MAT3253 Tutorial 3

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1 Review

- The complex exponential and trigonometric (sin, cos) functions are entire functions, i.e., they are defined and analytic on the entire complex plane.
- $exp(z) := 1 + \frac{z}{1!} + \frac{z^2}{2!} + \cdots + \frac{z^n}{n!} + \cdots$
- **Properties**
  1. Check its radius of convergence is ∞.
  2. $exp'(z) = exp(z)$
  3. *Addition Formula*: $e^{(a+b)} = e^a e^b$
     - It follows that the multiplicative inverse of e^z is e^{-z}, hence the exponential function never takes the value zero.
  4. For real y, $|e^{iy}| = 1$
- $cos(z) = \frac{e^{iz}+e^{iz}}{2}; sin(z) = \frac{e^{iz}-e^{iz}}{2i}$
- **Properties**
  1. cos, sin are real valued when z is real valued.
  2. $cos^2(z) + sin^2(z) = 1$
  3. $sin(a + b) = sin(a)cos(b) + cos(a)sin(b)$
  4. $cos(a + b) = cos(a)cos(b) - sin(a)sin(b)$
  5. $e^{iz} = cos(z) + isin(z)$
  6. D $cos(z) = -sin(z)$; D $sin(z) = cos(z)$

2 Exercises

1. Find the values of $sin\ i, cos\ i, tan\ (1 + i)$
2. The hyperbolic functions are defined by $coshz = (e^z + e^{-z})/2$; $sinhz = (e^z - e^{-z}/2$. Express them through $cosiz, siniz$. Derive the addition formulas.
3. The Periodicity
   - Show if e^z is a period function, i.e., if $\exists\ c, s.t., f(z) = f(z + c), \forall z$. Then c is a pure imaginary number.
   - Show e^z is periodic, moreover, its period are all integral multiples of $i\omega_0$. The number π is defined to be $\omega_0/2$
   - Show every nonzero complex number can be written in the form $z = re^{i\theta}$, where $r > 0, \theta \in R$. Moreover, this representation is unique up to adding to θ integral multiples of 2π.