STA4001: Stochastic Processes

Lecture 1

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Stochastic processes

- A stochastic process represents a system state that evolves randomly over time.
- We use $X(t)$ to denote the state at time t.

Google Stock Price

Open	1,252.21	Div yield	-
High	1,252.54	Prev close	1,254.44
Low	1,225.75	52-wk high	1,291.44
Mkt cap	858.64B	52-wk low	924.51
P/E ratio	33.08		

Examples of Google Stock Price State

- $X(t)$ is the Google price at time t.
- $X(t)$ is the Google price curve up to time t.

- When managing a portfolio of stocks, one might use

$$X(t) = \Big(X_{\text{Apple}}(t), X_{\text{Google}}(t), \dots, X_{\text{Baidu}}(t) \Big)$$

DIDI ride-hailing platform

Exmaples of DIDI ride-hailing platform state

- State

$$X(t) = \Big(D_1(t), \ldots, D_{K(t)}(t); \quad P_1(t), \ldots, P_{L(t)}(t)\Big),$$

- $K(t)$ number of drivers,
- $L(t)$ number of waiting passengers,
- $D_i(t)$ status of the ith driver: GPS location, current destination, occupancy status
- $P_j(t)$ status of the jth waiting passenger: GPS location, destination, waiting status, patience level (observable?).

Emergency department (ED) congestion

Hospital general ward

A ward consists of a group of beds to house in-patients of “similar” conditions.

Boarding patients

Boarding patient — a patient who finishes treatment in ED and waits to be transferred to the inpatient department (a ward)

State for modeling hospital inpatient operation

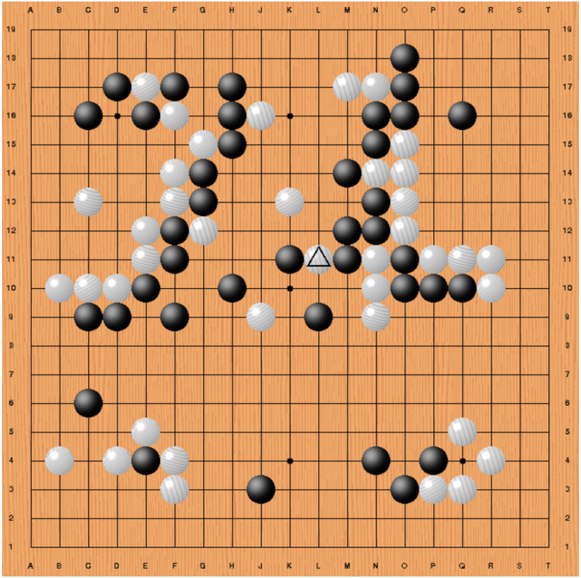
- State

$$X(t) = \Big(X_1(t), \ldots, X_J(t), Z_{11}(t), \ldots, Z_{1J}(t), \ldots, Z_{M1}(t), \ldots, Z_{MJ}(t)\Big)$$

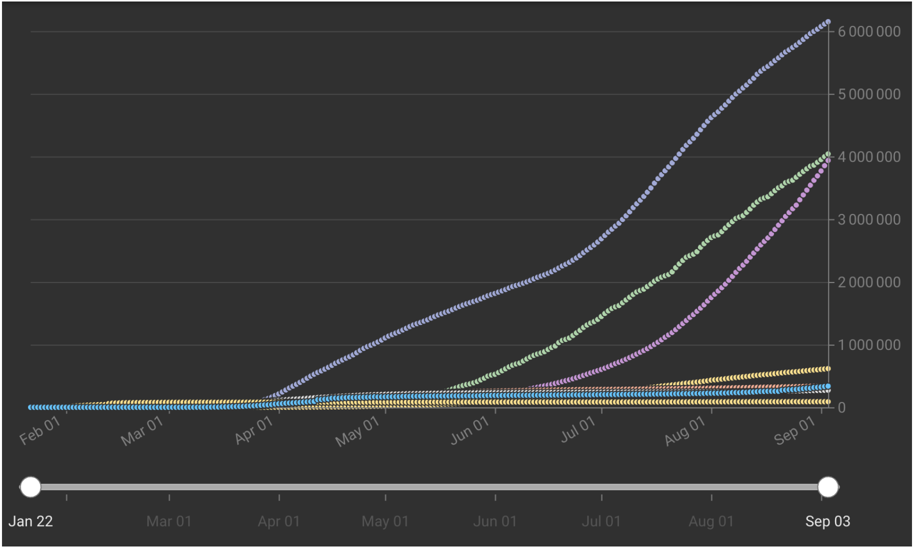
- J number of medical specialty
- $X_i(t)$ is the number of type i boarding patients
- $Z_{ij}(t)$ is the number of type i patients occuping ward j.

