STA4001 Stochastic Processes Spring 2021
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Homework 2
Due: Jan 26

1. Let D be a discrete random variable with the following p.m.f.

$$\mathbb{P}[D=k] = \begin{cases} 1/10 & \text{if } k=5 \\ 1/10 & \text{if } k=6 \\ 2/5 & \text{if } k=7 \\ 3/10 & \text{if } k=8 \\ 1/10 & \text{if } k=9 \\ 0 & \text{otherwise.} \end{cases}$$

   Find

   a) $\mathbb{E}[\min\{D,6\}]$.
   a) $\mathbb{E}[(6-D)^-]$.

   (Recall that for an $x \in \mathbb{R}$, $x^+ = \max\{x,0\}$ and $x^- = \max\{-x,0\}$).

2. Now assume that D is an exponential distribution with mean 5. Find

   a) $\mathbb{E}[\max\{D,5\}]$.
   a) $\mathbb{E}[(D-5)^-]$.

3. David buys fruits and vegetables wholesale and retails them at Davids Produce. One of the more difficult decisions is the amount of bananas to buy. Let us make some simplifying assumptions, and assume that David purchases bananas once a week at 10 cents per pound and retails them at 30 cents per pound during the week. Bananas that are more than a week old are too ripe and are sold for 5 cents per pound. Suppose the demand for the good bananas follows the same distribution as D given in the first question. Assume that David buys 8 pounds of banana every week.

   (a) What is the expected profit of David in a week ?
   (b) What is the variance of the profit per week?
   (c) What is the optimal order quantity each week to maximize the expected profit per week?

4. Suppose that David operates his business for 100 weeks and each week he orders 8 pounds of banana.

(a) What is your point estimate of his cumulative profit in 100 weeks?

(b) What is the 95% confidence interval of his cumulative profit in 100 weeks?

(c) Following the sample **Python** code in the 1st lecture, generate three sets of demands in **Python** following the distribution in the first question. Each set has 100 random numbers, representing demands for 100 consecutive weeks. Using the first set of demands, simulate the cumulative total profit in the first 100 weeks. Is your total profit within the 95% confidence interval? Using the 2nd set and 3rd set of demands, simulate the cumulative total profit in the 2nd and 3rd 100 weeks, respectively. Are they within the 95% confidence interval?

Besides submitting three numbers that represent the cumulative total profit in the 1st, 2nd, and 3rd 100 weeks, submit your code as well. Your code should be able to replicate these three numbers when a TA runs your code.
Note: **Matlab/R** code is also acceptable.

5. Suppose the demand for the good bananas follows the same distribution as D given in the second question.

(a) What is the expected profit of David in a week if he buys 5 pounds of banana?

(b) What is the optimal order quantity each week to maximize the expected profit for week?

(c) Assume the bananas must be purchased in non-negative integer pounds from the wholesaler, David cannot buy decimal pounds like 2.3 pounds or 4.5 pounds. In this case, what is the optimal order quantity each week to maximize the expected profit for week?

6. A store sells a perishable product. Suppose that the weekly demand D for the product is i.i.d with the following distribution.

d	5	10	20	30
$\Pr(D = d)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Sales happen from Monday-Friday. Unused items from one week *cannot* be used in the following week. Suppose that each item sells at \$200. Each item costs \$100 to order, and each leftover item by Friday evening has a salvage value of \$50. The store makes a weekly order from a supplier each Friday evening for the product to arrive early Monday morning (before sales commence).

(a) Find the optimal order quantity to maximize the expected profit per week.

(b) Compute the expected profit per week, given that the optimal order quantity from part (a) is ordered.

(c) Suppose that on the evening of a fixed Friday, the store receives as donation of 6 fresh items of product A, free of charge. Furthermore, there is now a $500 fixed cost to make an order from the supplier. Should the store place an order that Friday evening? If so, how many items should it order, in order to maximize the expected profit for the following week? Show your work to support your claim.