Theorem. *Any integer at least* 8 *can be formed by combination of* 3 *and* 5.

Proof. For $n \geq 8$, set

$$P(n): \text{ Any integer } m \in \{8, 9, \ldots, n\} \text{ can be formed by a combination of 3 and 5.}$$

Base case. If $n = 8$, then $m = 8 = 3 + 5$, so $P(8)$ is true.
Inductive step. Assume $P(t)$ is true for some $t \geq 8$. For $n = t + 1 \geq 9$, we consider the following:

① If $n = 9$, then $m = 8$ or 9. Hence we only need to consider the case when $m = 9$, which is equal to 3×3. So $P(t + 1)$ is true.

② If $n = 10$, then $m = 8, 9, 10$. So just need to consider the case when $m = 10$, which is equal to 5×2. So $P(t + 1)$ is true.

③ If $n \geq 11$, then $s = n - 3 \in \{8, \ldots, t\}$, which can be formed by combination of 3 and 5 by our assumption, say

$$s = 3a + 5b \qquad \text{for some } a, b \in \mathbb{N}$$

Then

$$n = s + 3 = 3(a + 1) + 5b$$

So $P(t + 1)$ is true.

Therefore, $P(n)$ is true for any $n \geq 8$. □

Alternatively, we can prove the theorem with an ordinary $P(n)$.

Proof. For $n \geq 8$, set

$$P(n): \text{ Any integer } n \text{ at least 8 can be formed by combination of 3 and 5.}$$

Base case. If $n = 8$, then $n = 3 + 5$, so $P(8)$ is true.
Inductive step. Assume $P(n)$ is true for any $n \in \{8, 9, \ldots, t\}$. For $n = t + 1 \geq 9$, we consider the following:

① If $n = 9$, then $n = 3 \times 3$. So $P(t + 1)$ is true.

② If $n = 10$, then $n = 5 \times 2$. So $P(t + 1)$ is true.

③ If $n \geq 11$, then $s = n - 3 \in \{8, \ldots, t\}$, which is a combination of 3 and 5 by assumption, say

$$s = 3a + 5b \qquad \text{for some } a, b \in \mathbb{N}$$

Then

$$n = s + 3 = 3(a + 1) + 5b$$

So $P(t + 1)$ is true.

Therefore, $P(n)$ is true for any $n \geq 8$. □