# MAT2006 Tutorial #10

**1.** (i) Assume $f(x)$ is differentiable on $(a,b)$ and $f'(x)$ is monotone there. Show that $f'(x)$ is continuous on $(a,b)$.

(ii) Assume $f$ is bounded and twice differentiable on $\mathbb{R}$. Show that there exists $\xi \in \mathbb{R}$ such that $f''(\xi) = 0$.

**2.** Assume that $f(x)$ is differentiable on $(0,1)$ and $f'(x)$ is bounded there.

(i) Show that the limit $\lim_{n\to\infty} f\left(\frac{1}{n}\right)$ exists.

(ii) Show that the limit $\lim_{x\to 0^+} f(x)$ exists.

**3.** If $f''(0)$ exists (and bounded), show that

$$f''(0) = \lim_{h\to 0} \frac{f(h) - 2f(0) + f(-h)}{h^2}.$$

**4.** (i) Assume $f$ is differentiable and unbounded in a bounded interval $(a,b)$. Show that $f'(x)$ is also unbounded.

(ii) Show that the converse of the above proposition is not true – assume $f$ is differentiable and $f'$ unbounded in $(a,b)$, $f$ is not necessarily unbounded.

**5.** (i) Let $f$ be uniformly continuous on all of $\mathbb{R}$, and define a sequence of functions by $f_n(x) = f(x + \frac{1}{n})$. Show that $f_n \to f$ uniformly.

(ii) Give an example to show that this proposition fails if $f$ is only assumed to be continuous and not uniformly continuous on $\mathbb{R}$.

— End —