Exercise 4

- There will be five exercises in this semester, which will account for 10% of the grade of this course.
- Exercise 4 includes 3 questions.
- Questions 1 is a True/False (T/F) question requiring explanations.
- Questions 2-3 are problem-solving questions requiring detailed solutions.
- Please show the details of your work leading to the solutions.
- The full mark of this assignment is 100.
- Submit a pdf file of your answers on Blackboard by Monday, Nov 23, 2020.

Question 1

In a two-way layout setting with k treatments and n blocks:

(a) The block effects in a randomized complete block design can be tested based on the ranks of the data within each treatment.

(b) If the data are incomplete and balanced (BIBD) with 5 treatments in each block and each treatment available in 10 blocks, then $n = 2k$ and there exists an integer m such that $m(k-1) = 40$.

(c) For incomplete data with empty cells, the ranks of missing data (in empty cells) do not affect the test for equal treatment effects against general alternatives.

(d) If 2 treatments are available in each block, then the vector $\mathbf{A} = [A_1 \ \cdots \ A_{k-1}]^{\mathsf{T}}$ in the Skillings-Mack statistic SM for arbitrary incomplete block data has elements

$$A_j = 2R_j - 3n\,, \quad j = 1,\ldots,k-1\,,$$

where R_j is the sum of in-block ranks within treatment j.

Question 2

In an incomplete block design with $c_{ij} \in \{0,1\}$ observation for block $i \in \{1,\dots,n\}$ and treatment $j \in \{1,\dots,k\}$, and s_i treatments in block i, the in-block ranks $\{r_{ij}\}$ have

$$\mathrm{E}[r_{ij}] = \frac{s_i+1}{2}, \quad \mathrm{Var}(r_{ij}) = \frac{(s_i+1)(s_i-1)}{12} I_{\{c_{ij}=1\}}, \quad \mathrm{Cov}(r_{iu}, r_{iv}) = -\frac{s_i+1}{12} I_{\{c_{iu}=c_{iv}=1\}} \text{ if } u \neq v.$$

Use the above equations to prove the following results:

(a) If the incomplete block design is balanced (BIBD), then the Durbin-Skillings-Mack statistic D has mean $\mathrm{E}[D] = k-1$.

(b) In BIBD, the covariance matrix of the random vector $\mathbf{R} = [R_1 \ \cdots \ R_{k-1}]^{\mathsf{T}}$ is

$$\mathrm{Var}_0(\mathbf{R}) = \frac{\lambda(s+1)}{12}\left(kI_{k-1} - \mathbf{1}\cdot\mathbf{1}^{\mathsf{T}}\right),$$

where $R_j = r_{1j} + \cdots + r_{nj}$, $j = 1,\dots,k$, I_{k-1} is the $(k-1)\times(k-1)$ identity matrix and $\mathbf{1}^{\mathsf{T}} = [1 \ \cdots \ 1]_{1\times(k-1)}$.

Question 3

Data from an incomplete block design are resented in the table below:

	X_{ij}				
Block	Treatment j				
i	1	2	3	4	5
1	16	18		32	
2	19			46	45
3		26	39		61
4			21	35	55
5		19		47	48
6	20		33	31	
7	13	13	34		
8	21		30		52
9	24	10			50
10		24	31	37	

(a) Test the null hypothesis of equal treatment effects against general alternatives by the Durbin-Skillings-Mack test using the large-sample approximation.

(b) Compare the effects of treatments by the Skillings-Mack two-sided all-treatment multiple comparison procedure for BIBD at level $\alpha = 0.10$ using the large-sample approximation with $q_{0.1} = 3.5$.