# Exercise 2

- There will be five exercises in this semester, which will account for 10% of the grade of this course.
- Exercise 2 includes 5 questions. Questions 1-3 are True/False (T/F) questions requiring explanations. Questions 4-5 are problem-solving questions requiring detailed solutions.
- Please show the details of your work leading to the solutions.
- Submit a pdf file of your answers on Blackboard by 11 p.m. Monday, Oct 26, 2020.

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*True/False questions part:*

## Question 1

If a discrete random variable $X$ has a symmetric distribution about a parameter $\theta$ with $\Pr(a \le X \le b) = 1$, $\Pr(X = a) > 0$ and $\Pr(X = b) > 0$, then

(a) $\theta = \mathrm{E}[X] = (a+b)/2$;

(b) $\Pr(X \le x) = \Pr(X \ge 2\theta - x)$ for all $x \in \mathbb{R} = (-\infty, \infty)$;

(c) $F(\theta + x) + F(\theta - x) = 1$ for all $x \in \mathbb{R}$, where $F(x) = \Pr(X \le x)$.

## Question 2

Let $T^+$ be the Wilcoxon signed rank statistic from a random sample $X_1, \ldots, X_8$ with median $\theta$, and $R_i$ the rank of $X_i$ for $T^+$. The following statements are true:

(a) $\Pr(T^+ = 7) = 5/256$ under $H_0: \theta = 0$.

(b) The distribution of $T^+$ is symmetric about 36.

(c) If $X_i < 0 < X_j$, then $R_i > R_j \Leftrightarrow X_i + X_j < 0$.

[Question 3-4 start from next page]

## Question 3

Based on two independent samples, if the Wilcoxon rank sum test rejects $\Delta = 0$ and the Ansari-Bradley test finds little evidence against $\gamma^2 = 1$, then:

(a) We can reasonably conclude that the two samples have a significant difference in location, but not in dispersion.

(b) The difference in location is justified, but not the equal dispersion.

The results of both tests are questionable and not well justified.

*Problem solving questions part:*

## Question 4

Let $X_1$ and $X_2$ be two independent continuous random variables, $T^+ = R_1\psi_1 + R_2\psi_2$ is the Wilcoxon signed rank test statistic, and $S = \psi_1 + 2\psi_2$ where $\psi_i = I_{\{X_i > 0\}}, i = 1, 2.$

(a) If $X_1 \sim U([-1,1])$ and $X_2 \sim U([-2,2])$, show that $\Pr(T^+ = i) = \Pr(S = i), i \in \{0,1,2,3\}$.

(b) Suppose that $X_1$ and $X_2$ have a common density $f(x) = 0.5e^{-x} I_{\{x \geq -\ln 2\}}$. Show that $X_1$ and $X_2$ have median 0, but $\Pr(T^+ = 1) \neq \Pr(S = 1)$.

[Question 5 starts from next page]

## Question 5

The following table is a copy of Table 4.4 on page 135 of the textbook. The data in the table are explained in Problem 5 on page 134.

**Table 4.4** Seconds Spent in Room after Witnessing Violence

| Olympics watchers $X$ | *Karate Kid* watchers $Y$ |
|---|---|
| 12 | 37 |
| 44 | 39 |
| 34 | 30 |
| 14 | 7 |
| 9 | 13 |
| 19 | 139 |
| 156 | 45 |
| 23 | 25 |
| 13 | 16 |
| 11 | 146 |
| 47 | 94 |
| 26 | 16 |
| 14 | 23 |
| 33 | 1 |
| 15 | 290 |
| 62 | 169 |
| 5 | 62 |
| 8 | 145 |
| 0 | 36 |
| 154 | 20 |
| 146 | 13 |

The question of interest is whether there exists a significant difference in variability (dispersion) between the two samples labelled by $X$ and $Y$.

(a) Find the scores $r_1,\ldots,r_N$ assigned to all $X_1,\ldots,X_m,Y_1,\ldots,Y_n$ with average scores for ties and calculate the Ansari-Bradley rank test statistic $C$ for dispersion.

(b) Assume the location-scale parameter model with equal location parameter for the data. Test $H_0:\gamma^2=1$ $(\text{Var}(X)=\text{Var}(Y))$ against $H_1:\gamma^2\neq 1$ $(\text{Var}(X)\neq\text{Var}(Y))$ by the large-sample approximate $p$-value of the Ansari-Bradley rank test.