STA3007: Tutorial 11

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Outline

Question 1(Textbook Problem 9.31)

Experimental geneticists use survival under stressful conditions to compare the relative fitness of different species. Dowdy and Wearden (1991) considered data relating to the survival of three species of Drosophila under increasing levels of organic phosphorus insecticide. Four batches of medium, identical except for the levels of insecticide they contained, were prepared. One hundred eggs from each of three Drosophila species were deposited on each of the four medium preparations and the level of insecticide (x) in parts per million (ppm) and number of Drosophila flies that survived to adulthood (y) for each combination are recorded in Table 9.7.

Question 1(Textbook Problem 9.31)

Test the hypothesis that the three species of Drosophila exhibit the same response to increasing levels of insecticide in the medium studied.

Table 9.7 Numbers of *Drosophila* Flies (Three Different Species) That Survive to Adulthood after Exposure to Various Levels (ppm) of an Organic Phosphorus Insecticide

Species	Level of insecticide (ppm)	Number survived to adulthood
Drosophila melanogaster	0.0	91
	0.3	71
	0.6	23
	0.9	5
Drosophila pseudoobscura	0.0	89
	0.3	77
	0.6	12
	0.9	2
Drosophila serrata	0.0	87
	0.3	43
	0.6	22
	0.9	8

Source: S. Dowdy and S. Wearden (1991).

Question 1(Textbook Problem 9.31)

Test statistic: Let

$$\overline{x}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij}, \quad i = 1,\ldots,k, \text{ and } \quad \overline{\beta} = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij} - \overline{x}_i)Y_{ij}}{\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij} - \overline{x}_i)^2} \tag{8.18}$$

Then define *aligned observations*:

$$Y_{ij}^* = Y_{ij} - \overline{\beta}x_{ij}, \quad j = 1,\ldots,n_i, \quad i = 1,\ldots,k. \tag{8.19}$$

Order $Y_{i1}^*,\ldots,Y_{in_i}^*$ increasingly (assuming no ties) and let r_{ij}^* denote the rank of Y_{ij}^* among $Y_{i1}^*,\ldots,Y_{in_i}^*$. Then compute for $i = 1,\ldots,k$,

$$T_i^* = \frac{1}{n_i + 1}\sum_{j=1}^{n_i}(x_{ij} - \overline{x}_i)r_{ij}^* \quad \text{and} \quad C_i^2 = \sum_{j=1}^{n_i}(x_{ij} - \overline{x}_i)^2 = \sum_{j=1}^{n_i} x_{ij}^2 - n_i\overline{x}_i^2 \tag{8.20}$$

The *Sen-Adichie statistic* V for testing H_0 against H_1 in (8.17) is defined by

$$V = 12\sum_{i=1}^{k}\left(\frac{T_i^*}{C_i}\right)^2 = 12\sum_{i=1}^{k}\frac{(T_i^*)^2}{C_i^2} \tag{8.21}$$

Asymptotic rejection rule: Reject H_0 if $V \geq \chi_{k-1,\alpha}^2$.

Question 2

Based on the independent variables $x_1, \cdots, x_{20}$ and the response variables $Y_1, \cdots, Y_{20}$ in the following table 1, compute the **running line smoother estimator** $\mu(x)$ for $x \in (15.5, 16.5)$. Then, estimate the response variable value for $x = 15.8$.

Table 1: Independent variables x_i and response variables Y_i

i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
x_i	1	2	3	4	5	6	7	8	9	10
Y_i	100	96	89	87	84	81	78	74	68	65
i	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
x_i	11	12	13	14	15	16	17	18	19	20
Y_i	61	56	52	48	45	41	38	34	30	25