Sheet 1 118010350

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E1.1 (a). True. (b). False. (c). True
(d). True (e) True. (f). False.
(g). True. (h). False (i). False. (j). True

A.1.1 l - length; w - width; h - height. ; d - depth
N - number of floors aboveground;
B - number of floors underground;

(a). minimize lwd,
$l, w, h, d, N, B \in \mathbb{R}$. N, B are integers.

subject to $w \le l \le 2w$,

$$l \le 40,$$

$$l \le h,$$

$$\frac{h+d}{N+B} \le 3.5$$

$$0.1 \le \frac{d}{d+h} \le 0.25.$$

$$(N+B) \cdot lw \ge 10000$$

$$1000(lw + 2 \cdot lh + 2wh) \le 5000000$$

$$l, w, h, d, N, B > 0$$

(b) Find a feasible point that
$l = 30$, $W = 20$, $h = 36$, $d = 12$ (m).
$N = 12$, $B = 4$

A 1.2.

	assemble	test	worth	value
type 1.	1/3 h	1/5 h	\$2	\$10
type 2.	1/4 h	1/4 h	\$1	\$8.

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(a) Introduce two variables here,

x_1: amount of type 1 produced each day;

x_2: amount of type 2 produced each day

$$\underset{x_1, x_2 \in \mathbb{R}}{\text{maximize}} \quad 8x_1 + 7x_2.$$

$$\text{subject to} \quad \frac{1}{3}x_1 + \frac{1}{4}x_2 \le 100$$
$$\frac{1}{5}x_1 + \frac{1}{4}x_2 \le 70.$$
$$x_1, x_2 \ge 0$$

(b) Standard form:

$$\underset{x_1, x_2, S_1, S_2 \in \mathbb{R}}{\text{minimize}} \quad -8x_1 - 7x_2$$

$$\text{subject to} \quad \frac{1}{3}x_1 + \frac{1}{4}x_2 + S_1 = 100$$
$$\frac{1}{5}x_1 + \frac{1}{4}x_2 + S_2 = 70$$
$$x_1, x_2, S_1, S_2 \ge 0.$$

(or in matrix form).

$$\underset{x}{\text{minimize}} \quad c^T x.$$

$$\text{subject to} \quad Ax = b$$
$$x \ge 0.$$

where $c = \begin{bmatrix} -8 \\ -7 \\ 0 \\ 0 \end{bmatrix}$, $A = \begin{bmatrix} \frac{1}{3} & \frac{1}{4} & 1 & 0 \\ \frac{1}{5} & \frac{1}{4} & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 100 \\ 70 \end{bmatrix}$

and $x = \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{bmatrix}$

(c). Define $t = \max\{0, \frac{1}{3}x_1 + \frac{1}{4}x_2 - 60\} = \left(\frac{1}{3}x_1 + \frac{1}{4}x_2 - 60\right)^+$

$$\underset{x_1, x_2, t \in \mathbb{R}}{\text{maximize}} \quad 8x_1 + 7x_2 - 7t$$

No.: Date: Subject to $t \geq \left(\frac{1}{3}x_1 + \frac{1}{4}x_2 - 60\right)^+$

$$\frac{1}{3}x_1 + \frac{1}{4}x_2 \leq 100$$

$$\frac{1}{5}x_1 + \frac{1}{4}x_2 \leq 70.$$

$$x_1, x_2, t \geq 0.$$

(d). MATLAB. solve (a).

$x_1 = 225$, $x_2 = 100$, optval $= 2.5 \times 10^3$.

A1.3. $v_1, v_2, v_3, v_4, v_5, v_6$

$v_1 \downarrow s$, $v_6 \downarrow t$.

(a). Let $\underset{(6\times 6)}{A} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{16} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{26} \\ c_{31} & c_{32} & c_{33} & \cdots & c_{36} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{61} & c_{62} & c_{63} & \cdots & c_{66} \end{bmatrix}$

$$= \begin{array}{cc} \begin{bmatrix} 0 & 11 & 8 & 0 & 0 & 0 \\ 0 & 0 & 10 & 12 & 0 & 0 \\ 0 & 1 & 0 & 0 & 11 & 0 \\ 0 & 0 & 4 & 0 & 0 & 15 \\ 0 & 0 & 0 & 7 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6. \end{matrix} \end{array}$$

