MAT 2002 Assignment 3

Deadline: Thursday 5:00 pm., 25th March

1. If $y_1(t), y_2(t)$ are the solutions of non-homogeneous equations

$$y^{(n)} + a_1(t)y^{(n-1)} + \cdots + a_n(t)y = r_1(t),$$
$$y^{(n)} + a_1(t)y^{(n-1)} + \cdots + a_n(t)y = r_2(t).$$

Proof that $y_1(t) + y_2(t)$ is the solution of

$$y^{(n)} + a_1(t)y^{(n-1)} + \cdots + a_n(t)y = r_1(t) + r_2(t).$$

2. Suppose (y_1, y_2), the foundamental set of solutions, is known, find the general solutions of the following non-homogeneous equations:

   (a) $y'' - y = \cos t, y_1 = e^t, y_2 = e^{-t}$;

   (b) $y'' + 4y = t \sin t, y_1 = \cos 2t, y_2 = \sin 2t$;

   (c) $y'' + \frac{t}{1-t}y' - \frac{1}{1-t}y = t - 1, y_1 = t, y_2 = e^t$.

3. Solve the following high-order differential equations.

   (a) $y^{(4)} - 5y'' + 4y = 0$;

   (b) $y''' - 3ay'' + 3a^2y' - a^3y = 0$;

   (c) $y^{(5)} - 4y''' = 0$.

4. Solve the following non-homogeneous equations with a proper guess of particular solution.

   (a) $y'' - 2y' + 2y = te^t \cos t$;

   (b) $y'' + 2y' + 5y = 4e^{-t} + 17 \sin 2t$;

   (c) $y'' + 9y = t \sin 3t$.

5. Solve the following non-homogeneous equations with variational parameters.

   (a) $y'' - y = \frac{2e^t}{e^t - 1}$.

   (b) $y'' + 4y' + 4y = \frac{e^{-2t}}{t^2}$;

   (c) $y'' - 2y' + y = e^t \sin t$;

   (d) $y''' - 3y'' + 3y' - y = \sqrt{t}e^t$.

6. (e^t, e^{-t}) is the fundamental set of solutions of $y'' - y = 0$. Try to find the standard fundamental set of solutions (i.e. $W(0) = 1$) which satisfies the initial condition

$$y(0) = 1, y'(0) = 0$$

and

$$y(0) = 0, y'(0) = 1.$$

And solve the equation under the initial condition

$$y(0) = y_0, y'(0) = y_0'.$$

7. Solve the following nonhomogeneous Euler equations with condition $t > 0$:

(a) $t^2y'' - 4ty' + 6y = \frac{42}{t^2}$;

(b) $t^2y'' - 2ty' + 2y = 5t^3 \cos t$;

(c) $t^3y''' + t^2y'' - 2ty' + 2y = t^3 \ln t$.