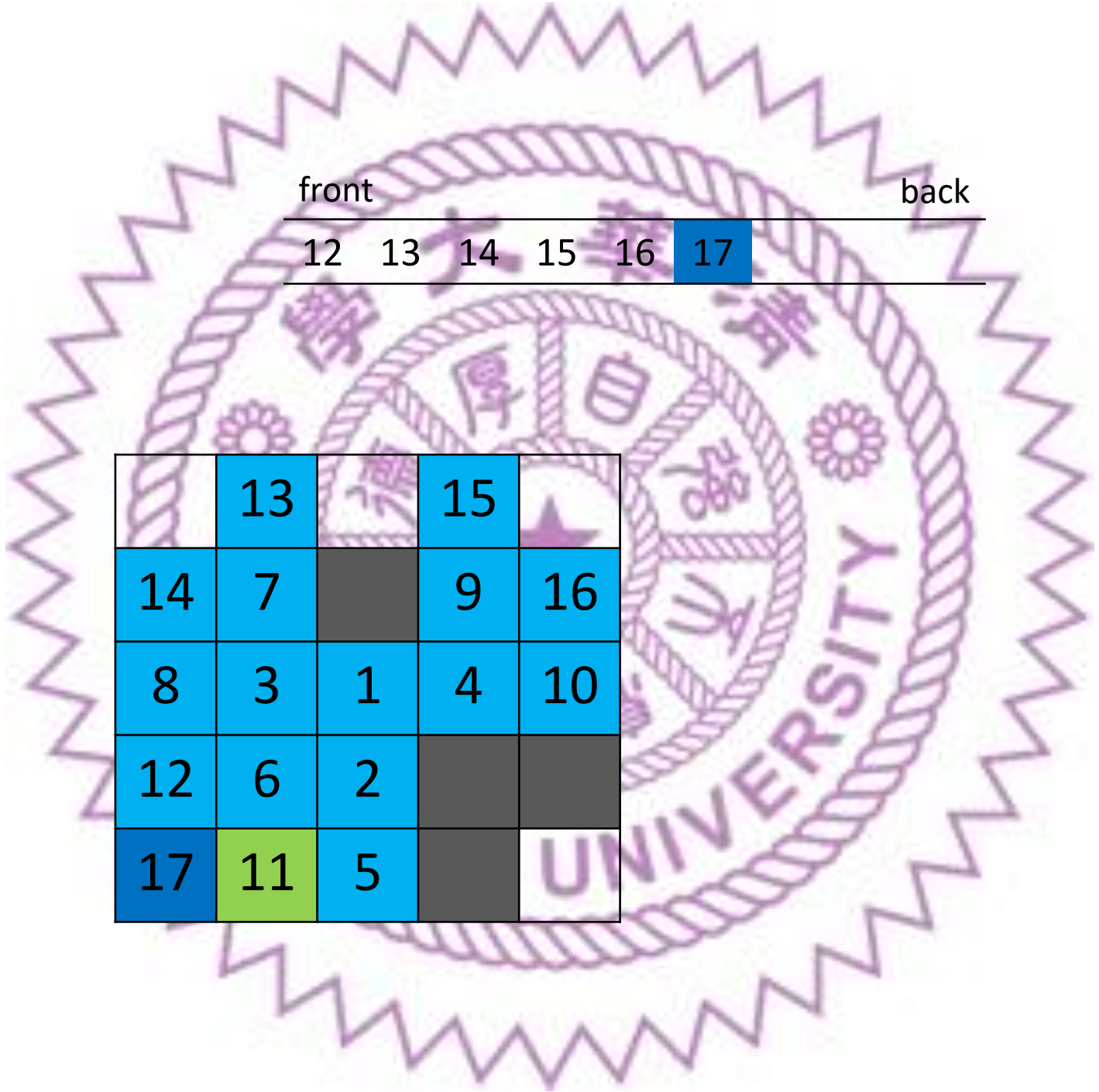
Demo

front back

11 12 13 14 15 16

	13		15	
14	7		9	16
8	3	1	4	10
12	6	2		
	11	5		

Demo



Demo

front back

13 14 15 16 17

	13		15	
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17	11	5		

Demo

front back

14	15	16	17	18	19

18	13	19	15	
14	7		9	16
8	3	1	4	10
12	6	2		
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Demo

front back

15 16 17 18 19

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Demo

front back

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Demo

front back

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Demo

front back

18 19 20

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Demo

front back

19 20

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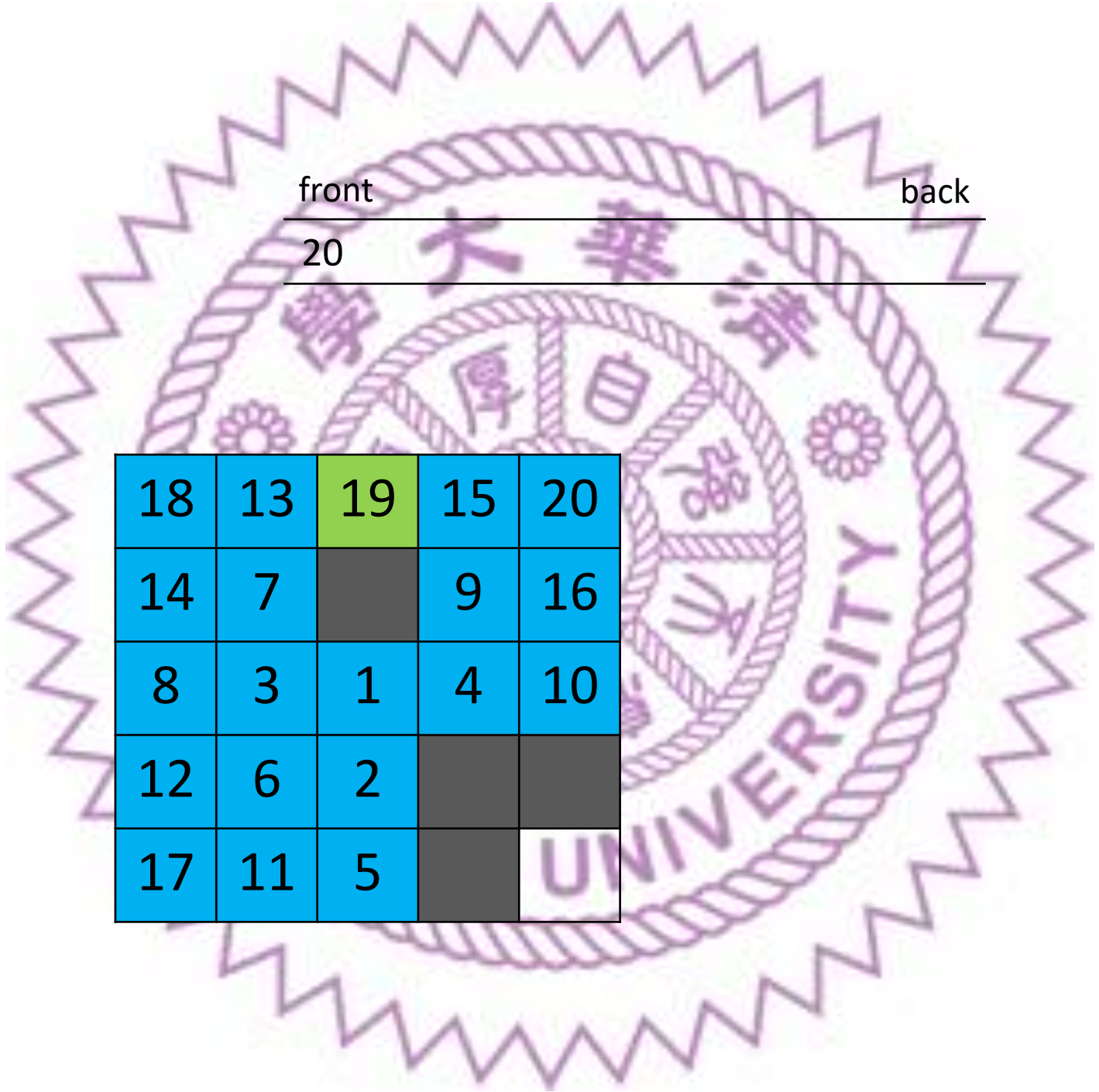
Demo

front back

20



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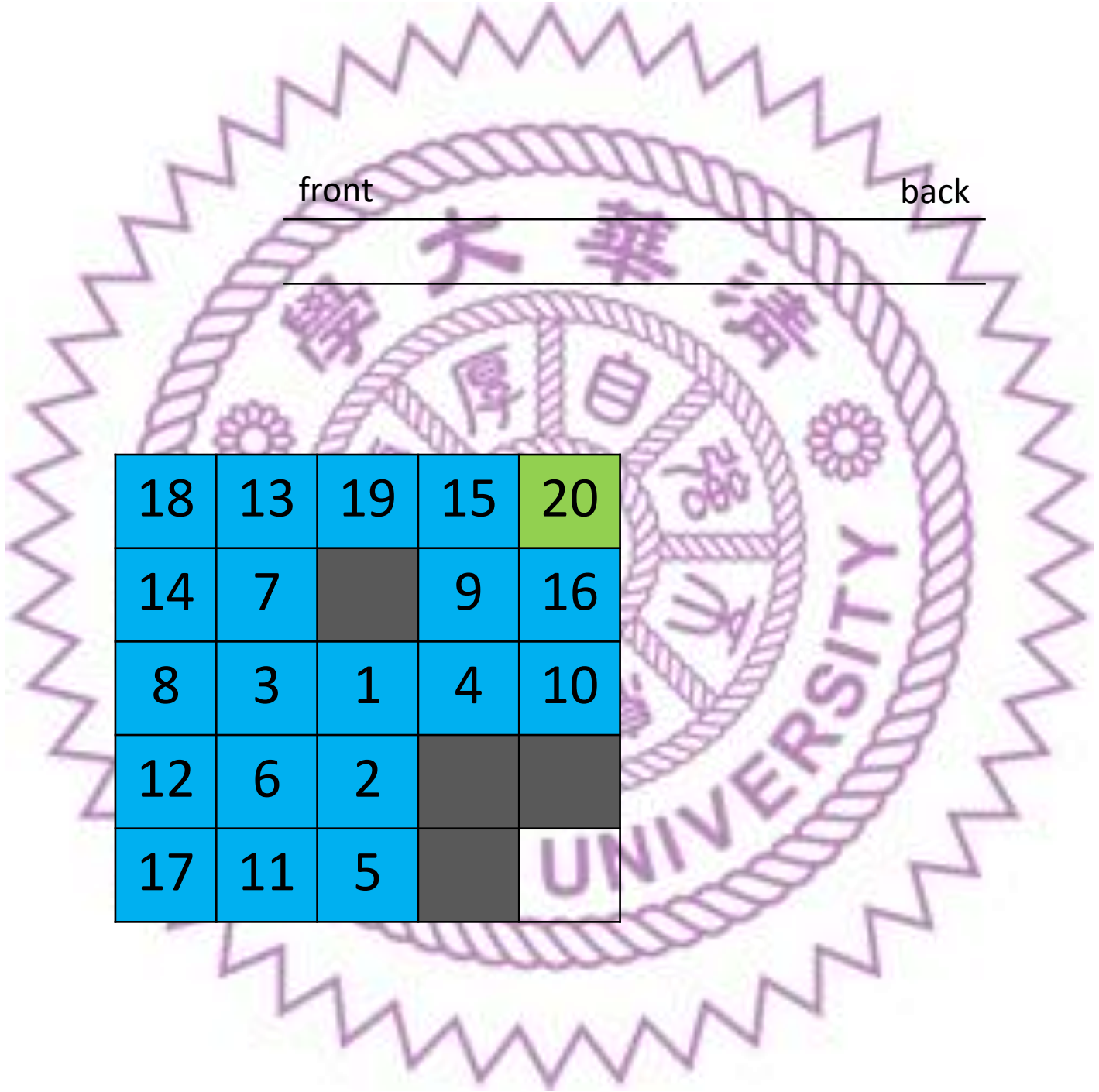


