Install Programme R

1. Go to

   https://www.r-project.org/

2. Click CRAN mirror

3. Choose one site from the list, for example,
   https://cran.ms.unimelb.edu.au/
   (School of Mathematics and Statistics, University of Melbourne)

4. Choose Download R for Windows (or for other system)

5. Open R and type in

   ```
   install.packages("NSM3")
   ```

6. Type

   ```
   library(NSM3)
   ```

R-commands for nonparametric statistics

library(NSM3)

One-sample location

Wilcoxon signed rank test

R-commands:

x <-c($x_1, x_2, \ldots, x_n$)

y <-c($y_1, y_2, \ldots, y_n$)

wilcox.test(x, y, paired=TRUE) for $H_1: \theta \neq 0$, $\theta = m_X - m_Y =$ median of $X - Y$

wilcox.test(x, y, paired=TRUE, alternative = "greater") for $H_1: \theta > 0$

wilcox.test(x, y, paired=TRUE, alternative = "less") for $H_1: \theta < 0$

Output: $V = T^+$ based on $Z = X - Y$, p-value against H_1

Example: $n = 9$

```
x <-c(1.83, 0.50, 1.62, 2.48, 1.68, 1.88, 1.55, 3.06, 1.30)
y <-c(0.878, 0.647, 0.598, 2.05, 1.06, 1.29, 1.06, 3.14, 1.29)
```

```
> wilcox.test(x, y, paired=TRUE)
data:  x and y
V = 40, p-value = 0.03906
alternative hypothesis: true location shift is not equal to 0
```

$T^+ = 40$, $p\text{-value} = 2\Pr(T^+ \geq 40) = 0.03906$ for $H_1: \theta \neq 0$

```
> wilcox.test(y,x, paired=TRUE, alternative = "less")
data:  y and x
V = 5, p-value = 0.01953
alternative hypothesis: true location shift is less than 0
```

$T^+ = 5$, $p\text{-value} = \Pr(T^+ \leq 5) = 0.01953$ for $H_1: \theta > 0$

Distribution of Wilcoxon signed-rank statistic T^+ (no ties)

psignrank(t, n ,lower.tail=TRUE) $\Rightarrow \Pr(T^+ \leq t)$

psignrank($a:b, n$,lower.tail=TRUE) $\Rightarrow \Pr(T^+ \leq t)$, $t = a, a+1, \ldots, b$

Example: $n = 9$

```
> psignrank(39,9,lower.tail=TRUE)
[1] 0.9804688
```

$\Pr(T^+ \leq 39) = 0.9804688 \Rightarrow \Pr(T^+ \geq 40) = 1 - \Pr(T^+ \leq 39) = 1 - 0.9804688 = 0.0195312$

```
> psignrank(39:42,9,lower.tail=TRUE)
[1] 0.9804688 0.9863281 0.9902344 0.9941406
```

$\Pr(T^+ \leq t)$, $t = 39, 40, 41, 42$.

Wilcoxon signed rank test conditional on ties

wilcox.test does not compute the exact *p*-value conditional on ties and gives a warning message.

Example:

```
x <-c(1.835, 0.507, 1.622, 2.483, 1.687, 1.880, 1.556, 3.060, 1.684)
y <-c(0.878, 0.647, 0.598, 2.343, 1.067, 1.292, 1.063, 3.541, 1.203)
```

```
> wilcox.test(x, y, paired=TRUE, alternative = "greater")
        Wilcoxon signed rank test with continuity correction

data:  x and y
V = 40.5, p-value = 0.01899
alternative hypothesis: true location shift is greater than 0

Warning message:
In wilcox.test.default(x, y, paired = TRUE, alternative = "greater") :
  cannot compute exact p-value with ties
```

R-command for ties

The following R-command computes exact $\Pr(T^+ \geq t)$ conditional on ties (although the numerical results are based on simulations and hence may not be exactly "exact"):

pPairedWilcoxon(x,y)

$\Rightarrow$ $T^+ = t$ and $\Pr(T^+ \geq t)$ based on $Z = Y - X$ (not $X - Y$ as in wilcox.test(x, y, paired=TRUE). Due to the symmetry of T^+, $\Pr(T^+ \geq t)$ is the same for $X - Y$ and $Y - X$.

Examples: For the same x, y as above,

```
> pPairedWilcoxon(x,y)
Number of X values: 9 Number of Y values: 9
Wilcoxon T+ Statistic: 4.5
Monte Carlo (Using 10000 Iterations) upper-tail probability: 0.9887
```

$T^+ = 4.5$ based on $Z = Y - X$, $\Pr(T^+ \geq 4.5) = \Pr(T^+ \leq 40.5) = 0.9887$ (approximate by Monte Carlo). From these we can also obtain $\Pr(T^+ \geq 41) = \Pr(T^+ > 40.5) = 1 - 0.9887 = 0.0113$

```
> pPairedWilcoxon(y,x)
Number of X values: 9 Number of Y values: 9
Wilcoxon T+ Statistic: 40.5
Monte Carlo (Using 10000 Iterations) upper-tail probability: 0.0155
```

$T^+ = 40.5$ based on $Z = X - Y$, $\Pr(T^+ \geq 40.5) = 0.0155$ based on $Z = X - Y$

When there are no ties, psignrank produced $\Pr(T^+ \geq 40.5) = \Pr(T^+ \geq 40) = 0.0195312$.

This is greater than $\Pr(T^+ \geq 40.5) = 0.0155$ conditional on ties in data x, y, confirming that ignoring ties leads to more conservative test results (more likely to accept H_0).

Two-sample location

Wilcoxon rank sum test

R-commands:

x <-c($x_1, x_2, \ldots, x_m$)

y <-c($y_1, y_2, \ldots, y_n$)

wilcox.test(y, x) for $H_1: \Delta \neq 0$ with $Y \sim X + \Delta$

wilcox.test(y, x, alternative = "greater") for $H_1: \Delta > 0$

wilcox.test(y, x, alternative = "less") for $H_1: \Delta < 0$

Output: $W = u$ (observed value of $U = W - n(n+1)/2$), p-value against H_1

Note: The test statistic in R is the Mann-Whitney statistic U, not the Wilcoxon rank sum W.

