Exercise 1

- There will be five exercises in this semester, which will account for 10% of the grades of this course.
- Assignment 1 includes 3 questions, all to be attempted.
- Show the details of your work leading to the solutions.
- Submit your solutions via Blackboard before 5 p.m. on Sep. 30 2020.

Question 1

Let $X_1,\ldots,X_n$ be independent continuous random variables with $\Pr(X_i<\theta)=0.5$ for a real number θ, $i=1,\ldots,n$. Based on the data observed from $X_1,\ldots,X_n$, please evaluate each of the following statements (T/F) and briefly explain the position you take:

(a) the Wilcoxon signed-rank test is better than the sign test for $H_0:\theta=0$.

(b) if $\Pr(X_i<\theta-x)+\Pr(X_i<\theta+x)=1$ for all $x\in(-\infty,\infty)$, then the Wilcoxon signed-rank statistic has a symmetric distribution.

(c) a nonparametric confidence interval of θ can be obtained from the order statistics of the sample $X_1,\ldots,X_n$.

Question 2

Given a sample of data $(X_1,\ldots,X_8)=(-11,6,-20,-9,-18,-22,16,-28)$, determine if there is sufficient evidence for median $\theta<0$ at the 5% level of significance by the following nonparametric tests:

(a) The sign test.

(b) The Wilcoxon signed rank test.

Question 3

The table below provides paired data from continuous random variables (X_i,Y_i) and the differences $Z_i=Y_i-X_i,\ \ i=1,\ldots,14.$

i	X_i	Y_i	$Z_i=Y_i-X_i$
1	270	525	255
2	150	570	420
3	270	190	–80
4	420	395	–25
5	202	370	168
6	255	210	–45
7	165	490	325
8	220	250	30
9	305	360	55
10	210	285	75
11	240	630	390
12	300	385	85
13	300	195	–105
14	70	295	225

Let θ denote the median of Z_i such that $\Pr(Z_i>\theta)=0.5$.

(a) Find the exact *p*-value of testing $H_0:\theta=0$ against $H_1:\theta>0$ by the sign test, and calculate its large-sample approximation. To get a closer approximation, use the *continuity correction*:

$$\Pr(B\ge b)=\Pr(B>b-0.5),\ \ b\in\{0,1,2,\ldots,n\},\text{ for }\ B\sim Bin(n,p).$$

(b) Estimate the median θ and obtain its exact and approximate confidence intervals with at least 95% confidence level based on the sign statistic.

(c) Calculate the approximate *p*-value of testing $H_0 : \theta = 0$ against $H_1 : \theta > 0$ by the Wilcoxon signed rank test.

(d) Estimate the median θ and obtain its approximate 95% confidence interval based on the Wilcoxon signed ranks.

(e) Compare and comment on the difference between the *p*-values of the sign test obtained in part (a) and the Wilcoxon signed rank test in part (c).

(f) Compare and comment on the difference between the confidence intervals of θ based on the sign statistic in part (b) and the Wilcoxon signed ranks in part (d).