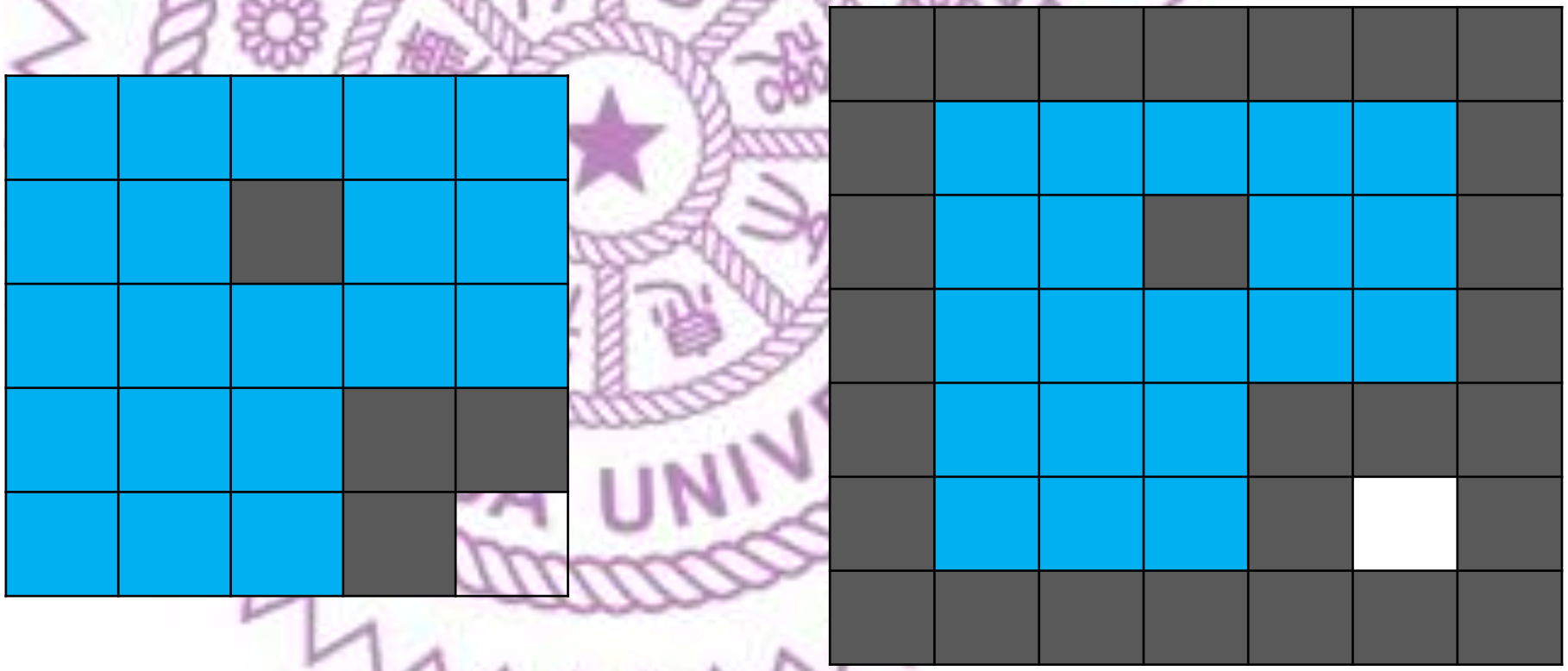
實作細節

- 有障礙物？就當作該格已經被填過了

- 同時有多個格子要倒水？
  - 一開始把所有有水的格子丟到queue中

實作細節

- 邊界好麻煩怎麼辦？
  - 弄一個外框，然後把外框都標記成障礙物



實作細節

```
void bfs(int x, int y) {
  std::queue<std::pair<int,int>> Q;
  Q.emplace(x, y);
  while(Q.size()) {
    std::tie(x, y) = Q.front();
    Q.pop();
    if(grid[x][y]) continue;
    grid[x][y] = true;
    Q.emplace(x+1, y);
    Q.emplace(x, y+1);
    Q.emplace(x-1, y);
    Q.emplace(x, y-1);
  }
}
```

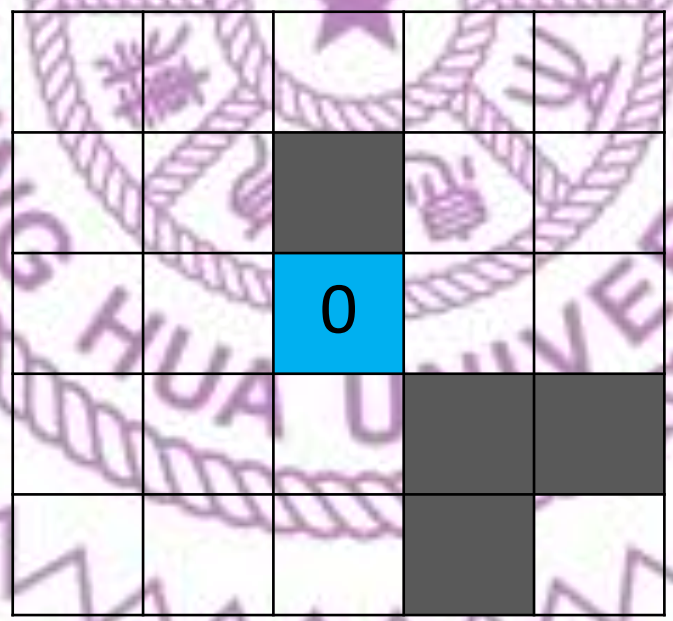
嶄新的複雜度

- 每一格最多只會被丟進queue一次
  - 進入queue內元素不超過格子個數=$O(NM)$
- 每個格子向鄰近的格子溢出只需要常數的操作$O(1)$
- 整體複雜度:$O(NM)$

同時擴散

- 觀察我們queue的執行結果，可以發現queue裡面「梯數」一定是非嚴格遞增的
- 只要在queue裡面多紀錄每個格子的「梯數」，然後每回合一口氣把同一個梯數的一起做完

同時擴散



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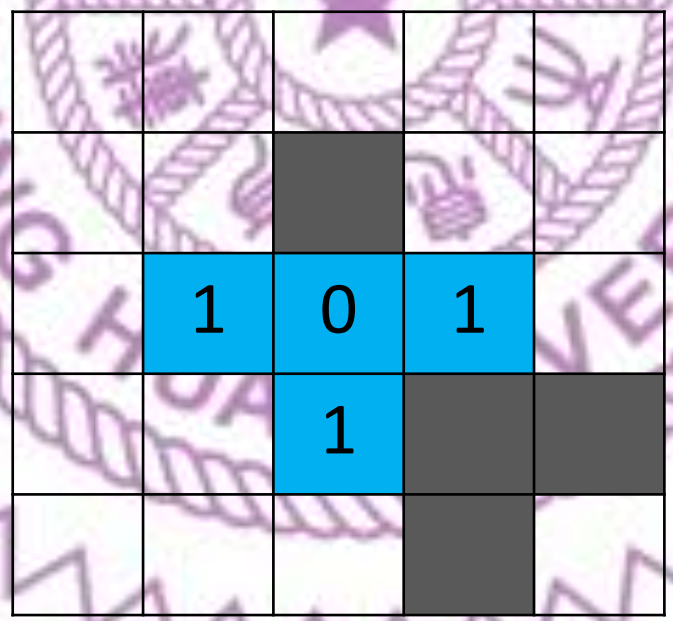

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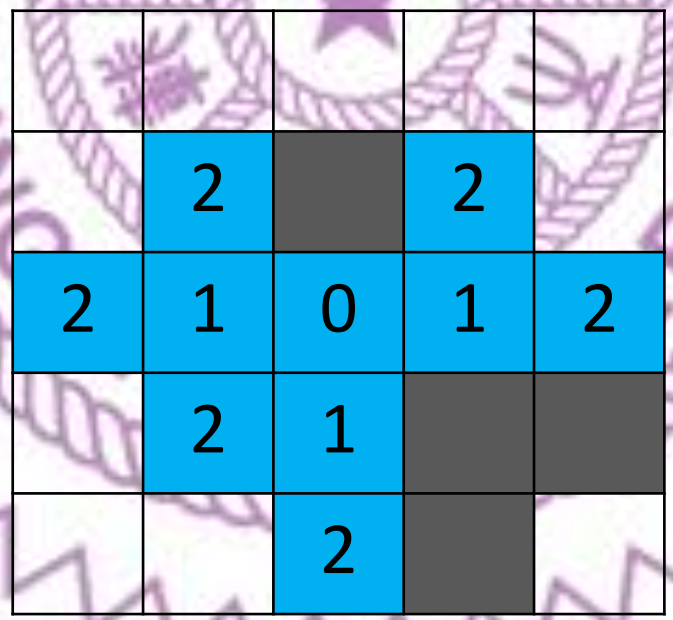
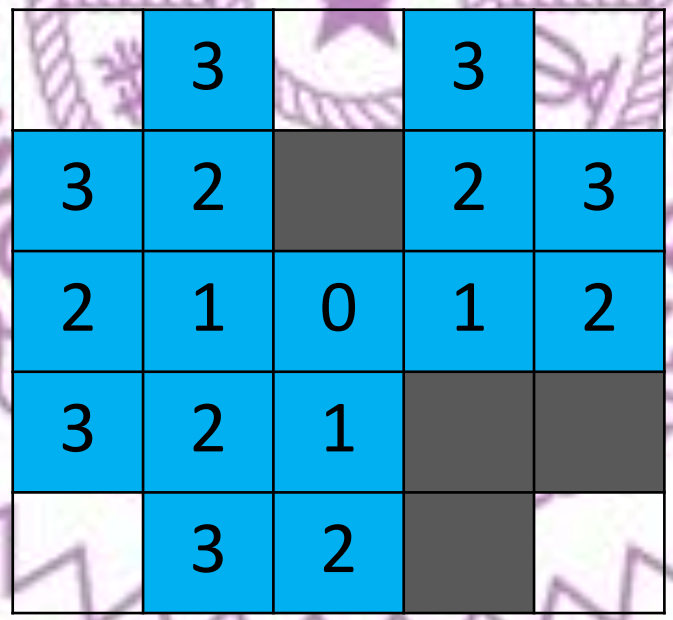


