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1. Consider the following graph:

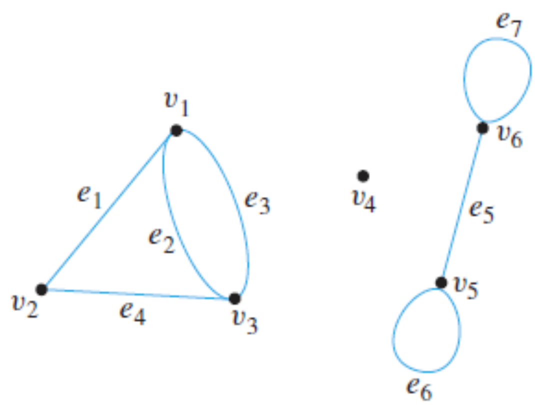


Figure 1: Question 1

(a) Write the vertex set and the edge set, and give a table showing the edge-endpoint function.

(b) Find all edges that are incident on v_1, all vertices that are adjacent to v_1, all edges that are adjacent to e_1, all loops, all parallel edges, all vertices that are adjacent to themselves, and all isolated vertices.

2. Draw the graph specified as follows (maybe more than one):

vertex set $= \{v_1, v_2, v_3, v_4\}$
edge set $= \{e_1, e_2, e_3, e_4\}$
edge-endpoint function:

Edge	Endpoints
e_1	$\{v_1, v_3\}$
e_2	$\{v_2, v_4\}$
e_3	$\{v_2, v_4\}$
e_4	$\{v_3\}$

3. Find the degree of each vertex of the graph G shown below. Then find the total degree of G.

$V_1 = 0$. $V_2 = 2$. $V_3 = 4$.

total $= 0+2+4 = 6$.

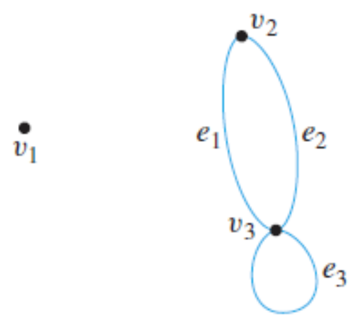


Figure 2: Question 3

4. Draw a graph with the specified properties or show that no such graph exists.

   (a) A graph with four vertices of degrees 1, 1, 2, and 3.

   (b) A graph with four vertices of degrees 1, 1, 3, and 3.

   (c) A simple graph with four vertices of degrees 1, 1, 3, and 3.

5. In the graph below, determine which of the following walks are trails, paths, circuits, or simple circuits.

   a. $v_1 e_1 v_2 e_3 v_3 e_4 v_3 e_5 v_4$ b. $e_1 e_3 e_5 e_5 e_6$ c. $v_2 v_3 v_4 v_5 v_3 v_6 v_2$

   d. $v_2 v_3 v_4 v_5 v_6 v_2$ e. $v_1 e_1 v_2 e_1 v_1$ f. v_1

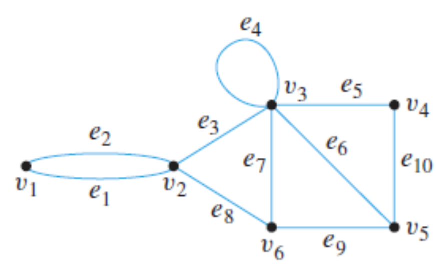


Figure 3: Question 5

6. Which of the following graphs are connected? (a).

7. Find all connected components of the following graph G.

3 connected components: G_1, G_2, G_3.

$V(G_1) = \{v_1, v_2, v_3\}$. $E(G_1) = \{e_1, e_2\}$.

$V(G_2) = \{v_4\}$. $E(G_2) = \Phi$.

$V(G_3) = \{v_5, v_6, v_7, v_8\}$. $E(G_3) = \{e_3, e_4, e_5\}$.

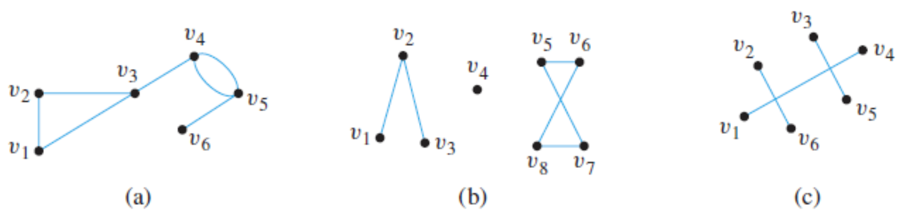


Figure 4: Question 6

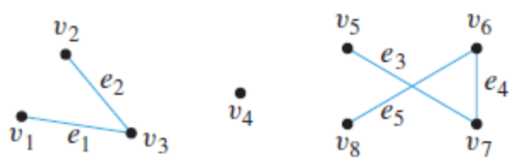


Figure 5: Question 7

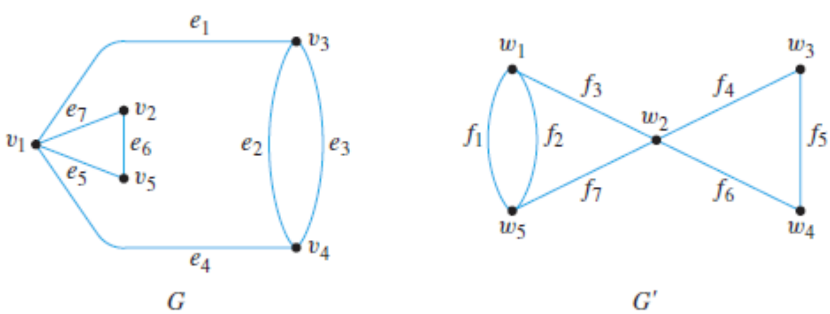


Figure 6: Question 8

8. Show that the following two graphs are isomorphic.

9. Give an example of a graph with five vertices and four edges that is not a tree.

10. (a) Prove that a tree with more than one vertex has at least two vertices of degree 1.

(b) Find all nonisomorphic trees with four vertices.

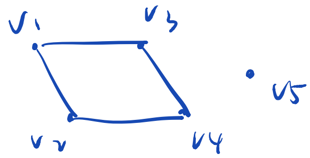
8. proof. There exists $f: V(G) \to V(G')$. which is 1-1.

$f(v_3) = w_1$, $f(v_4) = w_5$. $f(v_1) = w_2$.

$f(v_2) = w_3$. $f(v_5) = w_4$.

And $e_1 \to f_3$, $e_2 \to f_1$ $e_3 \to f_2$. $e_4 \to f_7$.

$e_5 \to f_6$. $e_6 \to f_5$. $e_7 \to f_4$.

8.  not connected.