1. Are the following sequences valid degree sequences of simple graphs? For each sequence, do one of the following: Construct a graph with the given degree sequence, or Show that the sequence cannot be a degree sequence of any simple graph.

   (a) (3,3,3,1)

   (b) (4,4,4,2,2)

   (c) (4,3,2,2,1)

   (d) (3,3,3,3,3,3)

2. For which values of m and n does the following graph contain an Eulerian Cycle?

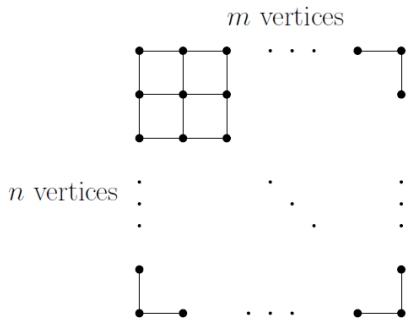

Figure 1: Question 2

3. For stable marriage problem, show an example that women can lie to get a better partner.

4. In the school of Athens, there are mentors and students. Each day, a mentor can meet at most one student and a student can meet at most one mentor. We know that each mentor supervises exactly d students and each student has exactly d mentors. We would like to find a schedule so that every student can meet with all his/her mentors in a minimum number of days.

   (a) Prove that the edges of a d-regular bipartite graph can be partitioned into d perfect matchings.

   (b) Show that there exists a meeting schedule such that every student can meet all his/her mentors in d days.

5. What is the chromatic number of the following graph?

   (a) K_n, complete graph with n vertices.

   (b) P_n, a path with n vertices.

   (c) C_n, a circle with n vertices.

6. A company manufactures n chemicals $C_1; C_2; ...; C_n$. Certain pairs of these chemicals are incompatible and would cause explosions if brought into contact with each other. As a precautionary measure the company wishes to partition its warehouse into compartments, and store incompatible chemicals in different compartments. What is the least number of compartments into which the warehouse should be partitioned? Model this problem as a graph coloring problem and explain your answer.

7. Suppose you run a day care for an office building and there are seven children $A; B; C; D; E; F; G$. You need to assign a locker where each child's parent can put the child's food. The children come and leave so they are not all there at the same time. You have 1 hour time slots starting 7:00 am to 12:00 am. A star in the following table means a child is present at that time. What is the minimum number of lockers you need to prepare? What's your plan to assign the lockers? Show your steps.

	A	B	C	D	E	F	G
7:00	*			*	*		
8:00	*	*	*				
9:00	*		*	*		*	
10:00	*		*			*	*
11:00	*					*	*
12:00	*				*		

Figure 2: Question 7

8. The following tours of garbage trucks in Boston are being considered (behind Mayor Meninos back).

   - Tour 1: The Prudential, the Garden, and the Charlestown Shipyard
   - Tour 2: Back Bay, the Charlestown Shipyard, the Prudential, and the Museum of Fine Arts
   - Tour 3: Symphony Hall, the Old North Church, and MIT
   - Tour 4: Quincy Market and the Charlestown Shipyard

- Tour 5: Quincy Market, Fanueil Hall and the Prudential
- Tour 6: Symphony Hall, Fenway Park and the Old North Church
- Tour 7: Fanueil Hall, Harvard and the Old North Church

Assuming the sanitation workers refuse to work more than three days a week, can these tours be partitioned so that no site is visited more than once on a given day?

9. Six reporters Asif (A), Becky (B), Chris (C), David (D), Emma (E) and Fred (F), are to be assigned to six news stories Business (1), Crime (2), Financial (3), Foreign(4), Local (5) and Sport (6). The table shows possible allocations of reporters to news stories. For example, Chris can be assigned to any one of stories 1, 2 or 4.

	1	2	3	4	5	6
A					✓	
B	✓			✓		
C	✓	✓		✓		
D					✓	
E			✓		✓	✓
F				✓		

Figure 3: Question 9

(a) Show these possible allocations on a bipartite graph.

(b) A possble matching is: A to 5, C to 1, E to 6, F to 4. Show this information, in a distinctive way, on a diagram.

(c) Use an appropriate algorithm to find a maximal matching. You should list any alternating paths you have used.

(d) Explain why it is not possible to find a complete matching.

10. Five coach drivers, Mihi, Pat, Robert, Sarah and Tony, have to be assigned to drive five coaches for the following school trips:

Adupgud Senior School is going to the Lake District.
Brayknee Junior School is going to the seaside.
Korry Stur Junior School is going to a concert.
Learnalott Senior School is going to the museum (two coaches needed)

Mihi and Sarah would like to drive senior school children. Robert and Pat would like to go on the seaside trip. Pat and Tony would like to attend the

concert. Robert and Pat would like to visit the museum. Pat and Tony would like to visit the Lake District.

The driver manager wishes to assign each driver to a trip they would like to do.

(a) Draw a bipartite graph to show the trips that the drivers would like to do. Initially Mihi and Pat are assigned to Learnalott Senior School, Robert is assigned to Brayknee Junior School and Tony is assigned to Adupgood Senior School.

(b) Starting from this matching use the maximum matching algorithm to find a complete matching. You must indicate clearly how the algorithm has been applied in this case. State your alternating path and complete matching