Sheet 3 118010350
A3.1 The linear program:
maximize 5X, + 2Xz +5Xz
Subject to 2X1+3X2+ X3=4
$ \begin{array}{c c} X_1, X_2, X_3 \geq 0 \\ \hline b = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, A = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, C = \begin{bmatrix} 4 \\ 7 \end{bmatrix}. $
(G) The dual problem of the LP:
minimize 4yi+7yz
Subject to 29, +42 25
341424222
y1+34225
71, 9230
(6)
0 3 2 5 7 7,
me can get y=2, y=1, thus min(44,+42)=15.
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(c) Use the complementarity conditions:
One find that X=1, X=0, X=2 is prinal fersible
@ ne find that y= 2, y=1 is dual fensible.
@ want to find X=[X1, X2, X3], y=[y1,y1]
Such that,
X1(24,+42-5)=0
X2(34)+242-2)=0
X3141+34x-5)=0
and 4, (2X1+3X2+X3-4)=0
12(X1+2X+3X3-7)=0.
Since X=[1,0,2] and y=[2,1] is satisfied D.O. O.
then we get that x, y are optimal solutions.
Thy, max (5X,+2X2+5X3) = 15.
A 2.2. The general linear program:
min cTX.
s.t. Ax < b. cx = d.
where $A \in \mathbb{R}^{m \times n}$, $C \in \mathbb{R}^{p \times n}$, $C \in \mathbb{R}^n$, $b \in \mathbb{R}^m$, $d \in \mathbb{R}^p$.
(a) The dual problem of the LP:
max bity
y a
st. [A] Ty = C
y; 50, ;=1, m
yj € [R, v= m+1, m+P.
$\frac{y_{j} \in [R, i=m+1, + m+p]}{\text{where } y \in [R]}$
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(6)	we trasform the dual problem in (a) again
	(a) (=> min [-b]], y
	9 [-d]
	S.t. A Ty=c
	450, in 1 m.
	yzEIR, izmtl, mtp.
	The dual problem of the dual problem:
	max cTx
	St. AXZ-b, cX=-d
	X > EIR. >= 1,
-	me let ==-X, then the dual problem:
	min C7
	5.t AZ = b, cZ = d.
	ZiEIR, iolina.
	Thus, the dual of dual is equivalent to problem (2).
A 3.3.	(a) Primal= maximize 2X1-X2
	subject to XI-XZE
	$-\chi_1 + \chi_2 \leq -2$
	$\chi_{11}\chi_{2}\gtrsim 0$
	Dunt: minimize y,-zyz
	Subject to y,-yz >2
	- y 1 + y 2 > -
	y., y 2 30
	

and the same of th	
(4)	Primal: minimize XI+2XI+XZ
	Subject to X1+X2=1
	Xz + Xz =
	X1, X2 20, X3 50.
	Dual: maximize yityz
	subject to yiel
	y, +y 2 = 2
	9)21
(0)	Primal = maximize SXIH 2Xx + 5X3
	subject to 2X1+3X2+X354
	X1+2X2+3X35]
	X1, X1, X30
	Dual = minimize 44,+74x
	Subject to 24, ty 25
ü	34, +24, 32
	71+3y,25
	y,, y = 30
(d)	Primal: minimize 2X1+X2
	X1, X2 EtR
	Subject to XI+ X2=1
	$-X_1+X_2$
	Pual: naximize 51+42
	911 92 EIR
	Subject to y,-yz=2
	y,+y2=

A3.4 (a) The linear program:
max t
χit
S.t. Ax > t1 0
$\mathcal{I}^{T}X$
$\chi \gtrsim 0$ (3)
10 Here XEIR4, Y= [X, X, X, X, I, X; denotes the probability of player.
calling out number is tEIR, t denote the lower bound of
player I's winning when player I calls out number 1, volisies.
The constraint 1 ensures that when player I calls out
number i, the player I's winning is no smaller than t
The constraint @ ensures that the sum of the probability of
calling out number is 1.
The constraint @ ensures that the probability of calling out
number i is nonnegative.
The objective function maximize t, get the max t so that
the strategy makes player I's winning reach maximum
whenever which number is carred by player I.
1 Using MATLAB
we get p* = -1.7266 x 10 9 50, t= -1.7266 x 10 9 and X= [X, X, X, X, X]
x1=0.1297, xx=0.2797 x3=0.3703, xy=0.1203.
(b) The primal problem written in a compact way:
max OT.X-t
s.t. [A -11]·[^x] ≥ 0
Maxleaf X > 0, t free

The dual problem of (a):
min m
y,m
S.t. [A] 1] [m] 20 0
[-17 07:[m] =1 S
40. m free B
1 Neing MATLAB.
ne get d*= 7.5470×10 ≈ 0.
