Tutorial 1

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Outline

Sample space and Event

Sample space

Sample space Ω is the set of all possible outcomes.

Event

If Ω is a discrete sample space, then every subset E of Ω is called an event; if Ω is a continuous sample space, $E \subset \Omega$ is a subset if and only if E is generated by subintervals of Ω through finite or countable set operations. (union, intersection and complement).

Mean

Expectation

$$E[x] = \int_{-\infty}^{+\infty} x dF\,(x) = \sum x f(x) \qquad discrete$$

$$= \int_{-\infty}^{+\infty} x f(x)\, dx \qquad continuous$$

$$E[g(x)] = \sum g(x) f(x)$$

$$= \int_{-\infty}^{+\infty} g(x) f(x)\, dx$$

F(x): cdf comulative distribution function
f(x): pdf/pmf probability density/mass function

Median and Variance, and common distribution

Median

$m = m_x$ is a median of x if
$P_r(x \leq m) \geq 0.5$ and $P_r(x \geq m) \geq 0.5$

Variance

$Var(X) = E[X - E(X))^2]$

Common Distribution

(1) Normal Distribution
(2) Binomial Distribution
(3) Poisson Distribution
(4) Uniform Distribution

Independent and Uncorrelated

Definition

(1) X & Y is independent if $F(x, y) = F_x(x)F_y(y)$
(2) X & Y is uncorrelated if $Cov(x, y) = 0$ or $E[XY] = E[X]E[Y]$

Relationship

Independent $\Rightarrow$ Uncorrelated
Uncorrelated $\not\Rightarrow$ Independent

Question1

Q1:

Assume that X and Y are jointly Gaussian random variables, i.e.

$$\begin{bmatrix} X \\ Y \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 & 6 \\ 6 & 18 \end{bmatrix} \right)$$

Find

(a) the probability density function (pdf) of X.

(b) $E(X + 2Y)$

(c) $Var(X - 2Y)$

(d) $E(X^2)$

(e) $E(XY)$

Question 2

Q2:

The following statements are correct ?

(1) It is possible that a random variable has exactly two medians $m_1 \neq m_2$.

(2) If (b_1, b_2) are randomly selected from $\{1, 2, 3, 4\}$ with replacement and ordered to $b_1 \leq b_2$, then the outcomes of (b_1, b_2) are equally likely to occur.

(3) In a test of the null hypothesis H_0 against the alternative H_1 , if the p-value of the test is below 0.05, then the probability to correctly accept H_1 is above 95%.