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CSC3001 **Discrete Mathematics**
Practice **Exercise 1** Sep. **20**20

Propositional Logic

1. **T/F** Which of the following are propositions?

F (a) $x < 30$.
T (b) "$x^2 = 23$" is a proposition ✓
T (c) You are Beautiful ✓
T (d) $15 > 20$. ✓
T (e) Every even integer greater than 2 can be expressed as the sum of two primes. ✓
F (f) $a^2 + b^2 = c^2$.
T (g) $\pi + e$ is rational. ✓

2. **SQ** Let h = "Peter is handsome", c = "Peter is clever", o = "Peter is optimistic". Rewrite the following in symbolic forms using $\neg$, $\vee$, $\wedge$. You don't have to simplify the formulas.

(a) Peter is handsome and clever but not optimistic. $h \wedge c \wedge (\neg o)$
(b) Peter is either clever or handsome or both. $h \vee c \vee (h \wedge c)$
(c) Peter is either clever or handsome but not both. $(h \wedge \neg c) \vee (\neg h \wedge c)$
(d) Peter is neither handsome, clever nor optimistic. $\neg(h \wedge c \wedge o)$
(e) Peter is not both handsome and clever, but he is optimistic. $\neg(h \wedge c) \wedge o$
(f) Peter is optimistic but not clever nor handsome. $\neg h \wedge \neg c \wedge o$

3. **SQ** Construct and then rewrite logical formulas for the following using only the $\neg$, $\vee$, $\wedge$ operators.(a)

(a) $p \oplus q$. $(p \wedge \neg q) \vee (\neg p \wedge q)$
(b) $p \rightarrow q$ $\neg p \vee q$
(c) $p \leftrightarrow q$. $(p \wedge q) \vee (\neg p \wedge \neg q)$
(d) $\neg(p \rightarrow q)$. $p \wedge \neg q$.

4. **SQ** Construct and then rewrite logical formulas for $f(p, q, r)$ using only the $\neg$, $\vee$, $\wedge$ operators.

(a)

p	q	r	$f(p, q, r)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

$f(p,q,r) = (p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$.

(b)

p	q	r	$f(p, q, r)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

$f(p,q,r) = \neg(p \wedge q \wedge \neg r) \wedge \neg(\neg p \wedge q \wedge \neg r) \wedge \neg(\neg p \wedge \neg q \wedge \neg r)$

(a) $(p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)$

(b) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q) \vee r$

$= (p \wedge q \wedge \neg r) \vee r \vee (p \wedge \neg q)$

$= ((r \vee p) \wedge (r \vee q)) \vee (p \wedge \neg q)$

$= r \vee ((p \wedge q) \vee (p \wedge \neg q)) = r \vee p$

5. **SQ** Proof or disprove the equivalence of the following logical formulas.

(a) $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$.

(b) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q) \vee r$ and $p \vee r$.

(c) $\neg(p \vee q \vee r)$ and $\neg p \wedge \neg q \wedge \neg r$.

(d) $p \wedge (p \vee q)$ and $p \vee (p \wedge q)$.

(e) $(p \wedge q) \vee (q \wedge r)$ and $q \vee (p \wedge r)$.

(c) $\neg(p \vee q \vee r) = \neg(p \vee (q \vee r))$

$= \neg p \wedge \neg(q \vee r)$

$= \neg p \wedge (\neg q \wedge \neg r)$

$= \neg p \wedge \neg q \wedge \neg r$

(d) $p \wedge (p \vee q) = (p \wedge p) \vee (p \wedge q)$

$= p \vee (p \wedge q)$

(e) $(p \wedge q) \vee (q \wedge r) = q \wedge (p \vee r)$

6. **SQ** Express the following using only the $\neg$, $\vee$, $\wedge$ operators.

(a) $\neg(p \rightarrow (p \rightarrow q))$ $p \wedge \neg q$.