@ Here y + 1R4, y = [y, y, y, y, y,], yi denotes the oppisite
- value of the probability of player I calling out number i; mER,
m denote the upper bound of (-1). player I's winning (i.e.
plenger I'S loss) when player I calls out number i, i=1,2,3,4
The constraint @ ensures that when player I calls out nuber in
the player I's loss is no larger than m
The constraint @ Ensures that the sum of probability of calling
out number i is 1
The constraint @ ensures that the probability of calling out
number à is nonnegative
The objective function minimize m, get mint so that the
strategy makes player I's loss reach minimum.
5 II 9
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(e) Define P = { x e R 4 : x > 0, 11 x = 1}.
Consider the primal problem and the dual problem,
The prinal: max t ; The dual: min m
S.t. XEP. AXZ+1 S.t. YEP, ATYEMIL
Then consider the expression max min yTAX.
Since max min yTAX: min max yTAX. Then, min yTAX < max min yTAX: min max yTAX < max yTA YEP (XXEP.) XEP YEP YEP XEP XEP XEP (EYE
Suppose the prinal problem: optimal value is p*, optimal solution is
the dual problem: optimal value is d*, optimal colution is y
Since AX* > p*1, then min yT p*11 < min yTAX*
Since ATy* < d*11, then y*TA < d*11
then max y*TAX = max d*11TX
- Pry Strong Duality Theorem, me have p*: d* =0
and 720, y20, and through the inequality above,
ne have min pt yT1 = max min yTAX = max dt 1 x
Since min P y 11 =0, and max d 11 X =0.
then we can get max min gTAX = 0
Since min pt yT11 =0, and max dt 11TX =0. then me can get max min yTAX = 0 therefore, pt = hax min yTAX = dt xep yep
rife .

Pry Strong Durly Theorem. We can get that $p^* = d^*$ and here we denote x^* . y^* as optimal solution. Since $p^* = d^*$, and $Ax^* > p^*1$. $A^*y^* = d^*1$. thus, we get that $Ax^* = A^*y^* = p^*1 = d^*1$. Consider max (min y^*Ax). The property of the solution of the consider player It's minnings. 1p: max t. s.t. (A) $y > t1$. Yet $y > 0$ Using Matcale, find that the optimal value is also 0 . Thus, the game is fair. Outson using number 1 and 2, now matrix A changes. A: $\begin{bmatrix} -2 & 2 \\ 3 & 4 \end{bmatrix}$ $\begin{bmatrix} -A > \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$. Consider player It's minnings. LP: max t. s.t. Axzer $x > 0$ Consider player It's minnings. LP: max t. s.t. Axze	
and here we denote x*, y* as optimal solution Since p*: d*, and Ax* > p*1. Ay* = d*1 thus, we get that Ax* = ATy* = p*1: d*1 Consider max (min y*Ax), Yep yep (d). We consider player It's winnings Lp: mex t. s.t (A)y > t1 y: t 1 y > 0 Using MATCAB, find that the optimal value is also 0 Thus, the game is fair. O when using number 1 and 2, now matrix A changes A: [-2 3] -A: [2-3] 3 y	By Strong Duly Theorem, we can get that px = dx
Since p*:d*, and Ax*: Ay*: d*1 thus, we get that Ax* = ATy* = p*1:d*1 Consider max (min y Ax), xep yep (d). We wasider player I's winnings. Lp: max t. S.t (-A)y:t1 y/t	
thus, we get that $Ax^* = A^*y^* = P^*1 - d^*1$ Consider note (min y Ax), Atp ytp (d). We wasider player I's winnings. Lp: note t. S.t (-A)y > t1 y; t Ty = 1 y > 0 Using MATCAB, find that the optimal value is also 0 Thus, the game is fair. (a) When using number 1 and 2, now metrix A changes. A: [-2 3] -A: [-2 3] 3 -4 [-3 y]. Consider player I's winnings. Lp: note t. S.t. Axzt II xit Tx=1 xit Tx=1 y= 0.083} x>0 Consider player I's minnings. Lp: note t. S.t. Ay > t1 yit Ty = 1 yit Thus the game is not fair, and player I is better in the yith yith yith yith yith yith yith yith	
Consider max (min y'Ax), rep ytp (d). We consider player I's vinnings. Lp: max t. s.t (A)y > tI yit Ty = 1 y > 0 Using MATCAB, find that the optimal value is also 0 Thus, the game is fair. (a) When using number 1 and 2, now matrix A changes A: [2 3] -A: [2 -3] 3 4] -A: [2 -3] Consider player I's minings. Lp: max t s.t Axz II x.t Tx=1 X>0 Consider player I's minings. Lp: max t s.t Ayz II xit Ty=1 y=1 dt:-0.083} We find that in this case, the optimal values are different than they they are is not foir, and player I is better in the	
(d). We consider player It's vinnings. Lp: mex t. S.t (-A)y > tI y: t I'y = 1 Y > 0 Using MATLAB, find that the optimal value is also 0. Thus, the game is fair. On when using number 1 and 2, now metrix A changes A: [-2 3] -A: [2-3] [3-4] -A: [2-3] (onsider player I's winnings, LP: max t S.t Axzt II x.t I'x=1 X>0 Consider player I's minnings. LP: max t S.t. Ayzt II y: ax t S.t.	
