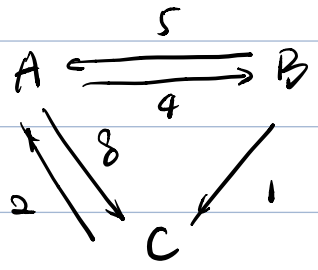
STA4001 Homework 8 118010350

1. (a) The rate diagram is given by



(b) The jump matrix is given by

$$J = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{5}{6} & 0 & \frac{1}{6} \\ 1 & 0 & 0 \end{bmatrix}$$

The holding time rates are $\lambda = [12, 6, 9]$

(c) See Python file.

2. (a)
$$P(0.2) = \begin{bmatrix} 0.2\,4984 & 0.180465 & 0.589649 \\ 0.2382896 & 0.2989325 & 0.4627779 \\ 0.1410591 & 0.0508258 & 0.8081151 \end{bmatrix}$$

(b)
$$P(1) = \begin{bmatrix} 0.1672999 & 0.1130718 & 0.7196283 \\ 0.1687556 & 0.1175811 & 0.7136634 \\ 0.1661992 & 0.1096633 & 0.7241378 \end{bmatrix}$$

(c) $P\{X(1.2)=C \mid X(0)=A\} = \sum_{k\in\{A,B,C\}} P\{X(1.2)=C, X(1)=k \mid X(0)=A\}$.

$$= \sum_{k\in\{A,B,C\}} P\{X(1.2)=C \mid X(1)=k\}\, P\{X(1)=k \mid X(0)=A\}$$

$$= \sum_{k\in\{A,B,C\}} P\{X(0.2)=C \mid X(0)=k\}\, P(X(1)=k \mid X(0)=A\}.$$

$$= 0.167 \times 0.589 + 0.113 \times 0.363 + 0.720 \times 0.808.$$

$$\approx 0.721$$

$$P\{X(3)=A \mid X(1)=B\} = P(X(2)=A \mid X(0)=B\}$$

$$= \sum_{k\in\{A,B,C\}} P\{X(2)=A,\ X(1)=k \mid X(0)=B\}$$

$$= \sum_{k\in\{A,B,C\}} P\{X(2)=A \mid X(1)=k\}\, P(X(1)=k \mid X(0)=B\}.$$

$$= \sum_{k\in\{A,B,C\}} P\{X(1)=A \mid X(0)=k\}\, P(X(1)=k \mid X(0)=B\}.$$

$$= 0.169 \times 0.167 + 0.118 \times 0.169 + 0.714 \times 0.166.$$

$$= 0.167.$$

(d) if $t=5$, then we get

$$P(5) = \begin{bmatrix} 0.166667 & 0.1111111 & 0.7222222 \\ 0.166666 & 0.1111111 & 0.7222222 \\ 0.166666 & 0.1111111 & 0.7222222 \end{bmatrix}$$

We observe that when t is large, $P(t)$ converges to the stationary distribution.

(e). By solving $\pi G = 0$, we can get the stationary distribution,

$$\pi = \left[\frac{9}{54}, \frac{6}{54}, \frac{39}{54}\right]$$

3. The CTMC has the following 5 states,

① Both A and B are working.

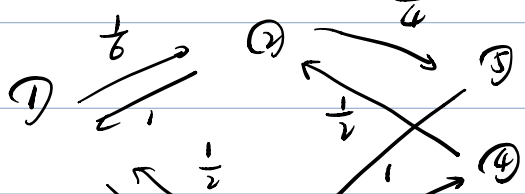
② A is working, B is down.

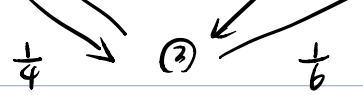
③ B is working, A is down.

④ A and B are down, A went down first.

⑤ A and B are down, B went down first.

Then the rate transition diagram is given by





4. The CTMC has 4 states, $\{0, A, B, 2\}$, where states 0 and 2 represents 0 and 2 customers in the system respectively. State A represents one customer in the system being served by the first server, and the state B represents one customer in the system being served by the second server.

The generater matrix is given by

$$G = \begin{bmatrix} -5 & 5 & 0 & 0 \\ 3 & -8 & 0 & 5 \\ 2 & 0 & -7 & 5 \\ 0 & 2 & 3 & -5 \end{bmatrix}$$

The rate diagram is given by