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Homework 9
Due: April 6, 2021

1. Consider a CTMC $X = \{X(t), t \geq 0\}$ on $S = \{A, B, C\}$ with generator G given by

$$G = \begin{bmatrix} -12 & 4 & 8 \\ 5 & -6 & 1 \\ 2 & 0 & -2 \end{bmatrix}$$

   (a) Draw the rate diagram.

   (b) Produce the jump matrix and holding time rates.

   (c) Starting from state B, use `Python` to simulate a sample path of the CTMC until the 6th jump of the CTMC occurs. Plot the sample path (by hand is ok). Please specify as much detail as possible.

2. Consider a CTMC $X = \{X(t), t \geq 0\}$ in Problem 1.

   (a) Use a computer software like `Python` or a good calculator to directly compute the transition probability matrix $P(t)$ at $t = 0.20$ minutes.

   (b) Do the previous part for $t = 1.0$ minute.

   (c) Using the results from parts (b) and (c), but without using a software package or calculator, find $P\{X(1.2) = C | X(0) = A\}$ and $P\{X(3) = A | X(1) = B\}$.

   (d) Do part (b) for $t = 5$ minutes. What phenomenon have you observed?

3. Consider a production system with 2 machines. When a machine is up and running, it takes a random amount of time to go down and these random "up times" for each machine are iid exponentially distributed. Machine A is an old machine. Its mean up time is 4 hour. Machine B is a new machine. Its mean up times is 6 hours. There is one repairman, John, on standby, who repairs machines one at a time following the order of breakdown. The repair times for machine A are iid exponentially distributed with mean 2 hours. The repair times for machine B are iid exponentially distributed with mean 1 hours.

   Describe a continuous time Markov chain to model the system and give the rate transition diagram.

4. Customers arrive at a two-server system according to a Poisson process with rate $\lambda = 5$ per hour. An arrival finding server 1 free will begin his service with him. An arrival finding server 1 busy, server 2 free will join server 2. An arrival finding both

servers busy goes away. Once a customer is served by either server, he departs the system. The service times at both servers are exponential random variables. Assume that the service rate of the first server is 3 per hour and the service rate of the second sever is 2 per hour.

Describe a continuous time Markov chain to model the system and give the rate transition diagram.