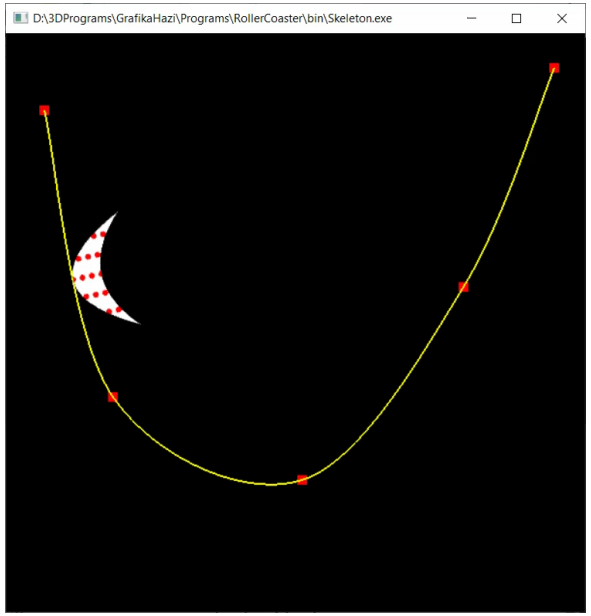
"Photographers don't take pictures. They create images."

Mark Denman

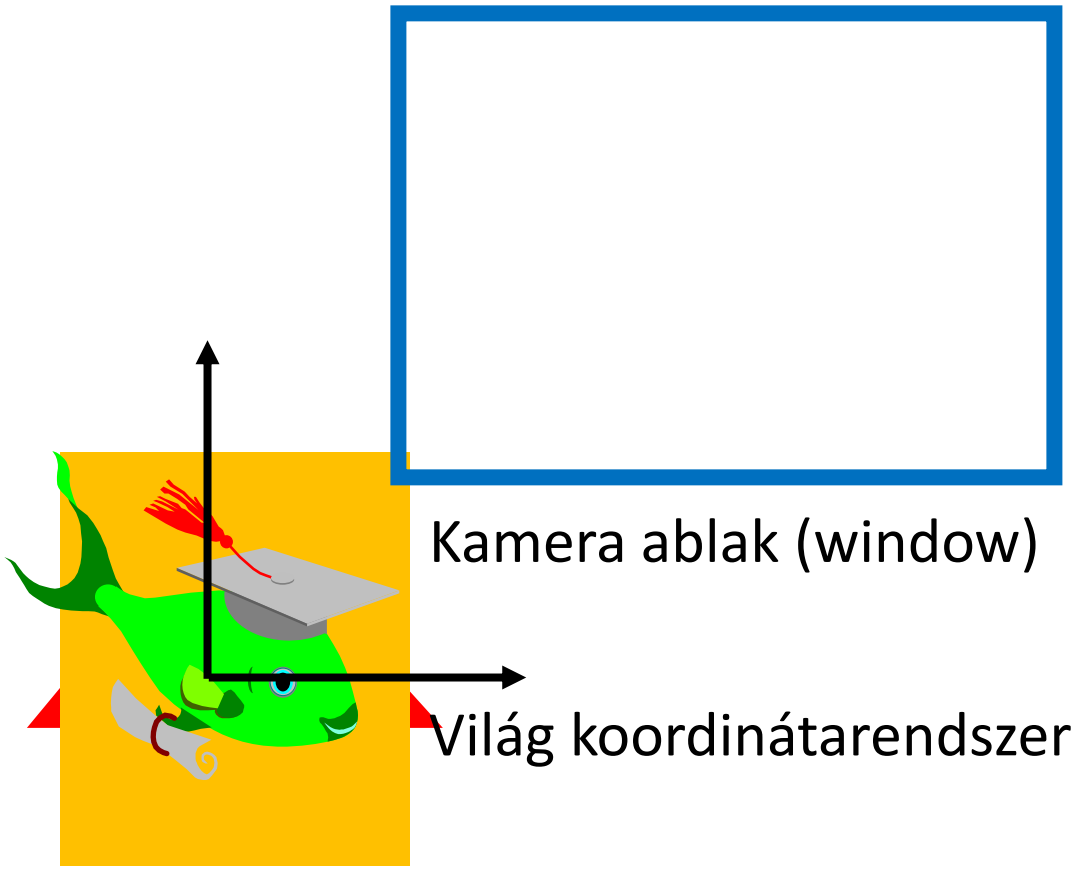
2D képszintézis

Szirmay-Kalos László

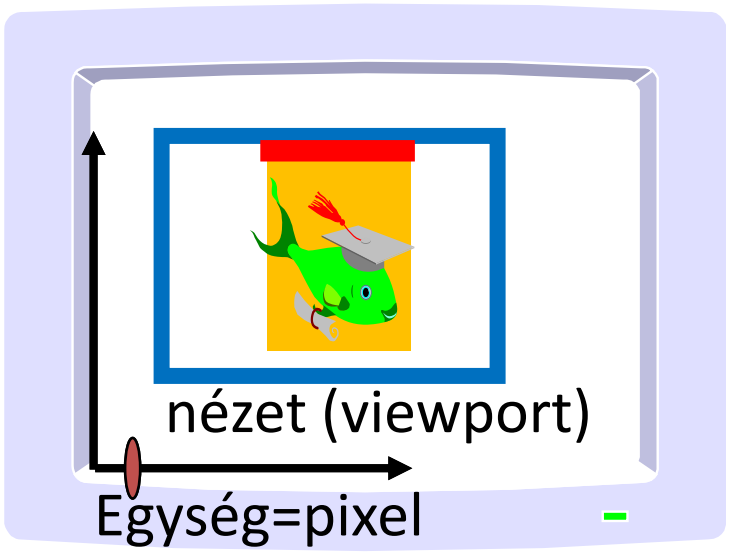


2D képszintézis