Example 1. $m = 6$, $n = 9$:

```
x <-c(0.878, 0.647, 0.598, 2.05, 1.06, 1.29)
y <-c(1.83, 0.50, 1.62, 2.48, 1.68, 1.88, 1.55, 3.06, 1.30)
```

```
> wilcox.test(y, x)

        Wilcoxon rank sum test with continuity correction

data:  y and x
W = 42, p-value = 0.08791
alternative hypothesis: true location shift is not equal to 0
```

$U = 42$, $W = U + n(n+1)/2 = 42 + 9(10)/2 = 42 + 45 = 87$

$p\text{-value} = 2\Pr(U \geq 42) = 2\Pr(W \geq 87) = 0.08791$ for $H_1: \Delta \neq 0$

```
> wilcox.test(y, x, alternative = "greater")

        Wilcoxon rank sum test with continuity correction

data:  y and x
W = 42, p-value = 0.04396
alternative hypothesis: true location shift is greater than 0
```

$U = 42$, $W = 87$, $p\text{-value} = \Pr(U \geq 42) = \Pr(W \geq 87) = 0.04396$ for $H_1: \Delta > 0$

Example 2. $m = 10$, $n = 5$:

```
x<-c(1.46, 0.80, 0.83, 1.64, 1.89, 1.04, 0.73, 1.91, 1.38, 1.45)
y<-c(0.88, 0.74, 1.15, 1.21, 0.90)
```

```
> wilcox.test(y, x, alternative = "less")

        Wilcoxon rank sum test

data:  y and x
W = 15, p-value = 0.1272
alternative hypothesis: true location shift is less than 0
```

$U = 15$, $W = 15 + 15 = 30$, $p\text{-value} = \Pr(U \leq 15) = \Pr(W \leq 30) = 0.1272$ for $H_1: \Delta < 0$

Distribution of the Mann-Whitney statistic U

$U = W - n(n+1)/2$, $\Pr(U \ge u) = \Pr(U \le mn - u)$, $\Pr(W \ge w) = \Pr(W \le n(m+n+1) - u)$

pwilcox($u, m, n,$ lower.tail=T) $\Rightarrow$ $\Pr(U \le u) = \Pr(W \le u + n(n+1)/2)$

pwilcox($a:b, m, n,$ lower.tail=T) $\Rightarrow$ $\Pr(U \le u)$, $u = a, a+1, \ldots, b$

qwilcox($\alpha, m, n,$ lower.tail=T) $\Rightarrow$ $q_\alpha : \Pr(U \le q_\alpha) = \alpha$ for achievable α

(The order of m and n does not matter in the above commands.)

$u_\alpha = mn - q_\alpha \Rightarrow \Pr(U \ge u_\alpha) = \Pr(U \le mn - u_\alpha) = \Pr(U \le q_\alpha) = \alpha$

$w_\alpha = u_\alpha + n(n+1)/2 \Rightarrow \Pr(W \ge w_\alpha) = \Pr(U \ge u_\alpha) = \alpha$

If α is not achievable, then $\Pr(U \le q_\alpha) > \alpha$ and $\Pr(U < q_\alpha) = \Pr(U \le q_\alpha - 1) < \alpha$

Example 1: $m = 2$, $n = 3$, $mn = 6$: $n(n+1)/2 = 3(4)/2 = 6$,

```
> pwilcox(0:6,2,3,lower.tail=T)
[1] 0.1 0.2 0.4 0.6 0.8 0.9 1.0
```

$\Pr(U \le u) = \Pr(W \le u + 6) = 0.1,\ 0.2,\ 0.4,\ 0.6,\ 0.8,\ 0.9,\ 1.0$ for $u = 0,1,2,3,4,5,6$.

```
> qwilcox(0.1,2,3,lower.tail=T)
[1] 0
```

$q_{0.1} = 0$: $\Pr(U \le 0) = \Pr(W \le 6) = \Pr(W \ge 12) = 0.1$, $u_{0.1} = mn - 0 = 6$, $w_{0.1} = 6 + 6 = 12$

```
> qwilcox(0.6,2,3,lower.tail=T)
[1] 3
```

$q_{0.6} = 3$: $\Pr(U \le 3) = \Pr(W \le 9) = \Pr(W \ge 9) = 0.6$, $u_{0.6} = 6 - 3 = 3$, $w_{0.6} = 3 + 6 = 9$

Example 2: $m = 6$, $n = 9$, $mn = 54$: $U = 42 \Rightarrow W = 42 + 9(10)/2 = 42 + 45 = 87$

$\Rightarrow \Pr(W \ge 87) = \Pr(U \ge 42) = \Pr(U \le 54 - 42) = \Pr(U \le 12)$

```
> pwilcox(12,6,9,lower.tail=T)
[1] 0.04395604
```

$\Pr(W \ge 87) = \Pr(U \le 12) = 0.04395604$ (matching p-value $= 0.04396$ for $H_1 : \Delta > 0$ on last page)

Example 3: $m = 10$, $n = 5$:

```
> pwilcox(6:10,10,5, lower.tail=T)
[1] 0.00965701 0.01398601 0.01998002 0.02763903 0.03762904
> pwilcox(6:10, 5,10, lower.tail=T)
[1] 0.00965701 0.01398601 0.01998002 0.02763903 0.03762904
```

$\Pr(U \le u)$, $u = 6,7,8,9,10$

```
> qwilcox(0.025, 10, 5, lower.tail=T)
[1] 9
```

$q_{0.025} = 9$: $\Pr(U \le 9) = 0.02764 > 0.025$, $\Pr(U < 9) = \Pr(U \le 8) = 0.01998 < 0.025$

Two-sample dispersion

Ansari-Bradley test

x <-c($x_1, x_2, \ldots, x_m$)

y <-c($y_1, y_2, \ldots, y_n$)

ansari.test(y,x, alternative = "greater")   for $H_1: \gamma^2 > 1$ or $\text{Var}(X) > \text{Var}(Y)$

ansari.test(y,x, alternative = "less")   for $H_1: \gamma^2 < 1$ or $\text{Var}(X) < \text{Var}(Y)$

ansari.test(y,x)   for $H_1: \gamma^2 \neq 1$ or $\text{Var}(X) \neq \text{Var}(Y)$

Output: $\text{AB} = c$ and p-value $= \Pr(C \geq c)$ for $H_1: \gamma^2 > 1$, p-value $= \Pr(C \leq c)$ for $H_1: \gamma^2 < 1$, p-value $= 2\min\{\Pr(C \geq c), \Pr(C \leq c)\}$ for $H_1: \gamma^2 \neq 1$.

Note: If use (x,y) instead of (y,x) in ansari.test, then
Output: $\text{AB} = X$-score and $c = TS - \text{AB}$, where $TS =$ total score.

Example: $m = 6$, $n = 9$, $N = 6 + 9 = 15$, $TS = (N+1)^2/4 = 16^2/4 = 64$

```
x <-c(0.87, 0.64, 0.59, 2.05, 1.06, 1.29)
y <-c(1.83, 0.50, 1.62, 2.48, 1.68, 1.88, 1.55, 3.06, 1.30)
```

```
> ansari.test(y,x, alternative = "greater")

        Ansari-Bradley test

data:  y and x
AB = 41, p-value = 0.3171
alternative hypothesis: true ratio of scales is greater than 1
```

$\text{AB} = c = 41$, p-value $= \Pr(C \geq c) = \Pr(C \geq 41) = 0.3171$

```
> ansari.test(y,x, alternative = "less")

        Ansari-Bradley test

data:  y and x
AB = 41, p-value = 0.76
alternative hypothesis: true ratio of scales is less than 1
```

$c = 41$, p-value $= \Pr(C \leq c) = \Pr(C \leq 41) = 0.76$

We can also find $\Pr(C = 41) = \Pr(C \leq 41) + \Pr(C \geq 41) - 1 = 0.76 + 0.3171 - 1 = 0.0771$

```
> ansari.test(y,x)

        Ansari-Bradley test

data:  y and x
AB = 41, p-value = 0.6342
alternative hypothesis: true ratio of scales is not equal to 1
```

p-value $= 2\min\{\Pr(C \geq 41), \Pr(C \leq 41)\} = 2\Pr(C \geq 41) = 2(0.3171) = 0.6342$

If use (x,y) instead of (y,x), then AB = 23 $\Rightarrow$ $c = TS - \text{AB} = 64 - 23 = 41$; p-values are the same.

