MAT2006 Tutorial #2

1. Show that every nonempty subset A of $\mathbb{R}$ that is bounded below has a greatest lower bound $\inf A$.

2. (a) Show that the Archimedean Property is equivalent to:
"for every positive real numbers x and ϵ, there exits $M \in \mathbb{N}$ such that $M\epsilon > x$."

(b) Recall that the Archimedean Property is a consequence of the AoC. Show that the Archimedean Property is also equivalent to:
"for any positive real number h and any real number x, there exists a unique integer k such that $(k-1)h \leq x < kh$."
Note. This is one step we used in lecture for proving $\mathbb{Q}$ is dense in $\mathbb{R}$. The Archimedean Property is stated as (a) in [Tao] and as (b) in [Zorich].

3. Prove that $\bigcap_{n=1}^{\infty}(0, \frac{1}{n}) = \emptyset$. Notice that this demonstrates that the intervals in the Nested Interval Property must be closed for the conclusion of the theorem to hold.

4. Show that the relation "has the same cardinality" satisfies the following property
(i) reflective: $A \sim A$;
(ii) symmetric: if $A \sim B$ then $B \sim A$;
(iii) transitive: if $A \sim B$ and $B \sim C$, then $A \sim C$.
Note. A binary relation satisfying the above three properties is called an *equivalence relation*.

5. Show that

$$\text{(i)} \quad (a,b) \sim (0,1); \qquad \text{(ii)} \quad (a,b) \sim \mathbb{R}; \qquad \text{(iii)} \quad [0,1] \sim (0,1).$$

— End —