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1. You are standing at 0. Each step you move one step to right (+1) or left (-1). Count the number of paths with following constraints.
   (a) Total number of steps is 6. $N=2^6$
   (b) Total number of steps is 8. After the 8 steps you go back to 0. $N=\frac{2^8}{4!4!}$
   (c) Total number of steps is 8. After the 8 steps you stand at 1. $N=0$.

2. Count the number of integer solutions with following constraints.
   (a) $x_1+x_2+x_3+x_4=20, x_1\geq 0, x_2\geq 0, x_3\geq 0, x_4\geq 0$ $N=\binom{23}{3}$
   (b) $x_1+x_2+x_3+x_4=20, 5\geq x_1\geq 0, x_2\geq 0, x_3\geq 0, x_4\geq 0$ $N=\binom{22}{3}+\binom{21}{3}+\binom{18}{3}+\binom{17}{3}+\binom{16}{3}+\binom{15}{3}$

3. What is the number of 'Hello's printed by the pseudocode below? $1\leq k<j<i\leq n$ $N=\binom{n}{3}$

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for i from 1 to n
  for j from 1 to i - 1
    for k from 1 to j - 1
      print 'Hello'
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4. 9 red balloons and 6 blue balloons are distributed to 4 children, how many distributions are possible under the following situations?
   (a) No constraints?
   (b) Every child must receive a balloon of red color?

5. How many possibilities are there for 8 non-attacking rooks on an 8-by-8 chessboard? How about having 8 different color rooks? How about having 1 red (R) rook, 3 blue (B) rooks , and 4 yellow (Y) rooks.

6. Derive the r-combination of n distinct objects with repetition.

7. Consider the word $MILLIMICRON$ in this question. How many distinguishable ways can the letters of the word be arranged in order? $N=\frac{11!}{2!2!3!}$

8. Consider the expansion of $(1+x-y)^{37}$. What is the coefficient of $x^{15}y^{13}$? $C=(-1)^{13}\times\frac{37!}{15!13!9!}$

9. n boys and m girls line up in a row. Count the number of ways for lining up with following constraints. Different orders among boys/girls are considered different ways.

(a) $n = 10, m = 15$

(b) $n = 5, m = 20$. There are no adjacent boys.