- $Y_j(n)$ the number of ward j patients that will be discharged in n days, $j = 1, \ldots, J$, $n = 1, 2, \ldots$

AlphaGo



Epidemic

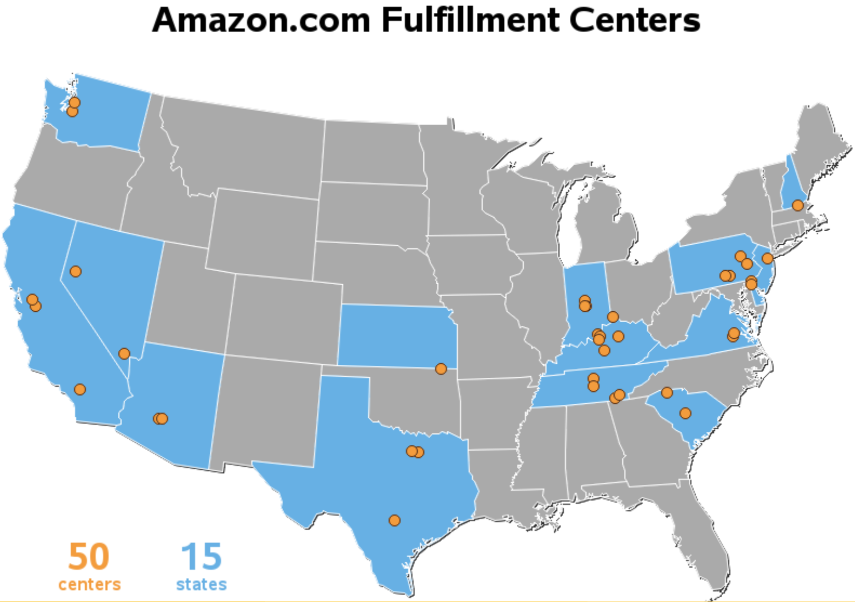


6 000 000
5 000 000
4 000 000
3 000 000
2 000 000
1 000 000
0
Feb 01
Mar 01
Apr 01
May 01
Jun 01
Jul 01
Aug 01
Sep 01
Jan 22
Mar 01
Apr 01
May 01
Jun 01
Jul 01
Aug 01
Sep 03

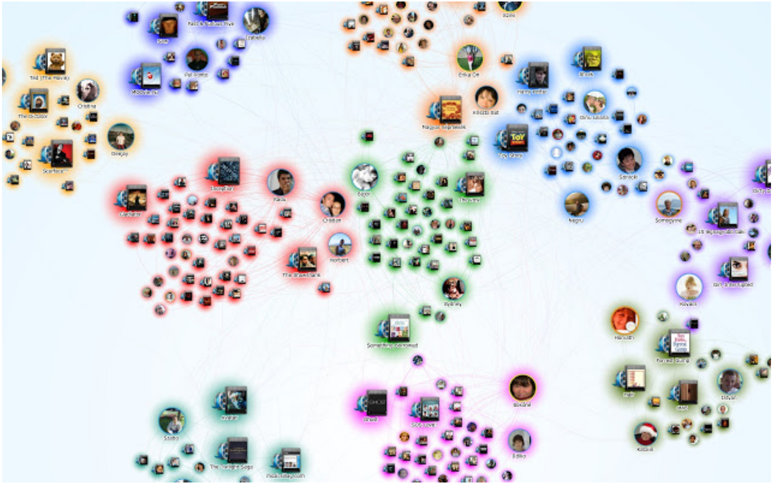
Data Center

State: status of (thousands or more of) servers, network switches, and locations of data storage.