$\underset{(6\times 6)}{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{16} \\ x_{21} & x_{22} & \cdots & x_{26} \\ \vdots & & \ddots & \vdots \\ x_{61} & x_{62} & \cdots & x_{66} \end{bmatrix}$ and $\underset{(1\times 6)}{B} = [0\ 0\ 0\ 1\ 1\ 1]$

and $\underset{(6\times 1)}{C} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

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$$\underset{x_{ij}\in\mathbb{R}}{\text{maximize}} \quad x_{4b}+x_{5b} \qquad (\text{sum}(\text{sum}(B X C)))$$

subject to $\quad x_{ij} \le C_{ij}, \ \forall (i,j)\in E. \qquad (X \le A)$

$$\sum_j x_{ji} = \sum_j x_{ij} \quad \forall i \ne s,t$$

$$x_{ij} \ge 0, \quad \forall (i,j)\in E.$$

(b) MATLAB solve (a).

$$X = \begin{bmatrix} 0 & 11 & 8 & 0 & 0 & 0 \\ 0 & 0 & 1.5988 & 10.2660 & 0 & 0 \\ 0 & 0.8648 & 0 & 0 & 9.3673 & 0 \\ 0 & 0 & 0.6333 & 0 & 0 & 15 \\ 0 & 0 & 0 & 5.3673 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

optval = 19.

A1.4 (a) linear problem:

$$\underset{x\in\mathbb{R}^n}{\text{minimize}} \quad c^T x$$

subject to $\quad a_i x - b_i \le \delta \quad \forall i = 1,2,\dots m$

$\quad a_i x - b_i \ge -\delta \quad \forall i = 1,2,\dots m$

$\quad x \ge 0.$

where a_i is the i-th row of A.

b_i is the i-th element of b.

proof. Since $\|Ax-b\|_\infty \le \delta$, let $y = Ax-b$.

then $\|y\|_\infty \le \delta$, and $\max_{1\le i\le m} |y_i| \le \delta$.

That is $|a_i x - b_i| \le \delta \iff a_i x - b_i \le \delta \ \forall i=1,\dots m$

and $a_i x - b_i \ge -\delta \ \forall i=1,\dots m$.

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(b) Introduce two variables here.

x_1: amount of fruit salad A processed.

x_2: amount of fruit salad B processed.

Linear program:

$$\begin{aligned} &\underset{x_1, x_2 \in \mathbb{R}}{\text{maximize}} && 10x_1 + 20x_2 \\ &\text{subject to} && \tfrac{1}{4}x_1 + \tfrac{1}{2}x_2 = 25 \\ &&& \tfrac{1}{8}x_1 + \tfrac{1}{4}x_2 = 10 \\ &&& 5x_1 + x_2 = 120 \\ &&& x_1, x_2 \geq 0. \end{aligned}$$

whic is equivalent to

standard form:

$$c = \begin{bmatrix} -10 \\ -20 \end{bmatrix}, \quad A = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{8} & \frac{1}{4} \\ 5 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 25 \\ 10 \\ 120 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{aligned} &\underset{x \in \mathbb{R}^2}{\text{minimize}} && c^T x \\ &\text{subject to} && Ax = b \\ &&& x \geq 0. \end{aligned}$$

(c) This linear program is not solvable

Since $A = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{8} & \frac{1}{4} \\ 5 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 25 \\ 10 \\ 120 \end{bmatrix}$

$$\left[\begin{array}{cc|c} \frac{1}{4} & \frac{1}{2} & 25 \\ \frac{1}{8} & \frac{1}{4} & 10 \\ 5 & 1 & 120 \end{array}\right] \Updownarrow \left[\begin{array}{cc|c} 0 & 0 & -5 \\ \frac{1}{8} & \frac{1}{4} & 10 \\ 5 & 1 & 120 \end{array}\right]$$

then it is easy to know that

$Ax = b$ has no solution.

