MAT3253 Homework 11

Due date: 16 Apr.

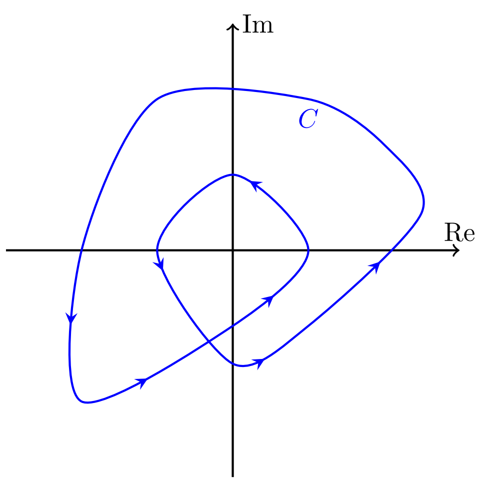
Question 1. (Bak&Newman Chapter 6, Ex.6) Suppose an analytic function f agrees with $\tan x$, $0 \le x \le 1$. (This means that for any real number x between 0 and 1, $f(x)$ is equal to $\tan(x)$.) Show that $f(z) = i$ has no solution. Could f be entire?

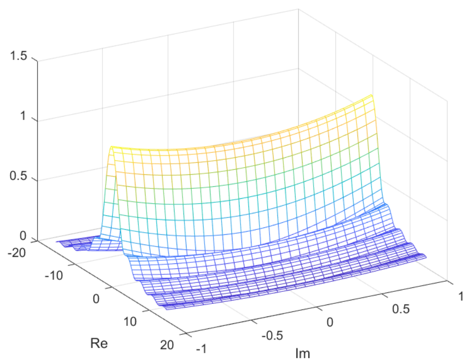
Question 2. (Bak&Newman Chapter 8, Ex.9) Define a function f analytic in the plane minus the non-positive real axis and such that $f(x) = x^x$ on the positive axis. Find $f(i)$, $f(-i)$. Show that $f(\bar{z}) = \overline{f(z)}$ for all z.

Question 3. Evaluate the complex integral

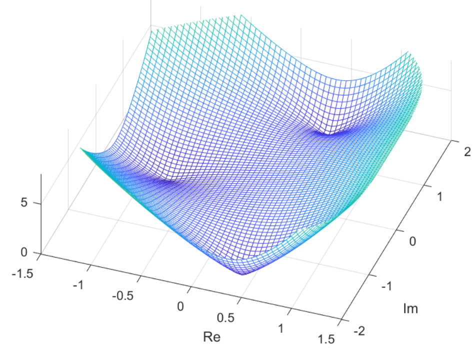
$$\int_C z^{-1/2}\, dz$$

over the following contour,

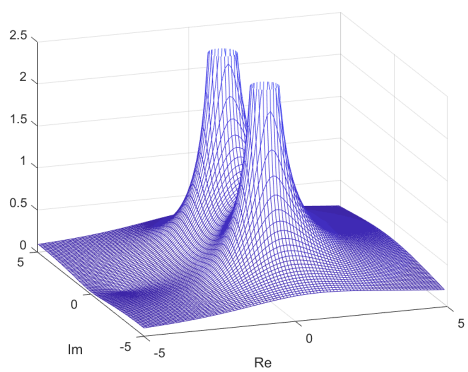




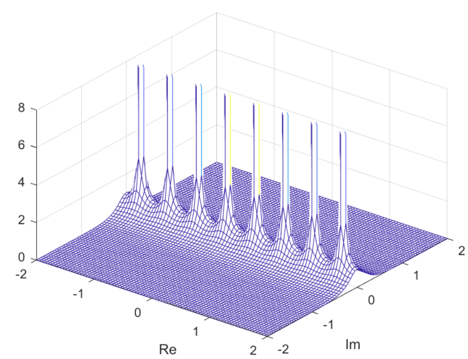
(a)



(b)



(c)



(d)

Figure 1: Plots of the modulus of functions in Question 4

Question 4.

Match the complex functions in (i) to (iv) with the plots of the modulus in Figure 1 in p.2.

(i) $f(z) = z^3 + 2z + 2$

(ii) $f(z) = \dfrac{\sin(z)}{z}$

(iii) $f(z) = \dfrac{z+4}{z^2+4}$

(iv) $f(z) = \dfrac{1}{\cos(2\pi z)}$

Question 5. For each function in Question 4, find all complex numbers $z \in \mathbb{C}$ such that z is a pole of the function. (If there is no pole, then just state that the function is analytic everywhere, or we have removable singularity.)

Question 6. Let z_0 be a nonzero complex number. Find a local primitive function in some small neighborhood of z_0 for

(a) $f(z) = \dfrac{1}{z^2}$

(b) $f(z) = \dfrac{1}{z}$

(b) $f(z) = \dfrac{\sin(z)}{z}$

(d) $f(z) = \dfrac{\cos(z)}{z}$

(A local primitive function is a function $F(z)$ that is analytic in a neighborhood of z_0 and $F'(z) = f(z)$ within the neighborhood.) You may use power series if the answer can be expressed more conveniently by power series. But your answer cannot be a multi-function. For example, $\log(z)$ is not an answer to part (b), unless you explicitly specify the branch of the log function.