MAT3253 Homework 13

Due date: 30 Apr.

Question 1. (Bak&Newman Chapter 9 Ex.13) Let $\{a_1, a_2, \ldots, a_k\}$ be a set of positive integers and

$$R(z) = \frac{1}{(z^{a_1} - 1)(z^{a_2} - 1)\cdots(z^{a_k} - 1)}.$$

Find the coefficient c_{-k} in the Laurent expansion for $R(z)$ about the point $z = 1$.

Question 2. (Bak&Newman Chapter 10 Ex.1) Determine the singularities and associated residues of

(a). $\dfrac{1}{z^4 + z^2}$

(b). $\cot z$

(c). $\csc z$

(d). $\dfrac{\exp(1/z^2)}{z-1} z$

(e). $\dfrac{1}{z^2 + 3z + 2}$

(f). $\sin(1/z)$

(g). $ze^{3/z}$

(h). $\dfrac{1}{az^2 + bz + c}$, $a \neq 0$.

Question 3. (Bak&Newman Chapter 10 Ex.2) Use the Residue Theorem to evaluate

(a). $\displaystyle\int_{|z|=1} \cot z \, dz$

(b). $\displaystyle\int_{|z|=2} \frac{1}{(z-4)(z^3-1)} \, dz$

(c). $\displaystyle\int_{|z|=1} \sin(1/z) \, dz$

(d). $\displaystyle\int_{|z|=2} ze^{3/z} \, dz$

Question 4. (Bak&Newman Chapter 10 Ex.4) Show that

$$\int_{|z|=1} (z + 1/z)^{2m+1}\, dz = 2\pi i \binom{2m+1}{m},$$

for any nonnegative integer m.

Question 5. (Bak&Newman Chapter 10 Ex.5) [Complex Lagrange interpolation] Let C be a simple closed curve with positive orientation enclosing the distinct points $\omega_1, \omega_2, \ldots, \omega_n$ and let

$$p(\omega) = (\omega - \omega_1)(\omega - \omega_2)\cdots(\omega - \omega_n).$$

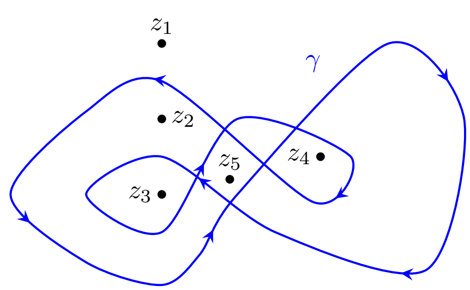
Suppose that $f(\omega)$ is analytic in a region that includes C. Show that

$$P(z) = \frac{1}{2\pi i}\int_C \frac{f(\omega)}{p(\omega)} \cdot \frac{p(\omega) - p(z)}{\omega - z}\, d\omega$$

is a polynomial of degree $n-1$, with $P(\omega_i) = f(\omega_i)$, for $i = 1, 2, \ldots, n$.

Question 6. (Bak&Newman Chapter 10 Ex.7) Suppose that f is entire and that $f(z)$ is real if and only if z is real. Use the Argument Principle to show that f can have at most one zero.

Question 7. Consider a closed curve γ show below.



Find the winding number of γ around the points z_1, z_2, z_3 , z_4 and z_5.