STA 3007 Exercise 4 118010350.

Q1. (a). False. The block effects in RCBD can be tested based on the ranks of the data within each block, rather than treatment

(b). True. Since the data are BIBD, then $S=5$, $p=10$, and $N=ns=kp$. $N=5\cdot n=10k$, thus $n=2k$. and $p(S-1)=\lambda(k-1)$, $10(5-1)=40=\lambda(k-1)$. thus there exists an interger m s.t. $m(k-1)=40$.

(c). False. For balanced incomplete data, the ranks of missing data do not affect the test for equal treatment effects against general alternatives, but for unbalanced incomplete data, the ranks of missing data do affect. must be defined by some ways

(d). False. If the data only has 2 treatments, then the data is complete, that is $S_i=2$, $i=1,\cdots,n$. $A_j=\sum_{i=1}^{n}\sqrt{\frac{12}{S_i+1}}\left(r_{ij}-\frac{S_i+1}{2}\right)$
$=\sum_{i=1}^{n}2\left(r_{ij}-\frac{3}{2}\right)=2R_j-3n$, $j=1,\cdots k-1$.
If the data has more than 2 treatnents, the the data is incomplete, that is $S_i\geq 2$, $i=1,\cdots,n$. A_j is not equal to $2R_j-3n$ definitely for $j=1,\cdots k-1$.

Q2. Proof (a) Since the data is BIBD, then $S_i=S$, $i=1,\cdots,n$.

$$E(r_{ij})=\frac{S+1}{2}\Rightarrow E(R_j)=\sum_{i=1}^{n}E(r_{ij})=\frac{P(S+1)}{2}. \quad ①$$

Since $r_{ij},\cdots r_{nj}$ are in different blocks, they are independent, hence $var(r_{ij})=\frac{(S+1)(S-1)}{12}\Rightarrow var(R_j)=\sum_{i=1}^{n}var(r_{ij})=\frac{P(S+1)(S-1)}{12}$, ②

By ① and ②, $E\left[\left(R_j-\frac{P(S+1)}{2}\right)^2\right]=E\left[(R_j-E(R_j))^2\right]$
$=var(R_j)=\frac{P(S+1)(S-1)}{12}$

Since $D=\frac{12}{\lambda k(S+1)}\sum_{j=1}^{k}\left(R_j-\frac{P(S+1)}{2}\right)^2$, then under H_0,

$$E(D)=\frac{12}{\lambda k(S+1)}\cdot\sum_{j=1}^{k}E\left[\left(R_j-\frac{P(S+1)}{2}\right)^2\right]$$
$$=\frac{12}{\lambda k(S+1)}\cdot k\cdot\frac{P(S+1)(S-1)}{12}$$
$$=\frac{P(S-1)}{\lambda}=k-1. \Rightarrow E(D)=k-1$$

(b). Since the data is BIBD, then $S_i=S$, $i=1,\cdots n$.

For $u \neq v$, Since $cov(r_{iu}, r_{iv}) = -\frac{s+1}{12} I\{c_{iu} = c_{iv} = 1\}$,

and $cov(r_{iu}, r_{tv}) = 0$. for $i \neq t$, then.

$$cov(R_u, R_v) = \sum_{i=1}^{n} \sum_{t=1}^{n} cov(r_{iu}, r_{tv}) = \sum_{i=1}^{n} cov(r_{iu}, r_{iv}) = -\frac{\lambda(s+1)}{12} \quad ③$$

under H_0, by ① and ③, the covariance matrix $Var(R)$ is

$$Var_0(R) = \frac{\lambda(s+1)}{12} \begin{bmatrix} k-1 & -1 & \cdots & -1 \\ -1 & k-1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & k-1 \end{bmatrix} = \frac{\lambda(s+1)}{12} (k I_{k-1} - 1 \cdot 1^T).$$

for $\frac{\lambda(s+1)}{12} \cdot (k-1) = \frac{\lambda(s+1)}{12} \cdot \frac{p(s-1)}{\lambda} = \frac{p(s+1)(s-1)}{12} = var(R_j)$.

