118010350

1. Determine which of the following graphs have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.

Note:
✓ - A circuit is a path which ends at the vertex it begins (so a loop is an circuit of length one).
✓ - An Euler circuit is a circuit that uses every edge in a graph with no repeats. Being a circuit, it must start and end at the same vertex.

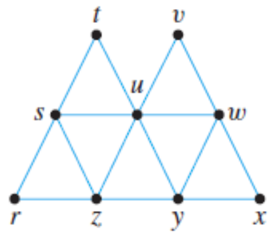

Figure 1: Question 1, (a)

(a). ✓. r → s → t → u → s → z → u → v → w → u → y → w → x → y → y → z → r.

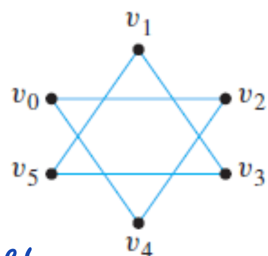

Figure 2: Question 1, (b)

(b). X . The graph are connected. V_1-V_3-V_5 and V_0-V_2-V_4 are seperated.

(c) X. The graph has 6 vertexes. we divide them into 2 groups (A,C,E) & (B,D,F). Each edge connects 2 groups. but there are 7 edges. we can not go back to the start side at end.

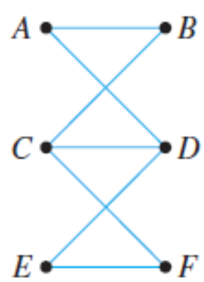

Figure 3: Question 1, (c)

2. Is it possible for a citizen of Knigsberg to make a tour of the city and cross each bridge exactly twice?

Possible. Since by adding edges to the original graph, there are zero vertex with odd degree.

3. Two jugs A and B have capacities of 3 quarts and 5 quarts, respectively. Can you use the jugs to measure out exactly 1 quart of water, while obeying the following restrictions? You may fill either jug to capacity from a water tap; you may empty the contents of either jug into a drain; and you may pour water from either jug into the other. (via graphs)

4. For each pair of simple graphs G and G′ of the following graphs, determine whether G and G′ are isomorphic. If they are, give a function g: V(G) → V(G′) that defines the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.

(a). Not isomorphic.

Since in G′, we can find 4 cycles consists of 4 vertexes.

i.e. u-v-w-x-u, w-x-y-z-w, x-y-z-u-x, z-u-v-w-z.

But we can not find them in G.

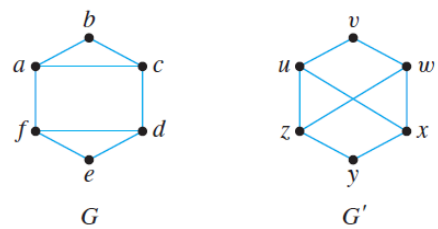

Figure 4: Question 4, (a)

(b). Isomorphic.

Define g: V(G) → V(G′).

g(a) = t, g(c) = u, g(e) = v.

g(g) = w, g(b) = x, g(d) = y.

g(f) = z.

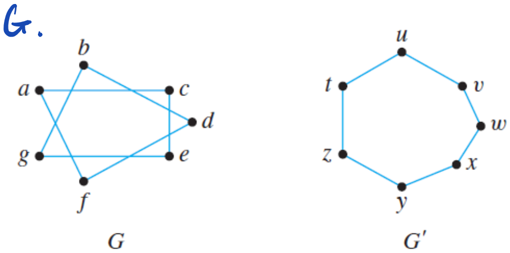

Figure 5: Question 4, (b)

(c). Isomorphic.

Define g: V(G) → V(G′)

g(a) = s, g(b) = t,

g(c) = u, g(d) = v,

g(e) = z, g(h) = w, g(f) = y, g(g) = x.

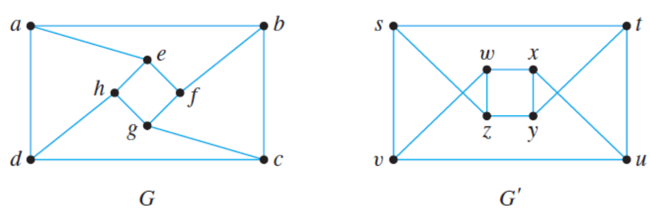

Figure 6: Question 4, (c)

5. Show that the following two graphs are not isomorphic by supposing they are isomorphic and deriving a contradiction.

Suppose G & G' are isomorphic

Since in G', we can find a vertex (w_5) of degree 3 connects to a vertex (w_6) of degree 1. but in G, we can not find a such connection.

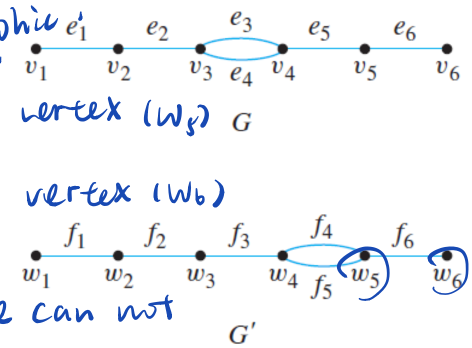

Figure 7: Question 5, (c)

eg. B_1 (G_1, G_2)
B_2 (G_2, G_1)
G_1 (B_2, B_1)
G_2 (B_1, B_2)

① B_1 — G_1
B_2 — G_2

② B_1 G_1
B_2 G_2

6. Construct an example in which there is more than one stable matching. (You only need two boys and two girls to do this.)

7. For the marrying procedure, suppose that the boys all have dierent favorite girls. How many steps does it take for the algorithm to converge?

8. Suppose that the boys have identical preferences. How many steps does it take for the algorithm to converge?

9. There are four people, Pat, Chris, Dana, and Leslie. They must pair o (each pair will share a two-bed suite). Each has preferences over which of the others they would prefer to have as a roommate.

   The preferences are:

   Leslie: Pat, Chris, Dana

   Chris: Leslie, Pat, Dana

   Pat: Chris, Leslie, Dana

   Dana: Chris, Leslie, Pat

   Show that no stable matching exists.

10. Suppose preferences are given by the following tables:

BOY	1	2	3	4	5
Adam	Beth	Amy	Diane	Ellen	Cara
Bill	Diane	Beth	Amy	Cara	Ellen
Carl	Beth	Ellen	Cara	Diane	Amy
Dan	Amy	Diane	Cara	Beth	Ellen
Eric	Beth	Diane	Amy	Ellen	Cara

Boys' Preferences

GIRL	1	2	3	4	5
Amy	Eric	Adam	Bill	Dan	Carl
Beth	Carl	Bill	Dan	Adam	Eric
Cara	Bill	Carl	Dan	Eric	Adam
Diane	Adam	Eric	Dan	Carl	Bill
Ellen	Dan	Bill	Eric	Carl	Adam

Girls' Preferences

Figure 8: Question 5, (c)

Find a stable matching using the Gale-Shapley algorithm with boys making proposals. Find a stable matching using the Gale-Shapley algorithm with girls making proposals.