MAT2006 Tutorial #12

1. (a) Recall that

$$[\log(1+x)]' = \frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots.$$

Derive the Taylor series expansion of $\log(1+x)$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots.$$

(b) What is the radius of convergence of the above Taylor series?
(c) Show that

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \log 2.$$

2. Find the domains of convergence of the following series

$$(a) \quad \sum_{n=1}^{\infty} n^{n^2} x^{n^3}; \qquad (b) \quad \sum_{n=1}^{\infty} \frac{x^{n^2}}{2^n}.$$

3. Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

4. Assume

$$\limsup_{n\to\infty} \sqrt[n]{|a_n|} = 1.$$

Show that the partial sum $S_n = \sum_{k=0}^{\infty} a_k$ also satisfies

$$\limsup_{n\to\infty} \sqrt[n]{|S_n|} = 1.$$

5. Find the sum of the series

$$\frac{x^2}{2\cdot 1} - \frac{x^3}{3\cdot 2} + \frac{x^4}{4\cdot 3} - \frac{x^5}{5\cdot 4} + \cdots$$

— End —