MAT2002: Ordinary Differential Equations

Assignment 1.   Deadline: 5pm, Feb 26th, 2021

Question 1.
Solve the following initial value problem. State the independent variable, dependent variable, the interval of definition.

(i)
$$y' = cos(t), \qquad y(0) = 0$$

(ii)
$$y' = 3 - 7t, \qquad y(0) = 1$$

Question 2.
Solve the following initial value problem.

(i)
$$t^4 y' + 5t^3 y = e^{-t}, \qquad y(-1) = 0 \quad for \quad t < 0$$

(ii)
$$y' = \frac{3y^2}{t}, \qquad y(1) = 2$$

(iii)
$$y' = \frac{1}{2} t y^3 (1 + t^2)^{-\frac{1}{2}}, \qquad y(0) = 1$$

(iv)
$$y' - y = 4te^{2t}, \qquad y(0) = 1$$

Question 3.
Solve the following equations.

(i)
$$\frac{dy}{dx} = \frac{2x^2 + xy + 3y^2}{x^2}$$

(ii)
$$\frac{dy}{dx} = \frac{2x^2 + 3y^2}{2xy}$$

(iii)

$$(t^2 + ty + y^2) - t^2 y' = 0 \qquad y(1) = 0 \qquad for \quad t > 0$$

Question 4.
Show that if a and λ are positive constants, and b is any real number, then every solution of the equation:

$$y' + ay = be^{-\lambda t}$$

has the property that $y \to 0$ as $t \to \infty$.

Question 5.
Find the value of y_0 for which the solution of the initial value problem

$$y' - y = 2 + 3sin(t), \qquad y(0) = y_0$$

remains finite as $t \to \infty$.

Question 6.
Solve the following initial value problem of y with respect ot x for $x \in (-\pi/2, 3\pi/2)$,

$$\frac{dy}{dx} \cos x + y = \cos^2 x$$

$$y(0) = -1$$

Question 7.
Consider the general first order linear equation $y' = p(t)y + g(t)$, show that

- (i) If $y_1(t)$ is a solution to $y' = p(t)y$, then $cy_1(t)$ is also a solution to $y' = p(t)y$ for $c \in \mathbb{R}$;
- (ii) If $y_2(t)$ is a solution to $y' = p(t)y + g(t)$, then $cy_1(t) + y_2(t)$ is also a solution to the equation $y' = p(t)y + g(t)$;
- (iii) All the solutions to $y' = p(t)y + g(t)$ is of the form $cy_1(t) + y_2(t)$

Question 8.
Determine whether the following ODEs are exact. Then solve these ODEs.

(i)

$$(4x + 3) + (5y - 1)\frac{dy}{dx} = 0$$

(ii)

$$(4x^2 - 2xy + 4) + (6y^2 - x^2 + 2)\frac{dy}{dx} = 0$$

(iii)

$$(3t^2y + 2ty + y^3) + (t^2 + y^2)\frac{dy}{dt} = 0 \qquad y(0) = 1$$

(iv)

$$(e^x siny - 3ysinx) + (e^x cosy + 3cosx)\frac{dy}{dx} = 0$$

Question 9.
Find the value of b for which the given equation is exact, and then solve it using that value of b.

(i)

$$(xy^2 + bx^2y) + (4x + y)x^2\frac{dy}{dx} = 0$$

(ii)

$$(ye^{2xy} + 5x) + bxe^{2xy}\frac{dy}{dx} = 0$$

Question 10.
Show that if $(N_x - M_y)/(xM - yN) = R$, where R only depends on xy, then the equation:

$$M + Ny' = 0$$

has an integrating factor of the form $v(xy)$. Find the formula of that integrating factor.