Modell

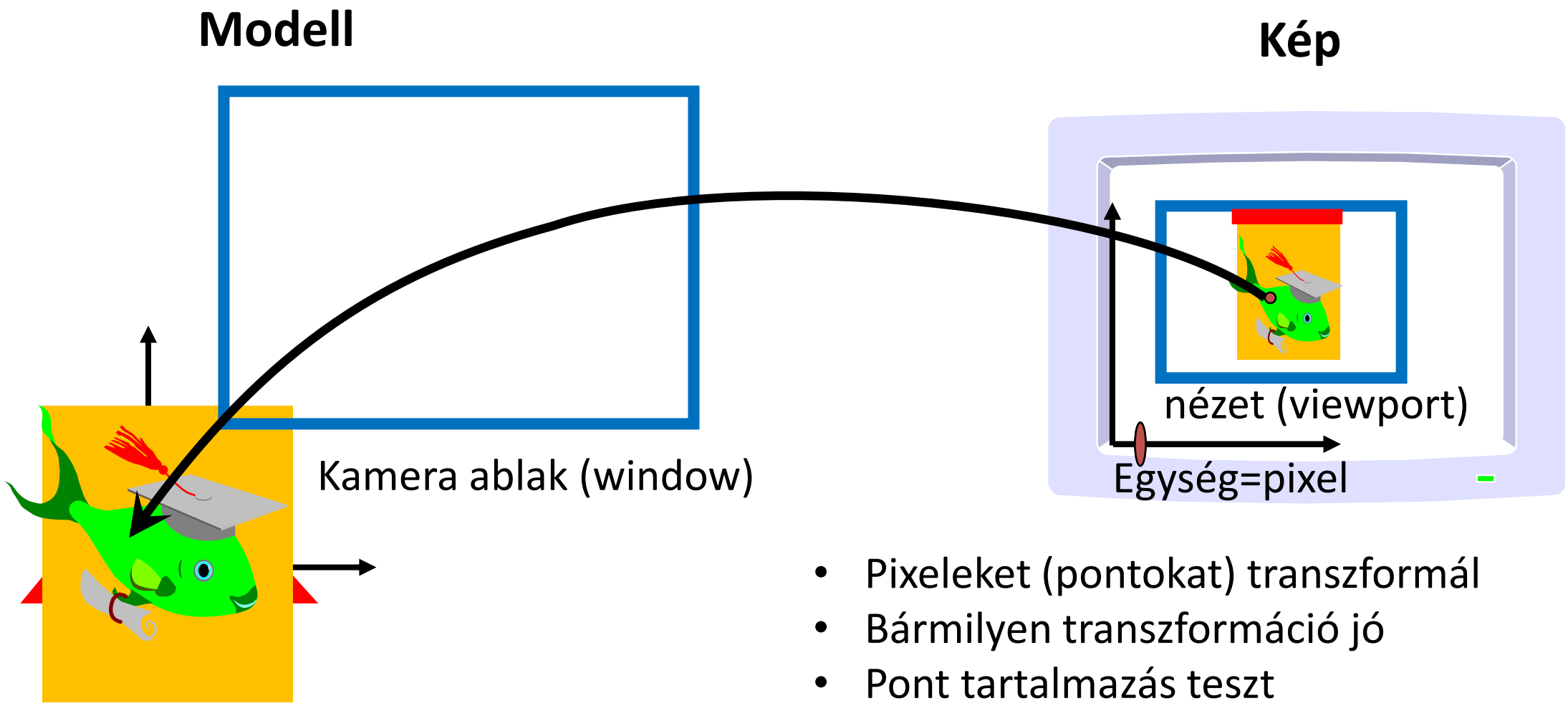


Kép



Saját színnel rajzolás

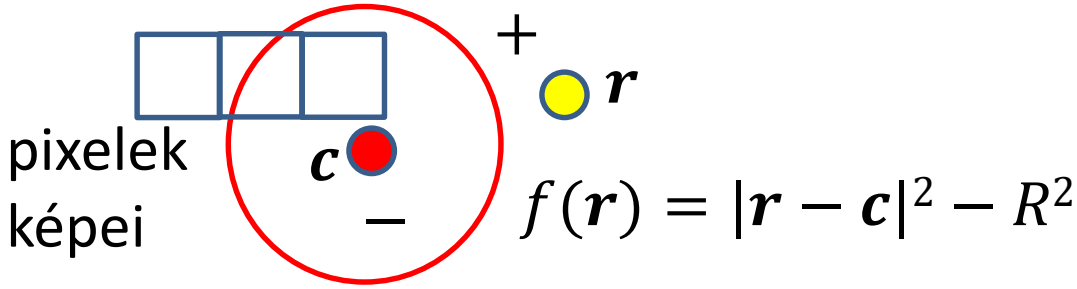
Pixel vezérelt 2D képszintézis



- Pixeleket (pontokat) transzformál
- Bármilyen transzformáció jó
- Pont tartalmazás teszt
- Lassú