Critical point of the Ansari-Bradley statistic C

cAnsBrad(α, m, n) $\Rightarrow$ $\Pr(C \le c_{1-\alpha} - 1) = \alpha$ and $\Pr(C \ge c_\alpha) = \alpha$

Example 1. $m = 6$, $n = 9$, target $\alpha = 0.025$, $N = m + n = 15$ is odd, $TS = 64$

```
> cAnsBrad(0.025,6,9)
Number of X values: 6 Number of Y values: 9
For the given alpha=0.025, the lower cutoff value is Ansari-Bradley C=29,
with true alpha level=0.017
For the given alpha=0.025, the upper cutoff value is Ansari-Bradley C=48,
with true alpha level=0.0144
```

For $\alpha = 0.017$, $\Pr(C \le 29) = 0.017$, $c_{1-0.017} - 1 = 29$, $c_{0.983} = 30$

For $\alpha = 0.0144$, $\Pr(C \ge 48) = 0.0144$, $c_{0.0144} = 48$

If take target $\alpha = 0.35$, then the output is shown below:

```
> cAnsBrad(0.35,6,9)
Number of X values: 6 Number of Y values: 9
For the given alpha=0.35, the lower cutoff value is Ansari-Bradley C=36,
with true alpha level=0.3323
For the given alpha=0.35, the upper cutoff value is Ansari-Bradley C=41,
with true alpha level=0.3171
```

This confirms $\Pr(C \ge 41) = 0.3171$ for $m = 6$, $n = 9$.

Example 2. $m = 6$, $n = 10$, target $\alpha = 0.025$, $N = m + n = 16$ is even,

$TS = N(N+2)/4 = 16(18)/4 = 72$, C is symmetric about $E_0[C] = n(N+2)/4 = 10(18)/4 = 45$.

```
> cAnsBrad(0.025,6,10)
Number of X values: 6 Number of Y values: 10
For the given alpha=0.025, the lower cutoff value is Ansari-Bradley C=35,
with true alpha level=0.0175
For the given alpha=0.025, the upper cutoff value is Ansari-Bradley C=55,
with true alpha level=0.0175
```

$\Pr(C \le 35) = 0.0175$, $c_{1-0.0175} - 1 = 35$, $c_{0.9825} = 36$, $\Pr(C \ge 36) = 1 - \Pr(C \le 35) = 0.9825$

$\Pr(C \ge 55) = 0.0175 = \Pr(C \le 35)$, $c_{0.0175} = 55$

Miller's Jackknife test

MillerJack(x,y) $\Rightarrow$ Q value

Example:

```
x <-c(0.87, 0.64, 0.59, 2.05, 1.06, 1.29)
y <-c(1.83, 0.50, 1.62, 2.48, 1.68, 1.88, 1.55, 3.06, 1.30)
```

```
> MillerJack(x,y)
[1] -0.2084661
```

$Q = -0.2084661$

Other two-sample problems

Lepage test

x <-c($x_1, x_2, \ldots, x_m$)

y <-c($y_1, y_2, \ldots, y_n$)

pLepage(x,y) $\Rightarrow$ Lepage test statistic $D = d$ and exact p-value $\Pr(D \geq d)$

pLepage(x,y, method="Asymptotic") $\Rightarrow$ $D = d$ and approximate p-value $\Pr(\chi_2^2 \geq d)$

Example 1

```
x <-c(12, 20, 32, 40, 60, 112)
y <-c(67, 90, 95, 120, 124, 135, 180, 190, 215, 399)
```

```
> pLepage(x,y)
Number of X values: 6 Number of Y values: 10
Lepage D Statistic: 9.3384
Exact upper-tail probability: 0.0035
```

$D = 9.3384$, $\Pr(D \geq 9.3384) = 0.0035$

Example 2

```
> pLepage(x,y, method="Asymptotic")
Number of X values: 6 Number of Y values: 10
Lepage D Statistic: 9.3384
Asymptotic upper-tail probability: 0.0094
```

$D = 9.3384$, $\Pr(\chi_2^2 \geq 9.3384) = 0.0094$

Critical points of Lepage test

cLepage(α, m, n) $\Rightarrow$ d_α: $\Pr(D \geq d_\alpha) = \alpha$

Example: $m = 6$, $n = 10$,

$\alpha = 0.05$:

```
> cLepage(0.05,6,10)
Number of X values: 6 Number of Y values: 10
For the given alpha=0.05, the upper cutoff value is Lepage D=5.61680672268908,
with true alpha level=0.05
```

$d_{0.05} = 5.6168$, $\Pr(D \geq 5.6168) = 0.05$

$\alpha = 0.0035$:

```
> cLepage(0.0035,6,10)
Number of X values: 6 Number of Y values: 10
For the given alpha=0.0035, the upper cutoff value is Lepage D=9.33837535014006,
with true alpha level=0.0035
```

$d_{0.0035} = 9.3384$, $\Pr(D \geq 9.3384) = 0.0035$

Kolmogorov-Smirnov (K-S) test

x <-c($x_1, x_2, \ldots, x_m$)

y <-c($y_1, y_2, \ldots, y_n$)

ks.test(x,y) for $H_0 : F(t) = G(t)$ against $H_1 : F(t) \neq G(t)$

Output: $D = \sup_{t \in \mathbb{R}} |F_m(t) - G_n(t)| = d_{\text{obs}}$ (observed value of D) and $\Pr(D \geq d_{\text{obs}})$

Example 1

```
x <-c(12, 20, 32, 40, 60, 112)
y <-c(67, 90, 95, 120, 124, 135, 180, 190, 215, 399)

> ks.test(x,y)

        Two-sample Kolmogorov-Smirnov test

data:  x and y
D = 0.83333, p-value = 0.003996
alternative hypothesis: two-sided
```

$D = d_{\text{obs}} = 0.83333$, $\Pr(D \geq 0.83333) = 0.003996$

Example 2

```
x <-c(-0.15, 8.60, 5.00, 3.71, 4.29, 7.74, 2.48, 3.25, -1.15, 8.38)
y <-c(2.55, 12.07, 0.46, 0.35, 2.69, 0.94, 1.73, 0.73, -0.35, -0.37)

> ks.test(x,y)

        Two-sample Kolmogorov-Smirnov test

data:  x and y
D = 0.6, p-value = 0.05245
alternative hypothesis: two-sided
```