(b) (Contrapositive) $\neg q \rightarrow \neg p$. $\neg(p \wedge \neg q)$

(c) (Converse) $q \rightarrow p$. $\neg(\neg p \wedge q)$

(d) (Inverse) $\neg p \rightarrow \neg q$. $\neg(\neg p \wedge q)$

(e) (Negation) $\neg(p \rightarrow q)$. $p \wedge \neg q$

7. **SQ** Prove or disprove the following arguments by truth tables. (Notice that the priority of operators= from the highest to the lowest: 1. $\neg$ 2. $\wedge$, $\vee$ 3. $\rightarrow$, $\leftrightarrow$)

(a)
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow p \\ \therefore p \vee q \end{array}$$

(b)
$$\begin{array}{l} p \rightarrow q \\ q \\ \therefore p \end{array}$$

(a)

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \vee q$
T	T	T	T	T
F	F	T	T	F
T	F	F	T	T
F	T	T	F	T

X (a).

(b)

p	q	$p \rightarrow q$
T	T	T
F	F	T
T	F	F
F	T	T

X (b)

(c)
p
$p \rightarrow q$
$\neg q \vee r$
$\therefore r$

(d)
$p \wedge q \rightarrow \neg r$
$p \vee \neg q$
$\neg q \rightarrow p$
$\therefore \neg r$

(e)
$p \rightarrow q \vee r$
$\neg q \vee \neg r$
$\therefore \neg p \vee \neg r$

(f)
$p \rightarrow q$
$\neg p$
$\therefore \neg q$

(g)
$p \rightarrow q \vee \neg r$
$q \rightarrow p \wedge r$
$\therefore p \rightarrow r$

(c) q r p $p \rightarrow q$ $\neg q \vee r$
T T T T T
F T T T T
F F T T T ← × (c)

(d) p q $\neg r$ $p \wedge q \rightarrow \neg r$ $p \vee \neg q$ $\neg q \rightarrow p$
T F F T T T ← × (d)

(e) p q r $p \rightarrow q \vee r$ $\neg q \vee \neg r$ $\neg p \vee \neg r$
T F T T T F ← × (e).

(f) p q $p \rightarrow q$ $\neg p$ $\neg q$.
F T T T F ← × (f).

(g) p q r $p \rightarrow q \vee \neg r$ $q \rightarrow p \wedge r$ $p \rightarrow r$
T F F T T F ← × (g).

08. **T/F**

F (a) $p \wedge \neg p$ is an valid argument.
$\therefore p$

F (b) $p \vee \neg p$ is an valid argument.
$\therefore p$

T (c) q
$\therefore p \vee \neg p$ is an valid argument.

F (d) The floor is dry
$\therefore$ Today is sunny is an valid argument.

T (e) $(p \rightarrow q) \vee (q \rightarrow p)$ is a tautology.

T (f) $((p \rightarrow q) \wedge (q \rightarrow p)) \wedge \neg(p \leftrightarrow q)$ is a contradiction.

B C / G H.
✗ ✓ ✗ ✓
✗ ✓ ✗ ✓/✗

B C / G H
✓ ✓ ✗ ✓
✗

9 **SQ** A detective has interviewed four witnesses to a crime. From their stories, the detective has concluded that

(a) If the butler is telling the truth, then so is the cook.

(b) The cook and the gardener cannot both be telling the truth.

(c) The gardener and the handyman are not both lying.

(d) If the handyman is telling the truth then the cook is lying.

Cook must be lying.

Deduce who MUST be lying? (There may be more than one liar.) Show your steps.

10 **SQ** Eve was killed by two person out of 4 suspects Alice, Bob, Carol and Dave. Detective Conan has the following observation

(a) If Dave didn't kill Eve, then Alice is ~~not~~ a murderer implies Carol is a murderer.

(b) Bob is a murderer only if Alice killed Eve.

(c) If Dave killed Eve, then it is impossible for Bob to be a murderer.

(d) Carol is not a murderer if Bob didn't kill Eve.

Deduce who are the murderers.

C and D.