# STA3007: Tutorial 6

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# Outline

# Question 1

## True/False

In a one-way layout with $k$ treatments, let

- $\tau_1,\ldots,\tau_k$ denote the effects of treatments $1,\ldots,k$, respectively, where a larger value of $\tau_j$ corresponds to a greater effect of treatment $j$, $j=1,\ldots,k$;
- $R_j$ the sum of ranks of the observations in treatment $j$, $j=1,\ldots,k$;
- $A_p^*$ the standardized Mack-Wolfe statistic for umbrella alternatives with known peak $p$ and $\hat{p}$ an estimate of the unknown $p$.

Then the following statements are valid:

(a) If $k=5$ and $n_1=\cdots=n_5=4$, then the Kruskal-Wallis test statistic for general alternatives can be calculated by

$$H=\frac{R_1^2+\cdots+R_5^2-8820}{140}$$

(b) $\Pr(A_{\hat{p}}^*\geq a)=\Pr(A_4^*\geq a)$ for any real value $a$ if $\hat{p}=4$.

# Question 2(Textbook Problem 6.29)

In many settings, a doseresponse relationship needs not be monotonic in the dosage. In in vitro mutagenicity assays, for example, experimental organisms may not survive the toxic side effects of high doses of the test agent, thereby actually reducing the number of organisms at risk of mutation and leading to a downturn (i.e., umbrella pattern) in the doseresponse curve. The date in Table 6.10 are a subset of the data considered by Simpson and Margolin (1986) in a discussion of the analysis of Ames test results. Plates containing Salmonella bacteria of strain TA98 were exposed to various doses of Acid Red 114. The tabled observations are the numbers of visible revertant colonies on the 18 plates in the study.

# Question 2(Textbook Problem 6.29)

Test the null hypothesis $H_0$ against the alternative that the peak of the doseresponse curve for Salmonella bacteria of strain TA98 exposure to Acid Red 114 occurs at dosage level $1000\mu g/ml$. (Use the Mack-Wolfe tests for umbrella alternatives with known peak at dosage level $1000\mu g/ml$ at a significance level of 0.05. )

**Table 6.10** Number of Revertant Colonies of Salmonella Bacteria of Strain TA98 under Exposure to Various Doses of Acid Red 114, with Hamster Liver Activation

| Dose, µg/ml | | | | | |
|---|---|---|---|---|---|
| 0 | 100 | 333 | 1000 | 3333 | 10,000 |
| 22 | 60 | 98 | 60 | 22 | 23 |
| 23 | 59 | 78 | 82 | 44 | 21 |
| 35 | 54 | 50 | 59 | 33 | 25 |

*Source:* D. G. Simpson and B. H. Margolin (1986).

# Question 3(Textbook Problem 6.38)

Consider the Acid Red 114 revertant colonies data in Table 6.10. Test the hypothesis of no differences in the number of revertant colonies over the dosage levels against a general umbrella alternative. (Use the Mack-Wolfe tests for umbrella alternatives with unknown peak at a significance level of 0.05.)

**Table 6.10** Number of Revertant Colonies of Salmonella Bacteria of Strain TA98 under Exposure to Various Doses of Acid Red 114, with Hamster Liver Activation

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# Question 3(Textbook Problem 6.38)

| $U_{1i}$ | $U_{2i}$ | $U_{3i}$ | $U_{4i}$ | $U_{5i}$ |
|---|---|---|---|---|
| $U_{12} = 9$ | | | | |
| $U_{13} = 9$ | $U_{23} = 6$ | | | |
| $U_{14} = 9$ | $U_{24} = 7$ | $U_{34} = 4$ | | |
| $U_{15} = 5.5$ | $U_{25} = 0$ | $U_{35} = 0$ | $U_{45} = 0$ | |
| $U_{16} = 3.5$ | $U_{26} = 0$ | $U_{36} = 0$ | $U_{46} = 0$ | $U_{56} = 2$ |

Table 1: Mann-Whitney statistics

# Question 4 (Textbook Problem 6.47, 6.72)

1. Apply the approximate procedure of the Steel-Dwass-Critchlow-Fligner (SDCF) two-sided all-treatment multiple comparison at 0.05 level of significance to the psychotherapeutic attraction data of Table 6.2.