同時擴散



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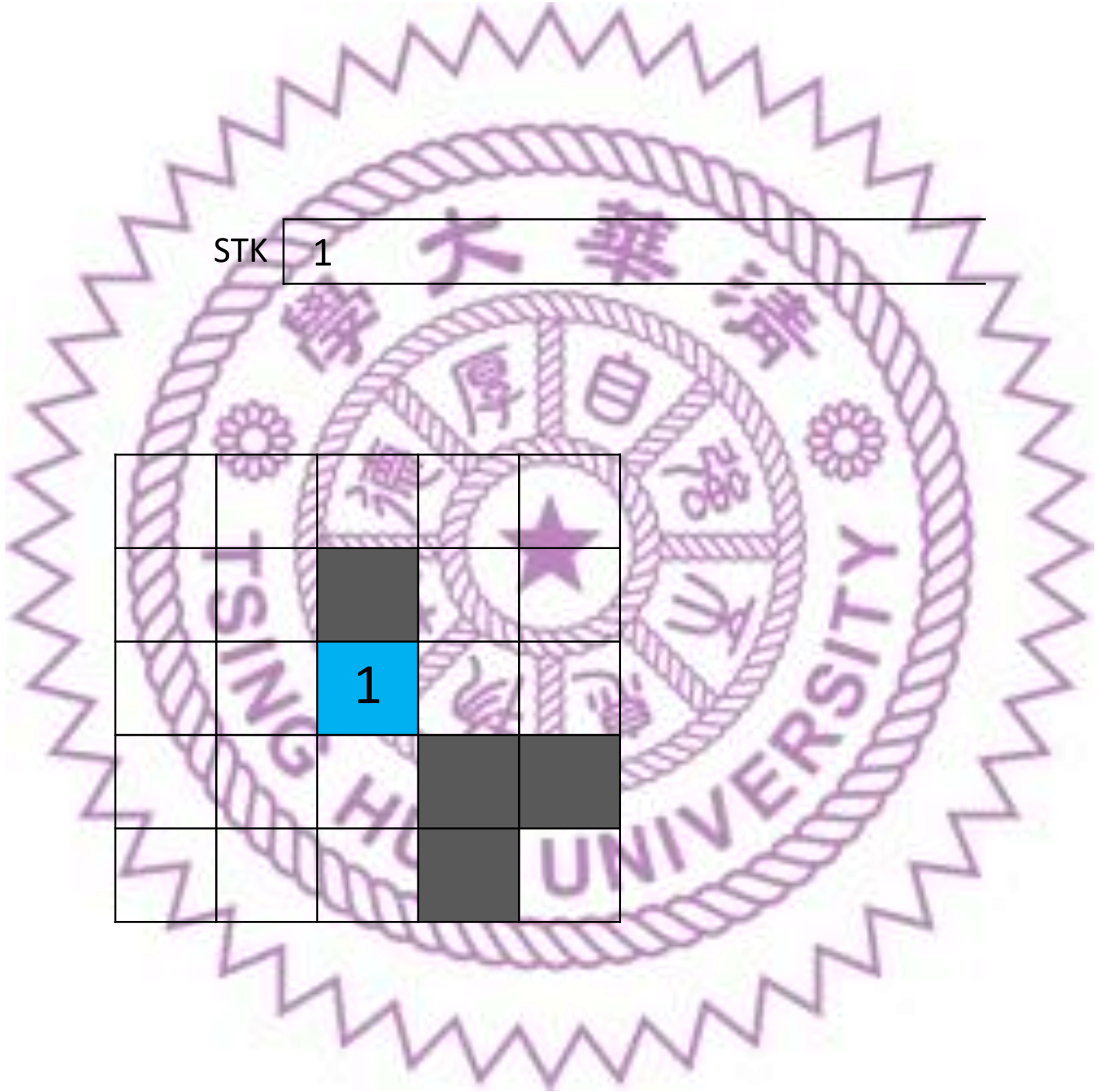
void bfs(int x, int y) {
  std::queue<std::pair<int, int>> Q;
  Q.emplace(x, y);
  int L = 0;
  while (Q.size()) {
    for (int Num = Q.size(); Num--;) {
      std::tie(x, y) = Q.front();
      Q.pop();
      if (grid[x][y])
        continue;
      grid[x][y] = true;
      Level[x][y] = L;
      Q.emplace(x + 1, y);
      Q.emplace(x, y + 1);
