MAT3253 Homework 1

Due date: 22 Jan.

In Question 5 and Question 6 parts b and c, we represent a complex number $x + iy$ by a pair (x, y). See the Construction I in Lecture 1.

Question 1. (Bak & Newman Ex.1.1) Express in the form $a + bi$:

a. $\frac{1}{6+2i}$

b. $\frac{(2+i)(3+2i)}{1-i}$

c. $\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^4$

d. i^2, i^3, i^4, i^5, ...

Question 2. (Bak & Newman Ex. 1.5) Suppose P is a polynomial with real coefficients. Show that $P(z) = 0$ if and only if $P(\bar{z}) = 0$ (i.e., zeroes of "real" polynomials come in conjugate pairs).

Question 3. (Bak & Newman Ex. 1.10) Prove:

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

and interpret the result geometrically.

Question 4. (Brown & Churchill Ex.1.2.2) Show that
a. $\mathrm{Re}(iz) = -\mathrm{Im}(z)$; b. $\mathrm{Im}(iz) = \mathrm{Re}(z)$.

Question 5. (Brown & Churchill Ex.1.2.11) Solve the equation $z^2+z+1 = 0$ for $z = (x, y)$ by writing

$$(x, y)(x, y) + (x, y) + (1, 0) = (0, 0)$$

and then solving a pair of simultaneous equations in x and y. (Hint: Use the fact that no real number x satisfies the given equation to show that $y \neq 0$.)

Question 6. (Brown & Churchill Ex.1.4.1) Locate the numbers $z_1 + z_2$ and $z_1 - z_2$ vectorially when

a. $z_1 = 2i,\ z_2 = \dfrac{2}{3} - i$; *b.* $(-\sqrt{3}, 1),\ z_2 = (\sqrt{3}, 0)$;

c. $z_1 = (-3, 1),\ z_2 = (1, 4)$; *d.* $z_1 = x_1 + iy_1,\ z_2 = x_1 - iy_1$.