STA 3007 Exercise 3 118010352.

Q1. (a). False. The known peak p may not equal to the estimate $\hat{p}$, thus we get $A_{\hat{p}}^* = A_p^*$ is not true

(b). False. The value of $\hat{p}$ varies in different rank allocations, even if $\hat{p} = p = 4$, $A_{\hat{p}}^*$ does not have the same distribution as A_4^* with known $p=4$. Thus, we get $Pr(A_{\hat{p}}^* \geq a) = Pr(A_4^* \geq a)$ for $\forall a \in \mathbb{R}$ is not true.

(c) False Since $\{[a_{uv}, b_{uv}), 1 \leq u < v \leq k\}$ are simultaneous $100(1-\alpha)\%$ intervals of $\{T_u - T_v, 1 \leq u < v \leq k\}$, then we get that $Pr(a_{uv} \leq T_u - T_v < b_{uv}, 1 \leq u < v \leq k) = 1-\alpha$, which works for $\{T_u - T_v\}$ simultaneously, but not seperately.

(d). True Since $Pr(a_{uv} \leq T_u - T_v < b_{uv}, 1 \leq u < v \leq k) = 1-\alpha$, then we get $Pr(T_u - T_v < a_{uv}$ or $T_u - T_v \geq b_{uv}$ for some $1 \leq u < v \leq k) = \alpha$.

Q2. (a). False Sum of ranks $= 1+2+\cdots+40$
$= (1+40) \times 40/2 = 820$. not equals to 800.

(b). True. Since $J = \sum_{u<v} U_{uv} = \sum_{v=2}^{8} \sum_{u=1}^{v-1} U_{uv}$. $1 \leq u < v \leq 8$,

then we have combinations, (1,2). (1,3) (1,4)... (1,8) 7
(2,3) (2,4), (2,5) (2,6), (2,7) (2,8) 6
(3,4) (3,5) (3,6) (3,7) (3,8) 5
(4,5) (4,6) (4,7) (4,8) 4
(5,6) (5,7) (5,8) 3
(6,7) (6,8) 2
(7,8) 1

$= (7+6+5+4+3+2+1) \times 5 \times 5 = 700$. values of 0 or 1.

(c). False. Since $A_4 = \sum_{u<v\leq 4} U_{uv} + \sum_{4 \leq u < v} U_{vu} = (U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}) + (U_{54} + U_{64} + U_{65} + \cdots + U_{87})$.

thus $\min A_4 = 0$, $\max A_4 = 16 \times 25 = 400$ not equals to 350.

Q3. (a). True. Since $N = 4 \times 5 = 20$, then by formula,

$$H = \frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} - 3(N+1)$$

$$= \frac{12}{20 \times 21} \left(\frac{R_1^2}{4} + \frac{R_2^2}{4} + \cdots + \frac{R_5^2}{4} \right) - 3 \times 21$$

$$= \frac{R_1^2 + R_2^2 + \cdots + R_5^2}{140} - 63$$

$$= \frac{R_1^2 + \cdots + R_5^2 - 8820}{140}$$

(b). False. $A_{\hat{p}}^*$ does not have the same distribution as A_p^* because $\hat{p}$ changes value with the assignment of ranks, whereas p is fixed.

(c). False. It is not sure what T_i $(i=1,2,3,4)$ truely means. large value of T_i may mean "more effictive". but can also mean "less effictive". thus it is not true to conclude that treatments 2, 3 are more effective than treatment 1

Q4. (a). Since $p=3$, $k=5$, then we have to get $U_{12}, U_{13}, U_{23}, U_{43}, U_{53}, U_{54}$

$U_{12} = 3 \times 3 = 9$. $U_{13} = 3+3+2 = 8$. $U_{23} = 3+2+0 = 5$.

$U_{43} = 3+3+2 = 8$. $U_{53} = 3 \times 3 = 9$, $U_{54} = 1+3+3 = 7$.

Then $A_3 = U_{12} + U_{13} + U_{23} + U_{43} + U_{53} + U_{54}$

$= 9+8+5+8+9+7$

$= 46$.

By R, we get $\alpha = 0.0086$, $a_{3,0.0086} = 45$. $Pr(A_3 \geq 45) = 0.0086$

Then the exact pvalue $= Pr(A_3 \geq 46) < 0.0086$

Thus we reject $H_0: \tau_1 = \cdots = \tau_5$ in favor of $H_1: \tau_1 \leq \tau_2 \leq \tau_3 \geq \tau_4 \geq \tau_5$ at the 5% level.

(b). $k=5$. $n_1 = n_2 = n_3 = n_4 = n_5 = 3$ and $N = 15$.

Similar to (a), we can get $U_{12}=9,\ U_{13}=8,\ U_{14}=6,\ U_{15}=4.$

$U_{23}=5,\ U_{24}=0,\ U_{25}=0.$

$U_{34}=1,\ U_{35}=0.$

$U_{45}=2.$

Then $U_{\cdot 1}=U_{21}+U_{31}+U_{41}+U_{51}=4\times(3\times 3)-U_{12}-U_{13}-U_{14}-U_{15}=9.$

$U_{\cdot 2}=U_{12}+U_{32}+U_{42}+U_{52}=9+3\times 9-U_{23}-U_{24}-U_{25}=31$

$U_{\cdot 3}=U_{13}+U_{23}+U_{43}+U_{53}=8+5+2\times 9-U_{34}-U_{35}=30$

$U_{\cdot 4}=U_{14}+U_{24}+U_{34}+U_{54}=6+0+1+9-U_{45}=14.$

$U_{\cdot 5}=U_{15}+U_{25}+U_{35}+U_{45}=4+0+0+2=6$

Since $n_1=\cdots=n_5$, we have $E_0[U_{\cdot 1}]=\cdots=E_0[U_{\cdot 5}]$, and $Var_0(U_{\cdot 1})=\cdots=Var_0(U_{\cdot 5})$, hence $U_{\cdot p}^{*}=\max\{U_{\cdot 1}^{*},\dots,U_{\cdot 5}^{*}\}$.

if and only if $U_{\cdot p}=\max\{U_{\cdot 1},\dots,U_{\cdot 5}\}=\max\{9,31,30,40,6\}=31.$

we get $\hat{p}=2$, $A_{\hat{p}}=A_2=U_{12}+U_{32}+U_{42}+U_{52}+U_{43}+U_{53}+U_{54}.$

$=9+6\times 9-U_{23}-U_{24}-U_{25}-U_{34}-U_{35}-U_{45}$

$=63-8=55.$

$N_1=n_1+n_2=3\times 2=6,\ N_2=n_2+n_3+n_4+n_5=3\times 4=12.\quad N=15.$

$E_0[A_2]=\frac{1}{4}\times(6^2+12^2-3^2\times 5-3^2)=31.5$

$Var_0[A_2]=\frac{1}{72}\times[2\times(6^3+12^3)+3\times(6^2+12^2)-5\times 3^2\times 9-3^2\times 9]$

$+\frac{1}{6}\times(3\times 6\times 12-3^2\times 15).$

$=54.75+13.5=68.25.$

Thus $A_{\hat{p}}^{*}=A_2^{*}=\dfrac{A_2-E_0[A_2]}{\sqrt{Var_0(A_2)}}=\dfrac{55-31.5}{\sqrt{68.25}}=2.8446$