MAT2006 Tutorial #3

1. (a) Let $C \subset [0,1]$ be uncountable. Show that there exists $a \in (0,1)$ such that $C \cap [a,1]$ is uncountable.

(b) Now let A be the set of all $a \in (0,1)$ such that $C \cap [a,1]$ is uncountable, and set $\alpha = \sup A$. Is $C \cap [\alpha, 1]$ an uncountable set?

(c) Does the statement in (a) remain true if "uncountable" is replaced by "infinite"?

2. Show that $2^{\mathbb{N}}$ and $\mathbb{R}$ have the same cardinality.
Hint. Consider the Schröder–Bernstein theorem.

3. Assume $\lim_{n\to\infty} a_n = a$. Show that $\lim_{n\to\infty} |a_n| = |a|$.

4. Show that

$$\text{(i)} \quad \lim_{n\to\infty} \sqrt[n]{p} = 1, \quad \text{where } p > 0.$$

$$\text{(ii)} \quad \lim_{n\to\infty} \sqrt[n]{n} = 1.$$

$$\text{(iii)} \quad \lim_{n\to\infty} \sqrt[2n+1]{n^2+n} = 1.$$

— End —