(d) Now the linear program is

$$\underset{x_1, x_2 \in \mathbb{R}}{\text{maximize}} \quad 10x_1 + 20x_2$$

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subject to $\frac{1}{4}x_1 + \frac{1}{2}x_2 \geq 20$

$\frac{1}{4}x_1 + \frac{1}{2}x_2 \leq 30$ ✓

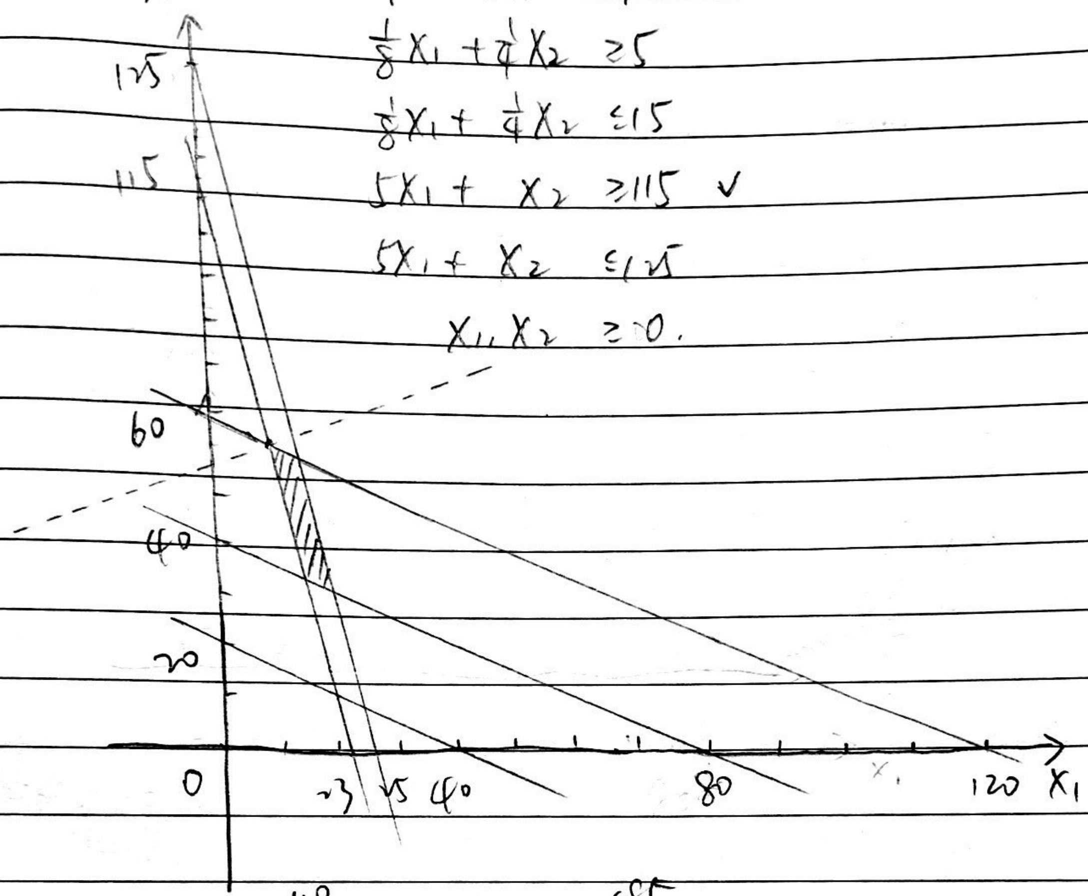
$\frac{1}{8}x_1 + \frac{1}{4}x_2 \geq 5$

$\frac{1}{8}x_1 + \frac{1}{4}x_2 \leq 15$

$5x_1 + x_2 \geq 115$ ✓

$5x_1 + x_2 \leq 125$

$x_1, x_2 \geq 0.$



Here $x_1 = \frac{110}{9} \approx 13$, $x_2 = \frac{485}{9} \approx 53$ profit $= 1190$.

The constraints $\frac{1}{4}x_1 + \frac{1}{2}x_2 \leq 30$ and $5x_1 + x_2 \geq 115$ are active.

Mango: $\frac{1}{4} \times 13 + \frac{1}{2} \times 53 \approx 30$

Pineapple: $\frac{1}{8} \times 13 + \frac{1}{4} \times 53 \approx 15$

Strawberry: $5 \times 13 + 1 \times 53 = 118$.

total: $30 + 15 + 118 = 163$.

E1.2 Assume $x \in \mathbb{R}^2$ is the center of the circle and $r \in \mathbb{R}$ is the radius of the circle.

$$\underset{x \in \mathbb{R}^2, r \in \mathbb{R}}{\text{minimize}} \quad r$$

subject to $\|y_i - x\|_2 \leq r$, $\forall i = 1, 2, \dots, K$

$r > 0$