

Claim. *The marrying procedure is boy optimal.*

Proof. Suppose that the marrying procedure is not boy optimal. Then there exists some boy not matched with his best valid girl.

Let x be the first such boy during the procedure. Then x must be rejected by some valid girl, say the first one is A . Then A is matched with another boy y .

Since x is the first boy being rejected by his valid girls, it follows that y is not yet rejected by any other of his valid girls. So y prefers A rather than any other of his valid girls. In the meantime, A rejects x , so A prefers y rather than x .

Consider that A is valid for x , so (x, A) should be a couple in another stable matching. However, both y, A have incentive to leave their partners in this stable matching, so (y, A) is an unstable pair in this matching, which is a contradiction. \square

Claim. *The marrying procedure is girl pessimal.*

Proof. Suppose that the marrying procedure is not girl pessimal. Then there exists a boy x married to a girl A , while A has a worse valid boy y .

This means that A prefers x rather than y . On the other hand, since marrying procedure is boy optimal, A is the best valid partner for x . That is, x prefers A rather than any other of his valid girls.

Note that y is valid for A , so (y, A) should be a couple in another stable matching. However, both x, A have incentive to leave their partners in this stable matching, so (x, A) is an unstable pair in this matching, which is a contradiction. \square