Pixel vezérelt megközelítés: Tartalmazás (objektum, pont)

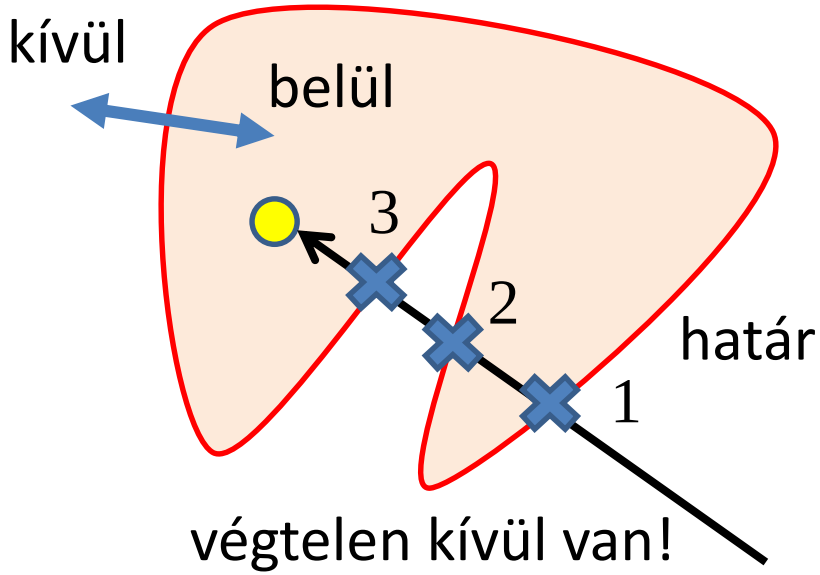
- Határ implicit görbe:

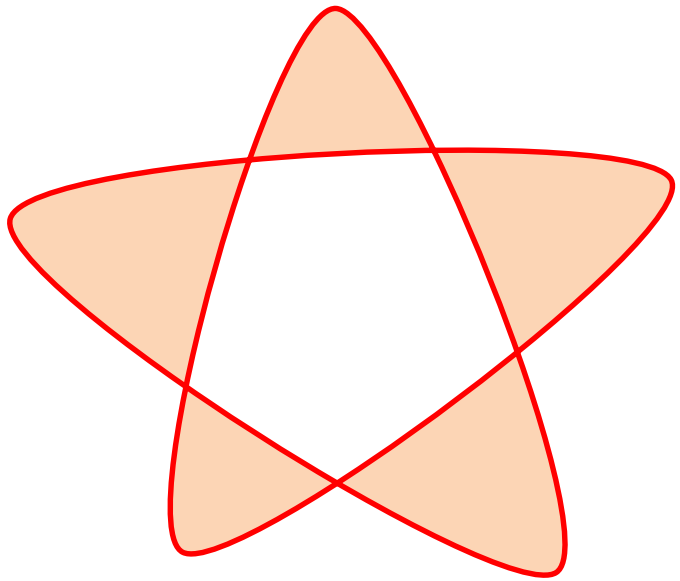


$$f(\boldsymbol{r}) = |\boldsymbol{r} - \boldsymbol{c}|^2 - R^2$$

> 0 : egyik oldalon
$f(x, y) = 0$: határon
< 0 : másik oldalon
(ált. belül)

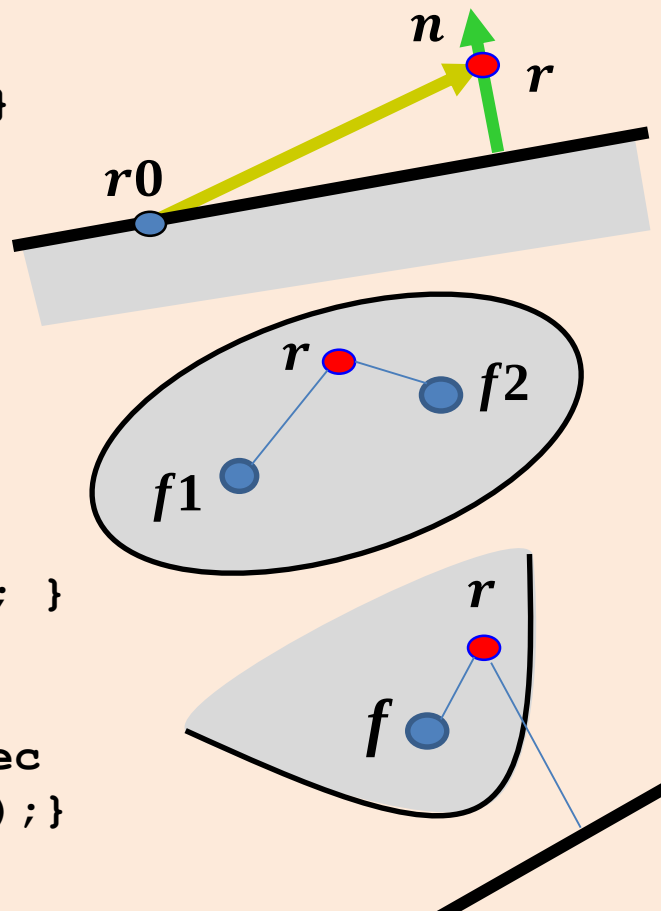
- Határ parametrikus görbe:





Pixel vezérelt rendering

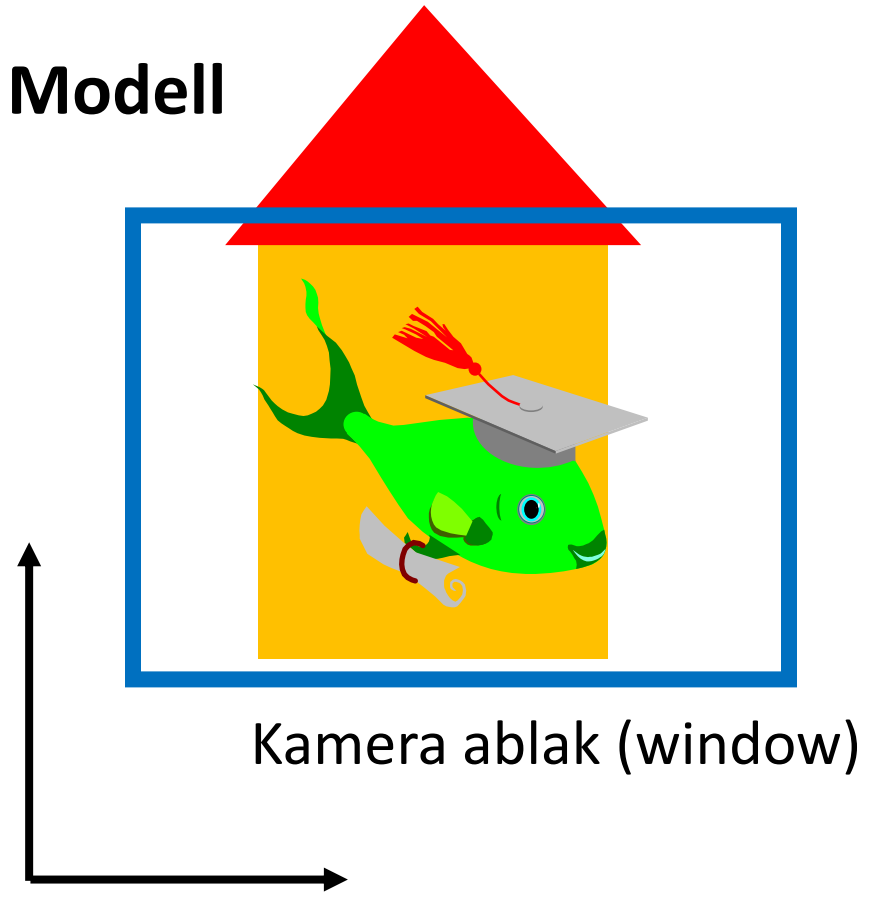
```
struct Object { // base class
   vec3 color;
   virtual bool In(vec2 r) = 0; // containment test
};
struct Circle : Object {
   vec2 center;
   float R;
   bool In(vec2 r) { return (dot(r-center, r-center) < R*R); }
};
struct HalfPlane : Object {
   vec2 r0, n; // position vec, normal vec
   bool In(vec2 r) { return (dot(r-r0, n) < 0); }
};
struct GeneralEllipse : Object {
   vec2 f1, f2;
   float C;
   bool In(vec2 r) { return (length(r-f1) + length(r-f2) < C); }
};
struct Parabola : Object {
   vec2 f, r0, n; // f=focus, (r0,n)=directrix line, n=unit vec
   bool In(vec2 r) { return (fabs(dot(r-r0, n)) > length(r-f));}
};
```

