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Out of gas: quantifying fatigue in MLB relievers

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Abstract: As relief pitcher usage in Major League Baseball has spiked in recent years, optimal bullpen decision-making has become increasingly vital for team managers. Throughout the season, managers must be mindful to avoid overusing their most talented relievers, due to the risks of injury and ineffectiveness. Despite the substantial amount of attention given to pitcher arm health and injury prevention, the effect of workload on pitcher fatigue is poorly understood. As a result, many of these overuse decisions are driven by feel and intuition. In this paper, we borrow ideas from toxicology to provide a framework for estimating the effect of recent workload on short-term reliever effectiveness, as measured by fastball velocity. Treating a thrown pitch as a fatigue-inducing "toxin" administered to a player's arm, we develop a Bayesian hierarchical model to estimate the pitcher-level dose-response relationship, the rate of recovery, and the relationship between pitch count and fatigue. Based on the model, we find that the rate of reliever fatigue rises with increasing pitch count. When relief pitchers throw more than 15 pitches in an appearance, they are expected to suffer small, short-term velocity decreases in future games; upon crossing the 20 pitch threshold, this dip is further amplified. For each day that passes after the appearance, we estimate that the effect on a player's velocity is cut roughly in half. Finally, we identify the relievers most affected by fatigue, along with those most resilient to its effects.

Keywords: baseball; Bayesian; dose-response; pitcher; recovery.

1 Introduction

In the 2016 World Series, Chicago Cubs relief ace Aroldis Chapman made three appearances in 4 days, throwing 42 pitches in Game 5, 20 pitches in Game 6 after a day of rest, and 35 pitches in Game 7. By the final game of the seven-game series, Chapman's trademark fastball had dropped from his season average of 100.9 miles-per-hour (mph) to an average of 98.9 mph, and he gave up a game-tying home run off of a 97.8 mph fastball in the 8th inning. Pundits and experts alike questioned Cubs manager Joe Maddon's usage of Chapman – including Chapman himself Witz (2016). Though equipped with player feedback, medical expertise, and pitch-tracking data, managers like Maddon are required to make bullpen decisions that are largely judgment calls. Could Maddon have predicted the drop in Chapman's fastball velocity in the winner-take-all Game 7, based on previous usage? In this paper, we aim to develop a rigorous framework for quantifying velocity loss based on cumulative usage in preceding days.

Reliever usage in the MLB has seen a steady increase in the last few decades, including a spike in last few years (see Figure 1). This has coincided with higher velocity fastballs thrown by relievers and a widening gap between reliever and starter effectiveness Keri and Paine (2014). Because most relief appearances are one inning or less, relievers can exert near-maximum effort on every pitch, resulting in higher strikeout rates and fewer runs allowed. By 2016, starters were replaced by relievers at never-before-seen rates, especially in the playoffs Cameron (2016a). Relievers became valued more than ever, as contracts for free-agent relievers rose dramatically during the past two offseasons Cameron (2016b), Stephen (2017).

Increased usage of relief pitchers throwing at maximum exertion comes at a cost, however. It is well accepted that throwing a baseball at high velocity puts strain on a player's elbow and shoulder, leading to greater risk of significant injury Andrews and Bruce (2014), Fleisig et al. (1996), Bushnell et al. (2017), Wilk et al. (2009), particularly when the number of days between consecutive appearances is low Whiteside et al. (2016). In addition, fatigued pitchers may change the biomechanics of their deliveries Escamilla et al. (2007), which can put even more stress on the arm and further increase injury risk Fortenbaugh, Fleisig, and Andrews (2009). The difficulty reaching and sustaining peak performance associated with athlete fatigue Murray et al. (2001) may manifest itself in pitchers via decreased velocity, spin rates, and/or command.