(d). We consider player II's winnings. 1p: mex t. s.t (-A)y > tI y = 1 y 20 Using MATLAB, find that the optimal value is also 0. Then, the game is fair. (a) When using number 1 and 2, now metrix A changes. A= [-2, 3] -A= [2-3] 3 y -A= [2-3] 4 y -2 y Consider player I's minnings. LP: max t s.t AxxII x.t ITx=1 P*= 0.081} x>0 Consider player I's minnings. LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083}	ueb neb
(d). We consider player II's winnings. 1p: mex t. s.t (-A)y > tI y = 1 y 20 Using MATLAB, find that the optimal value is also 0. Then, the game is fair. (a) When using number 1 and 2, now metrix A changes. A= [-2, 3] -A= [2-3] 3 y -A= [2-3] 4 y -2 y Consider player I's minnings. LP: max t s.t AxxII x.t ITx=1 P*= 0.081} x>0 Consider player I's minnings. LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083} y>0 LP: max t s.t AyzII yit Iy = 1 d*:-0.083}	
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Using MATCAB, find that the optimal value is also a Thing, the game is fair. Thing the game is fair. When using number 1 and 2, now matrix A changes A: [-2 3] -A: [2-3] 3-4] [-3 4]. Consider player I's winnings. LP: max t St Axzt II X:t IX=1 P*= 0.083} X > 0 Consider player I's minnings. LP: wax t St. Ay > 1 Y=1 d*=-0.083} Y=0 We find that in this case, the optimal values out different they they are different they they are different they they are different they they are is not fair, and player I is better in the	LP: max to at (-1) in a contraction
Using MATCAB, find that the optimal value is also 0 Thus, the game is fair. (a) When using number 1 and 2, now matrix A changes A=\[\frac{1}{2} \frac{3}{3} \] \[\frac{1}{2} \] \[\frac{1} \] \[\frac{1}{2} \] \[\frac	yit sty
Using MATCAB, find that the optimal value is also 0. Thus, the game is fair. O when using number 1 and 2, now matrix A changes. A: [-2 3] -A: [2-3] [3-4]	V
Thus, the game is fair. O When using number 1 and 2, now metrix A changes A: [-2 3] -A: [2-3] 3-4] [-3 4]. Consider player I's winnings. LP: max t St Ax3t11 x.t 2 ^T x=1 P*= 0.083} x>0 Consider player I's minnings. LP: max t St. Ay >t I yit 1y=1 d*:-0.083} y>0 we find that in this case, the optimal values are different than the game is not fair, and player I is better in the	
when using number 1 and 2, now metrix A changes A: [-2 3] -A: [2-3] [3-4] [-3 4]. Consider player I's winnings. LP: max t St Axzt II x.t I'x=1 P*= 0.083} X > 0 Consider player I's winnings. LP: max t St. Ay > t I yit Iy=1 At: -0.083} We find that in this case, the optimal values are different than the game is not fair, and player I is better in the	
A: $\begin{bmatrix} -2 & 2 \\ 3 & 4 \end{bmatrix}$ -A: $\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$. Consider player I's minnings. LP: max t St Ax>t IT X=1 P*= 0.083} $x \ge 0$ Consider player I's minnings. LP: max t St. Ay>t I yit Iy=1 dt:-0.083} Y>0 Ne find that in this case, the optimal values are different than the game is not fair, and player I is better in the game.	
A: $\begin{bmatrix} -2 & 2 \\ 3 & 4 \end{bmatrix}$ -A: $\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$. Consider player I's minnings. LP: max t St Ax>t IT X=1 P*= 0.083} $x \ge 0$ Consider player I's minnings. LP: max t St. Ay>t I yit Iy=1 dt:-0.083} Y>0 Ne find that in this case, the optimal values are different than the game is not fair, and player I is better in the game.	When using number 1 and 2, now metrix A changes
Consider player I's winnings, LP: max t st Axzt. 1 x.t	$A \geq \begin{bmatrix} -2 & 3 \\ 2 & 4 \end{bmatrix} - A \geq \begin{bmatrix} 2 & -3 \\ 2 & 4 \end{bmatrix}$
LP: max t st $Ax > 1$ xit $Tx = 1$ $X \ge 0$ Consider player I's minnings. LP: max t st. $Ay > 1$ $Y = 1$ $Y = 1$ $Y = 1$ $Y = 0.083$ $Y \ge 0$ LP: max t st. $Y = 1$ $Y = 1$ $Y = 1$ LP: max t st. $Y = 1$ $Y = 1$ $Y = 1$ LP: max t st. $Y = 1$ $Y = 1$ LP: max t st. $Y = 1$ $Y = 1$ LP: max t st. $Y = 1$ $Y = 1$ LP: max t st. $Y = 1$ $Y = 1$ LP: max t st. $Y = 1$ LP: m	L 3 4 J .
