MAT2006 Tutorial #5

1. For each of the following sequences, does it converge? If so, find the limit.

$$(a) \quad x_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}.$$

$$(b) \quad x_n = \sum_{k=n^2}^{(n+1)^2} \frac{1}{\sqrt{k}}.$$

2. Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \sin(nx).$$

3. Suppose that given any $p \in \mathbb{N}$, it always hold

$$\lim_{n\to\infty} (a_{n+1} + a_{n+2} + \cdots + a_{n+p}) = 0.$$

Does the series $\sum_{n=1}^{\infty} a_n$ converge? Why?

4. Assume that $a_n \geq 0$ for all $n \in \mathbb{N}$. Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ also converges.

5. Discuss the convergence (absolutely, conditionally or not convergent) for the series:

$$(a) \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[n]{n}}; \qquad (b) \quad \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+2019};$$

6. Show that the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

converges whenever $|x| < 1$, and the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}$$

converges for any $x \in \mathbb{R}$.

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