Prior attempts to quantify pitcher response to workload have primarily focused on within-game fatigue and its cumulative contribution over the course of a season. Authors in Bradbury and Forman (2012) developed a linear model to estimate the performance effect of

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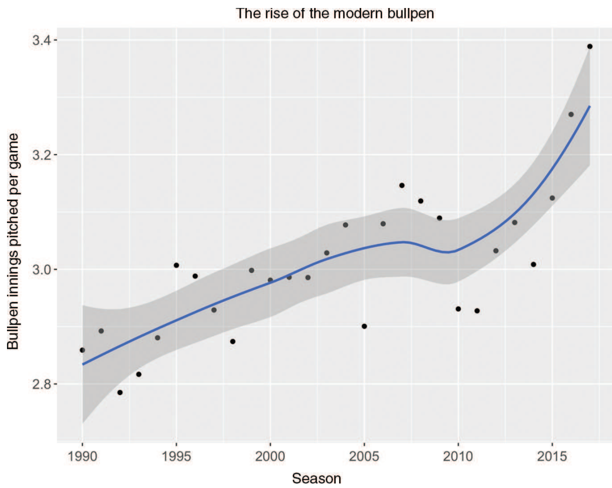

Figure 1: Average bullpen workload, 1990–2017. The solid curve and standard error ribbons correspond to the fitted values and 95% confidence intervals, respectively, obtained via local regression (LOESS).

cumulative workload, though their response variable, ERA, is a volatile measure of in-game performance for relievers. Sonne and Keir (2016) utilized a three-compartment biomechanical model of the arm to estimate muscle fatigue when the time between pitches is varied, arguing that implementing a pitch clock may impair recovery from fatigue. However, the impact of short-term workload on a pitcher's performance has not been studied extensively in the literature, even though this may have a large impact on a manager's bullpen decisions.

Since fastball velocity is recognized by both scouts and analysts as an important component of MLB pitcher success Fast (2010), we focus on estimating the relationship between short-term reliever workload and fastball velocity. We estimate this relationship via a principled, Bayesian hierarchical framework, borrowing modeling ideas from toxicology. By treating pitches as a fatigue-inducing "toxin" administered to a pitcher, we estimate (1) the relationship between the number of pitches thrown in a game and the toxicity of an outing, (2) the rate at which the toxin exits the pitcher's system, and (3) the relationship between the dose of the toxin and mean fastball velocity for each pitcher.

In Section 2, we motivate our approach and detail the Bayesian model used to assess the relationship between short-term reliever workload and fastball velocity. Section 3 describes and interprets the model results in context. A discussion and conclusion, including directions for future research, follow in Sections 4 and 5.

2 Methods

2.1 Data

To answer our research question, we scraped pitch-level Statcast data from MLB.com, collecting available information on every regular-season pitch thrown between 2015 and 2017. We choose to focus on pitchers with exclusive relief roles, defined as those without any starts over the 3-year period. For each reliever, in order to avoid the confounding influence of injuries, we only included years in which he threw at least 200 pitches and games in which he pitched with fewer than 10 days of rest. Figures 2 and 3 display the typical workload of the 324 unique relief pitchers in our study. Most successful relief outings tend to last for an inning (≈15 pitches), but can be extended to multiple innings in some situations. Although relievers throw far fewer pitches per outing than do starters, they pitch far more frequently, with over half of appearances coming with fewer than 2 days of rest.

A reliever's mean fastball velocity in a game is the response variable of interest in our study. Although Statcast classifies the type of each pitch, not all relievers throw a four-seam fastball. As such, for each reliever, we define his fastball as the fastest pitch among four-seam fastballs, two-seam fastballs, sinkers, cut-fastballs, splitters, or sliders that he throws at least 10% of the time. Since some relief pitchers throw both a four-seam and two-seam fastball, we label both pitches as fastballs if they average within two mph of each other. Because a pitcher's fastball velocity is slightly left-skewed, we only consider relief appearances in which at least three fastballs are thrown, so that the sampling distribution of a pitcher's mean fastball velocity in each game is nearly normal. After final

