Tutorial 5

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Outline

Question 1

Here we consider the exact distribution of the Two-sample **Kolmogorov-Smirnov test** statistic J for given independent random variables $X_1, \cdots, X_m$ and $Y_1, \cdots, Y_n$ with $m = 2$ and $n = 3$.

1. Assume that there are no ties in the data values.

Question 1

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1. Assume that there are no ties in the data values.
2. Assume that $(X_1, X_2) = (3.2, 6.3)$ and $(Y_1, Y_2, Y_3) = (1.9, 1.9, 6.3)$.

Question 2

The following table provides the ranks $r_{i,j}$ of observed data $X_{i,j}$, $i = 1, \cdots, 8$ $j = 1, 2, 3$, from $k = 3$ populations in a one-way layout:

	r_{i1}	r_{i2}	r_{i3}
	5	1	22
	7	10	12
	8	9	21
	2	23	14
	3	13	16
	6	24	17
	11	15	18
	4	19	20
Sum	46	114	140

1. Test $H_0 : \tau_1 = \tau_2 = \tau_3$ against general alternatives that τ_1, τ_2, τ_3 are not all equal by the **Kruskal-Wallis test** at the 0.5% level of significance.

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Sum	46	114	140

1. Test $H_0 : \tau_1 = \tau_2 = \tau_3$ against general alternatives that τ_1, τ_2, τ_3 are not all equal by the **Kruskal-Wallis test** at the 0.5% level of significance.
2. Let U_{uv} denote the number of pairs $\{(r_{su}, r_{tv}) : s, t = 1, \cdots, 8\}$ such that $r_{su} < r_{tv}$ for $1 \leq u < v \leq 3$. Test $H_0 : \tau_1 = \tau_2 = \tau_3$ against ordered alternatives $H_1 : \tau_1 \leq \tau_2 \leq \tau_3$ with at least one strict inequality by the **Jonckheere-Terpstra test** at 0.1% level of significance.

Question 3 (Textbook Problem 6.1, 6.11)

Pretherapy training of clients has been shown to have beneficial effects on the process and outcome of counseling and psychotherapy. Sauber (1971) investigated four different approaches to pretherapy training:

1. Control (no treatment).
2. Therapeutic reading (TR) (indirect learning).
3. Vicarious therapy pretraining (VTP) (videotaped, vicarious learning).
4. Group, role induction interview (RII) (direct learning).

Treatment conditions $2-4$ were expected to enhance the outcome of counseling and psychotherapy as compared with a control group, those subjects receiving no prior set of structuring procedures. One of the major variables of the study was that of psychotherapeutic attraction. The basic data in Table 6.2 consist of the raw scores for this measure according to each of the four experimental conditions.

Question 3 (Textbook Problem 6.1, 6.11)

1. Apply the large-sample approximation method of **Kruskal-Wallis test** for general alternatives, with the correction for ties, at 5% level of significance.

Table 6.2 Raw Scores Indicating the Degree of Psychotherapeutic Attraction for Each Experimental Condition

Control	Reading (TR)	Videotape (VTP)	Group (RII)
0	0	0	1
1	6	5	5
3	7	8	12
3	9	9	13
5	11	11	19
10	13	13	22
13	20	16	25
17	20	17	27
26	24	20	29

Source: S. R. Sauber (1971).

Question 3 (Textbook Problem 6.1, 6.11)

1. Apply the large-sample approximation method of **Kruskal-Wallis test** for general alternatives, with the correction for ties, at 5% level of significance.
2. Apply the large-sample approximation method of **Jonckheere-Terpstra test** to the psychotherapeutic attraction data of Table 6.2 using the postulated ordering $\tau_1 \leq \tau_2 \leq \tau_3 \leq \tau_4$. Compare and contrast this result with that obtained for the KruskalWallis test at 5% level of significance.

Table 6.2 Raw Scores Indicating the Degree of Psychotherapeutic Attraction for Each Experimental Condition

Control	Reading (TR)	Videotape (VTP)	Group (RII)
0	0	0	1
1	6	5	5
3	7	8	12
3	9	9	13
5	11	11	19
10	13	13	22
13	20	16	25
17	20	17	27
26	24	20	29

Source: S. R. Sauber (1971).

Question 3 (Textbook Problem 6.1, 6.11)

Control	**Reading**	**Videotape**	**Group**
0(2)	0(2)	0(2)	1(4.5)
1(4.5)	6(11)	5(9)	5(9)
3(6.5)	7(12)	8(13)	12(19)
3(6.5)	9(14.5)	9(14.5)	13(21.5)
5(9)	11(17.5)	11(17.5)	19(27)
10(16)	13(21.5)	13(21.5)	22(31)
13(21.5)	20(29)	16(24)	25(33)
17(25.5)	20(29)	17(25.5)	27(35)
26(34)	24(32)	20(29)	29(36)
R_1=125.5	R_2=168.5	R_3=156	R_4=216
$R_{.1}$=13.944	$R_{.2}$=18.722	$R_{.3}$=17.333	$R_{.4}$=24

Table: Ascending order and rank sum

Question 3 (Textbook Problem 6.1, 6.11)

tied group	element	size
1	0	3
2	1	2
3	3	2
4	5	3
5	9	2
6	11	2
7	13	4
8	17	2
9	20	3

Table: Size of tied groups