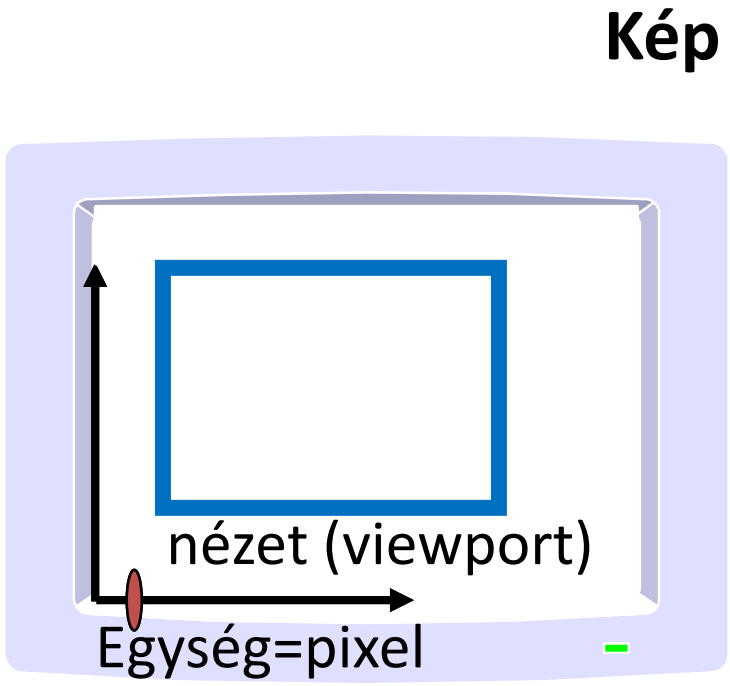


Pixel vezérelt rendering

```
class Scene {                    // virtual world
   list<Object *> objs;          // objects with decreasing priority
   Object *picked = nullptr;     // selected for operation
public:
   void Add(Object * o) { objs.push_front(o); picked = o; }
   void Pick(int pX, int pY) { // pX, pY: pixel coordinates
      vec2 wPoint = Viewport2Window(pX, pY); // transform to world
      picked = nullptr;
      for(auto o : objs) if (o->In(wPoint)) { picked = o; return; }
   }
   void BringToFront() {
      if (picked) { // move to the front of the priority list
         objs.erase(find(objs.begin(), objs.end(), picked));
         objs.push_front(picked);
      }
   }
   void Render() {
      for(int pX = 0; pX < xmax; pX++) for(int pY = 0; pY < ymax; pY++) {
         vec2 wPoint = Viewport2Window(pX, pY); // wPoint.x = a * pX + b * pY + c
         for(auto o : objs) if (o->In(wPoint)) { image[pY][pX] = o->color; break; }
      }
   }
};
```

