MAT3253 Homework 6

Due date: 12 Mar.

Question 1. (Bak&Newman Ex.2.9) Find the radius of convergence of the following power series (a). $\sum_{n=0}^{\infty} z^{n!}$; (b). $\sum_{n=0}^{\infty}(n+2^n)z^n$.

Question 2. (Bak&Newman Ex.2.10) Suppose $\sum_n c_n z^n$ has radius of convergence R. Find the radius of convergence of
(a). $\sum_{n=0}^{\infty} n^p c_n z^n$; (b). $\sum_{n=0}^{\infty} |c_n| z^n$;
(c). $\sum_{n=0}^{\infty} c_n^2 z^n$.

Question 3. (Bak&Newman Ex.2.17) Suppose $\sum_{k=0}^{\infty} a_k = A$ and $\sum_{k=0}^{\infty} b_k = B$. Suppose further that each of the series is absolutely convergent. Show that if

$$c_k := \sum_{j=0}^{k} a_j b_{k-j}$$

then

$$\sum_{k=0}^{\infty} c_k = AB.$$

Outline: Use the fact that $\sum |a_k|$ and $\sum |b_k|$ converge to show that $\sum d_k$ converges, where

$$d_k := \sum_{j=0}^{k} |a_j||b_{k-j}|.$$

In particular

$$d_{n+1} + d_{n+2} + \cdots \to 0 \text{ as } n \to \infty.$$

Note then that if

$$\begin{aligned} A_n &:= a_0 + a_1 + \cdots + a_n \\ B_n &:= b_0 + b_1 + \cdots + b_n \\ C_n &:= c_0 + c_1 + \cdots + c_n, \end{aligned}$$

$A_n B_n = C_n + R_n$, where $|R_n| \le d_{n+1} + d_{n+2} + \cdots + d_{2n}$, and the result follows by letting $n \to \infty$.

Question 4. (Bak&Newman Ex.3.15) Verify the identities for complex number z:

(a). $\sin 2z = 2\sin z \cos z$,

(c). $(\sin z)' = \cos z$.

Question 5. (Bak&Newman Ex.3.20) Show that

$$\sin(x + iy) = \sin x \cosh y + i \cos x \sinh y.$$

Question 6. Solve

$$\sin(z) = i.$$

(You may check your answer by substituting it into the identity in Question 5.)