Homework 6
Due: March 16, 2021

1. Consider a jewelry store that only sells diamond rings and operates as follows. Each week, the store is open from Monday-Friday. The weekly (5-day) demand is random, and has distribution

$$D = \begin{cases} 0, & \text{w.p. } 1/6 \\ 1, & \text{w.p. } 1/6 \\ 2, & \text{w.p. } 1/6 \\ 3, & \text{w.p. } 1/6 \\ 4, & \text{w.p. } 1/6 \\ 5, & \text{w.p. } 1/6. \end{cases}$$

Assume that each ring sells for \$100, and any rings unsold by the end of Friday require cleaning on Saturday, which costs \$10 per ring. After a week's worth of sales, the store owner reviews inventory on Saturday morning, and decides how many rings to order. The ring supplier offers two shipping options: standard or express shipping. Standard shipping costs \$15 per ring, and the order arrives on the following Friday evening (a week after it is placed) after the store closes. Express shipping costs \$35 per ring, but the order arrives on the evening of the next day (Sunday).

Consider the following ordering policy: each Saturday morning, the store owner looks at the inventory and sees x rings. She then orders $(3 - x)^+$ rings via standard shipping, and then places an express order to ensure she starts out on Monday with 5 rings (if she has more than 5 rings on Saturday morning, no order is placed). Let X_n be the number of rings in inventory on the morning of the nth Saturday, $n = 0, 1, 2, 3 \ldots$

   (a) Compute the stationary distribution of this DTMC.
   (b) Calculate the long-run weekly profit of the store.

2. Consider the following transition matrix:

$$P = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.7 & 0 & 0.3 \\ 0.8 & 0 & 0.2 & 0 \end{pmatrix}$$

(a) Is the Markov chain periodic? Give the period of each state.

(b) Is $(\pi_1, \pi_2, \pi_3, \pi_4) = (33/96, 27/96, 15/96, 21/96)$ the stationary distribution of the Markov Chain?

(c) Is $P_{11}^{100} = \pi_1$? Is $P_{11}^{101} = \pi_1$? Give an expression for π_1 in terms of P_{11}^{100} and P_{11}^{101}.

3. Suppose that you type "healthy food store" into a search engine. Each of the five web pages, A, B, C, D, E, contains the relevant information on the subject. Suppose that

> A has links to B and C,
> B has links to A and D,
> C has link to D and E,
> D has link to A, B, and C,
> E has link to B.

Compute the "PageRank" of these five web pages.