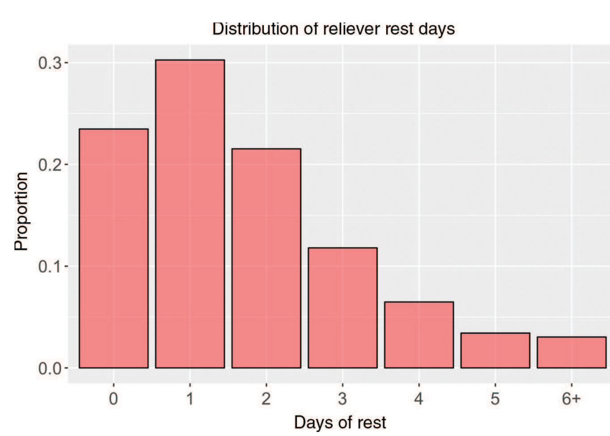

Figure 2: Relief pitchers have high frequency workloads, often pitching consecutive days or every other day.

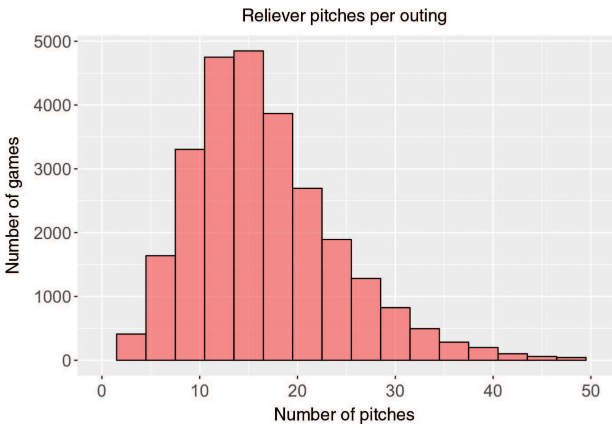

Figure 3: The distribution of pitch counts per game over all relief appearances. Appearances with more than 50 pitches (70 total) are not displayed.

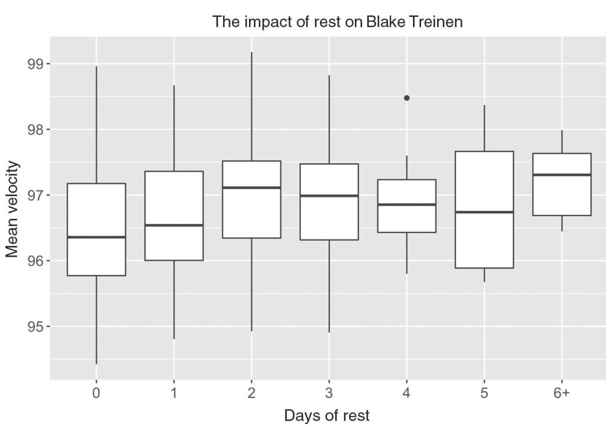

Figure 4: The relationship between days of rest and average fastball velocity for Blake Treinen, the closer for the Oakland Athletics.

processing, a total of 268,966 fastballs spread over 26,774 unique appearances were included in the 3-year study.

From exploratory data analysis, there appears to be a small, but noticeable trend in the relationship between the number of days of rest that a player receives and his average fastball velocity in the next game. Blake Treinen is an illustration of this phenomenon (Figure 4); he steadily gains a few tenths of a mph on his fastball when he is allowed an extra day to recover from an outing. After about 2 days of rest, there appear to be diminishing returns to increasing days of rest. In addition, preliminary nonparametric additive modeling approaches find a negative relationship between pitch count in the previous outing and mean fastball velocity. Affirming intuition, the strength and magnitude of this relationship diminishes with an increasing number of days of rest.

