STA4001 T02 118010350

1. $E(6-2X) = 6-2E(X) = 6-2\times2 = 2.$

$var(6-2X) = var(2X) = 4var(X) = 36.$

$E((X-3)/4) = E(X/4) - 3/4 = -1/4.$

$var((X-3)/4) = var(X/4) = var(X)/16 = 9/16.$

2. (a) $2c+4c = 6c = 1, \Rightarrow c = 1/6.$

(b) $E(X) = 2\cdot Pr(X=2) + 4\cdot Pr(X=4) = 2\cdot2\cdot(1/6) + 4\cdot4\cdot(1/6) = 10/3.$

(c) $E(X^2) = 4\cdot Pr(X=2) + 16\cdot Pr(X=4) = 4\cdot2\cdot(1/6) + 16\cdot4\cdot(1/6) = 12.$

(d) $var(X) = E(X^2) - E(X)^2 = 12 - (10/3)^2 = 8/9.$

(e). when $i=2$, $(i-4)^+ = 0$. $Pr(X=i) = 1/3.$

when $i=4$, $(i-4)^+ = 0$. $Pr(X=i) = 2/3.$

$\Rightarrow E[(X-4)^+] = 0.$

3. (a) $Pr(X=k) = 2^k\cdot e^{-2}/k!$, $k = 0,1,2.$

(b). $E(X) = 2.$

(c) $var(X) = 2.$

(d) $Y = \min(X,3)$, $y =$ 0, 1, 2, 3.

$Pr(Y=y) =$ e^{-2}, $2\cdot e^{-2}$, $2\cdot e^{-2}$, $1-5\cdot e^{-2}$

(e) $E(Y) = 0\cdot Pr(Y=0) + 1\cdot Pr(Y=1) + 2\cdot Pr(Y=2) + 3\cdot Pr(Y=3)$

$= 2\cdot e^{-2} + 2\cdot2\cdot e^{-2} + 3\cdot(1-5\cdot e^{-2})$

$= 3 - 9\cdot e^{-2}.$

4. (a) $\int_0^\infty c\cdot e^{-2s}ds = c/2 = 1 \Rightarrow c = 2.$

(b) $E(Y) = 2\int_0^\infty s\cdot e^{-2s}ds = 2\cdot\frac{1}{4}(-2s\cdot e^{-2s} - e^{-2s})\Big|_0^\infty = 1/2.$

$E(Y^2) = 2\int_0^\infty s^2 \cdot e^{-2s} ds = 2\cdot\left(-\frac{1}{2}s^2 \cdot e^{-2s} + \frac{1}{4}(-2s\cdot e^{-2s} - e^{-2s})\right)\Big|_0^\infty = 1/2$

$var(Y) = E(Y^2) - E(Y)^2 = 1/2 - (1/2)^2 = 1/4$

$var(Y) / (E(Y)^2) = 1$

(c) $Pr(Y>4) = 1 - Pr(Y \le 4) = 1 - \int_0^4 2\cdot e^{-2s} ds = e^{-8}$

(d) $Pr(Y>6) = 1 - Pr(Y \le 6) = 1 - \int_0^6 2\cdot e^{-2s} ds = e^{-12}$

$Pr(Y>2) = 1 - Pr(Y \le 2) = 1 - \int_0^2 2\cdot e^{-2s} ds = e^{-4}$

$Pr(Y>6 \mid Y>2) = Pr(Y>6) / Pr(Y>2) = e^{-12}/e^{-4} = e^{-8}$

(e) $Pr(Y > x^*) = 2/3, \Rightarrow Pr(Y \le x^*) = 1/3.$

$\int_0^{x^*} 2\cdot e^{-2s} ds = (-1)\cdot e^{-2s}\Big|_0^{x^*} = 1 - e^{-2x^*} = 1/3$

$\Rightarrow e^{-2x^*} = 2/3 \quad \Rightarrow \quad x^* = -\ln(2/3)/3.$

5. (a). $Pr(X=Y) = 0.$

(b). $Pr(\min(X,Y) > 1/3) = Pr(X > 1/3, Y > 1/3) = \int_{1/3}^\infty \int_{1/3}^\infty 2\cdot e^{-(2s+t)} ds dt = e^{-1}$

(c). $Pr(X \le Y) = \int_0^\infty \int_0^t 2\cdot e^{-(2s+t)} ds dt = 2/3$

(d). $f_X(x) = \int_0^\infty 2\cdot e^{-(2x+t)} dt = 2\cdot e^{-2x}$

(e) $E(XY) = \int_0^\infty \int_0^\infty st2e^{-(2s+t)} ds dt = \int_0^\infty te^{-t} dt \int_0^\infty 2se^{-2s} ds = 1/2$

6. (a) Poisson

(b) Exponential

(c) Geometric

(d) Bernoulli

(e) Binomial

(f) Normal

7. Define X_i as the processing time for item i.

x_i follows the exponential distribution with rate $1/2$.

Let $Y = X_1 + X_2 + \cdots + X_{100}$, Y follows the gamma distribution with shape 100 and rate $1/2$.

$Pr(Y \le 195) = 0.4134$.