      Q.emplace(x - 1, y);
      Q.emplace(x, y - 1);
    }
    L += 1;
  }
}
```

想一下

- 剛剛的淹水演算法中，用的資料結構是queue
- 如果用stack代替queue，會發生什麼事呢？

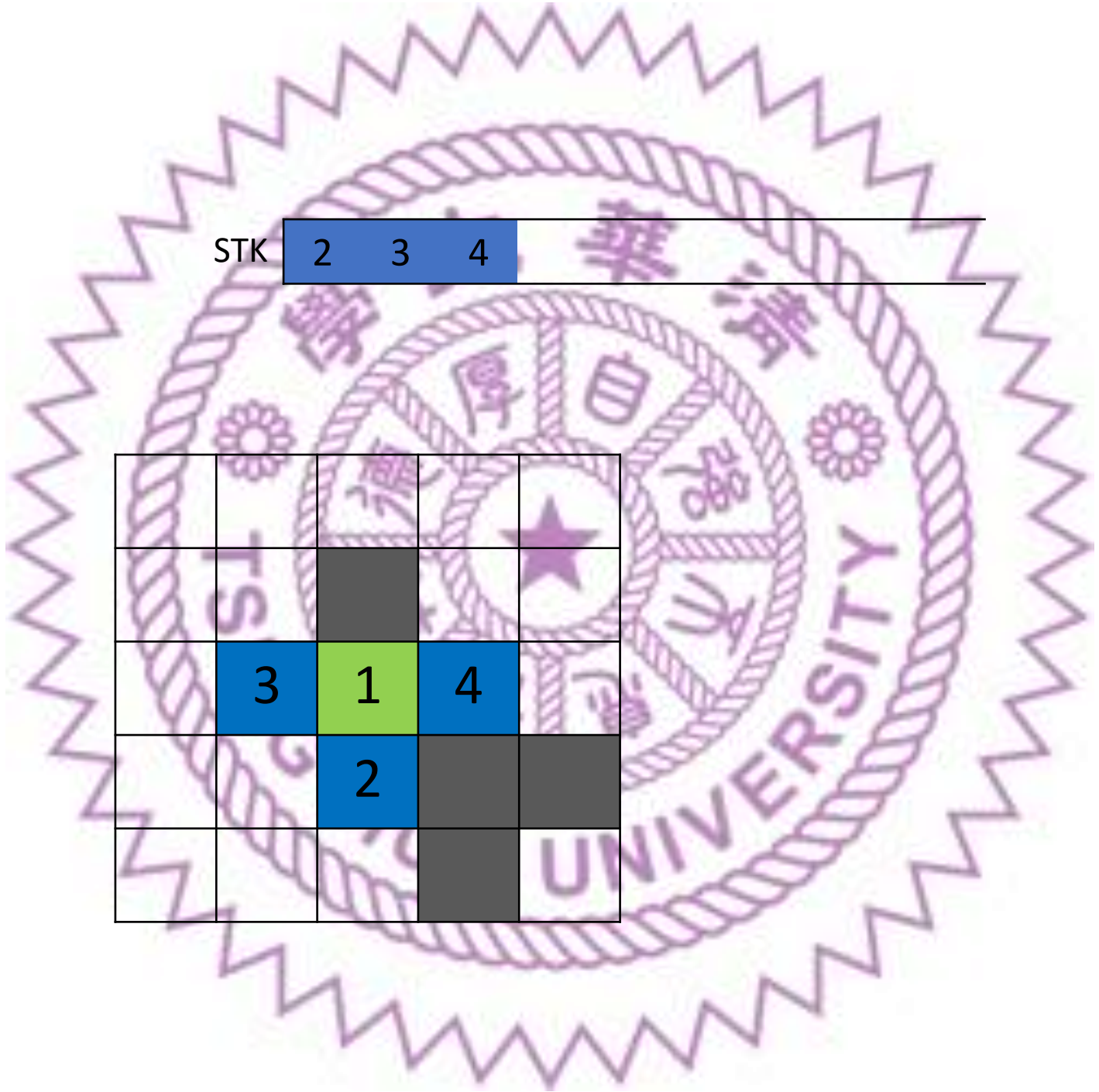
Demo

stack：1



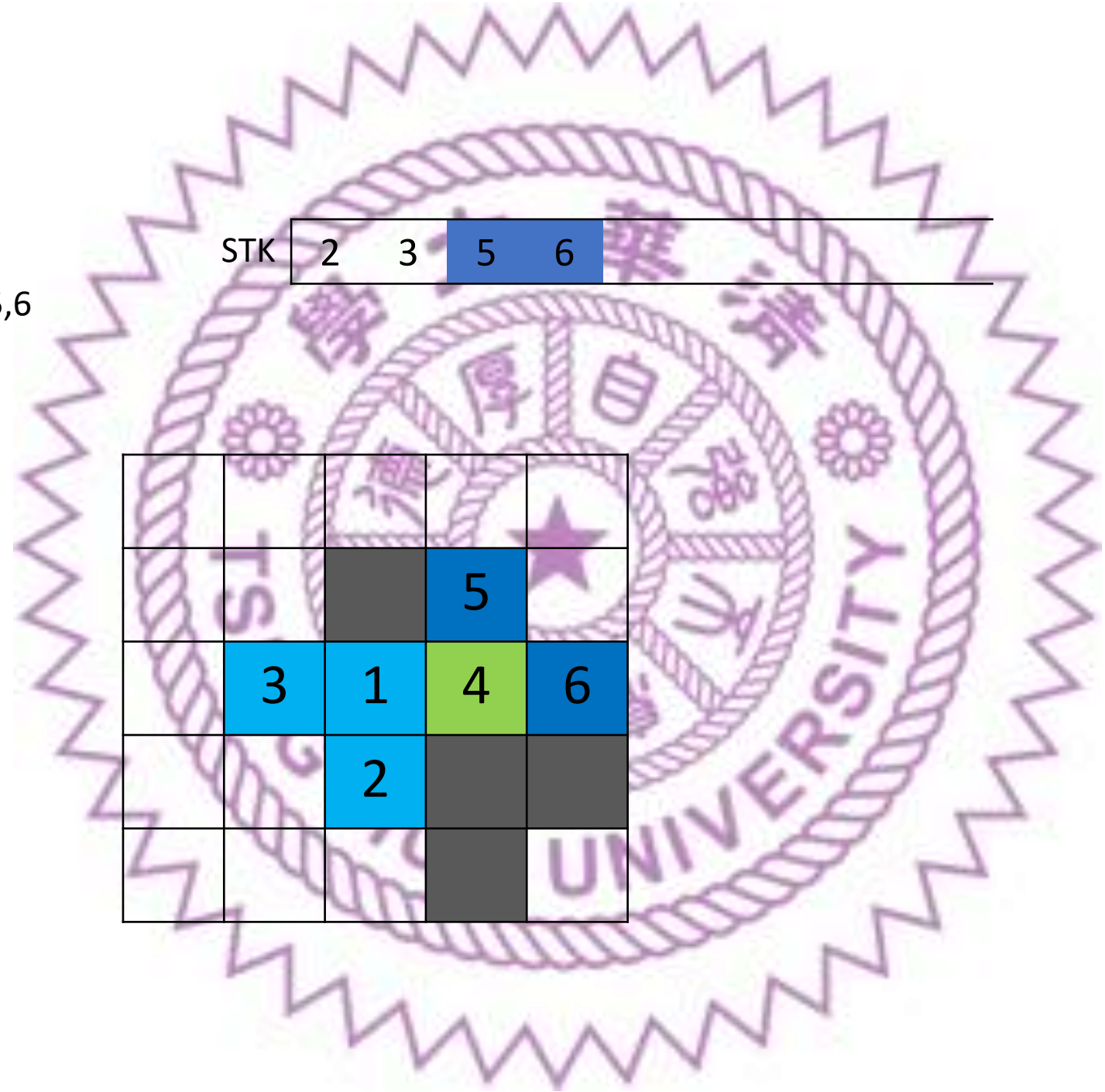
Demo

stack：2,3,4



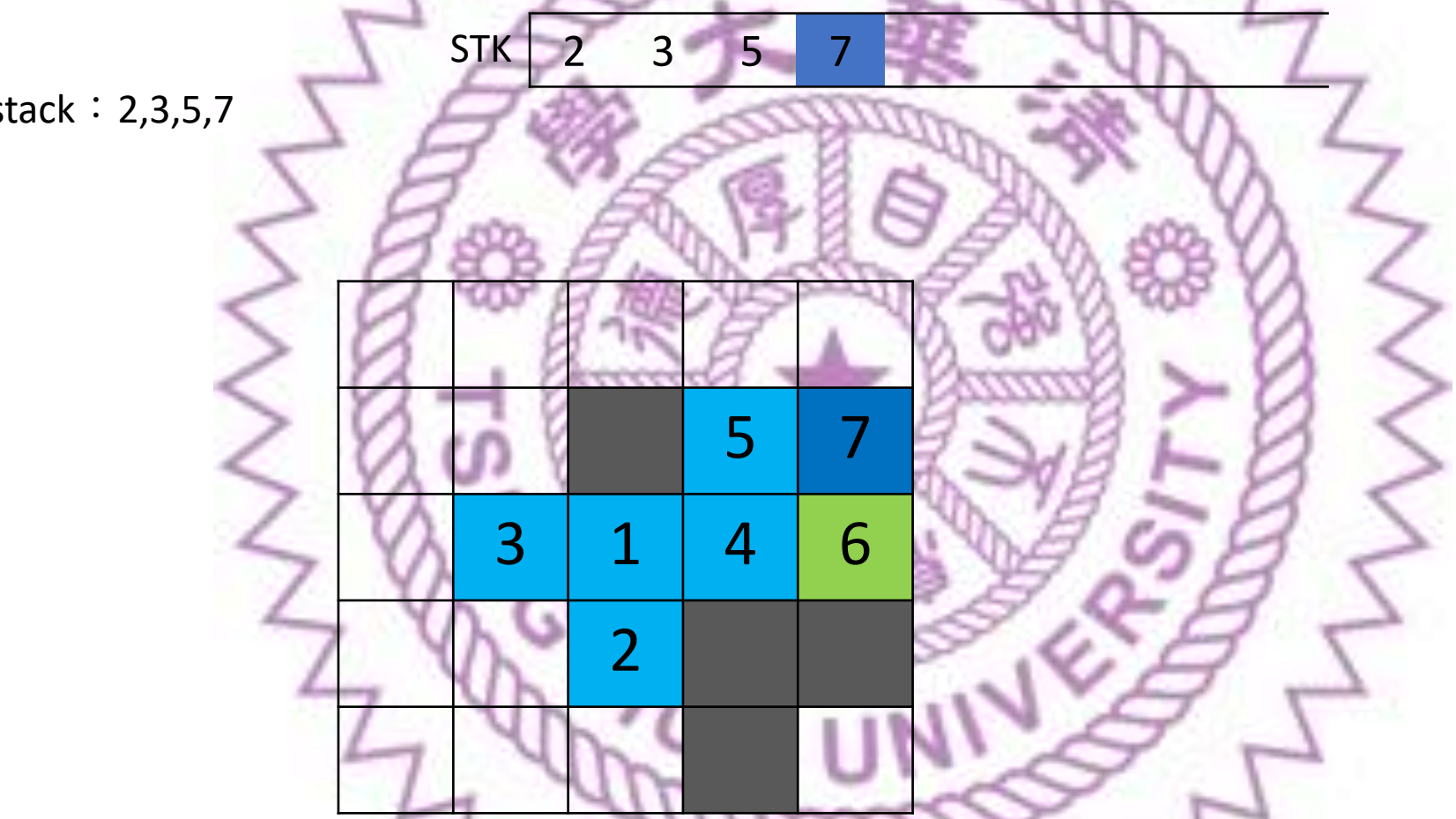
Demo

stack：2,3,5,6



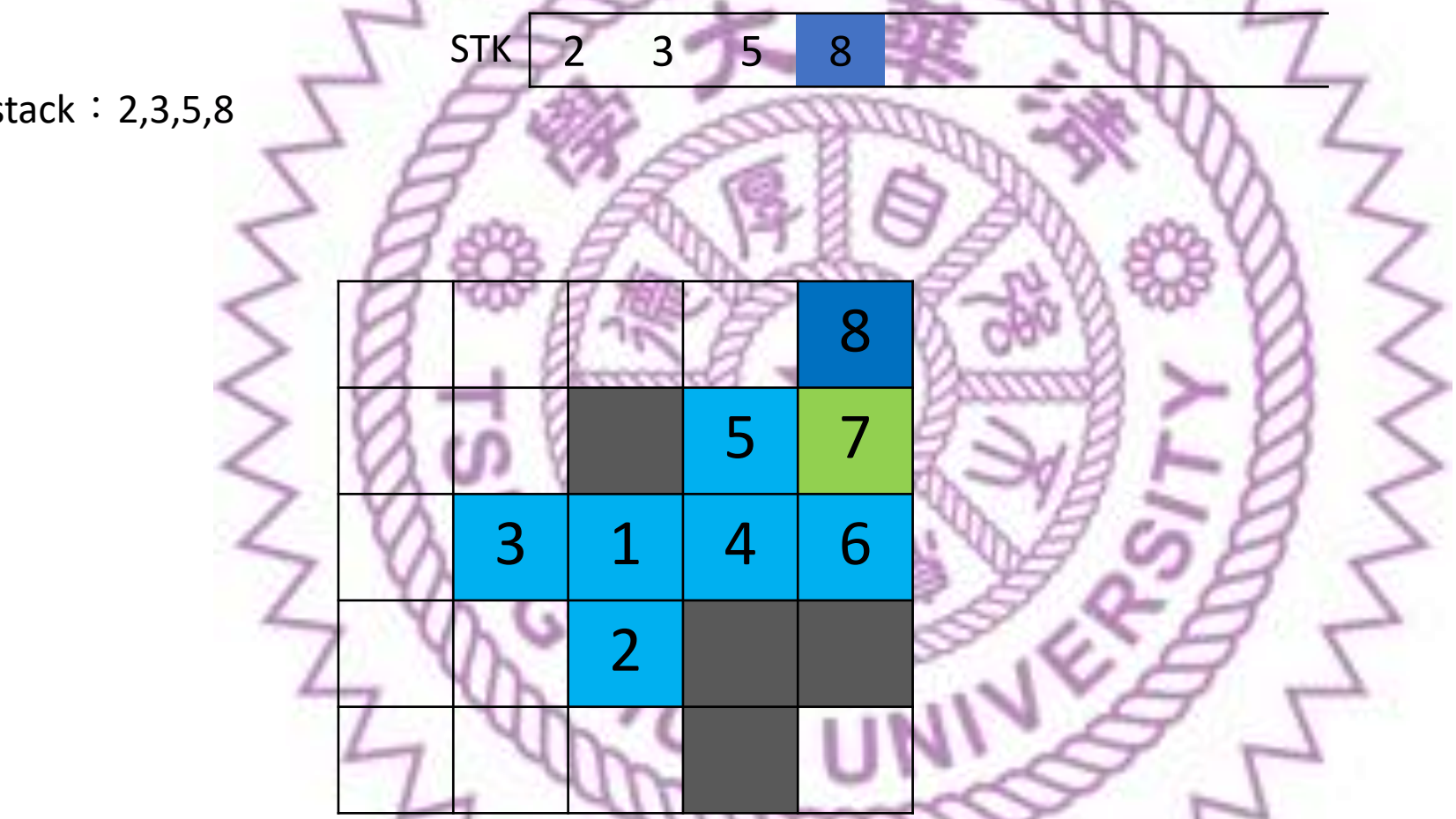
Demo

stack：2,3,5,7



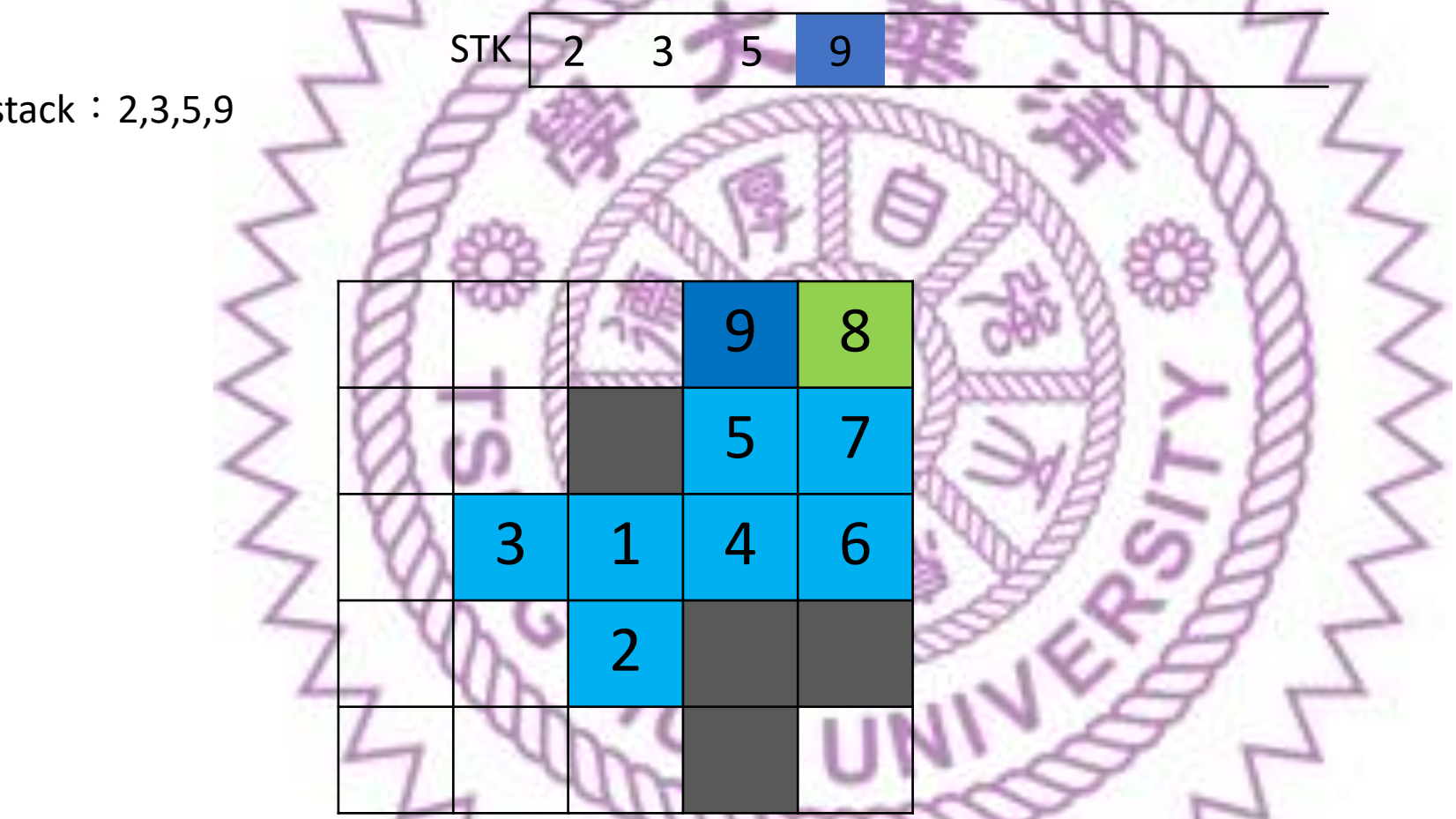
Demo

