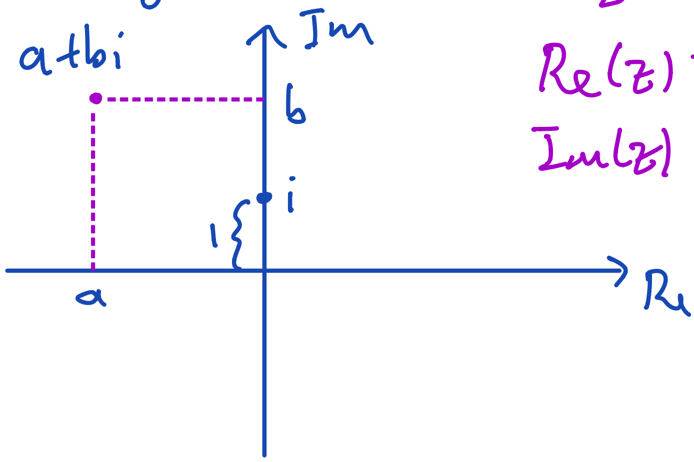
MAT 3253 Lecture 1

Complex number is a number in the form

$a+bi$, $a, b \in \mathbb{R}$

$i^2=-1$ — set of real numbers

Complex plane / Argand plane



$z = a+bi$

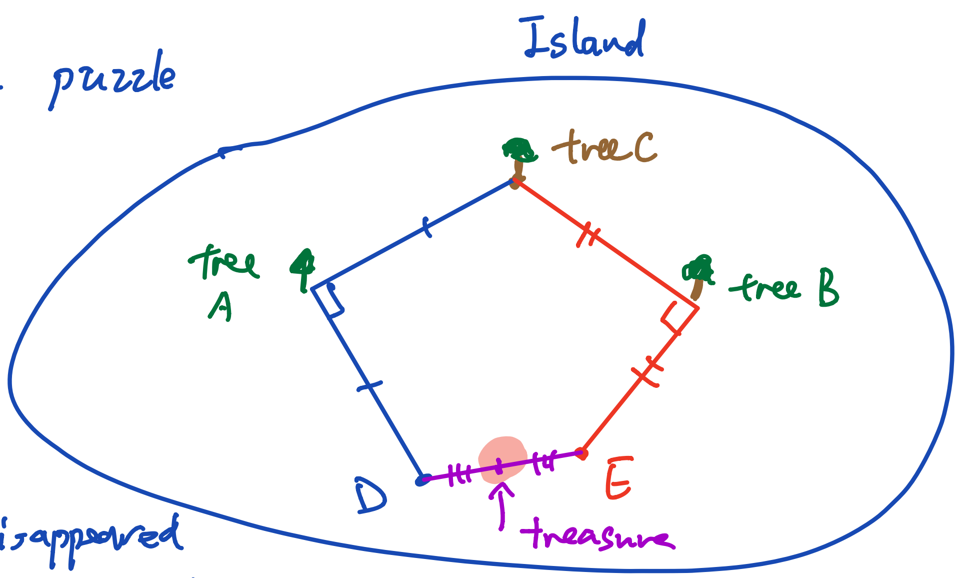
$\mathrm{Re}(z) = a$

$\mathrm{Im}(z) = b$

$$(a+bi)+(c+di) = (a+c)+(b+d)i$$

$$(a+bi)\cdot(c+di) = (ac-bd) + (ad+bc)i$$

Geometric puzzle



tree C disappeared

How to locate the treasure?

Def

A number system $(F, +, \cdot)$ is called a field if

(closedness) $a+b \in F \quad \forall a,b \in F$

(associative) $(a+b)+c = a+(b+c) \quad \forall a,b,c \in F$

(commutative) $a+b = b+a \quad \forall a,b \in F$

$\exists\, 0 \in F$ s.t. $0+a = a+0 = a \quad \forall a$

$\forall a \in F, \ \exists\, a' \in F$ s.t. $a+a' = 0$

(closedness) $a \cdot b \in F \quad \forall a,b \in F$

$(a\cdot b)\cdot c = a\cdot(b\cdot c) \quad \forall a,b,c \in F$

$a\cdot b = b\cdot a \quad \forall a,b \in F$

$\exists\, 1 \quad 1\cdot a = a\cdot 1 = a \quad \forall a \in F$

$\forall a \in F\setminus\{0\} \ \exists\, a'' \in F$ s.t. $a\cdot a'' = 1$

(distributive)
$a\cdot(b+c) = a\cdot b + a\cdot c \quad \forall a,b,c \in F$

Example $\mathbb{Q}$ is a field

$\mathbb{R}$ is a field

A field F is a complex field if it contains $\mathbb{R}$ as subfield and

$\exists\, I$ s.t. $I^2 + 1 = 0$

additive identity

multiplicative identity

Construction 1

Define $F = \{ (x, y) : x, y \in \mathbb{R} \}$

$$(x_1, y_1) + (x_2, y_2) \triangleq (x_1 + x_2, y_1 + y_2)$$

$$(x_1, y_1) \cdot (x_2, y_2) \triangleq (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$$

definition

Construction 2

Define $F = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b \in \mathbb{R} \right\}$

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} + \begin{bmatrix} c & -d \\ d & c \end{bmatrix} = \begin{bmatrix} a+c & -b-d \\ b+d & a+c \end{bmatrix}$$

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} c & -d \\ d & c \end{bmatrix} = \begin{bmatrix} ac-bd & -ad-bc \\ ad+bc & ac-bd \end{bmatrix}$$

$a+bi$ is a notation for (a, b) or $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

Use

"Multiply by i is rotation $90°$ counterclockwise"

to solve the geometric puzzle.