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Homework 9

Due by April 13, 2021

1. A small barber shop, operated by a single barber, has room for only two costumers. Potential costumers arrive at a Poisson rate of 3 per hour, and the successive serving times are independent exponential random variables of mean 1/4 hour.
   (a) What is the average number of customers in the shop?
   (b) What is the proportion of potential costumers that enters the shop?
   (c) If the barber could work 3 times as fast, how much more businesss would he do (in average)? (Hint: Compare potential costumers that enters the shop.)

2. A call center has two agents, John and Mary. It has four phone lines. An incoming call that finds all lines are occupied receives a busy signal and is rejected. An accepted call finds all agents busy waits in the queue. While waiting (listening to music), the caller may abandon without service. When a caller's waiting time exceeds her patience time, the caller abandons without service. The patience times are iid exponentially distributed with mean 4 minutes. John's processing times of calls are iid exponentially distributed with mean 2 minutes. Mary's processing times of calls are iid exponentially distributed with mean 1 minute. The arrival process is Poisson with rate $\lambda = 1$ call per minute. An arrival to an empty system always is always processed by John.
   (a) Describe a CTMC so it can be used to compute the probability in the next part. You need to specify all input data for this CTMC including the state space.
   (b) Using Matlab/Python to compute the probability that there are three calls in system at time $t = 2$ minutes given that the system is empty at $t = 0$.
   (c) Find the expected number of calls in system at time $t = 2$ given the system is empty at $t = 0$.
   (d) Long-run average utilization of each agent.
   (e) Long-run average queue size.
   (f) The rate at which calls are leaving agents.

3. Consider a call center that is staffed by K agents with three phone lines. Call arrivals follow a Poisson process with rate 2 per minute. An arriving call that finds all lines busy is lost. An arriving call that finds all agents busy will wait in a phone line until it enters into service. Call processing times are exponentially distributed with mean 1 minute.

(a) Find the throughput and average waiting time when $K = 1$.
(b) Find the throughput and average waiting time when $K = 2$.
(c) Find the throughput and average waiting time when $K = 3$.

4. For problem 2, please find

   (a) long-run fraction of calls gets busy signal.
   (b) average waiting time among those calls that enter into system.
   (c) fraction of calls that enter into system, but later abandon without service.

5. Consider a CTMC on state space S with generator G. Prove that a distribution π on S is a *stationary distribution* of the CTMC if and only if for any "cut" (partition) (A, A^c) with $A \subset S$:

$$\sum_{i \in A} \sum_{j \in A^c} \pi(i) G_{ij} = \sum_{i \in A^c} \sum_{j \in A} \pi(i) G_{ij}.$$