**Table 6.2** Raw Scores Indicating the Degree of Psychotherapeutic Attraction for Each Experimental Condition

| Control | Reading (TR) | Videotape (VTP) | Group (RII) |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 1 | 6 | 5 | 5 |
| 3 | 7 | 8 | 12 |
| 3 | 9 | 9 | 13 |
| 5 | 11 | 11 | 19 |
| 10 | 13 | 13 | 22 |
| 13 | 20 | 16 | 25 |
| 17 | 20 | 17 | 27 |
| 26 | 24 | 20 | 29 |

*Source:* S. R. Sauber (1971).

# Question 4 (Textbook Problem 6.47, 6.72)

1. Apply the approximate procedure of the Steel-Dwass-Critchlow-Fligner (SDCF) two-sided all-treatment multiple comparison at 0.05 level of significance to the psychotherapeutic attraction data of Table 6.2.
2. Estimate the simple contrast $\theta = \tau_4 - \tau_1$ for the psychotherapeutic attraction data in Table 6.2.

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# Question 4 (Textbook Problem 6.47, 6.72)

1. Apply the approximate procedure of the Steel-Dwass-Critchlow-Fligner (SDCF) two-sided all-treatment multiple comparison at 0.05 level of significance to the psychotherapeutic attraction data of Table 6.2.
2. Estimate the simple contrast $\theta = \tau_4 - \tau_1$ for the psychotherapeutic attraction data in Table 6.2.
3. Find a set of approximate simultaneous 95% confidence intervals for the simple contrast $\tau_2 - \tau_1$.

**Table 6.2** Raw Scores Indicating the Degree of Psychotherapeutic Attraction for Each Experimental Condition

| Control | Reading (TR) | Videotape (VTP) | Group (RII) |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 1 | 6 | 5 | 5 |
| 3 | 7 | 8 | 12 |
| 3 | 9 | 9 | 13 |
| 5 | 11 | 11 | 19 |
| 10 | 13 | 13 | 22 |
| 13 | 20 | 16 | 25 |
| 17 | 20 | 17 | 27 |
| 26 | 24 | 20 | 29 |

*Source:* S. R. Sauber (1971).

# Question 4 (Textbook Problem 6.47, 6.72)

| Control | Reading |
|---|---|
| 0(1.5) | 0(1.5) |
| 1(3) | 6(7) |
| 3(4.5) | 7(8) |
| 3(4.5) | 9(9) |
| 5(6) | 11(11) |
| 10(10) | 13(12.5) |
| 13(12.5) | 20(15.5) |
| 17(14) | 20(15.5) |
| 26(18) | 24(17) |

Table 2: Data and ranks of population 1&2

| tied group | element | size |
|---|---|---|
| 1 | 0 | 2 |
| 2 | 3 | 2 |
| 3 | 13 | 2 |
| 4 | 20 | 2 |

Table 3: Size of tied groups

# Question 4(Textbook Problem 6.47, 6.72)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (1)-(9) | -26 | -20 | -19 | -17 | -17 | -15 | -13 | -13 | -11 |
| (10)-(18) | -10 | -10 | -8 | -7 | -6 | -6 | -6 | -6 | -5 |
| (19)-(27) | -4 | -4 | -4 | -3 | -3 | -3 | -2 | -2 | -1 |
| (28)-(36) | -1 | 0 | 0 | 1 | 1 | 2 | 3 | 3 | 3 |
| (37)-(45) | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 6 | 6 |
| (46)-(54) | 6 | 6 | 7 | 7 | 7 | 7 | 8 | 8 | 8 |
| (55)-(63) | 8 | 9 | 10 | 10 | 10 | 10 | 10 | 11 | 11 |
| (64)-(72) | 12 | 13 | 14 | 15 | 15 | 17 | 17 | 17 | 17 |
| (73)-(81) | 19 | 19 | 19 | 20 | 20 | 21 | 21 | 23 | 24 |

Table 4: Ordered pairwise differences for population 1 & 2