2.2 Toxicology modeling

In order to incorporate both volume and frequency of workload in a principled, coherent framework, we borrow ideas from toxicology to motivate an interpretable latent variable model for athlete fatigue. Our final model consists of a nesting of three separate models: a *volume-dose model*, a *pharacokinetic model* and a *dose-response model*, of which the latter two are frequently used in toxicology and clinical pharmacology Holford (2006). Below, we highlight the importance of each of these models in the context of quantifying reliever fatigue.

- The *volume-dose model* represents the monotonically increasing relationship between the workload x_{jt} of player j on day t and the *dose* d_{jt} of fatigue-inducing toxin administered to the player on that day. In our context, x_{jt} is the number of pitches thrown by a reliever during an appearance on day t. This is a very important relationship for managers to know, since they need to make decisions about how long to leave in a pitcher to keep him fresh for future games.
- The *pharmacokinetic model* estimates the *concentration* c_{jt} of the toxin in player j's body on day t remaining from the doses administered during previous days $\{d_{j(t-k)},\ k = 1, 2, \ldots, \infty\}$. This enables a manager to equate varying amounts of pitch counts and rest when pacing a reliever over the course of the season.
- The *dose-response model* estimates the monotonically decreasing relationship between the concentration of the toxin c_{jt} and the player's corresponding *response* v_{jt}. In our context, the response is mean fastball velocity. This represents the degree to which a pitcher's velocity is affected by his level of fatigue, allowing a manager to make better bullpen decisions.

In this approach, neither the dose nor concentration are directly observed. Thus, a model constructed in this way is not identifiable without introducing some simplifying assumptions; these choices may vary by application. In particular, we assume that both the volume-dose model and pharmacokinetic model are deterministic and do not vary across players. Moreover, we also assume that each of the three relationships can be well described by a prespecified parametric model, as is typical in toxicology Ritz et al. (2016). We describe and motivate our selected parametric representations below.

We assume that the volume-dose relationship can be characterized by a continuous, piecewise linear function, also known as a linear spline. The linear spline, one of the simplest generalized additive models Hastie and Tibshirani (2004), is preferred for its combination of flexibility, interpretability, and desirable behavior Wold (1974). Because the relationship between volume and dose is monotone increasing, we constrain the slope of each line to be non-negative. In addition, we select a total of four knots at the 25th, 50th, 75th, and 90th percentiles of reliever pitch counts (11, 15, 21, and 27 pitches), creating a total of five pieces. Explicitly,

$$d_{jt} = \begin{cases} \beta_1 x_{jt} & x \leq 11 \\ 11\beta_1 + \beta_2(x_{jt} - 11) & 11 < x \leq 15 \\ 11\beta_1 + 4\beta_2 + \beta_3(x_{jt} - 15) & 15 < x \leq 21 \\ 11\beta_1 + 4\beta_2 + 6\beta_3 \\ \quad + \beta_4(x_{jt} - 21) & 21 < x \leq 27 \\ 11\beta_1 + 4\beta_2 + 6\beta_3 \\ \quad + 6\beta_4 + \beta_5(x_{jt} - 27) & x > 27 \end{cases} \tag{1}$$

We assume that the behavior of the fatigue-inducing toxin (the pharmacokinetic model) is well described by a monocompartmental model, which is the simplest way to describe the behavior of drug distribution and elimination in the human body Shargel, Wu-Pong, and Yu (2006). The monocompartmental model assumes that the body acts as a single compartment and that the toxin is eliminated at a constant rate. Suppose an initial dose d_0 is administered to a player at time $t = 0$. The concentration of the toxin remaining in the player's body at time t is predicted to be

$$c(t) = d_0 e^{-\phi t}, \tag{2}$$

where ϕ is an elimination rate constant. Since the monocompartmental model is additive, the total concentration of fatigue in pitcher j's system before an appearance on day t can be expressed as the sum of all concentrations remaining from the dosages administered during each of his previous games:

$$c_{jt} = \sum_{k=1}^{\infty} d_{j(t-k)} e^{\phi(t-k)}. \tag{3}$$

Because 5 days of rest is considered to be a significant amount of rest for a starting pitcher, we approximate (3) by only considering games played 6 days prior to an appearance. This assumes that that any outing more than 6 days ago has no effect on a pitcher's velocity in a game, which is approximately true for most reasonable values of ϕ.

