Assignment 5

Deadline: 10pm, May 5th, 2021

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Question 1

Use Laplace transform to solve the initial value problem

$$4y^{''} + y = g(t), y(0) = 3, y^{'}(0) = -7$$

Question 2

Calcualte the Laplace transform of $f(t) = sin(\omega t + \theta)$

Question 3

If $L[f(t)] = F(s)$, prove that $L[\frac{f(t)}{t}] = \int_s^{+\infty} F(u)du$

Question 4

If $L[f(t)] = F(s)$, calculate the laplace transform of $\int_0^t f(\tau)d\tau$

Question 5

If $f^{'}(t) + \int_0^t f(\tau)d\tau = 1$ and $f(0) = 1$, use Laplace transform to get $f(t)$

Question 6

Find the inverse Laplace transform of $F(s) = \frac{2s+1}{s^2+6s+13}$

Question 7

Find the inverse Laplace transform of $F(s) = \frac{1}{(s^2+2s+2)^2}$
Hint: $sin(\alpha)sin(\beta) = -\frac{1}{2}(cos(\alpha + \beta) - cos(\alpha - \beta))$

Question 8

Find the inverse Laplace transform of $F(s) = \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s-4}$ and write it as a right continuous piecewise-defined function.

Question 9

Use the Laplace transform to solve the following initial value problem:

$$y' + 2y = f(t), \quad y(0) = 1,$$

where

$$f(t) = \begin{cases} 0 & \text{if } 0 \le t < 1 \\ t & \text{if } 1 \le t < \infty \end{cases}.$$

Question 10

Use Laplace transform to solve the following initial value problem

$$y' - y = \int_0^t (t-\lambda)e^{\lambda} d\lambda, \quad y(0) = -1$$

Question 11

Use Laplace transform to solve the initial value problem

$$y'' + 2y' + y = \delta(t-2), \quad y(0) = 0, y'(0) = 1, 0 \le t \le 6 \ .$$

Graph the solution on the indicated interval.