E-commerce Distribution Network



Friends network



- $X(t) = (X_1(t), X_2(t), \ldots, X_{14}(t))$,
- $X_1(t)$ is the number of boxes (of milk) on day t that will expire in one day.
- $X_i(t)$ is the number of boxes (of milk) on day t will expire in i days.

Stochastic models and analysis

- Stochastic processes
- Markov chains (this semester, both mathematics and modeling)

Outline

News Vendor Problems

- A store sells perishable product, say, paper version of New York Times.
- Selling price $c_p = \$1.00$
- Variable cost $c_v = \$0.25$
- Salvage value $c_s = \$0.00$
- How many copies should the store order from the publisher the previous night?

Demand distribution

- Suppose the demand D has the following distribution

d	10	15	20	25	30
$\mathbb{P}(D = d)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$

Python simulation

- Suppose $q = 20$ copies are ordered.

day	demand	profit
1	30	15
2	30	15
3	10	5
4	30	15
5	25	15
6	10	5
7	15	10
8	25	15
9	30	15
10	30	15

- Day 1 profit = 20 (1) - 20(.25) = 20 - 5 =15.
- Day 7 profit = 15(1)-20(.25) = 10.

Profit formulas

- Day i profit $= \min(20, D_i) - 5$.
- Day i profit $= \min(q, D_i)c_p - qc_v$.
- If $c_s = .1$, Day 7 profit $=$

Profit formulas

- Day i profit $= \min(20, D_i) - 5$.
- Day i profit $= \min(q, D_i)c_p - qc_v$.
- If $c_s = .1$, Day 7 profit $=$
- Day i profit $= \min(q, D_i)c_p - qc_v + \max(q - D_i, 0)c_s$.

How did Python generate the demands

day	8-sided die	demand	profit
1	7	30	15
2	8	30	15
3	2	10	5
4	8	30	15
5	6	25	15
6	1	10	5
7	3	15	10
8	5	25	15
9	8	30	15
10	8	30	15

Python code

```
% Monte Carlo method for newsvendor problem
import numpy as np
import random
n=10000
x=[10,10,15,20,25,25,30,30]
d=np.array(random.choices(x,k=n))
cv=.25
cs=0
cp=1.0
q=20
p=np.minimum(d,q)*cp-q*cv+ np.maximum(q-d,0)*cs
print(d);print(p)
np.mean(p)
```

How many to order?

- Order $q = 20$ copies every day for $n = 100000$ days

$$\text{total } 1188870, \qquad \text{average } 11.8887.$$

- Order $q = 22$ copies every day for $n = 100000$ days

$$\text{total } 1239199, \qquad \text{average } 12.3920.$$

Predict average profit per day

- 5, 10, 15

Predict average profit per day

- 5, 10, 15
- 5/4+10/8 +15*5/8 =11.875

Optimal order quantity

- Objective: maximize the expected profit for a day

$$h(q) = \mathbb{E}[\text{Profit}(q, D)] = c_p \mathbb{E}[\min(q, D)] - c_v q + c_s \mathbb{E}[\max(q - D, 0)]. \tag{1}$$

- $\mathbb{E}[\min(20, D)] \neq \min(20, \mathbb{E}[D])$

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$$h(q) = \mathbb{E}[\text{Profit}(q, D)] = c_p \mathbb{E}[\min(q, D)] - c_v q \\ + c_s \mathbb{E}[\max(q - D, 0)]. \tag{1}$$

- $\mathbb{E}[\min(20, D)] \neq \min(20, \mathbb{E}[D])$

- $\mathbb{E}[\min(20, D)] = 16.875.$

- Notation

$$x \wedge y = \min(x, y), \quad a^+ = \max(a, 0).$$

- An identity

$$q = D \wedge q + (q - D)^+. \tag{2}$$