STA 4001 Stochastic Processes Spring 2021
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Homework 7
Due: March 30, 2021

1. Suppose there are two tellers taking customers in a bank. Service times at a teller are independent, exponentially distributed random variables, but the first teller has a mean service time of 2 minutes while the second teller has a mean of 5 minutes. There is a single queue for customers awaiting service. Suppose at noon, 3 customers enter the system. Customer A goes to the first teller, B to the second teller, and C queues. To standardize the answers, let us assume that T_A is the length of time in minutes starting from noon until Customer A departs, and similarly define T_B and T_C.

   (a) What is the probability that Customer A will still be in service at time 12:05?

   (b) What is the expected length of time that A is in the system?

   (c) What is the expected length of time that A is in the system if A is still in the system at 12:05?

   (d) How likely is A to finish before B?

   (e) What is the mean time from noon until a customer leaves the bank?

   (f) What is the average time until C starts service?

2. Nortel in Canada operates a call center for customer service. Assume that each caller speaks either English or French, but not both. Suppose that the arrival for each type of calls follows a Poisson process. The arrival rates for English and French calls are 2 and 1 calls per minute, respectively. Assume that call arrivals for different types are independent.

   (a) What is the probability that the 2nd English call will arrive after minute 5?

   (b) Find the probability that, in first 2 minutes, there are at least 2 English calls and exactly 1 French call?

   (c) What is the expected time for the 1st call that can be answered by a bilingual operator (that speaks both English and French) to arrive?

   (d) Suppose that the call center is staffed by bilingual operators that speak both English and French. Let $N(t)$ be the total number of calls of both types that arrive during $(0, t]$. What kind of a process is $N(t)$? What is the mean and variance of $N(t)$?

   (e) What is the probability that no calls arrive in a 10 minute interval?

(f) What is the probability that at least two calls arrive in a 10 minute interval?

(g) What is the probability that two calls arrive in the interval 0 to 10 minutes and three calls arrive in the interval from 5 to 15 minutes?

(h) Given that 4 calls arrived in 10 minutes, what is the probability that all of these calls arrived in the first 5 minutes?

(i) Find the probability that, in the first 2 minutes, there are at most 1 call, and, in the first 4 minutes, there are exactly 3 calls?

3. Suppose customer arrive to a bank according to a Poisson process but the arrival rate fluctuates over time. From the opening time at 9 a.m. until 11, customers arrive at a rate of 10 customers per hour. From 11 to noon, the arrival rate increases linearly until it reaches 20 customers per hour. From noon to 1pm, it decreases linearly to 15 customers per hour, and remains at 15 customers per hour until the bank closes at 5 p.m. For notation, let $N(t)$ be the number of arrivals in the t hours since the bank opened and $\lambda(t)$ the arrival rate at t hours after opening.

   (a) What is the arrival rate at 12:30pm?

   (b) What is the average number of customers per day?

   (c) What is the probability of k arrivals between 11:30 and 11:45?

   (d) What is the probability of k arrivals between 11:30 and 11:45 given that there were 7 arrivals between 11:00 and 11:30?

   (e) Consider the first customer that arrives after noon and let T be the length of time since noon to when that customer arrives. What is the probability that this customer arrives after 12:10, after 12:20? Is T exponentially distributed?

4. Assume the following five numbers are generated independently, following uniform$(0, 1)$ distribution:

$$0.15386, \quad 0.89607, \quad 0.15657, \quad 0.61391, \quad 0.80390. \tag{1}$$

   Using these fives numbers to generate arrival times of five customers following (homogeneous) Poisson arrival process with rate 10 per hour.

5. Using the five numbers in (1) and the time-change method to generate five arrival times that follow a non-homogeneous Poisson process with rate function $\{\lambda(t) = t, t \geq 0\}$.

6. Assume that another five numbers are generated independently, following uniform$(0, 1)$ distribution:

$$0.58126, \quad 0.27798, \quad 0.15661, \quad 0.26974, \quad 0.22975 \tag{2}$$

And they are independent of the numbers generated in (1). Using accept-rejection method to generate as many arrival times as possible that follow a non-homogeneous Poisson process with rate function $\{\lambda(t) = t, t \geq 0\}$.