$D = d_{\text{obs}} = 0.6$, $\Pr(D \geq 0.6) = 0.05245$

Alternative R-command

pKolSmirn(x,y) $\Rightarrow J = dD = d \sup_{t \in \mathbb{R}} |F_m(t) - G_n(t)| = j$ ($d = \gcd(m, n)$) and $\Pr(J \geq j)$

Example

```
x <-c(12, 20, 32, 40, 60, 112)
y <-c(67, 90, 95, 120, 124, 135, 180, 190, 215, 399)

> pKolSmirn(x,y)
Number of X values: 6 Number of Y values: 10
Kolmogorov-Smirnov J Statistic: 1.6667
Exact upper-tail probability: 0.004
```

$J = 1.6667$, $\Pr(J \geq 1.6667) = 0.004$

$m = 6$, $n = 10$, $d = 2$, $J = dD = 2(0.83333) = 1.6667$

Critical points of K-S test

$J = \frac{mn}{d} D = \frac{mn}{d} \sup_{t \in \mathbb{R}} |F_m(t) - G_n(t)|$, where $d = \gcd(m,n)$ (differ from the J in pKolSmirn)

cKolSmirn(α, m, n) $\Rightarrow j_\alpha$: $\Pr(J \ge j_\alpha) = \alpha$

Example 1

$m = 6$, $n = 10$, $\alpha = 0.004$

```
> cKolSmirn(0.004,6,10)
Number of X values: 6 Number of Y values: 10
For the given alpha=0.004, the upper cutoff value is Kolmogorov-Smirnov J=25,
with true alpha level=0.004
```

$j_{0.004} = 25$, $\Pr(J \ge 25) = 0.004$

Compare with the results from ks.test:

$m = 6$, $n = 10$, $d = 2$

$D = 0.83333 = \frac{5}{6} \Rightarrow J = \frac{6(10)}{2} \cdot \frac{5}{6} = 25$, $\Pr(J \ge 25) = \Pr(D \ge 0.83333) = 0.003996 = 0.004$

Example 2

$m = n = 10$, $\alpha = 0.06$

```
> cKolSmirn(0.06,10,10)
Number of X values: 10 Number of Y values: 10
For the given alpha=0.06, the upper cutoff value is Kolmogorov-Smirnov J=6,
with true alpha level=0.0524
```

$j_{0.0524} = 6$, $\Pr(J \ge 6) = 0.0524$

Compare with the results from ks.test:

$m = 10$, $n = 10$, $d = 10$

$D = 0.6 \Rightarrow J = \frac{10(10)}{10}(0.6) = 10(0.6) = 6$, $\Pr(J \ge 6) = \Pr(D \ge 0.6) = 0.05245$

Approximate critical points

qKolSmirnLSA(α) $\Rightarrow q_\alpha^*$: $Q(q_\alpha^*) = \alpha$, $\Pr(J^* \ge q_\alpha^*) \approx \alpha$

Example

```
> qKolSmirnLSA(0.01)
[1] 1.627
```

$q_{0.01}^* = 1.627$, $Q(1.627) = 0.01$, $\Pr(J^* \ge 1.627) \approx 0.01$

$q_{0.01}^* \approx \sqrt{-0.5 \ln(0.01/2)} = 1.628$

One-way Layout

Kruskal-Wallis test

cKW($\alpha, c(n_1, \ldots, n_k)$) $\Pr(H \geq h_\alpha) = \alpha$

> cKW(0.05,c(3,2,3,2,3))

Monte Carlo Approximation (with 10000 Iterations) used:

Group sizes: 3 2 3 2 3
For the given alpha=0.05, the upper cutoff value is Kruskal-Wallis H=8.06593407,
with true alpha level=0.049

$\alpha = 0.049$ (may differ each time using R), $h_{0.049} = 8.0659$, $\Pr(H \geq 8.0659) = 0.049$

Jonckheere-Terpstra test

cJCK($\alpha, c(n_1, \ldots, n_k)$) $\Pr(J \geq j_\alpha) = \alpha$

> cJCK(0.05,c(6,5,7))
Group sizes: 6 5 7
For the given alpha=0.05, the upper cutoff value is Jonckheere-Terpstra J=75,
with true alpha level=0.0443

$\alpha = 0.0443$, $j_{0.0443} = 75$, $\Pr(J \geq 75) = 0.0443$

Mack-Wolfe test, known peak

cUmbrPK($\alpha, c(n_1, \ldots, n_k), p$) $\Pr(A_p \geq a_{p,\alpha}) = \alpha$

> cUmbrPK(0.001,c(7,3,5,4,4,3),4)
Group sizes: 7 3 5 4 4 3
For the given alpha=0.001, the upper cutoff value is Mack-Wolfe Peak Known A 4=137,
with true alpha level=8e-04

$\alpha = 0.0008$, $a_{4,0.0008} = 137$, $\Pr(A_4 \geq 137) = 0.0008$

> cUmbrPK(0.01, c(3,3,3,3,3),3)
Group sizes: 3 3 3 3 3
For the given alpha=0.01, the upper cutoff value is Mack-Wolfe Peak Known A 3=45,
with true alpha level=0.0086

$\alpha = 0.0086$, $a_{3,0.0086} = 45$, $\Pr(A_3 \geq 45) = 0.0086$

Mack-Wolfe test, unknown peak

cUmbrPU($\alpha, c(n_1, \ldots, n_k)$) $\Pr(A^*_{\hat{p}} \geq a^*_{\hat{p},\alpha}) = \alpha$

> cUmbrPU(0.036, c(3,3,3,3,3))

Monte Carlo Approximation (with 10000 Iterations) used:

Group sizes: 3 3 3 3 3
For the given alpha=0.036, the upper cutoff value is Mack-Wolfe Peak Unknown
A*(p-hat)=2.3533936217, with true alpha level=0.0342

$\alpha = 0.0342$, $a^*_{\hat{p},0.0342} = 2.353$, $\Pr(A^*_{\hat{p}} \geq 2.353) = 0.0342$

Two-sided multiple comparisons

$w_\alpha^*:\ \Pr\left(|W_{uv}^*| < w_\alpha^*, 1 \le u < v \le k\right) = 1-\alpha$

cSDCFlig($\alpha, c(n_1,\ldots,n_k)$)

Example 1 $k=3$, $(n_1,\ldots,n_k)=(6,6,6)$

> cSDCFlig(0.1,c(6,6,6))

Monte Carlo Approximation (with 10000 Iterations) used:

Group sizes: 6 6 6

For the given experimentwise alpha=0.1, the upper cutoff value is Dwass, Steel, Critchlow-Fligner W=2.94392028877595, with true experimentwise alpha level=0.0997