stack：2,3,5,8



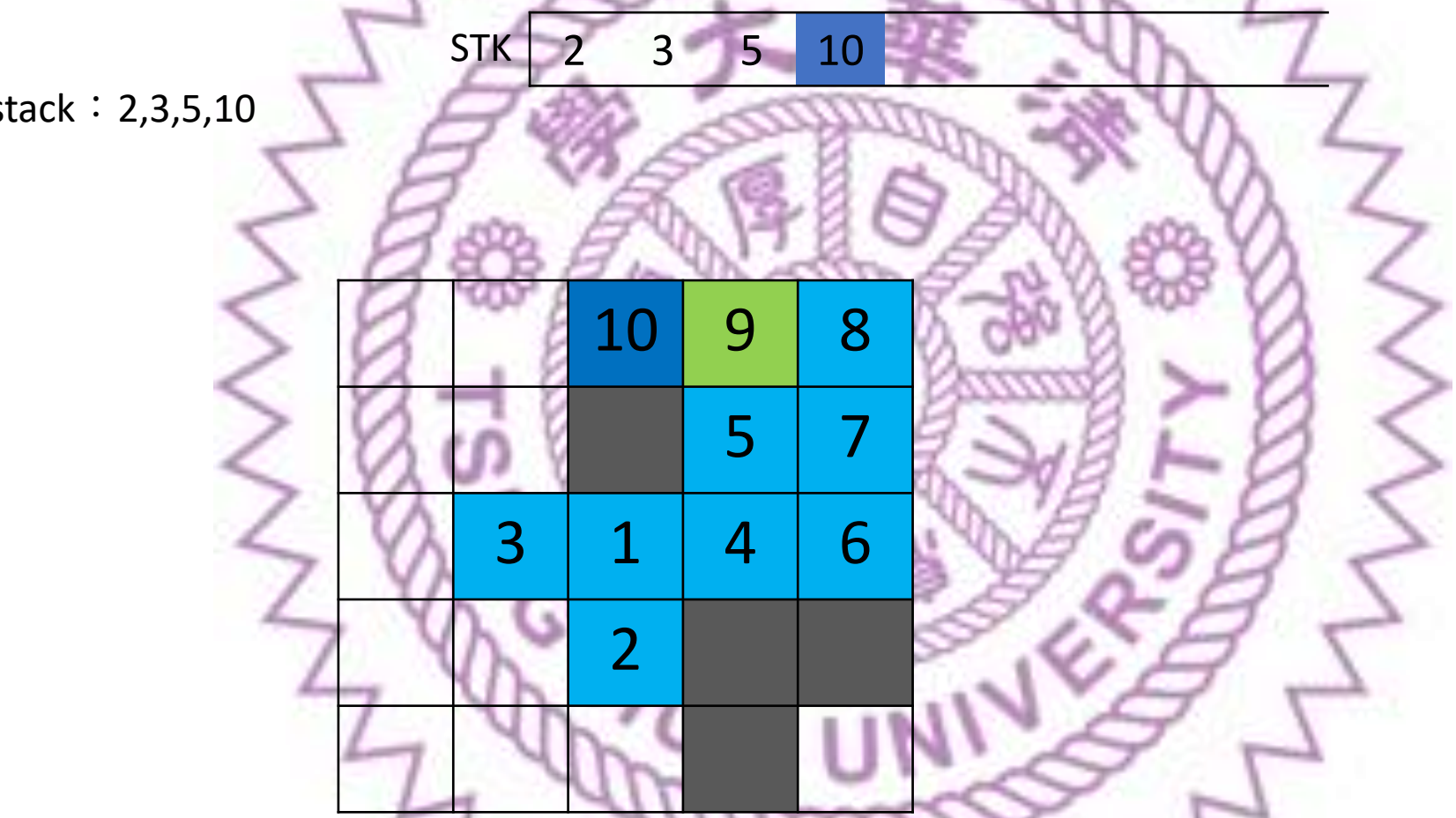
Demo

stack：2,3,5,9



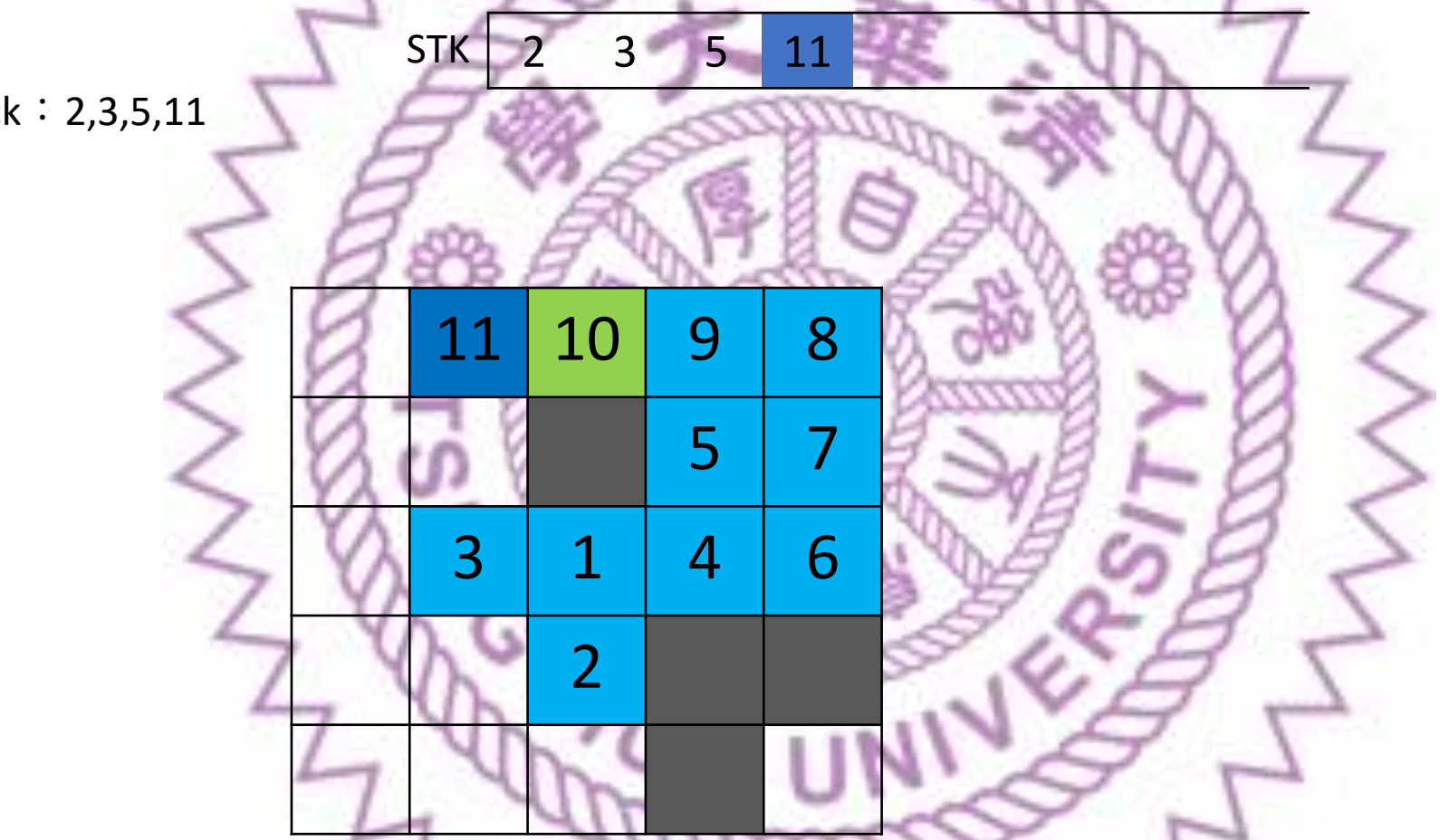
Demo

stack：2,3,5,10



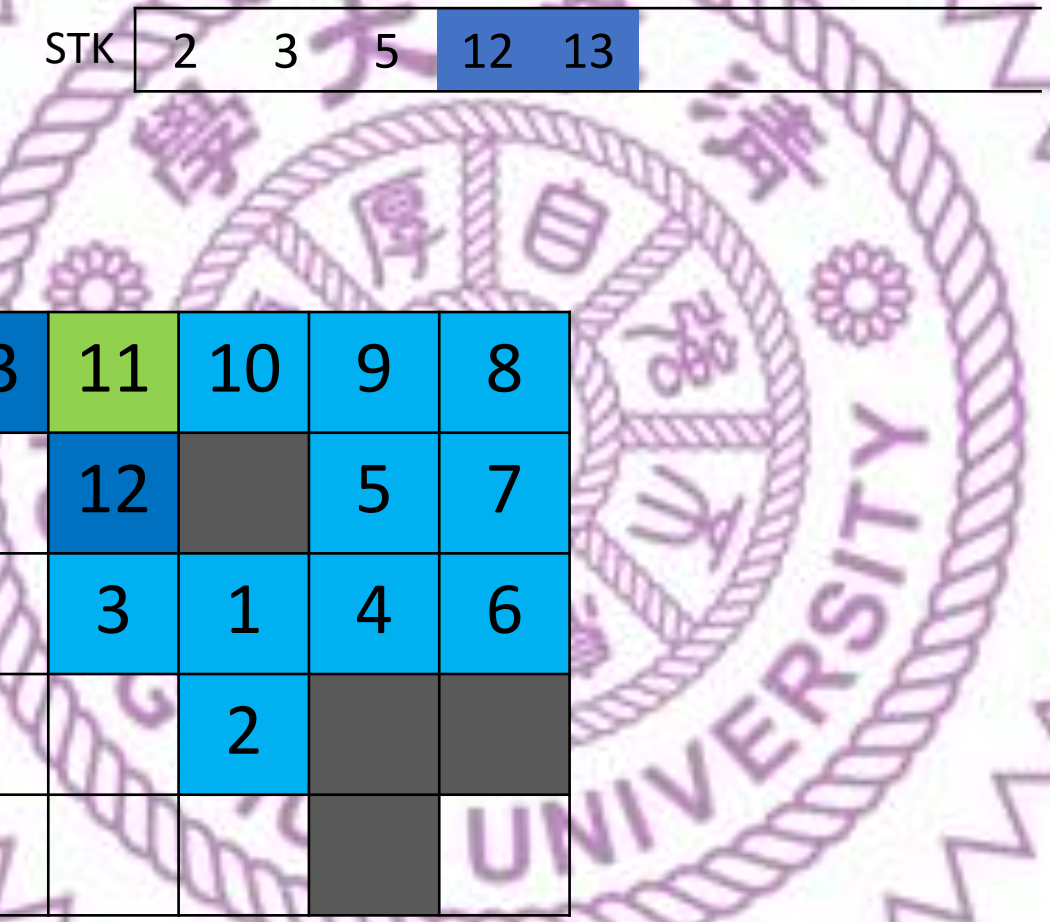
Demo

stack：2,3,5,11



Demo

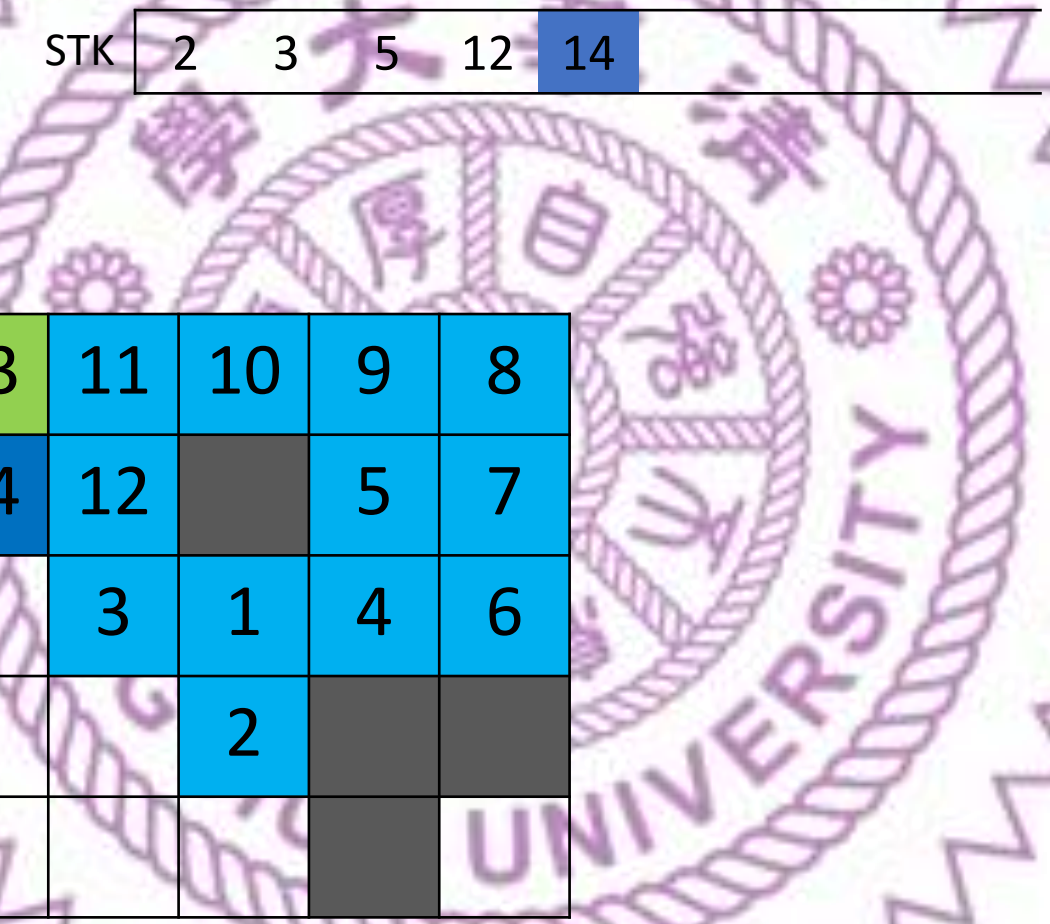
stack：2,3,5,12,13



13	11	10	9	8
	12		5	7
	3	1	4	6
		2		

Demo

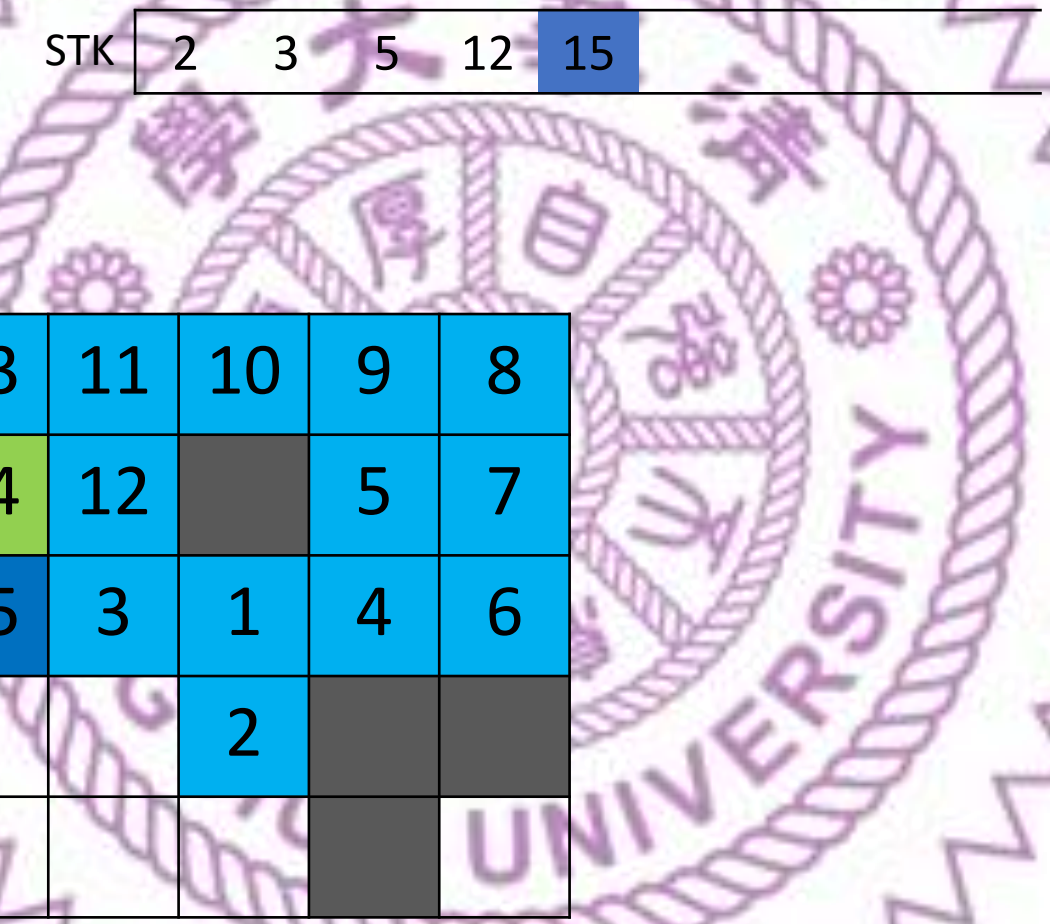
stack：2,3,5,12,14



13	11	10	9	8
14	12		5	7
	3	1	4	6
		2		

Demo

