STA3007: Tutorial 8

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Outline

Question 1

1. Apply the procedure of Wilcoxon-Nemenyi-McDonald-Thompson two-sided all-treatment multiple comparisons to the data of Table 7.2.
2. Estimate the contrast $\theta = -\tau_1 - 2\tau_2 + 3\tau_3$ for the ozone exposure data in Table 7.2.

Table 7.2 Effect of Experimental Ozone Exposures on Airway Resistance (cm $H_2O l/s$)

Subject	After .1 ppm	After .6 ppm	After 1.0 ppm
1	−.08	.01	.06
2	.21	.17	.19
3	.50	−.11	.34
4	.14	.07	.14

Source: J. R. Goldsmith and J. A. Nadel (1969).

Question 2 (Textbook Prob. 7.56 & 7.58)

Given the experimental measurements in the following table,

1. Apply the large-sample approximation method of BIBD with approximate significant level $\alpha \approx 0.05$, to test the hypothesis of interest versus general alternatives.
2. Compare the effects of treatments using SKillings-Mack two-sided all-treatment multiple comparison procedure for BIBD with $\alpha = 0.05$.

Table 7.13 Reactions of Male Rats to Chemical Substances

	Chemical Substance						
Rat	A	B	C	D	E	F	G
1	10.2	6.9		14.2			
2			9.9	12.9		14.1	
3		12.1	11.7		8.6		
4				14.3	9.1		7.7
5		8.8				16.3	8.6
6	13.1				9.2	15.2	
7	11.3		9.7				6.2

Source: W. Mendenhall (1968).

Question 3

In a randomized complete block design with k treatments and n blocks, the means, variances and covariances of block ranks $\{r_{ij}\}$ under the null hypothesis H_0 of equal treatments effects are given by

$$E[r_{ij}] = E[r_{11}] = \frac{k+1}{2}, \quad \mathrm{Var}[r_{ij}] = \mathrm{Var}[r_{11}] = \frac{(k+1)(k-1)}{12},$$
$$\mathrm{Cov}(r_{iu}, r_{iv}) = \mathrm{Cov}(r_{11}, r_{12}) = -\frac{k+1}{12} \quad \text{for} \quad u \neq v,$$
$$\mathrm{Cov}(r_{iu}, r_{tv}) = 0 \quad \text{for} \quad i \neq t, \quad u, v \in \{1, \cdots, k\}.$$

Prove the following results under H_0.

1. $E_0[S] = k-1$ for the Friedman, Kendall-Babington Smith test statistic $S = \frac{12}{nk(k+1)} \sum_{j=1}^{k} \left(R_j - \frac{n(k+1)}{2} \right)^2$, where $R_j = r_{1j} + r_{2j} + \cdots + r_{nj}$, $j = 1, \cdots, k$.
2. The covariance matrix of the random vector $\mathbf{A} = [A_1 \cdots A_{k-1}]^T = \sqrt{\frac{12}{k+1}} [R_1 \cdots R_{k-1}]^T$ is given by $\Sigma_0 = n(k\mathbf{I}_{k-1} - \mathbf{1} \cdot \mathbf{1}^T)$, where $\mathbf{I}_{k-1}$ is the $(k-1) \times (k-1)$ identity matrix and $\mathbf{1} = [1, \cdots, 1]^T$ is the $(k-1) \times 1$ column vector of 1's.