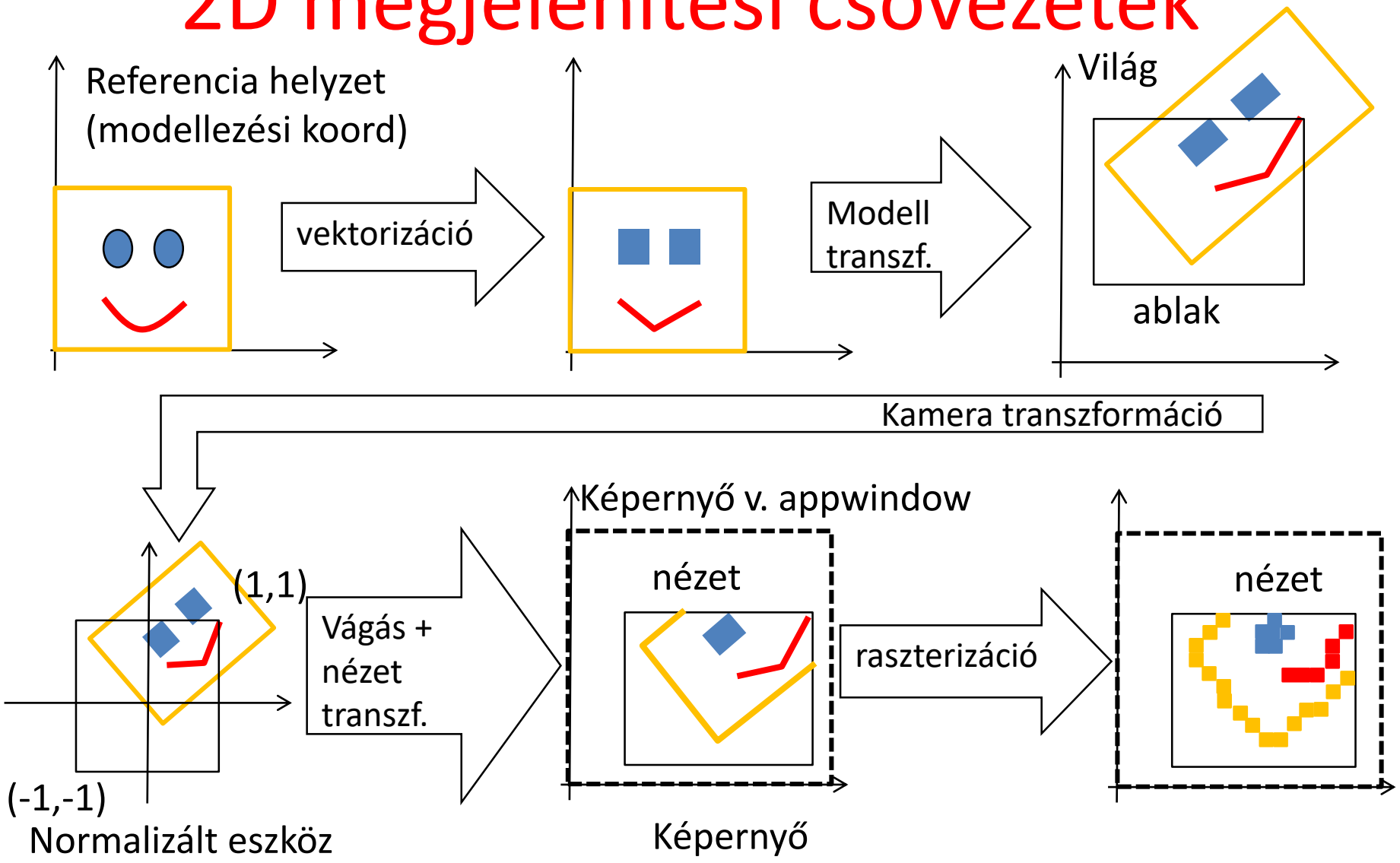
Objektum vezérelt 2D képszintézis





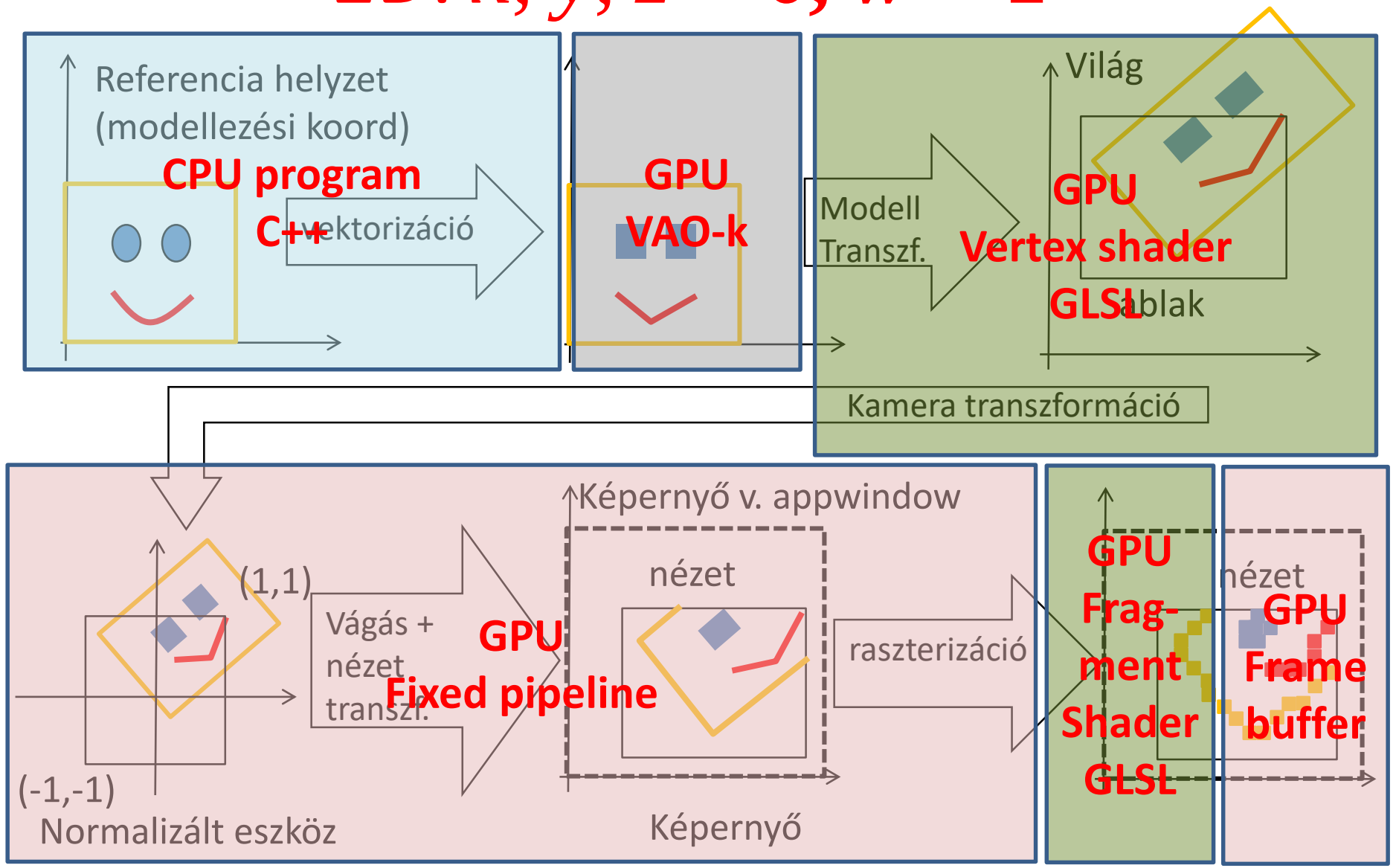
Saját színnel rajzolás
a kis prioritásúakkal kezdve