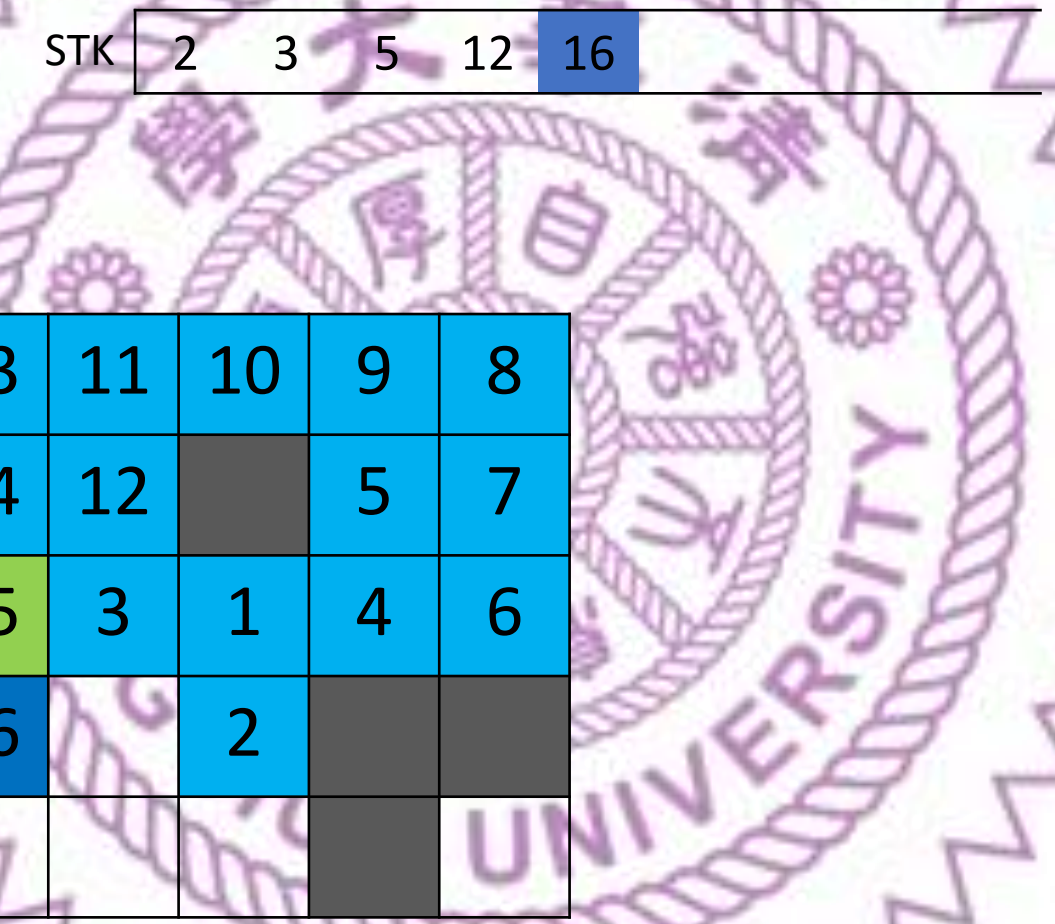
stack：2,3,5,12,15



13	11	10	9	8
14	12		5	7
15	3	1	4	6
		2		

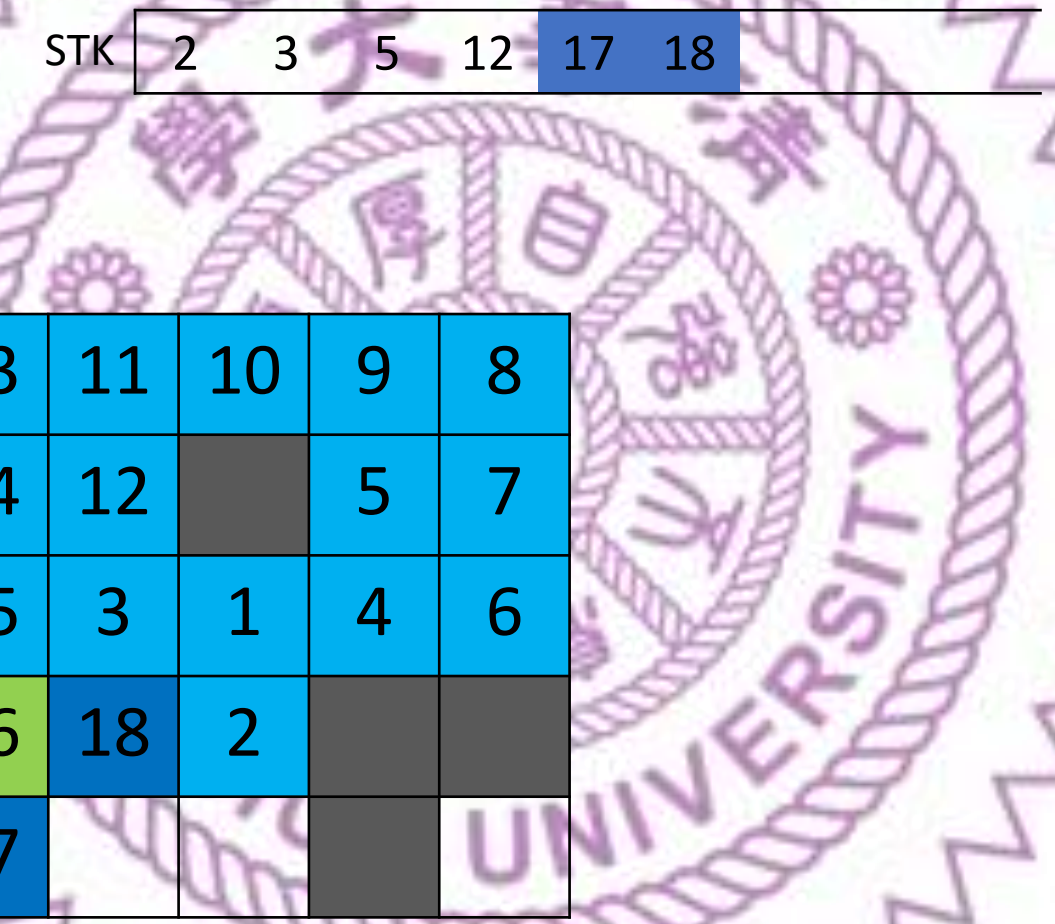
Demo

stack：2,3,5,12,16



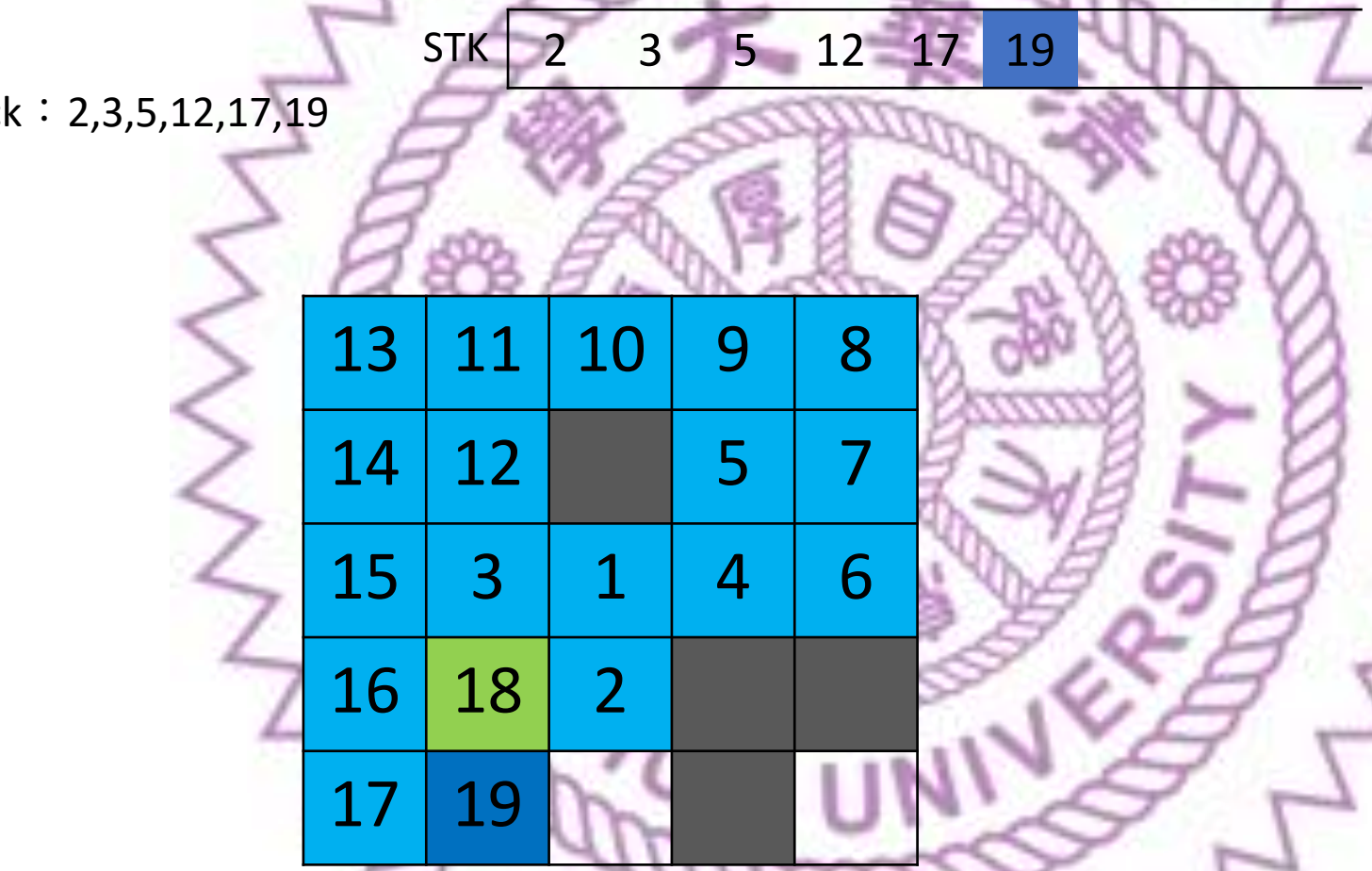
Demo

stack：2,3,5,12,17,18



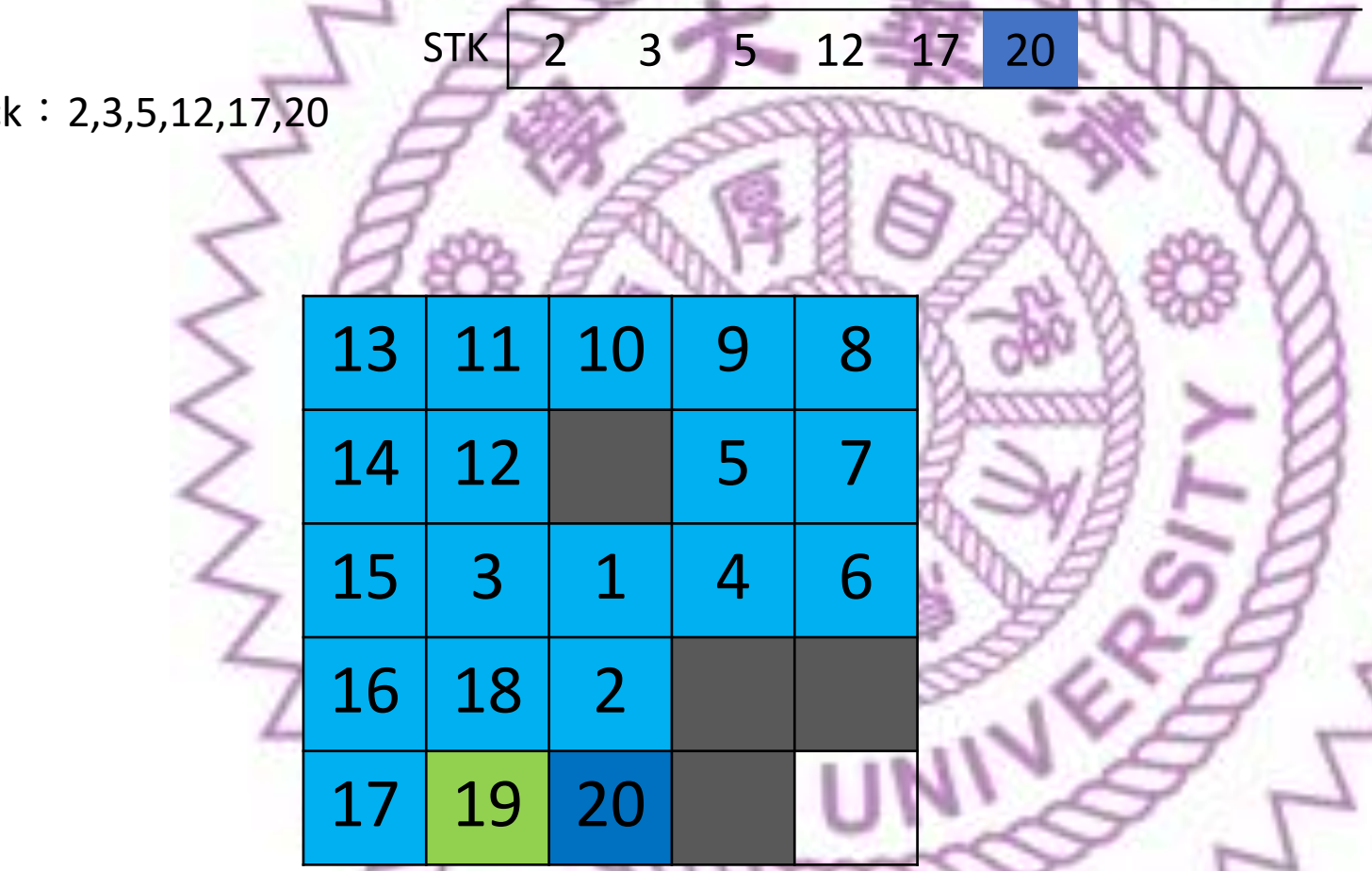
Demo

stack：2,3,5,12,17,19



Demo

stack：2,3,5,12,17,20



Demo

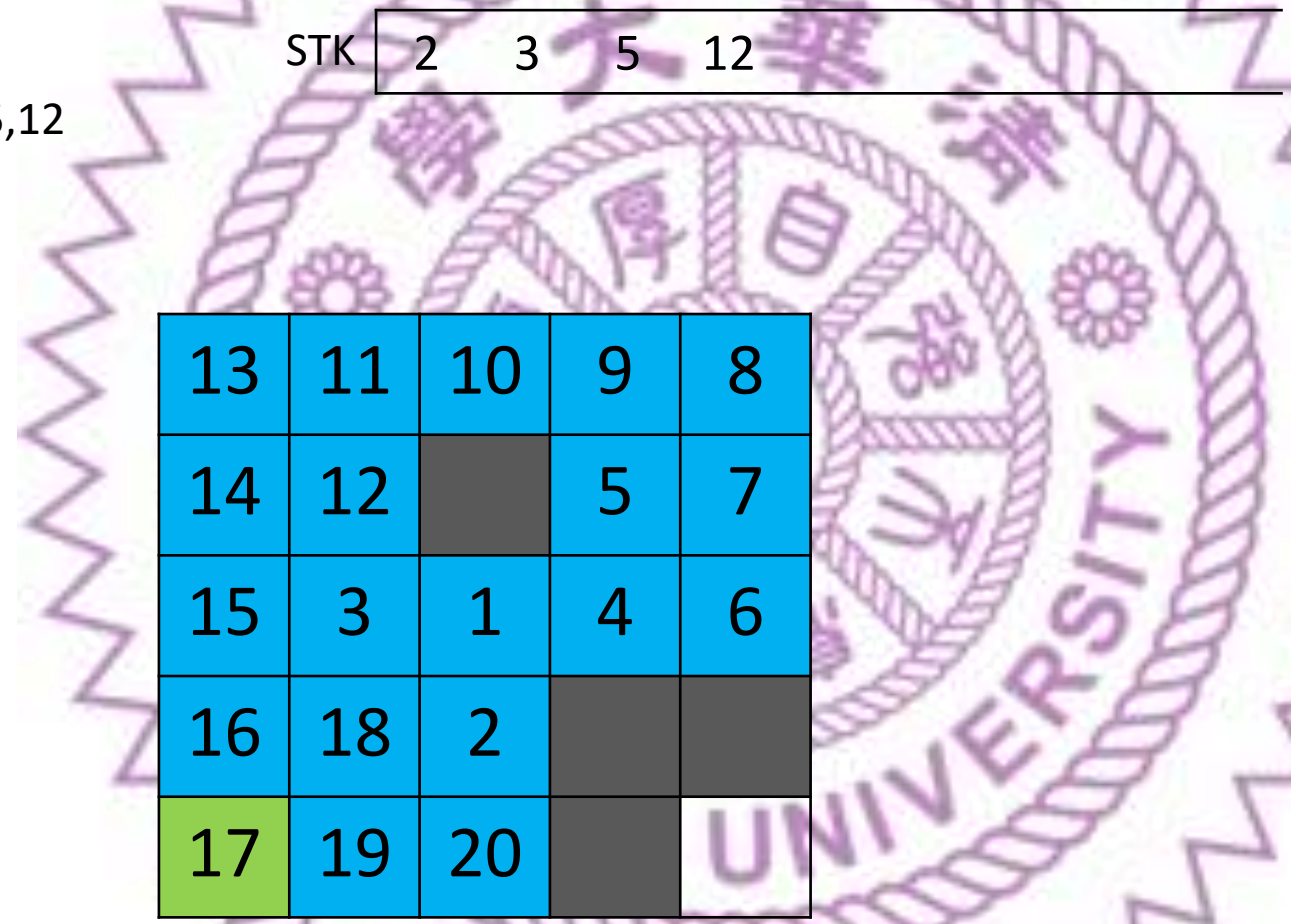
stack：2,3,5,12,17

STK | 2 | 3 | 5 | 12 | 17 |

13	11	10	9	8
14	12		5	7