$\alpha = 0.0997$, $w_{0.0997}^* = 2.9439$, $\Pr\left(|W_{uv}^*| < 2.9439, 1 \le u < v \le 3\right) = 1-0.0997 = 0.9003$

Example 2 $k=4$, $(n_1,\ldots,n_k)=(10,10,10,10)$

> cSDCFlig(0.01,c(10,10,10,10))

Monte Carlo Approximation (with 10000 Iterations) used:

Group sizes: 10 10 10 10

For the given experimentwise alpha=0.01, the upper cutoff value is Dwass, Steel, Critchlow-Fligner W=4.27617987059879, with true experimentwise alpha level=0.0076

$\alpha = 0.0076$, $w_{0.0076}^* = 4.276$, $\Pr\left(|W_{uv}^*| < 4.276, 1 \le u < v \le 4\right) = 1-0.0076 = 0.9923$

Approximation

$w_\alpha^* \approx q_\alpha:\ \Pr\left(\max\{Z_1,\ldots,Z_k\} - \min\{Z_1,\ldots,Z_k\} \ge q_\alpha\right) = \alpha$, $Z_1,\ldots,Z_k \sim$ i.i.d. $N(0,1)$

cRangeNor(α, k)

$\alpha = 0.1$, $k=3$:

> cRangeNor(0.1,3)
[1] 2.903

$q_{0.1} = 2.903$ (compare to $w_{0.0997}^* = 2.9439$ in Example 1 above)

$\alpha = 0.01$, $k=4$:

> cRangeNor(0.01,4)
[1] 4.404

$q_{0.01} = 4.404$ (compare to $w_{0.0076}^* = 4.276$ in Example 2 above)

$\alpha = 0.025$, $k=4$:

> cRangeNor(0.025,4)
[1] 3.985

$q_{0.025} = 3.985$

One-sided multiple comparisons

c_α^*: $\Pr(W_{uv}^* < c_\alpha^*, 1 \le u < v \le k) = 1 - \alpha$

cHaySton($\alpha, c(n_1, \ldots, n_k)$)

Example 1 $k = 3$, $(n_1, n_2, n_3) = (3, 4, 6)$

```
> cHaySton(0.05,c(3,4,6))
Monte Carlo Approximation (with  10000  Iterations) used:

Group sizes: 3 4 6
For the given experimentwise alpha=0.05, the upper cutoff value is Hayter-Stone W*=3.0151134458, with
true experimentwise alpha level=0.0303
```

$\alpha = 0.0303$, $c_{0.0303}^* = 3.015$, $\Pr(W_{uv}^* < 3.015, 1 \le u < v \le 3) = 1 - 0.0303 = 0.9697$

Example 2 $k = 4$, $(n_1, \ldots, n_k) = (10, 10, 10, 10)$

```
> cHaySton(0.01,c(10,10,10,10))
Monte Carlo Approximation (with  10000  Iterations) used:

Group sizes: 10 10 10 10
For the given experimentwise alpha=0.01, the upper cutoff value is Hayter-Stone W*=4.0623708771,
with true experimentwise alpha level=0.0091
```

$\alpha = 0.0091$, $c_{0.0091}^* = 4.062$, $\Pr(W_{uv}^* < 4.062 \le u < v \le 4) = 1 - 0.0091 = 0.9919$

Approximation

$c_\alpha^* \approx d_\alpha$: $\Pr(D > d_\alpha) = \alpha$, where

$$D = \max_{1 \le i < j \le k} \frac{Z_j - Z_i}{\sqrt{(n_i + n_j)/(2 n_i n_j)}}, \text{ independent } Z_i \sim N(0, 1/n_i),\ i = 1, \ldots, k.$$

cHayStonLSA(α, k)

$\alpha = 0.03$, $k = 3$:

```
> cHayStonLSA(0.03,3)
[1] 3.237
```

$d_{0.03} = 3.237$ (compare to $c_{0.0303}^* = 3.015$ in Example 1 above), $\Pr(D > 3.237) = 0.03$

$\alpha = 0.01$, $k = 4$:

```
> cHayStonLSA(0.01,4)
[1] 4.098
```

$d_{0.01} = 4.098$ (compare to $c_{0.0091}^* = 4.062$ in Example 2 above), $\Pr(D > 4.098) = 0.01$

One-sided treatments-versus-control multiple comparisons

y_{α}^{*}: $\Pr\left(N^{*}(R_{.u}-R_{.1})<y_{\alpha}^{*}, u=2,\ldots,k\right)=1-\alpha$

cNDWol($\alpha, c(n_1,\ldots,n_k)$)

$\alpha=0.1$, $k=3$, $(n_1,n_2,n_3)=(6,6,6)$

> cNDWol(0.1, c(6,6,6))

Monte Carlo Approximation (with 10000 Iterations) used:

Control group size: 6 Treatment group size(s): 6 6

For the given experimentwise alpha=0.1, the upper cutoff value is Nemenyi, Damico-Wolfe Y*=30, with true experimentwise alpha level=0.0983

$\alpha=0.0983$, $y_{0.0983}^{*}=30$, $\Pr\left(6(R_{.2}-R_{.1})<30, 6(R_{.3}-R_{.1})<30\right)=1-0.0983=0.9017$

Approximation

For $n_1=b$, $n_2=\cdots=n_k=n$

$m_{\alpha,\rho}^{*}$: $\Pr\left(\max\{Z_2,\ldots,Z_k\}\geq m_{\alpha,\rho}^{*}\right)=\alpha$, $Z_2,\ldots,Z_k \sim N_{k-1}(0,\ldots,0;1,\ldots,1;\rho)$

Then

$$y_{\alpha}^{*}\approx m_{\alpha,\rho}^{*}N^{*}\sqrt{\frac{N(N+1)}{12}\left(\frac{1}{b}+\frac{1}{n}\right)} \quad \text{with} \quad \rho=\frac{n}{b+n}$$

cMaxCorrNor($\alpha, k-1, \rho$)

For $\alpha=0.05$, $k=5$, $\rho=0.5$

> cMaxCorrNor(0.05,4,0.5)
[1] 2.16

$m_{0.05,0.5}^{*}=2.16$, $\Pr\left(\max\{Z_2,\ldots,Z_5\}\geq 2.16\right)=0.05$

For $\alpha=0.1$, $k=3$, $b=n=6$, $N=3(6)=18$, $N^{*}=6$, $\rho=6/(6+6)=0.5$

> cMaxCorrNor(0.1,2,0.5)
[1] 1.57

$m_{0.1,0.5}^{*}=1.57$, $\Pr\left(\max\{Z_2,Z_3\}\geq 1.57\right)=0.1$

$$y_{0.1}^{*}\approx 1.57(6)\sqrt{\frac{18(19)}{12}\left(\frac{1}{6}+\frac{1}{6}\right)}=29.04$$

