

Cook's measure:

$$D_i = \frac{(\hat{\beta}_{(i)} - \hat{\beta})^T X^T X (\hat{\beta}_{(i)} - \hat{\beta})}{p \cdot MS_{Res}} \quad (1)$$

$$= \frac{1}{p} \cdot r_i^2 \cdot \frac{h_{ii}}{1-h_{ii}} \quad (2)$$

Recall:

$$\star X_{(i)}^T X_{(i)} = X^T X - \underline{x}_i \cdot \underline{x}_i^T \quad (3)$$

\underline{x}_i^T is the i -th row of $X_{n \times p}$
 $\rightarrow 1 \times p$
 $\underline{x}_i = (\underline{x}_i^T)^T$ is the i -th column of $X_{n \times p}$
 $\rightarrow p \times 1$

★ Apply matrix inversion lemma:

$$\left(X_{(i)}^T X_{(i)} \right)^{-1} = \left(X^T X \right)^{-1} + \frac{\left(X^T X \right)^{-1} x_i x_i^T \left(X^T X \right)^{-1}}{1 - h_{ii}}$$

$$h_{ii} = x_i^T \left(X^T X \right)^{-1} x_i \quad \text{scalar}$$

$$\star \quad X_{(i)}^T \begin{pmatrix} y \\ -y_{(i)} \end{pmatrix} = \underline{X^T y} - \underline{x_i y_i} \quad \text{outer-product view.} \quad (5)$$

$$\underline{\hat{\beta}_{(i)}} = \underline{\hat{\beta}} \left[- \left(X^T X \right)^{-1} x_i y_i + \frac{\left(X^T X \right)^{-1} x_i x_i^T \left(X^T X \right)^{-1} \left(X^T y - x_i y_i \right)}{1 - h_{ii}} \right]$$

$$\Rightarrow \underline{\hat{\beta}} - \underline{\hat{\beta}_{(i)}} = \frac{\left(X^T X \right)^{-1} x_i e_i}{1 - h_{ii}} \quad (6)$$

$$X(\hat{\beta} - \hat{\beta}_{(i)}) = \frac{X(X^T X)^{-1} \underline{x}_i e_i}{1 - h_{ii}} \quad (7)$$

$$\begin{aligned} & \frac{(\hat{\beta} - \hat{\beta}_{(i)})^T X^T X (\hat{\beta} - \hat{\beta}_{(i)})}{=} \\ & \frac{\underline{x}_i^T (X^T X)^{-1} X^T X (X^T X)^{-1} \underline{x}_i \cdot e_i^2}{(1 - h_{ii})^2} \end{aligned}$$

$$= \frac{\underline{x}_i^T (X^T X)^{-1} \underline{x}_i \cdot e_i^2}{(1 - h_{ii})^2}$$

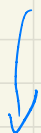
$$= \frac{h_{ii}}{(1 - h_{ii})^2} \cdot e_i^2 \quad (8)$$

$$\text{Finally, } D_i = \frac{h_{ii}}{(1 - h_{ii})^2} \cdot e_i^2 \cdot \frac{1}{p \cdot MS_{res}}$$

$$(8-b) \quad r_i = \left(\frac{e_i}{\sqrt{MS_{res} (1 - h_{ii})}} \right)^2 \cdot \frac{h_{ii}}{1 - h_{ii}} \cdot \frac{1}{p}$$

DF FITs measure :

$$\text{DF FIT}_i = \frac{\hat{y}_i - \hat{y}_{(i)}}{\sqrt{S_{(i)}^2 h_{ii}}} \quad (9)$$



$$= t_i \sqrt{\frac{h_{ii}}{1-h_{ii}}} \quad (10)$$

$$\hat{\beta} - \hat{\beta}_{(i)} = \frac{(X^T X)^{-1} x_i e_i}{1-h_{ii}}$$

$$\hat{y}_i - \hat{y}_{(i)} = x_i^T (\hat{\beta} - \hat{\beta}_{(i)})$$

$$= \frac{x_i^T (X^T X)^{-1} x_i}{1-h_{ii}} \cdot e_i = \frac{h_{ii}}{1-h_{ii}} e_i$$

(11)

Inserting ⑪ to ⑨, we get

$$\frac{\hat{y}_i - \hat{y}_{(i)}}{\sqrt{S_{(i)}^2 h_{ii}}} = \frac{h_{ii} \cdot e_i}{(1-h_{ii}) \sqrt{S_{(i)}^2 h_{ii}}}$$

$$= \frac{e_i}{\sqrt{S_{(i)}^2 (1-h_{ii})}} \cdot \frac{\sqrt{h_{ii}}}{\sqrt{1-h_{ii}}}$$

R-studentized
residual ←

$$= t_i \cdot \sqrt{\frac{h_{ii}}{1-h_{ii}}}$$

⑫

Connection :

a. Ignore $\frac{1}{p}$ in (8-b), then

b. $D_i \approx (\text{DFITS}_i)^2$, if $t_i \approx r_i$
