Proof: $\underline{e}$ and $\hat{\underline{y}}$ are uncorrelated.

Random Vector $\underline{V} = \begin{bmatrix} \underline{V}_a \\ \underline{V}_b \end{bmatrix}$, where $\underline{V}$ is a column vector of size $n \times 1$, and $\underline{V}_a$ is of size $n_a \times 1$, $\underline{V}_b$ of size $n_b \times 1$

$$\mathrm{cov}(\underline{V}) = \mathrm{Cov}\left(\begin{bmatrix} \underline{V}_a \\ \underline{V}_b \end{bmatrix}\right) = \begin{bmatrix} \mathrm{Cov}(\underline{V}_a), & \mathrm{Cov}(\underline{V}_a, \underline{V}_b) \\ \mathrm{Cov}(\underline{V}_b, \underline{V}_a), & \mathrm{Cov}(\underline{V}_b, \underline{V}_b) \end{bmatrix}$$

Independence of $\hat{\underline{e}} = \underline{y} - \hat{\underline{y}}$ and fitted values $\hat{y}$

$$\underline{e} = y - \hat{y} = (I - H)\underline{y}$$

$$\hat{\underline{y}} = H\underline{y}$$

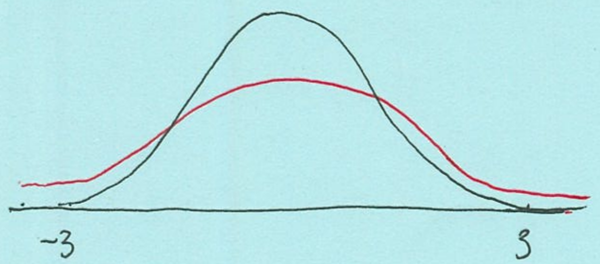
Let $\underline{V} = \begin{bmatrix} \underline{e} \\ \hat{\underline{y}} \end{bmatrix} = \begin{bmatrix} (I-H)\underline{y} \\ H\underline{y} \end{bmatrix} = \begin{bmatrix} I-H \\ H \end{bmatrix} \underline{y}$

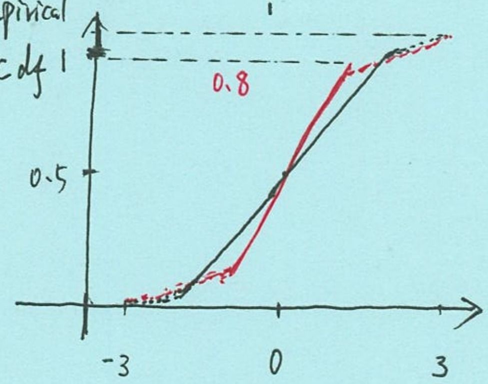
$$\mathrm{Cov}(\underline{V}) = \begin{bmatrix} \mathrm{Cov}[(I-H)\underline{y}], & \mathrm{cov}[(I-H)\underline{y}, H\underline{y}] \\ \mathrm{Cov}[H\underline{y}, (I-H)\underline{y}], & \mathrm{cov}[H\underline{y}] \end{bmatrix} = \sigma^2 \begin{bmatrix} I-H, & \underline{0} \\ \underline{0}, & H \end{bmatrix}$$

$$\mathrm{Cov}[H\underline{y}, (I-H)\underline{y}] = \underline{0} = \mathrm{Cov}[(I-H)\underline{y}, H\underline{y}]$$

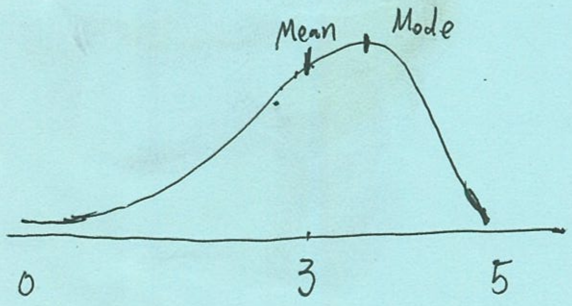
Gaussian

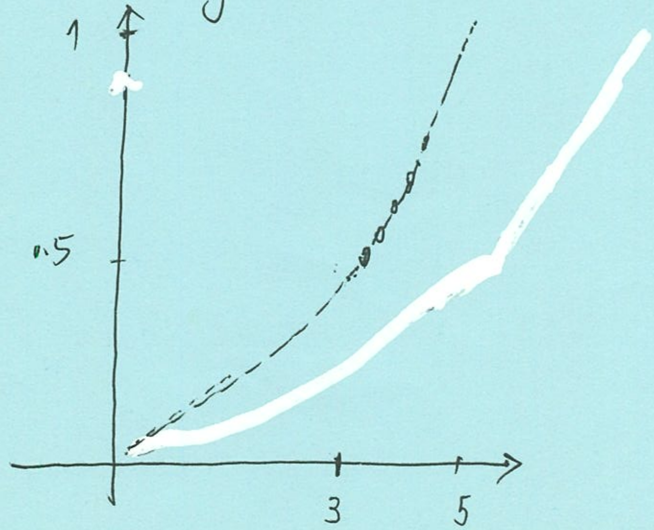
Heavy-tailed, e.g. t_ν. with small ν





Negative skewed (Left skewed) Distribution: has a long left tail. The mean is left to the mode





positive-skewed (right skewed) distribution: has a long right tail. The mean is right to the mode