Exercise 3

- There will be five exercises in this semester, which will account for 10% of the grade of this course.
- Exercise 3 includes 4 questions.
- Questions 1-3 are True/False (T/F) questions requiring explanations.
- Questions 4 is problem-solving questions requiring detailed solutions.
- Please show the details of your work leading to the solutions.
- The full mark of this assignment is 100.
- Submit a pdf file of your answers on Blackboard by Monday, Nov 9, 2020.

Question 1

In a one-way layout with k treatments, let

- A_p^* denote the standardized Mack-Wolfe statistic for umbrella alternatives with known peak p and $\hat{p}$ a unique estimate of the unknown p;
- $\{[a_{uv}, b_{uv}), 1 \le u < v \le k\}$ be simultaneous $100(1-\alpha)\%$ confidence intervals of simple contrasts $\{\tau_u - \tau_v, 1 \le u < v \le k\}$.

Then the following equalities hold:

(a) $A_{\hat{p}}^* = A_p^*$;

(b) $\Pr\left(A_{\hat{p}}^* \ge a\right) = \Pr\left(A_4^* \ge a\right)$ for any real value a if $\hat{p} = 4$;

(c) $\Pr\left(a_{uv} \le \tau_u - \tau_v < b_{uv}\right) = 1 - \alpha$ for all $1 \le u < v \le k$;

(d) $\Pr\left(\tau_u - \tau_v < a_{uv} \text{ or } \tau_u - \tau_v \ge b_{uv} \text{ for some } 1 \le u < v \le k\right) = \alpha$.

Question 2

In a one-way layout with $n_1 = \cdots = n_8 = 5$ and no ties:

(a) the sum of all ranks $\{r_{ij}, i = 1,\ldots,5; j = 1,\ldots,8\}$ in the Kruskal-Wallis test statistic for general alternatives is 800;

(b) the Jonckheere-Terpstra test statistic J for ordered alternatives is a sum of 700 values that are either 0 or 1;

(c) the Mack-Wolfe test statistic A_4 for umbrella alternatives takes an integer value between 0 and 350.

Question 3

In a one-way layout with k treatments, let

- $\tau_1,\ldots,\tau_k$ denote the effects of treatments $1,\ldots,k$, respectively, where a larger value of τ_j corresponds to a greater effect of treatment j, $j = 1,\ldots,k$;
- R_j the sum of ranks of the observations in treatment j, $j = 1,\ldots,k$;
- A_p^* the standardized Mack-Wolfe statistic for umbrella alternatives with known peak p and $\hat{p}$ an estimate of the unknown p.

Then the following statements are valid:

(a) If $k = 5$ and $n_1 = \cdots = n_5 = 4$, then the Kruskal-Wallis test statistic for general alternatives can be calculated by

$$H = \frac{R_1^2 + \cdots + R_5^2 - 8820}{140}$$

(b) $\Pr\left(A_{\hat{p}}^* \geq a\right) = \Pr\left(A_4^* \geq a\right)$ for any real value a if $\hat{p} = 4$.

(c) If one-sided treatments-versus-control multiple comparisons with $k = 4$ decide $\tau_2 > \tau_1, \tau_3 > \tau_1$ and $\tau_4 = \tau_1$ at the $\alpha = 10\%$ level, then we can claim that treatments 2 and 3 are more effective than the control treatment 1, whereas treatment 4 has the same effect as treatment 1, with at least 90% probability of making right claims.

Question 4

A set of data in the one-way layout with 5 treatments are listed below:

Treatment				
1	2	3	4	5
17	73	86	20	22
45	54	71	39	15
18	47	36	28	25

Denote the effects of treatments $1,\ldots,5$ by $\tau_1,\ldots,\tau_5$ respectively.

(a) Test the null hypothesis of equal treatment effects $H_0:\tau_1=\cdots=\tau_5$ versus umbrella alternatives $H_1:\tau_1\le\tau_2\le\tau_3\ge\tau_4\ge\tau_5$ with at least one strict inequality for known peak $p=3$ by the Mack-Wolfe test at the 1% level.

(b) If the peak p of umbrella alternatives is unknown, estimate p and calculate the estimated Mack-Wolfe test statistic $A_{\hat{p}}^*$.