Since $6(R_{.u}-R_{.1})<29.04 \Leftrightarrow 6(R_{.u}-R_{.1})<30$,

$y_{0.1}^{*}\approx 29.04 \Rightarrow y_{0.1}^{*}\approx 30$ (compare to $y_{0.0983}^{*}=30$)

Two-way Layout

Complete block design

Friedman, Kendall-Babington Smith test for general alternatives

$s_\alpha :\ \Pr(S \ge s_\alpha) = \alpha$

cFrd(α, k, n)

> cFrd(0.025,5,7)

Monte Carlo Approximation (with 10000 Iterations) used:

Number of blocks: n=7
Number of treatments: k=5
For the given alpha=0.025, the upper cutoff value is Friedman, Kendall-Babington Smith S=10.6285714286, with true alpha level=0.0243

$\alpha = 0.0243$, $s_{0.0243} = 10.629$, $\Pr(S \ge 10.629) = 0.0243$

Page test for ordered alternatives

$l_\alpha :\ \Pr(L \ge l_\alpha) = \alpha$

cPage(α, k, n)

> cPage(0.01,5,3)

Monte Carlo Approximation (with 10000 Iterations) used:

Number of blocks: n=3
Number of treatments: k=5
For the given alpha=0.01, the upper cutoff value is Page L=155, with true alpha level=0.01

$\alpha = 0.01$, $l_{0.01} = 155$, $\Pr(L \ge 155) = 0.01$

Two-sided multiple all-treatment comparisons

Wilcoxon-Nemenyi-Macdonald-Thompson procedure

$r_\alpha :\ \Pr(|R_u - R_v| < r_\alpha, 1 \le u < v \le k) = 1 - \alpha$

cWNMT(α, k, n)

> cWNMT(0.01,3,22)

Monte Carlo Approximation (with 10000 Iterations) used:

Number of blocks: n=22
Number of treatments: k=3
For the given alpha=0.01, the upper cutoff value is Wilcoxon, Nemenyi, McDonald-Thompson R=20, with true alpha level=0.0087

$\alpha = 0.0087$, $r_{0.0087} = 20$, $\Pr(|R_u - R_v| < 20, 1 \le u < v \le 3) = 1 - 0.0087 = 0.9913$

> cRangeNor(0.01,3)

[1] 4.121

$q_{0.01} = 4.121$

One-sided treatments-versus-control multiple comparisons

Nemenyi-Wilcoxon-Wilcox-Miller procedure

r_α^* : $\Pr(|R_u - R_1| < r_\alpha^*, u = 2,\ldots,k) = 1-\alpha$

cNWWM(α, k, n)

> cNWWM (0.05,5,8)

Monte Carlo Approximation (with 10000 Iterations) used:

Number of blocks: n=8
Number of treatments: k=5
For the given alpha=0.05, the upper cutoff value is Nemenyi, Wilcoxon-Wilcox, Miller R*=14, with true alpha level=0.05

$\alpha = 0.05$, $r_{0.05}^* = 14$, $\Pr(|R_u - R_1| < 14, u = 2,3,4,5) = 1-0.05 = 0.95$

BIBD

Durbin-Skillings-Mack test statistic D

$\Pr(D \ge d_{\alpha,s}) = \alpha$

cDurSkiMa(α , **obs.mat**)

obs.mat = matrix of $c_{ij} = 0$ or 1

obs.mat=matrix(c(1,1,0,1,0,0,0,1,0,1,0,1,0,0,0,0,1,1,0,0,1,1,0,0,0,0,1,1,0,1,1,0,0,1,0,0,1,0,0,1,0,1,0,0, 0,1,1,1,0), nrow=7, ncol=7, byrow = TRUE)

```
obs.mat
     [,1] [,2] [,3] [,4] [,5] [,6] [,7]
[1,]    1    1    0    1    0    0    0
[2,]    1    0    1    0    1    0    0
[3,]    0    0    1    1    0    0    1
[4,]    1    0    0    0    0    1    1
[5,]    0    1    1    0    0    1    0
[6,]    0    1    0    0    1    0    1
[7,]    0    0    0    1    1    1    0
```

> cDurSkiMa(0.25, **obs.mat**)
Number of blocks: n=7
Number of treatments: k=7
Number of treatments per block: s=3
Number of observations per treatment: p=3
Number of times each pair of treatments occurs together within a block: lambda=1
For the given alpha=0.25, the upper cutoff value is Durbin, Skillings-Mack D=8.57142857142857, with true alpha level=0.2305

$\alpha = 0.2305$, $d_{0.2305,3} = 8.5714$, $\Pr(D \ge 8.5714) = 0.2305$

> cRangeNor(0.2,7)

[1] 3.39

$q_{0.2} = 3.39$

Arbitrary incomplete block design

$sm_\alpha :\ \Pr(SM \geq sm_\alpha) = \alpha$

cSkilMack(*α*, **obs.mat**)

```
> obs.mat=matrix(c(1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,1,1,1), nrow=8, ncol=3, byrow = TRUE)

obs.mat
     [,1] [,2] [,3]
[1,]    1    1    1
[2,]    1    1    1
[3,]    1    1    1
[4,]    1    0    1
[5,]    1    1    1
[6,]    1    1    1
[7,]    1    1    1
[8,]    1    1    1

> cSkilMack(0.01, obs.mat)
Monte Carlo Approximation (with 10000 Iterations) used:

Number of blocks: n=8
Number of treatments: k=3
Number of treatments per block: s=3
Number of treatments per block: s=3
Number of treatments per block: s=3
Number of treatments per block: s=2
Number of treatments per block: s=3
Number of treatments per block: s=3
Number of treatments per block: s=3
Number of treatments per block: s=3
For the given alpha=0.01, the upper cutoff value is Skillings-Mack SM=8.52805, with true alpha
level=0.0098
```

$\alpha = 0.0098,\ \ sm_{0.0098} = 8.528,\quad \Pr(SM \geq 8.528) = 0.0098$

Block design with replications

$ms_\alpha :\ \Pr(MS \geq ms_\alpha) = \alpha$

cMackSkil(α, k, n, c)

```
> cMackSkil(0.01,4,3,3)

Monte Carlo Approximation (with 10000 Iterations) used:

Number of blocks: n=3
Number of treatments: k=4
For the given alpha=0.01, the upper cutoff value is Mack-Skillings MS=10.53846, with true alpha
level=0.0098
```

$\alpha = 0.0098,\ \ ms_{0.0098} = 10.54,\quad \Pr(MS \geq 10.54) = 0.0098$

Independence

Kendall statistic

```
x<-c(12,16,24,35,38,44,57,63,65,69,74,92)
y<-c(6,5,11,3,15,9,45,18,60,25,33,48)

> cor(x, y, method="kendall")
[1] 0.6363636
```

$\hat{\tau} = 0.6363636 \quad n = 12 \quad N = n(n-1)/2 = 12(11)/2 = 66 \quad K = N\hat{\tau} = 42$