Finally, we assume that the dose-response relationship is linear, so that the average velocity drop in a game is proportional to the concentration of the toxin in the pitcher's body at the start of the game. Many other well-studied dose-response models are possible, including the generalized log-logistic, log-normal, and Weibull models Ritz et al. (2016). We specify pitcher-level random intercepts to serve as a baseline for pitcher fastball velocity and random slopes to allow pitchers to respond to the toxin differently. Assuming normality of the sampling distribution, our dose-response model is

$$v_{jt} \sim \text{Normal}(\mu_{jt}, \sigma^2/n_{jt}) \tag{4}$$

$$\mu_{jt} = \alpha_j + m_j c_{jt}, \tag{5}$$

where μ_{jt} represents the true mean fastball velocity for pitcher j on day t, n_{jt} is the number of fastballs thrown by pitcher j on day t, and σ^2 is the variance of fastball velocity around the true mean.

2.3 Bayesian model specification

Putting equations (1–5) together, we have the following model:

$$v_{jt} \sim \text{Normal}(\mu_{jt}, \sigma^2/n_{jt}) \qquad \mu_{jt} = \alpha_j + m_j c_{jt}$$

$$c_{jt} = \sum_{k=1}^{6} d_{j(t-k)} e^{\phi(t-k)}$$

$$d_{jt} = \begin{cases} \beta_1 x_{jt} & x \leq 11 \\ 11\beta_1 + \beta_2(x_{jt} - 11) & 11 < x \leq 15 \\ 11\beta_1 + 4\beta_2 + \beta_3(x_{jt} - 15) & 15 < x \leq 21 \\ 11\beta_1 + 4\beta_2 + 6\beta_3 \\ \quad + \beta_4(x_{jt} - 21) & 21 < x \leq 27 \\ 11\beta_1 + 4\beta_2 + 6\beta_3 + 6\beta_4 \\ \quad + \beta_5(x_{jt} - 27) & x > 27 \end{cases} \tag{6}$$

We choose to model the pitcher-level intercepts α_j and response magnitudes m_j hierarchically, fixing the global mean of m_j at 1 in order to make the model identifiable. As such, we specify the diffuse priors

$$\alpha_j \sim \text{Normal}(\mu_\alpha, \tau_\alpha^2) \qquad \mu_\alpha \sim \text{Normal}(93, 3^2)$$
$$\tau_\alpha \sim \text{HC}(0, 1) \tag{7}$$

$$m_j \sim \text{Normal}_{[0,\infty)}(1, \tau_m^2) \qquad 1/\tau_m^2 \sim \text{Gamma}(10, 1), \tag{8}$$

where HC(0, 1) corresponds to the half-Cauchy distribution and $\text{Normal}_{[0,\infty)}$ represents the truncated normal distribution, constrained to positive values. The priors for the global parameters of interest are given as

$$\beta_k \sim \text{Normal}_{[0,\infty)}(0, 10^2), \qquad k = 1, 2, 3, 4, 5. \tag{9}$$

$$\phi \sim \text{Gamma}(0.1, 0.1) \qquad 1/\sigma^2 \sim \text{Gamma}(0.1, 0.1). \tag{10}$$

The joint posterior distribution of the random parameters found in equations (7–10) was estimated via Gibbs sampling Gelman et al. (2014), Plummer (2003). Each of three Markov chains with overdispersed initial values were ran for 10,000 iterations after a burn-in of 1000 iterations, thinned every 10 iterations. The model was validated using Markov Chain Monte Carlo diagnostics and posterior predictive checks.