15	3	1	4	6
16	18	2		
17	19	20		

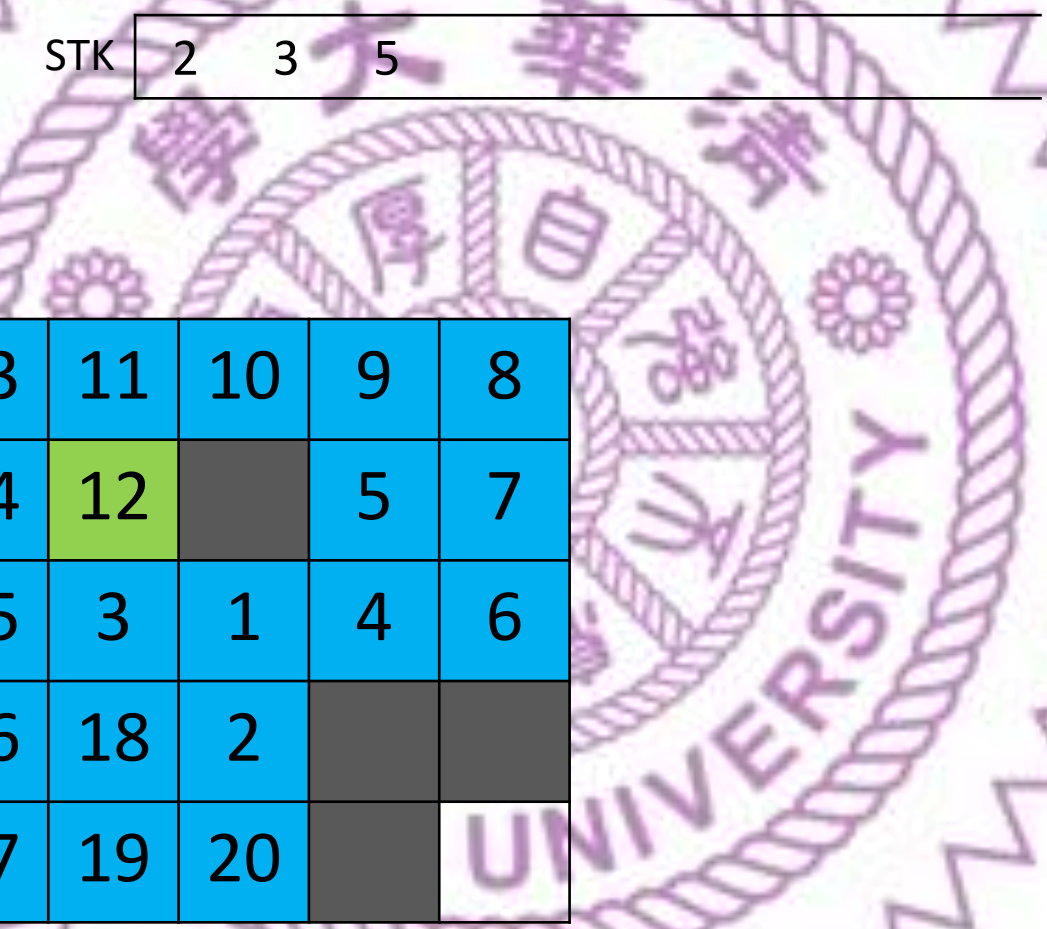
Demo

stack：2,3,5,12



Demo

stack：2,3,5



13	11	10	9	8
14	12		5	7
15	3	1	4	6
16	18	2		
17	19	20		

Demo

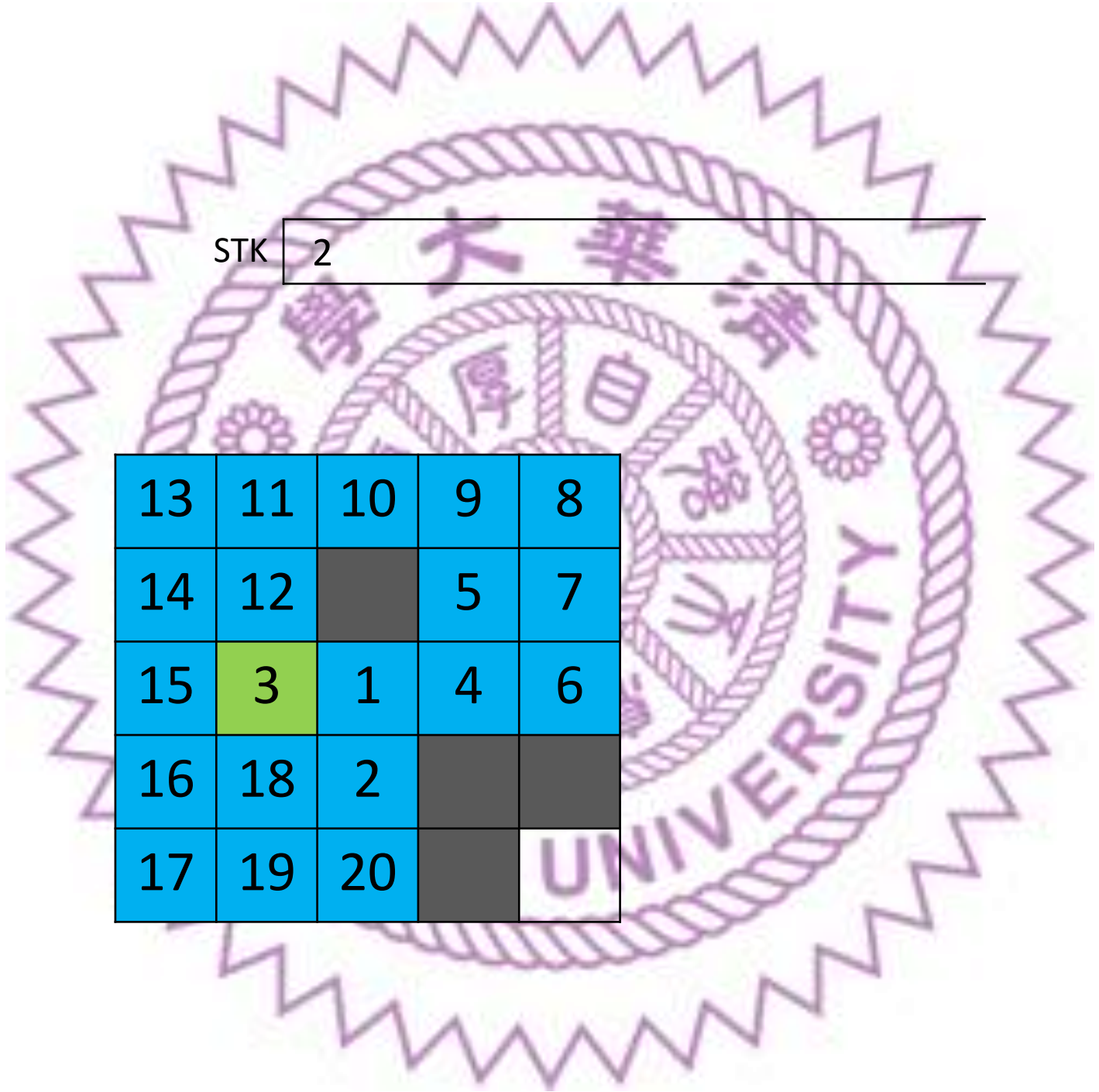
stack：2,3

STK 2 3

13	11	10	9	8
14	12		5	7
15	3	1	4	6
16	18	2		
17	19	20		

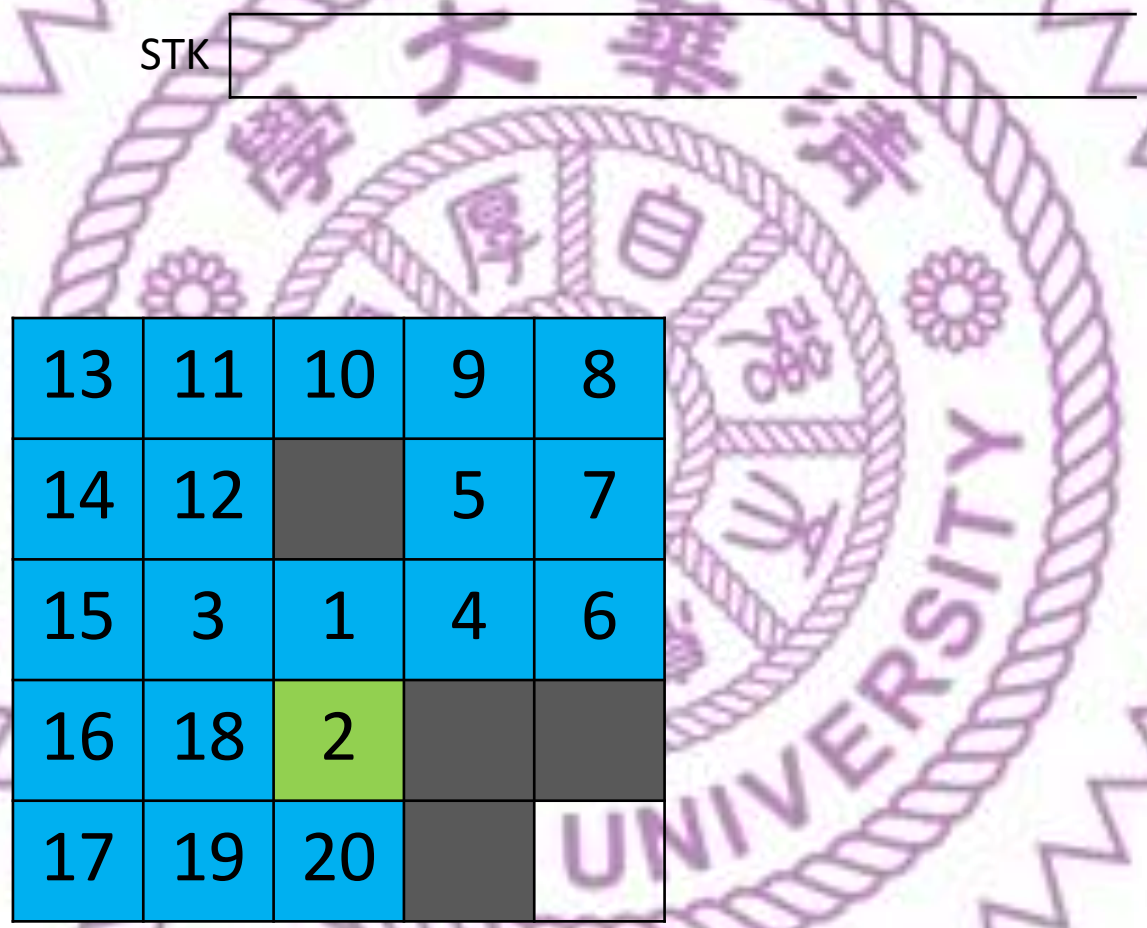
Demo

stack：2



Demo

STK



13	11	10	9	8
14	12		5	7
15	3	1	4	6
16	18	2		
17	19	20		

Stack 版本