Kendall test

```
> cor.test (x, y, method="kendall", alt="greater")

        Kendall's rank correlation tau

data:  x and y
T = 54, p-value = 0.001591
alternative hypothesis: true tau is greater than 0
sample estimates:
tau
0.6363636
```

$T = \#\{(u,v) : u < v, S_u < S_v\}$, $K = T - (N - T) = 54 - (66 - 54) = 54 - 12 = 42 \qquad k_{0.001591} = 0.6363636$

$\Pr(\hat{\tau} \geq 0.6363636) = \Pr(\bar{K} \geq 0.6363636) = \Pr(K \geq 42) = 0.001591$

```
x<-c(1,2,3,4,5,6)
y<-c(3,2,1,4,5,6)

> cor.test (x, y, method="kendall", alt="greater")

Kendall's rank correlation tau

data:  x and y
T = 12, p-value = 0.06806
alternative hypothesis: true tau is greater than 0
sample estimates:
tau
0.6
```

$N = 6(5)/2 = 15 \qquad K = 12 - (15 - 12) = 12 - 3 = 9 \qquad \hat{\tau} = K/15 = 12/15 = 0.6$

$\Pr(\hat{\tau} \geq 0.6) = \Pr(\bar{K} \geq 0.6) = \Pr(K \geq 9) = 0.06806 \qquad k_{0.06806} = 0.6$

Confidence interval of τ

```
x<-c(1,2,3,4,5,6,7,8,9)
y<-c(1,7,2,5,3,4,9,8,6)

> kendall.ci (x, y, alpha=0.1, type="t")

1 - alpha = 0.9 two-sided CI for tau:
0.027, 0.862
```

Approximate 90% confidence interval of τ : (0.027, 0.862)

Distribution of Kendall statistic

library(SuppDists)

pKendall($\bar{k} = k/N$, N= n , lower.tail=T) $\Rightarrow$ $\Pr(\bar{K} \le \bar{k}) = \Pr(K \le k) = \Pr(K \ge -k)$

qKendall(p= α , N= n , lower.tail=T)

$\Rightarrow$ $-\bar{k}$: $\Pr(\bar{K} \le -\bar{k}) = \Pr(\bar{K} \ge \bar{k}) = \Pr(K \ge N\bar{k}) \ge \alpha$ and $\Pr(K \ge Nk_\alpha + 2) \le \alpha$

Example: For $n = 9$, $N = 9(8)/2 = 36$, $\alpha = 0.10$:

> pKendall(-12/36, N=9, lower.tail=T)
[1] 0.1297591 $\Rightarrow$ $\Pr(K \le -12) = \Pr(K \ge 12) = 0.1297591$

> pKendall(-14/36, N=9, lower.tail=T)
[1] 0.09009039 $\Rightarrow$ $\Pr(K \ge 14) = 0.09009039$

> pKendall(-16/36,N=9, lower.tail=T)
[1] 0.05971947 $\Rightarrow$ $\Pr(K \ge 16) = 0.05971947$

> qKendall(p=0.10, N=9, lower.tail=T)

[1] -0.3333333

$\Pr(\bar{K} \le -1/3) = \Pr(K \ge 36/3) = \Pr(K \ge 12) = 0.12976 > 0.10$ $\Pr(K \ge 14) = 0.09009 < 0.10$

Distribution of Spearman rank correlation coefficient

library(SuppDists)

pSpearman($-x$, r= n) $\Rightarrow$ $\Pr(r_s \le -x) = \Pr(r_s \ge x)$ $\Rightarrow$ $r_{s,\alpha} = x$ for $\alpha = \Pr(r_s \ge x)$

qSpearman(α , r= n) $\Rightarrow$ $-r_{s,\alpha}$ (approximate): $\Pr(r_s \le -r_{s,\alpha}) = \Pr(r_s \ge r_{s,\alpha}) \approx \alpha$

Example: For $n = 7$, $\alpha = 0.01$:

> pSpearman(-0.7, r=7)

[1] 0.04404762

$\Pr(r_s \le -0.7) = \Pr(r_s \ge 0.7) = 0.04404762$ $r_{s,0.044} = 0.7$

> qSpearman(0.01, r=7)

[1] -0.7857143

$\Pr(r_s \ge 0.7857143) \approx 0.01$

> pSpearman(-0.7857143, r=7)

[1] 0.02400794

$\Pr(r_s \ge 0.7857143) = 0.02400794$

Spearman rank correlation coefficient

```
x<-c(12,16,24,35,38,44,57,63,65,69,74,92)
y<-c(6,5,11,3,15,9,45,18,60,25,33,48)

> cor(x, y, method="pearson")
[1] 0.8251748
```

$r_s = 0.8251748$

Spearman test

```
> cor.test(x, y, method="spearman", alt="greater")

        Spearman's rank correlation rho

data:  x and y
S = 50, p-value = 0.0008593
alternative hypothesis: true rho is greater than 0
sample estimates:
rho
0.8251748
```

$r_s = 0.8251748 \quad r_{s,0.0008593} = 0.8251748 \quad \Pr(r_s \geq 0.8251748) = 0.0008593$

$$n = 12 \quad \sum_{i=1}^{n} D_i^2 = \sum_{i=1}^{12} (R_i - S_i)^2 = S = 50 \quad r_s = 1 - \frac{6S}{n(n^2-1)} = 1 - \frac{6(50)}{12(144-1)} = \frac{118}{143} = 0.8251748$$

With ties

```
x<-c(1.5,1.5,3,4,5,6,7)
y<-c(2.5,4,2.5,1,5,6,7)

> cor.test (x, y, method="spearman", alt="greater")

        Spearman's rank correlation rho

data:  x and y
S = 16.8, p-value = 0.03996
alternative hypothesis: true rho is greater than 0
sample estimates:
rho
0.7

Warning message:
In cor.test.default(x, y, method = "spearman", alt = "greater") :
  Cannot compute exact p-value with ties
```

$S = 16.8 \quad r_s = 0.7 \quad r_{s,0.03996} = 0.7 \quad \Pr(r_s \geq 0.7) = 0.03996$ (not accurate)

($S = 16.8$ is not precise in this case. The precise value is $S = 1 + 2.5^2 + 0.5^2 + 3^2 = 16.5$)

More accurate p-value: $\Pr(r_s \geq 0.7) = 0.04404762$ using pSpearman(-0.7, r=7)

Regression

Theil test

```
x<-c(1,2,3,4,5)
y<-c(1.26, 1.27, 1.12, 1.16, 1.03)

> theil(x, y, beta.0=0,type="l")
Alternative: beta less than 0
C = -6, C.bar = -0.6, P = 0.117
beta.hat = -0.056
alpha.hat = 1.316

1 - alpha = 0.95 lower bound for beta:
-0.13, Inf
```

$C = -6$ $\quad \overline{C} = -0.6$ $\quad p$-value $= \Pr(C \le -6) = 0.117$ $\quad \hat{\beta} = -0.056$ $\quad \hat{\alpha} = 1.316$

Confidence interval of slope

```
> theil(x, y, alpha=0.1, beta.0=0,type="t")
Alternative: beta not equal to 0
C = -6, C.bar = -0.6, P = 0.233
beta.hat = -0.056
alpha.hat = 1.316

1 - alpha = 0.9 two-sided CI for beta:
-0.13, 0.01
```

Approximate 90% confidence interval of β: (-0.13, 0.01)

The exact level of this confidence interval $(S_{(2)}, S_{(9)}) = (-0.13,\ 0.01)$ can be calculated via the Kendall's statistic K: $\Pr(S_{(2)} < \beta < S_{(9)}) = \Pr(-8 < K < 8) = 1 - 2\Pr(K \ge 8)$.