3 Results

We can explore the results of our model by examining the posterior distribution of relevant parameters and functions of those parameters. Figure 5 displays the estimated volume-dose relationship, determined by the posterior distribution of β_k, $k=1, \ldots, 5$. Posterior means, along with 95% credible intervals are given in the figure. As the figure indicates, pitches thrown at high pitch counts have a much more negative impact on future fastball velocity than those thrown at low pitch counts. In particular, an outing of fewer than 11 pitches does not contribute substantially to near-future fastball velocity loss. A 20-pitch outing has roughly twice the estimated impact on pitcher fatigue as a 15-pitch outing, and a 30-pitch outing has 3 times the impact as a 20-pitch outing. Overall, we suggest that if a manager wants a reliever to be available the next day without a noticeable dip in performance, he should limit that pitcher to 15 pitches. For each pitch past 21 pitches, the estimated amount of fatigue that carries over to future games increases dramatically.

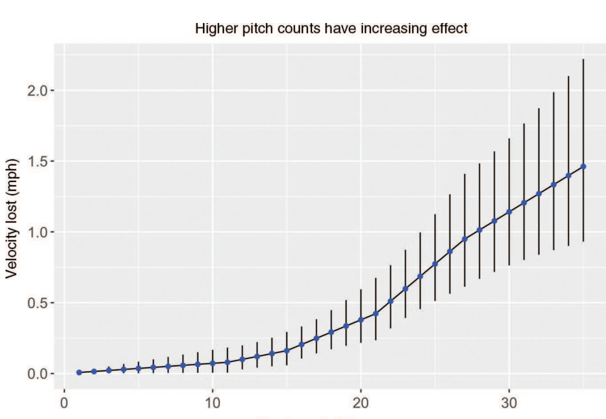

Figure 5: The posterior volume-dose relationship, modeled as a linear spline at knots {11, 15, 21, 27}. The rate of fatigue dosage, in terms of mph lost, begins to noticeably increase after 15 pitches and rises substantially after 21 pitches.

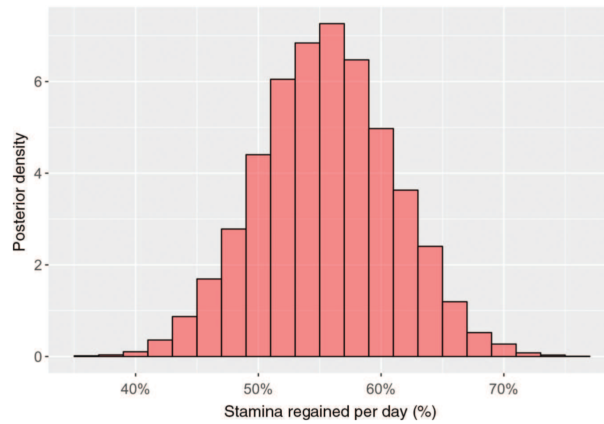

Figure 6: The posterior distribution of $(1 - e^{\phi})$. Pitchers regain between 45% and 65% of their stamina per day of rest.

The population-level elimination rate constant ϕ represents the rate at which the fatigue toxin is eliminated from the body under the monocompartmental model. The function $e^{-\phi}$ estimates the effect of a day of rest on the amount of fatigue, so $1 - e^{-\phi}$ represents the percentage of stamina regained by a day of rest, with posterior distribution shown in Figure 6. If a player is expected to experience a 1.14 mph fastball velocity loss due to a 30 pitch outing on Monday (e.g. from Figure 5) and the estimated recovery rate is 0.55, then on Tuesday we expect his velocity to lower by $1.14(1 - 0.55) = 0.513$ mph. If he rests on Tuesday and pitches on Wednesday, we expect his velocity to be lower by $1.14(1 - 0.55)^2 = 0.231$ mph. The proportion of stamina regained per day has a posterior mean of 0.55, though we see it can reasonably be as low as 0.45 or as high as 0.65 (roughly the 95% credible interval). A quick take-away for managers is that pitches 2 days ago have about half the effect as pitches yesterday.

