The Chinese University of Hong Kong, Shenzhen
SDS · School of Data Science
Andre Milzarek · Summer Semester 2020

MAT 3007 – Optimization

Exercise Sheet 4

Assignment A4.1 (Sensitivity in Linear Programming): (approx. *25* points)
Consider the following linear program:

$$\begin{array}{lll} \text{maximize} & 3x_1 + 4x_2 + 3x_3 + 6x_4 & \\ \text{subject to} & 2x_1 + x_2 - x_3 + x_4 & \geq 12 \\ & x_1 + x_2 + x_3 + x_4 & = 8 \\ & -x_2 + 2x_3 + x_4 & \leq 10 \\ & x_1, x_2, x_3, x_4 & \geq 0. \end{array} \tag{1}$$

a) Rewrite problem (1) in standard form and find an initial basic feasible solution. Apply the simplex method to solve the reformulated problem.

b) Verify that the solution obtained by the simplex method is the unique optimal solution of the problem.

c) Derive the dual problem of the linear program (1) and calculate a dual solution. Investigate whether the dual solution is unique.

d) Do the optimal solution and the objective function value change if we

- decrease the objective function coefficient for x_3 to 1?
- increase the objective function coefficient for x_3 to 12?
- decrease the objective function coefficient for x_1 to 1?
- increase the objective function coefficient for x_1 to 7?

e) Find the possible range for adjusting the coefficient 8 of the second constraint such that the current basis is kept optimal.

Assignment A4.2 (Insurance Company): (approx. *25* points)
An insurance company is introducing three products: special risk insurance, mortgage insurance, and long-term care insurance. The expected profit and the work requirements are summarized in the following tables.

Profit per Unit	
Special Risk Insurance	3300 RMB
Mortgage Insurance	2000 RMB
Long-Term Care Insurance	5000 RMB

Department	Working Hours per Unit			Working Hours Available
	Special Risk	Mortgage	Long-Term Care	
Underwriting	2	1	1	250
Administration	3	1	2	150
Claims	1	2	4	160

The management team wants to establish sales quotas for each product to maximize the total expected profit.

a) Formulate this problem as a linear optimization problem. Specify the decision variables, the objective function, and constraints. Write a `MATLAB` program to solve the problem.

b) After solving the problem, the final simplex tableau (for the standard form) is given as below (the variables correspond to the natural order as in the description of the problem):

B	0	500	0	0	820	840	257400
4	0	$\frac{1}{2}$	0	1	$-\frac{11}{30}$	$\frac{1}{10}$	161
1	1	0	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	28
3	0	$\frac{1}{2}$	1	0	$-\frac{1}{10}$	$\frac{3}{10}$	33

Derive the dual solution corresponding to the services of the three departments. Use the dual solution and its interpretation as shadow prices to explain why the mortgage insurance is not sold.

c) Find the possible range of "working hours available" for the service "claims" that keeps the current basis optimal.

d) Find the range for the expected profit of the special risk insurance such that the current basis remains optimal.

Sheet 4 is due on **Jul, 12th**. Submit your solutions before **Jul, 12th**, **11:00 am**.