To find $\Pr(K \ge 8)$, take (x_i, y_i), $i = 1, \ldots, 5$, such that $K = 8$; then use R-command cor.test:

```
x<-c(1,2,3,4,5)
y<-c(2,1,3,4,5)

> cor.test (x, y, method="kendall", alt="greater")
        Kendall's rank correlation tau

data:  x and y
T = 9, p-value = 0.04167
alternative hypothesis: true tau is greater than 0
sample estimates:
tau
0.8
```

The results show $\Pr(K \ge 8) = \Pr(\overline{K} \ge 0.8) = 0.04167$. Thus the exact level of $(S_{(2)}, S_{(9)})$ for β is

$$\Pr(S_{(2)} < \beta < S_{(9)}) = 1 - 2\Pr(K \ge 8) = 1 - 2(0.04167) = 0.9167 = 91.67\%$$

Sen-Adichie test of equal slope

```
x1 <- x2 <- x3 <- x4 <- c(0, 1.5, 3, 4.5, 6)
y1 <- c(0, 33.019, 111.314, 196.205, 230.658)
y2 <- c(0, 131.831, 181.603, 230.07, 258.119)
y3 <- c(0, 33.351, 97.463, 196.615, 217.308)
y4 <- c(0, 8.959, 105.384, 211.392, 255.105)

z <- list(cbind(x1, y1), cbind(x2, y2), cbind(x3, y3), cbind(x4, y4))
> z
[[1]]
      x1      y1
[1,] 0.0   0.000
[2,] 1.5  33.019
[3,] 3.0 111.314
[4,] 4.5 196.205
[5,] 6.0 230.658

[[2]]
      x2      y2
[1,] 0.0   0.000
[2,] 1.5 131.831
[3,] 3.0 181.603
[4,] 4.5 230.070
[5,] 6.0 258.119

[[3]]
      x3      y3
[1,] 0.0   0.000
[2,] 1.5  33.351
[3,] 3.0  97.463
[4,] 4.5 196.615
[5,] 6.0 217.308

[[4]]
      x4      y4
[1,] 0.0   0.000
[2,] 1.5   8.959
[3,] 3.0 105.384
[4,] 4.5 211.392
[5,] 6.0 255.105

> sen.adichie(z)

Null: all slopes are equal
V = 1.5, P = 0.682
```

Multiple regression

```
library(Rfit)

x1 <- c(0, 1, 2, 3, 4, 5, 6, 7)
x2 <- c(5, 2, 8, 3, 4, 10, 15, 12)
x3 <- c(2, 6, 3, 8, 5, 1, 8, 9)
y <- c(5, 7, 12, 11, 16, 21, 19, 25)

> rfit(y ~ x1 + x2 + x3)
Call:
rfit.default(formula = y ~ x1 + x2 + x3)

Coefficients:
(Intercept)          x1          x2          x3
  6.9617639   3.2034567  -0.1334766  -0.6462207
```

HM test

```
r.01 <- rfit(y ~ x1, intercept=F)
f.01 <- rfit(y ~ x1 + x2 + x3)

> drop.test(f.01, r.01)

Drop in Dispersion Test
F-Statistic     p-value
    3.1181      0.1527
```

$HM = 3.1181$

$\Pr\left(F_{q,n-p-1} \geq 3.1181\right) = \Pr\left(F_{2,4} \geq 3.1181\right) = 0.1527 \quad (n = 8,\ p = 3,\ q = 2)$

```
h.01 <- drop.test(f.01, r.01)

> h.01$RD
         [,1]
[1,] 5.508773

> h.01$tauhat
[1] 1.766714
```

$D_J^* = 5.508773 \quad \hat{\tau} = 1.766714 \qquad HM = \dfrac{2D_J^*}{q\hat{\tau}} = \dfrac{2(5.508773)}{2(1.766714)} = 3.118090$

Matrix

Matrix input

A <- matrix(c(1,2,3,4,5,6,7,8,9), 3)

```
> A
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
```

A <- matrix(c(1,2,3,4,5,6,7,8,9), 3, byrow = T)

```
> A
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
```

A <- matrix(c(1,2,3,4,5,6,7,8), nrow=2, ncol=4)

```
> A
     [,1] [,2] [,3] [,4]
[1,]    1    3    5    7
[2,]    2    4    6    8
```

A <- matrix(c(1,2,3,4,5,6,7,8), nrow=2, ncol=4, byrow = T)

```
> A
     [,1] [,2] [,3] [,4]
[1,]    1    2    3    4
[2,]    5    6    7    8
```

Combine matrices

A1<- matrix(c(1,2,3,4,5,6,7,8), nrow=2, ncol=4, byrow = T)

```
> A1
     [,1] [,2] [,3] [,4]
[1,]    1    2    3    4
[2,]    5    6    7    8
```

A2<- matrix(c(11,12,13,14,15,16,17,18), nrow=2, ncol=4, byrow = T)

```
> A2
     [,1] [,2] [,3] [,4]
[1,]   11   12   13   14
[2,]   15   16   17   18
```

A<- matrix(c(A1,A2), nrow=2, ncol=8)

```
> A
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
[1,]    1    2    3    4   11   12   13   14
[2,]    5    6    7    8   15   16   17   18
```

Matrix multiplication

```
A <- matrix(c(1,2,3,4,5,6,7,8,9), 3)
> A
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9

B <- matrix(c(1,2,3,4,5,6,7,8,9), 3, byrow = T)
> B
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9

C <- A%*%B
> C
     [,1] [,2] [,3]
[1,]   66   78   90
[2,]   78   93  108
[3,]   90  108  126
```

Matrix inverse

```
A <- matrix(c(8, -2, -3, -2, 6 , -2, -3, -2, 7), 3)
> A
     [,1] [,2] [,3]
[1,]    8   -2   -3
[2,]   -2    6   -2
[3,]   -3   -2    7

> solve(A)
          [,1]      [,2]      [,3]
[1,] 0.1919192 0.1010101 0.1111111
[2,] 0.1010101 0.2373737 0.1111111
[3,] 0.1111111 0.1111111 0.2222222
```