$\frac{\lambda(s+1)}{12} \cdot (-1) = -\frac{\lambda(s+1)}{12} = cov(R_u, R_v)$. $u \neq v$.

Q3. (a)

	Treatment j.				
Block i	1	2	3	4	5.
1	16 (1)	18 (2)	\	32 (3)	\
2	19 (1)	\	\	46 (3)	45 (2)
3	\	26 (1)	29 (2)	\	61 (3)
4	\	\	21 (1)	35 (2)	55 (3)
5	\	19 (1)	\	47 (2)	48 (3)
6	20 (1)	\	33 (3)	31 (2)	\
7	13 (1.5)	13 (1.5)	34 (3)	\	\
8	21 (1)	\	30 (2)	\	52 (3)
9	24 (2)	10 (1)	\	\	50 (3)
10.	\	24 (1)	31 (2)	37 (3).	\
	$R_1 = 7.5$	$R_2 = 7.5$	$R_3 = 13$	$R_4 = 15$	$R_5 = 17$.

H_0: $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5$, against H_1: $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5$ are not all equal

Since $k = 5$, $p = 6$, $s = 3$, $\lambda = 3$, then the test statistic

$$D = \frac{12}{\lambda k(s+1)} \sum_{j=1}^{k} \left(R_j - \frac{p(s+1)}{2}\right)^2 = \frac{12}{3 \cdot 5 \cdot 4} \sum_{j=1}^{5} R_j^2 - \frac{3 \cdot 4 \cdot 6^2}{3} = 15.1$$

we can get $\chi^2_{k-1, \alpha} = \chi^2_{4, 0.05} = 9.4877$.

Since $D = 15.1 > 9.4877$, then there is strong evidence to indicate significant difference between treatments. So we reject H_0: $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5$ and accept H_1 at 5% level.

(b). The Skilling-Mack two-sided all-treatment multiple comparison for BIBD is: for each pair (T_u, T_v), with $u<v$.

Decide $T_u \neq T_v$ if $|R_u - R_v| \geq q_\alpha \sqrt{\frac{(s+1)(ps-p+\lambda)}{12}}$

Otherwise accept $T_u = T_v$.

Since $q_\alpha \cdot \sqrt{\frac{(s+1)(ps-p+\lambda)}{12}} = 3.5 \cdot \sqrt{\frac{4\cdot(6\cdot3-6+3)}{12}} = 7.8262$.

then $|R_1 - R_2| = 0 < 7.8262$, $\Rightarrow$ Accept $T_1 = T_2$. at 10% level

$|R_1 - R_3| = 5.5 < 7.8262 \Rightarrow$ Accept $T_1 = T_3$. at 10% level

$|R_1 - R_4| = 7.5 < 7.8262$. $\Rightarrow$ Accept $T_1 = T_4$ at 10% level.

$|R_1 - R_5| = 9.5 > 7.8262 \Rightarrow$ Decide $T_1 \neq T_5$ at 10% level.

$|R_2 - R_3| = 5.5 < 7.8262 \Rightarrow$ Accept $T_2 = T_3$ at 10% level.

$|R_2 - R_4| = 7.5 < 7.8262 \Rightarrow$ Accept $T_2 = T_4$. at 10% level.

$|R_2 - R_5| = 9.5 > 7.8262 \Rightarrow$ Decide $T_2 \neq T_5$ at 10% level.

$|R_3 - R_4| = 2 < 7.8262 \Rightarrow$ Accept $T_3 = T_4$ at 10% level

$|R_3 - R_5| = 4 < 7.8262 \Rightarrow$ Accept $T_3 = T_5$ at 10% level

$|R_4 - R_5| = 2 < 7.8262 \Rightarrow$ Accept $T_4 = T_5$ at 10% level.