Objektum vezérelt megközelítés: 2D megjelenítési csővezeték



GPU megjelenítési csővezeték

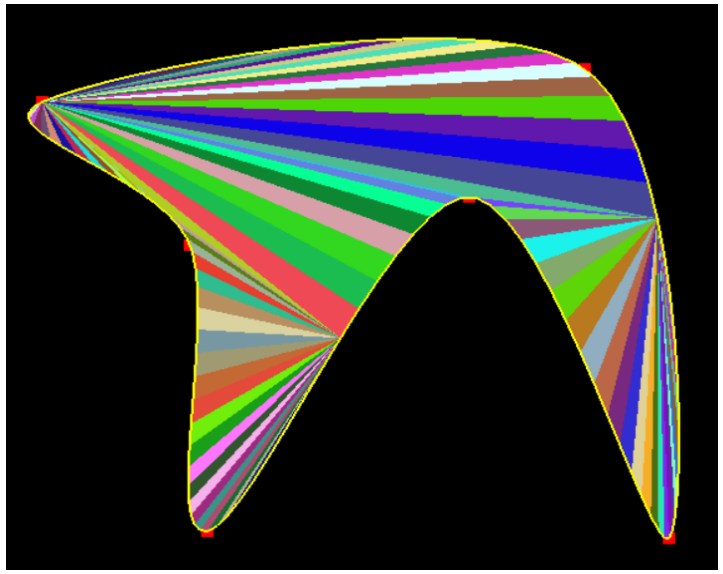
2D: $x, y, z = 0, w = 1$



"μή μου τοὺς κύκλους τάραττε."
Ἀρχιμήδης

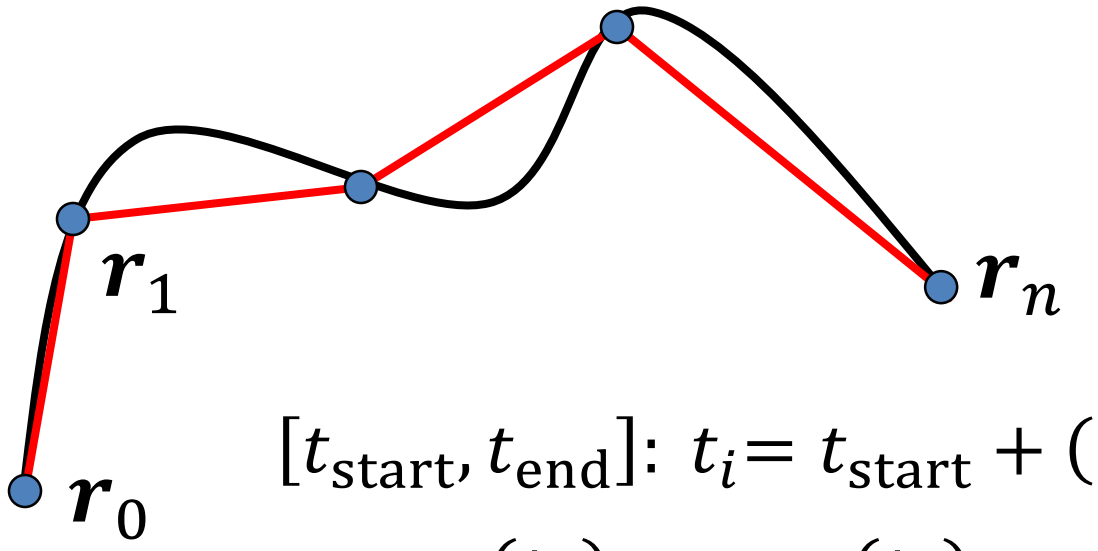
2D képszintézis
2. Vektorizáció és háromszögesítés

Szirmay-Kalos László



Vektorizáció (CPU)

$\boldsymbol{r}(t), t \in [t_{\text{start}}, t_{\text{end}}]$



GL_LINE_STRIP
GL_LINE_LOOP

$[t_{\text{start}}, t_{\text{end}}]$: $t_i = t_{\text{start}} + (t_{\text{end}} - t_{\text{start}})i/n$

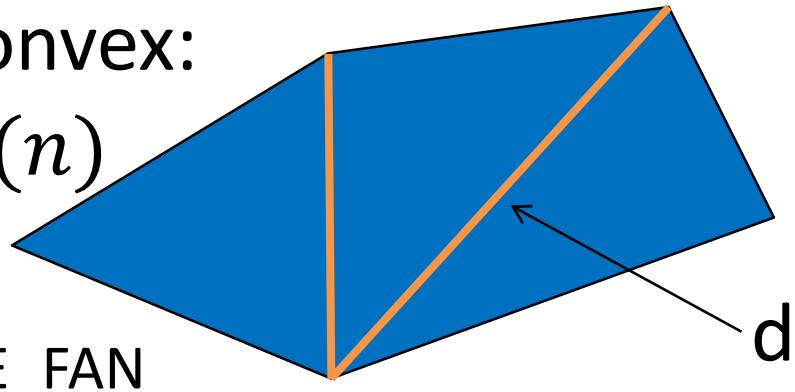
$\boldsymbol{r}_0 = \boldsymbol{r}(t_0), \boldsymbol{r}_1 = \boldsymbol{r}(t_1), \dots, \boldsymbol{r}_n = \boldsymbol{r}(t_n)$

		Hw érdekében
Görbe	→ nyílt töröttvonal	→ szakaszok
Terület határa	→ zárt töröttvonal = poligon	→ háromszögek

Poligon háromszögekre bontása

Diagonál mentén értelmes vágni, mert az csökkenti a csúcspontok számát!