```
void dfs(int x, int y){
  std::stack<std::pair<int,int>> STK;
  STK.emplace(x, y);
  while(STK.size()){
    std::tie(x, y) = STK.top();
    STK.pop();
    if(grid[x][y]) continue;
    grid[x][y] = true;
    STK.emplace(x+1, y);
    STK.emplace(x, y+1);
    STK.emplace(x-1, y);
    STK.emplace(x, y-1);
  }
}
```

分類

- 用queue:
  - 廣度優先搜索(Breadth first search,BFS)
- 用stack:
  - 深度優先搜索(Depth first search,DFS)
- 事實上DFS不會像那樣實作

遞迴來實做DFS

- 在進入遞迴的時候，其實函數的資訊都會被記錄在系統提供的stack中
- 因此可以用遞迴實作DFS

```
void dfs(int x, int y) {
  if(grid[x][y]) return;
  grid[x][y] = true;
  dfs(x+1, y);
  dfs(x, y+1);
  dfs(x-1, y);
  dfs(x, y-1);
}
```

+1 -1 很麻煩? 用 for 快速枚舉

```cpp
void bfs(int x, int y) {
  queue<pair<int, int>> Q;
  Q.emplace(x, y);
  int L = 0;
  while (Q.size()) {
    for (int Num = Q.size(); Num--;) {
      tie(x, y) = Q.front();
      Q.pop();
      if (grid[x][y]) continue;
      grid[x][y] = true;
      Level[x][y] = L;
      for (auto [dx, dy] : Dxy)
        Q.emplace(x + dx, y + dy);
    }
    L += 1;
  }
}
```

```cpp
pair<int, int> Dxy[4] =
    {{1, 0}, {0, 1}, {-1, 0}, {0, -1}};
```

```cpp
void dfs(int x, int y) {
  if (grid[x][y]) return;
  grid[x][y] = true;
  for (auto [dx, dy] : Dxy)
    dfs(x + dx, y + dy);
}
```

找出步驟數最少的解

BFS 經典應用

經典題

- https://leetcode.com/problems/water-and-jug-problem/
- 給你兩個容器，容量分別為 jug1Capacity 和 jug2Capacity
- 旁邊有無限供應的水源
- 一開始容器是空的，目標要讓容器的水量總和等於 targetCapacity
- 你只能做以下三件事
  - 將某個容器裝滿水
  - 將某個容器的水倒光
  - 將A容器的水倒到B容器中，直到B容器滿了或是A空了為止

題目問法：

- 給你 jug1Capacity, jug2Capacity, targetCapacity
- 問法一：問你有沒有解(原本題目)
- 問法二：問你有解的話最少需要幾個步驟

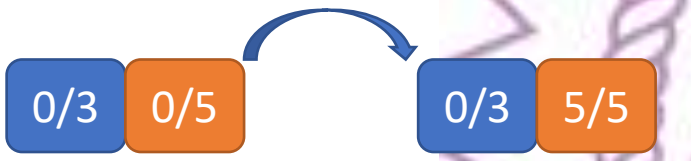
範例測資

- Input: jug1Capacity = 3, jug2Capacity = 5, targetCapacity = 4
- Output: true

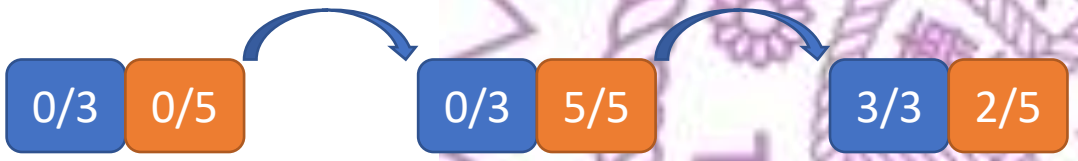
模擬一遍 targetCapacity = 4

0/3 0/5

模擬一遍 targetCapacity = 4

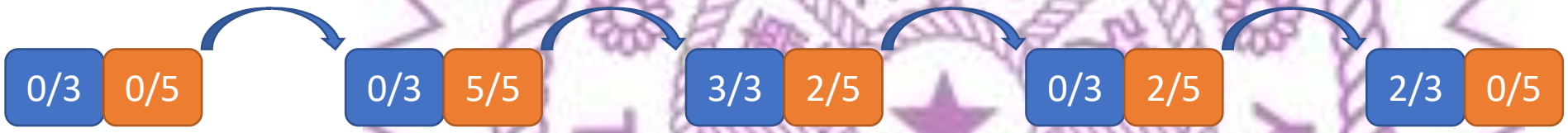


模擬一遍 targetCapacity = 4

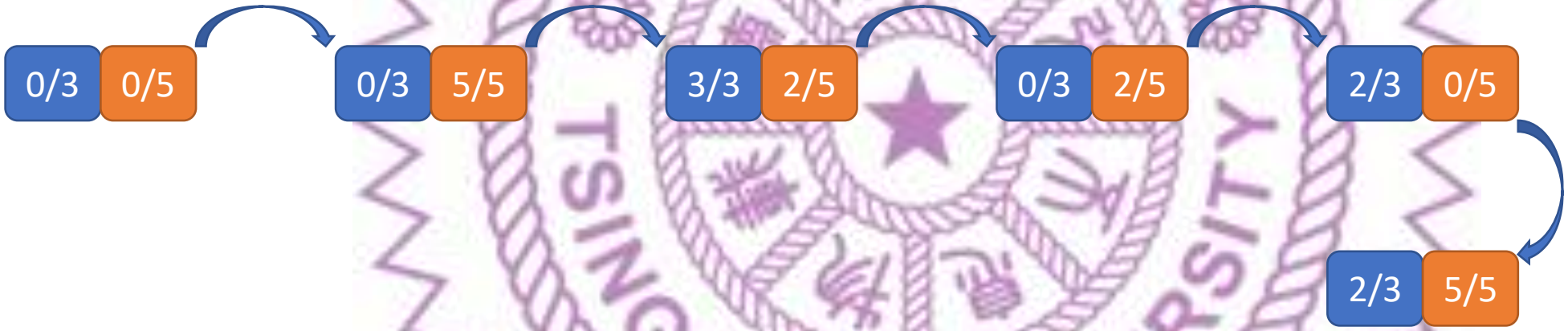


模擬一遍 targetCapacity = 4

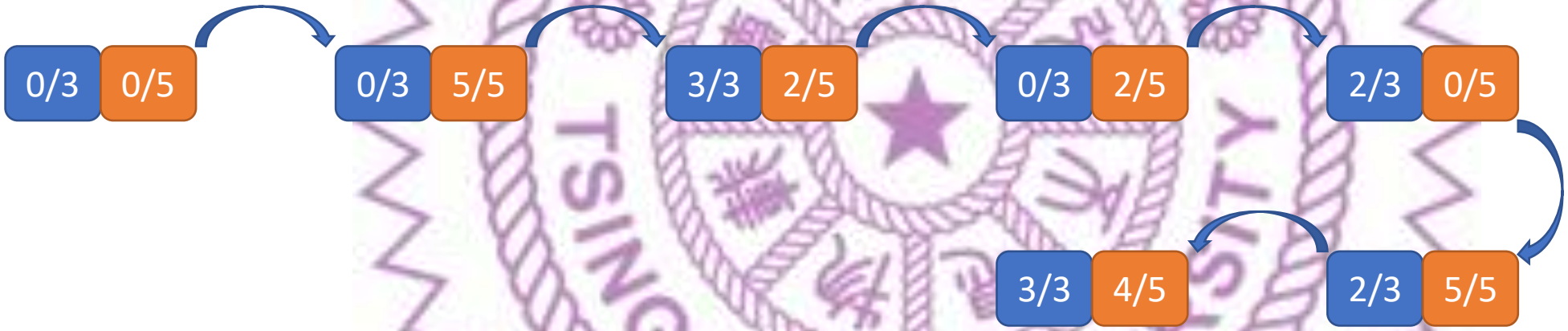
模擬一遍 targetCapacity = 4



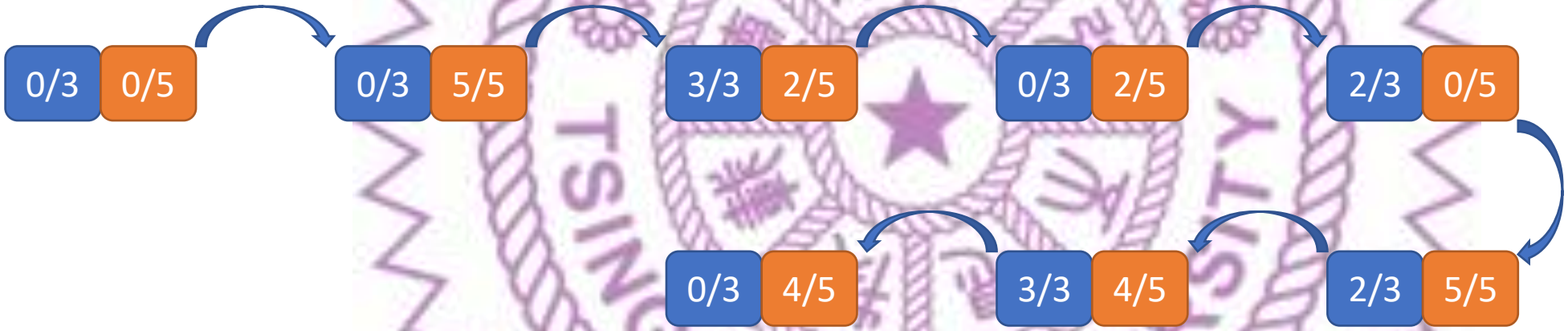
模擬一遍 targetCapacity = 4



模擬一遍 targetCapacity = 4



模擬一遍 targetCapacity = 4



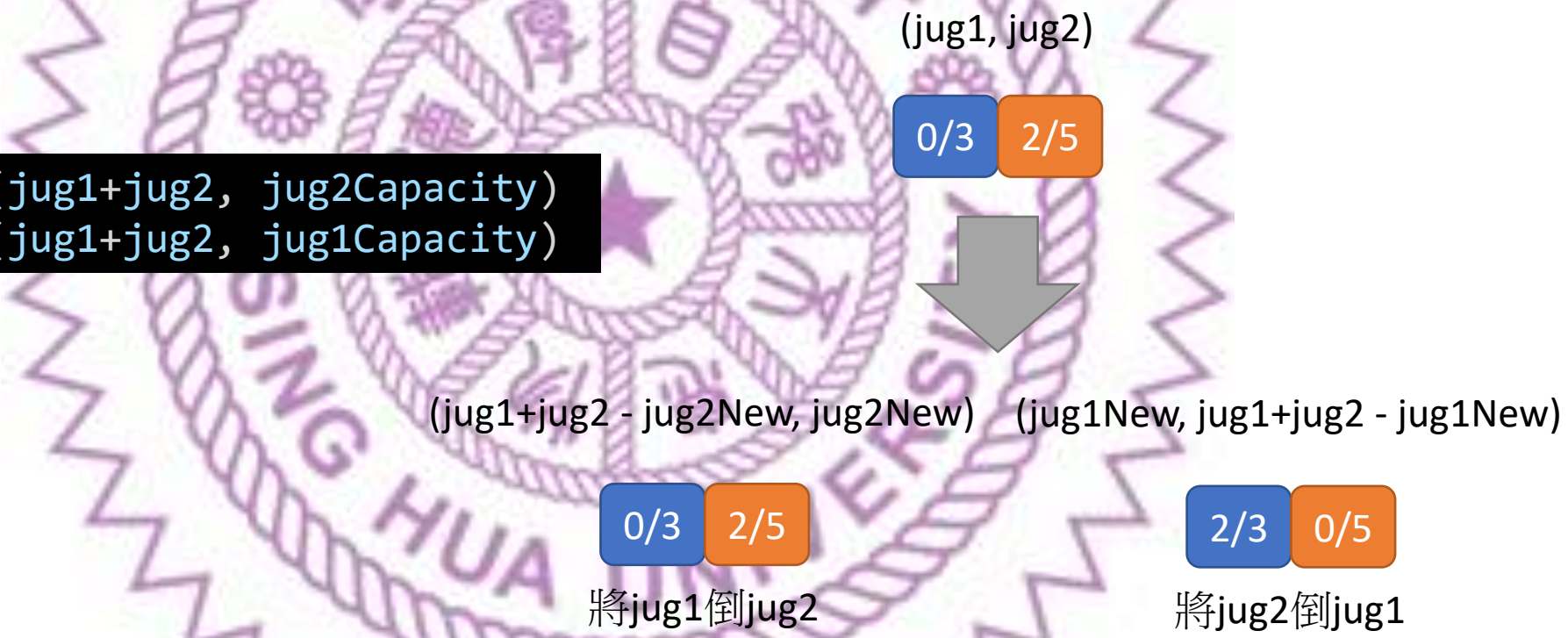
想法：每個狀態枚舉所有操作

(jug1, jug2)

(jug1Capacity, jug2)	(jug1, jug2Capacity)	(0, jug2)	(jug1, 0)	(?, ?)	(?, ?)
3/3 2/5	0/3 5/5	0/3 2/5	0/3 0/5	0/3 2/5	2/3 0/5
將jug1倒滿	將jug2倒滿	將jug1倒光	將jug2倒光	將jug1倒jug2	將jug2倒jug1

想法：每個狀態枚舉所有操作

```
jug2New = min(jug1+jug2, jug2Capacity)
jug1New = min(jug1+jug2, jug1Capacity)
```



TLE 的程式碼

```cpp
bool canMeasureWater(int jug1Capacity, int jug2Capacity, int targetCapacity) {
  queue<pair<int, int>> Q;
  set<pair<int, int>> Visited;
  Q.emplace(0, 0);
  while (Q.size()) {
    auto [jug1, jug2] = Q.front(); Q.pop();
    if (Visited.count({jug1, jug2})) continue;
    if (jug1 + jug2 == targetCapacity) return true;
    Visited.emplace(jug1, jug2);
    Q.emplace(jug1Capacity, jug2); // jug1 倒滿
    Q.emplace(jug1, jug2Capacity); // jug2 倒滿
    Q.emplace(0, jug2); // jug1 倒光
    Q.emplace(jug1, 0); // jug2 倒光
    int jug2New = min(jug1 + jug2, jug2Capacity);
    int jug1New = min(jug1 + jug2, jug1Capacity);
    Q.emplace(jug1 + jug2 - jug2New, jug2New); // jug1 倒入 jug2
    Q.emplace(jug1New, jug1 + jug2 - jug1New); // jug2 倒入 jug1
  }
  return false;
}
```