While population effects are useful for rule-of-thumb decisions, we also estimate player-specific dose-response relationships. The parameter m_j denotes the magnitude of the linear response that player j experiences for a given concentration of fatigue. Because the response is linear, a response magnitude of 1 indicates that the player has a completely average level of response to fatigue, whereas a value of 2 suggest that a player is twice as sensitive to the effects of fatigue. The estimates for m_j experience shrinkage towards the mean of 1 (a consequence of the prior) and partial pooling of information between players.

Table 1 lists the players such that the posterior probability of above-average response is greater than 0.8. Similarly, Table 2 lists the players such that the posterior probability of below-average response. We see that Neftali Feliz experiences the biggest velocity drop from fatigue by a

Table 1: The relievers most affected by fatigue.

	Pitcher name	Magnitude	$\Pr(m_j > 1)$
1	Neftali Feliz	1.955	0.985
2	Jose Ramirez	1.541	0.894
3	Tommy Hunter	1.544	0.890
4	Scott Alexander	1.409	0.840
5	Hunter Strickland	1.414	0.838
6	Tommy Kahnle	1.427	0.836
7	Addison Reed	1.405	0.832
8	Aaron Loup	1.378	0.800

Table 2: The relievers least affected by fatigue.

	Pitcher Name	Magnitude	$\Pr(m_j < 1)$
1	Bryan Shaw	0.357	0.980
2	Corey Knebel	0.594	0.876
3	Jim Johnson	0.583	0.875
4	Elvis Araujo	0.602	0.863
5	Simon Castro	0.631	0.843
6	Tyler Clippard	0.637	0.842
7	Brad Ziegler	0.651	0.834
8	Matthew Bowman	0.656	0.825
9	Scott Oberg	0.672	0.814

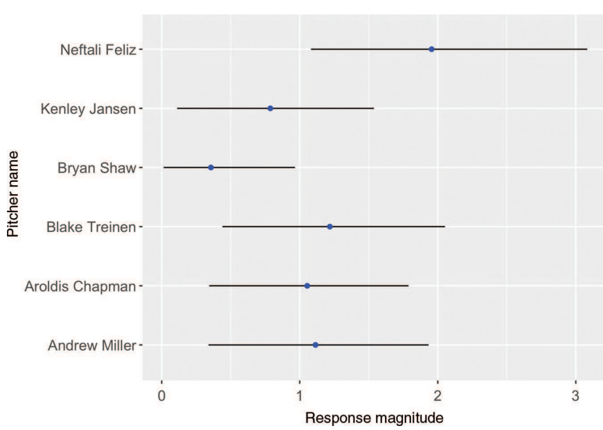

Figure 7: Posterior distribution of response magnitudes for selected pitchers. The differences between these pitchers are minimal, with the exception of a few noticeable cases.

wide margin; Bryan Shaw, a lauded workhorse Bastian (2016), has by far the lowest magnitude.

Figure 7 displays posterior estimates of m_j with uncertainty for a select few number of pitchers. Because of wide uncertainty bands, Shaw and Feliz are the only players in this figure without overlapping 95% credible intervals. However, despite the high amount of uncertainty, these estimates may be useful to managers making borderline decisions.

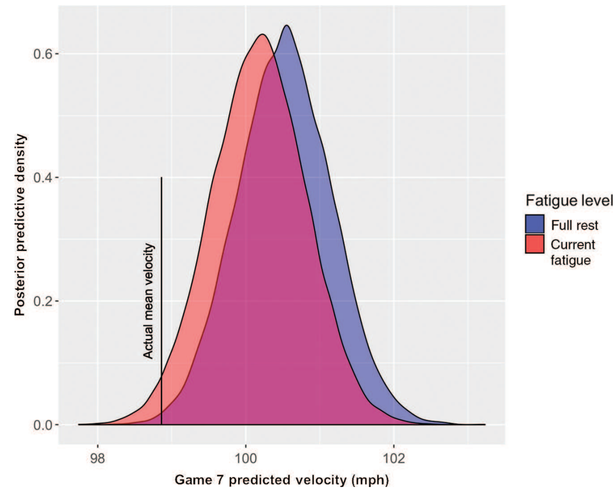

Figure 8: Posterior predictive density for Aroldis Chapman's mean velocity in Game 7 of the 2016 World Series, based on previous usage in the series. Even after accounting for a particularly fatigued Chapman, the observed velocity drop was in the lower tail of the distribution.

