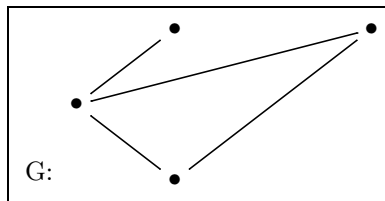


## DEGREE SEQUENCE

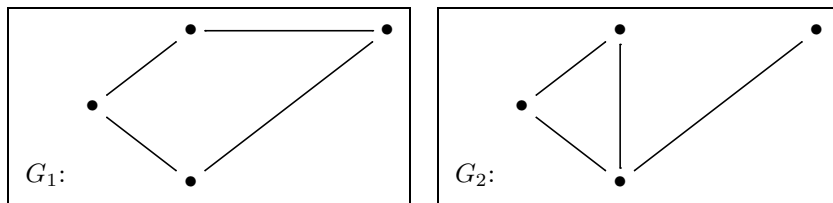
The *degree sequence* of a graph is the sequence of the degrees of the vertices, with these numbers put in ascending order, with repetitions as needed. Thus



has degree sequence

$(1, 2, 2, 3)$ .

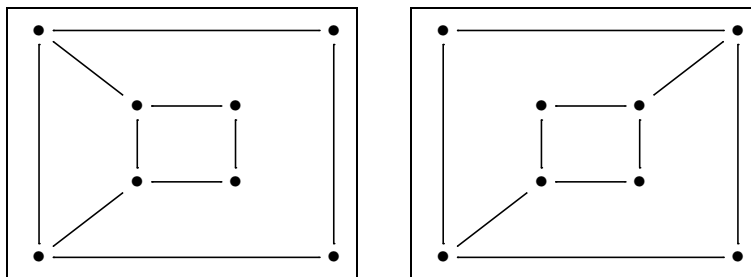
Two graphs with different degree sequences cannot be isomorphic. For example, these two graphs are not isomorphic,



since one has four vertices of degree 2 and the other has just two. Their degree sequences are

$(2, 2, 2, 2)$  and  $(1, 2, 2, 3)$ .

It is common for even simple connected graphs to have the same degree sequences and yet be non-isomorphic. For example, we saw in class that these



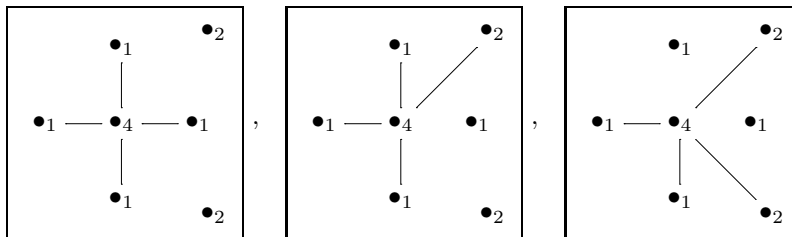
are not isomorphic, but they both have the degree sequence

$(2, 2, 2, 2, 3, 3, 3, 3)$ .

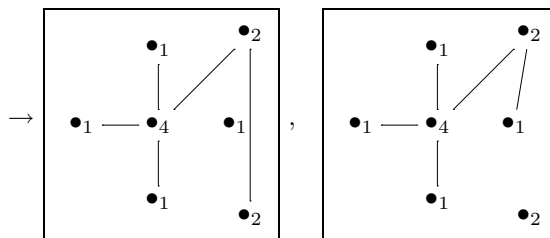
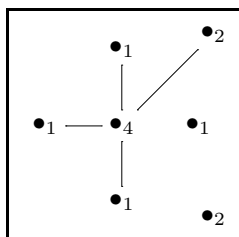
**Example 0.1.** Up to isomorphism, find all simple graphs with degree sequence  $(1, 1, 1, 1, 2, 2, 4)$ .

**Solution:**

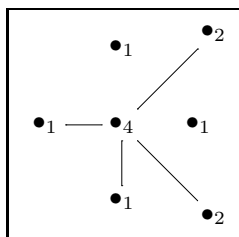
The degree 4 vertex must be adjacent to 0, 1 or 2 of the vertices of degree 2, so we have three cases to consider:

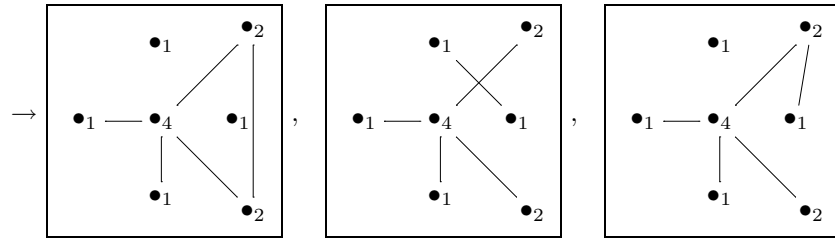


In the first case, we need to add two edges between the same two vertices, which is not allowed. In the second case, we can add an edge in two ways

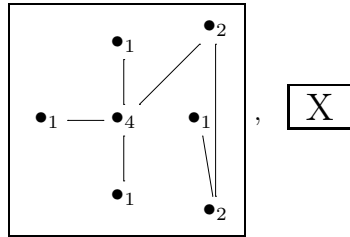


and there are three ways in the third case:

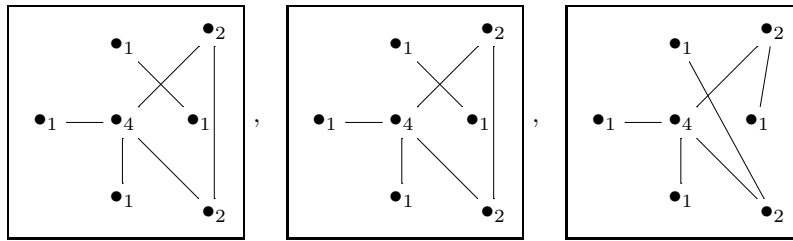




In all five cases, the last edge is forced, or not possible, so we get



and



Removing duplicates, and making the diagrams nicer, we get our answer as:

