

MAT2006 Tutorial #4

1. Show that the sequence

$$\sqrt{2}, \quad \sqrt{2\sqrt{2}}, \quad \sqrt{2\sqrt{2\sqrt{2}}}, \quad \dots,$$

is convergent and find its limit.

2. (i) Show that

$$\limsup_{n \rightarrow \infty} (a_n + b_n) \leq \limsup_{n \rightarrow \infty} a_n + \limsup_{n \rightarrow \infty} b_n$$

and

$$\liminf_{n \rightarrow \infty} (a_n + b_n) \geq \liminf_{n \rightarrow \infty} a_n + \liminf_{n \rightarrow \infty} b_n$$

(ii) Give examples that “=” do not hold in the above inequalities.

3. Assume the Bolzano–Weierstrass Theorem is true and use it to construct a proof of the Monotone Convergence Theorem without making any appeal to the Archimedean Property.

4 (Decimal representations of real numbers). Show that the following sequence is convergent

$$\{y_n\}_{n=1}^{\infty} \quad \text{with} \quad y_n = P_0 + \frac{P_1}{10} + \frac{P_2}{10^2} + \dots + \frac{P_n}{10^n},$$

where $P_0 \in \mathbb{Z}$ and $P_n \in \{0, 1, 2, \dots, 9\}$ for $n \in \mathbb{N}$.

————— If time allows, discuss the following question —————

5. Show that the sequence

$$2, \quad 2 + \frac{1}{2}, \quad 2 + \frac{1}{2 + \frac{1}{2}}, \quad 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}, \quad \dots$$

is convergent and find its limit.

— End —