4 Discussion

Using the results of the above model, we revisit the motivating example given in Section 1: was Joe Maddon's decision to pitch Chapman in Game 7 of the 2016 World Series defensible, or should he have expected a large velocity drop due to fatigue from previous outings? Figure 8 illustrates the posterior predictive distribution of Chapman's Game 7 fastball velocity, given the information up to the start of the game. Our model expects an average drop of 0.5 mph off of Chapman's fastball velocity due to the effects of fatigue from previous outings. Although an average fastball velocity lower than 99.0 mph is unlikely ($\approx$3%) even given our estimated levels of fatigue, it is 7 times more likely than at full rest. However, in our view, Chapman's expected decline in performance was not severe enough to consider resting him in such a high-leverage situation with no games in the near-future.

Although our approach is unique and provides actionable insight for bullpen decision-making, it has a few shortcomings in its simplicity. For instance, a one-parameter monocompartmental pharmacokinetic model may not capture the long-term effects of high pitch counts and high velocities, since the estimated effect decays to nearly zero within a week. As a result, we do not incorporate the cumulative effects of seasonal workload, which may be a substantial contributor to reliever fatigue. Our model also ignores other potential causes of fatigue, such as travel, high-leverage situations, and pitch-level characteristics. Moreover, the model cannot distinguish between a faster recovery rate and a lower response to fatigue.

These shortcomings can be addressed by additional model complexity, although an expanded model may be more computationally expensive and less interpretable.

In addition, the framework described in the article assumes that a player's daily fatigue is administered in a single dosage. Future work can relax this assumption to consider variation of fastball velocity within a game, so that a manager might be able to use this framework to project in-game velocity decreases. Returning to a previous example, Chapman's Game 7 fastball velocity dropped by around 1 mph between the 8th and the 9th innings of Game 7. A combined look at within-game and across-game fatigue could yield additional insights on the relationship between workload and performance.

Future work could also investigate other potential responses throughout all of athletics, such as tennis serve velocity Kovalchik (2017), basketball player movement, and other measures of player performance. Within baseball, a multivariate dose-response model could be implemented for a variety of pitch types to predict spin rates, velocities, and release points, using a common underlying model for player fatigue. This may provide additional takeaways for managers looking to optimize bullpen decision-making, further motivating the use of this methodology in other sports.

5 Conclusion

In this paper, we aimed to quantify the relationship between workload and short-term fastball velocity in relief pitchers. Borrowing ideas from toxicology to construct a nested hierarchical model, we used Markov Chain Monte Carlo techniques to conduct Bayesian inference on the relationship between the number of pitches thrown in previous outings and the mean velocity in the upcoming game. We estimated a piecewise linear volume-dose relationship, finding that high pitch count outings have a disproportionately large impact on fatigue. From this analysis, we recommend that pitchers who throw more than 15 pitches in an appearance be given at least 1 day of rest in order to perform close to their best in future games.

Moreover, we estimated that relief pitchers recover roughly 55% of their stamina for each day of rest they receive. We also found a small amount of evidence that some pitchers respond differently to fatigue than others, though the magnitude of this effect is somewhat small, and there are only a few pitchers for which this effect differs significantly from average. While there remain some interesting and important extensions that include cumulative effects, within-game effects, and additional fatigue predictors, we believe our model can be a valuable tool for managers making critical bullpen decisions.

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