Consider player It's minnings. LP: max t S.t. Ay >+ I yet Ty = 1 dt: -0.08} We find that in this case, the optimal values are different than the game is not fair, and player I is better in the game.	Consider player I's winnings,
Consider player It's minnings. LP: max t S.t. Ay >+ I yet Ty = 1 dt: -0.08} We find that in this case, the optimal values are different than the game is not fair, and player I is better in the game.	LP: max t st Axztil
Consider player It's minnings. LP: max t St. Ay > 12 yit	$2^{T}x=1$ $P^{*}=0.083$
LP: max t St. Ay ≥ t I yit Dy = 1 dt: -0.08}} Ne find that in this case, the optimal values are differently thus the game is not fair, and player. I is better in the game is not fair, and player. I is better in the game is not fair, and player. I is better in the game is not fair, and player. I is better in the game is not fair, and player.	X ≥0
LP: max t St. Ay ≥ t I yit Dy = 1 dt: -0.08}} Ne find that in this case, the optimal values are differently thus the game is not fair, and player. I is better in the game is not fair, and player. I is better in the game is not fair, and player. I is better in the game is not fair, and player. I is better in the game is not fair, and player.	Consider player I's minnings.
My = 1 dt = -0.08}} Ne find that in this case, the optimal values are different than the game is not fair, and player. I is better in the game is not fair, and player. I is better in the game is not fair, and player.	V
y≥0. Ne find that in this case, the optimal values are differently thus the game is not fair, and player. I is better in the	$y_1 + y_2 = 1 \qquad d^* = -0.08$
ve find that in this case, the optimal values are different thus the game is not fair, and player. I is better in the	U
thus the game is not fair, and player. I is better in the	V
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hir	minimize 1/AX-61/00, AEIRMXM, beIRM
A3.5	XtIR"
[0]	The linear program of (4):
	minimize t
	Xit
	Subject to AX-b < t1
	-Ax+bst1
(b)_	The prinal problem can be rensitten as:
	minimize O.X+t
	α it
	subject to $\begin{bmatrix} A & -1 \end{bmatrix} \begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} b \\ -A & -1 \end{bmatrix} \begin{bmatrix} t \end{bmatrix} \begin{bmatrix} -b \end{bmatrix}$
-	
	The dual problem of the primal problem:
	maximize 5-y-5.7.
	14 3
	Subject to [AT -AT] [7]
	$\begin{bmatrix} -1^{7} - 1^{7} \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} \leq 1$
	yeo, 7:0
(0)	Consider the dual problem in 16).
	Let KERM, and Kiyt
	Then the dual problem can be rewritten as:
-	maximize btk.
	K
	Subject to ATK = 0
	since 11411, + 11211, =1, and y=0, 2=0
	then 1 KI = 11 y-21, = 11 y 11 of 11 211, = 1.
	+hs, 11 k11, <1.

Thuy	the dual problem is equivalent to:
, ,	max bty yell
	y e IR
	s.t. ATy=0, 11y11,=1
(d). we k	some min 1/Ax-bl/ so is equivalent to the prime probler
in (c), and max by is equivalent to the dual prob
in t	b) here y= }y \ ATy = 0, 11y 11, \(1 \)
2 Sin	Le me can always find y=0, such that y + y,
thus	the fensible set y is nonempty, there exists a
basi	ic feasible solution of dual problem.
	o, we can easily know that the dual problem is
bon	nded, due to the constriants.
by	5. the dual problem has an optimal solution.
~ ~ ~	strong duality theorem, the primal problem also has
<i>(</i> (a optimal solution, and the optimal values of them
<i>\text{\tin}\text{\tint{\text{\tetx{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\tin}\text{\ti}}\\ \tittt{\text{\text{\text{\texi}\text{\text{\text{\text{\text{\texi}\text{\text{\text{\texi}\text{\texi}\text{\text{\texi}\text{\text{\texi}\tilit{\text{\texi}\tiint{\texit{\texit{\texi}\titt{\texitit}}\\tittt{\text{\texi}\text{\texit{\texitit}}</i>	e the same, that is
	min IIAX-bl/ as = wax bty.
(e) using	MATLAB.
Run	the wodes of the prince and the due, we can find the
	ine is similar to each other
u	
Maxleaf	· · · · · · · · · · · · · · · · · · ·