Konvex:
$O(n)$

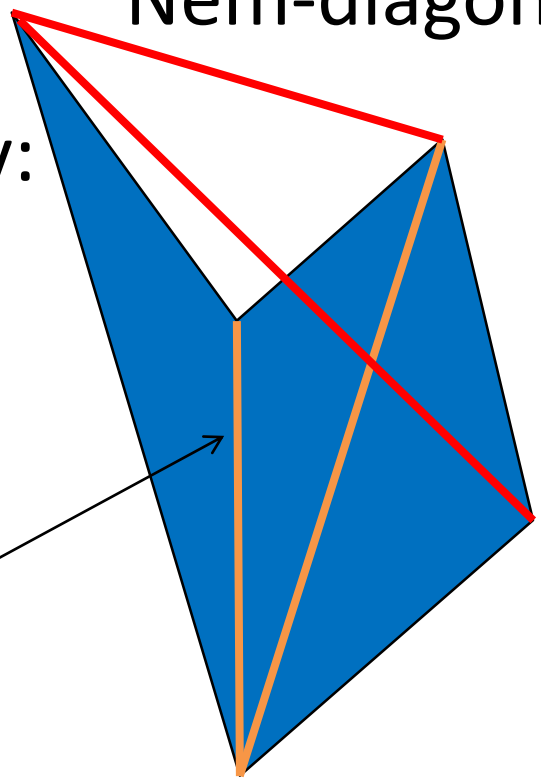


GL_TRIANGLE_FAN

diagonál

Nem-diagonál

Konkáv:
$O(n^4)$



Tétel: Minden 4+ csúcsú **egyszerű** sokszögnek van diagonálja, azaz mindegyik felbontható diagonálok mentén.

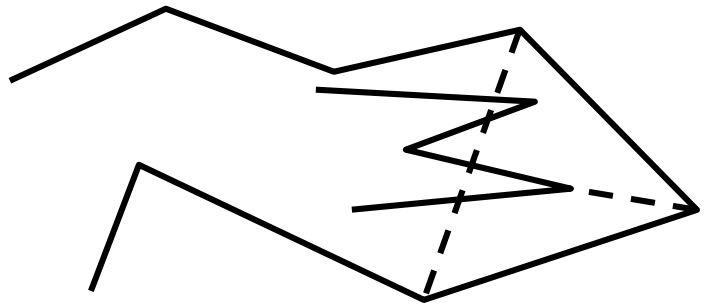
Diagonál mer
csökkenti a cs

Ko
$O($

Egyszerű sokszög

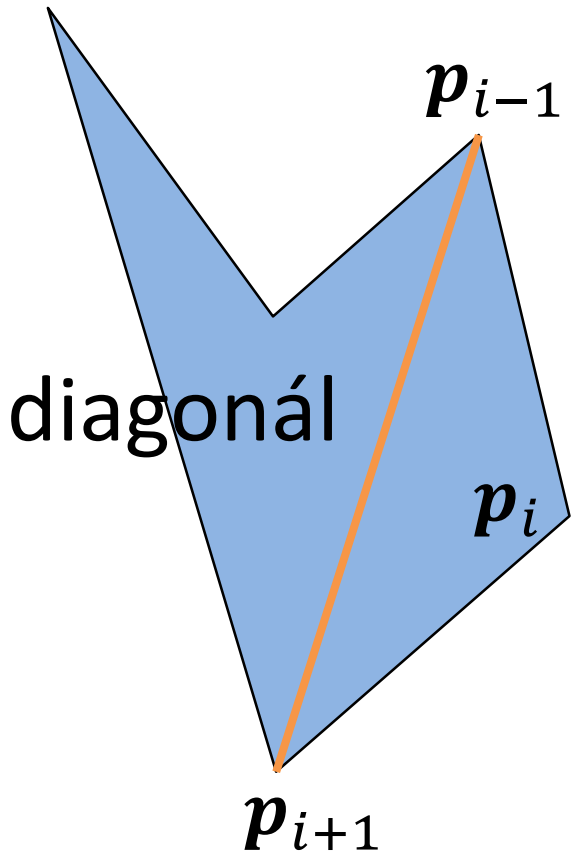
Nem egyszerű sokszög

Tétel: Minden 4+ csúcsú **<u>egyszerű</u>** sokszögnek van diagonálja, azaz mindegyik felbontható diagonálok mentén.



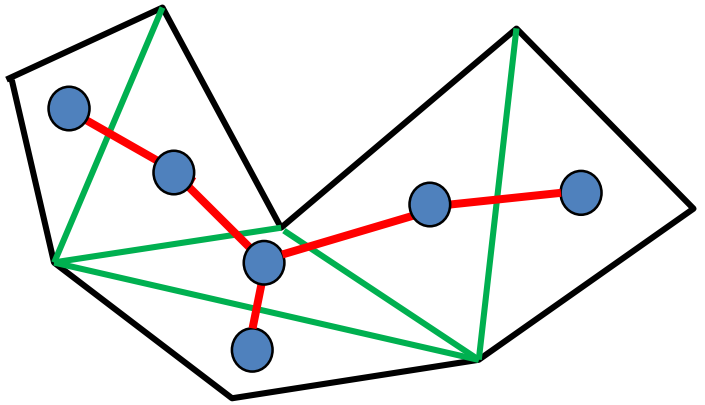
Konvex csúcs

Fül



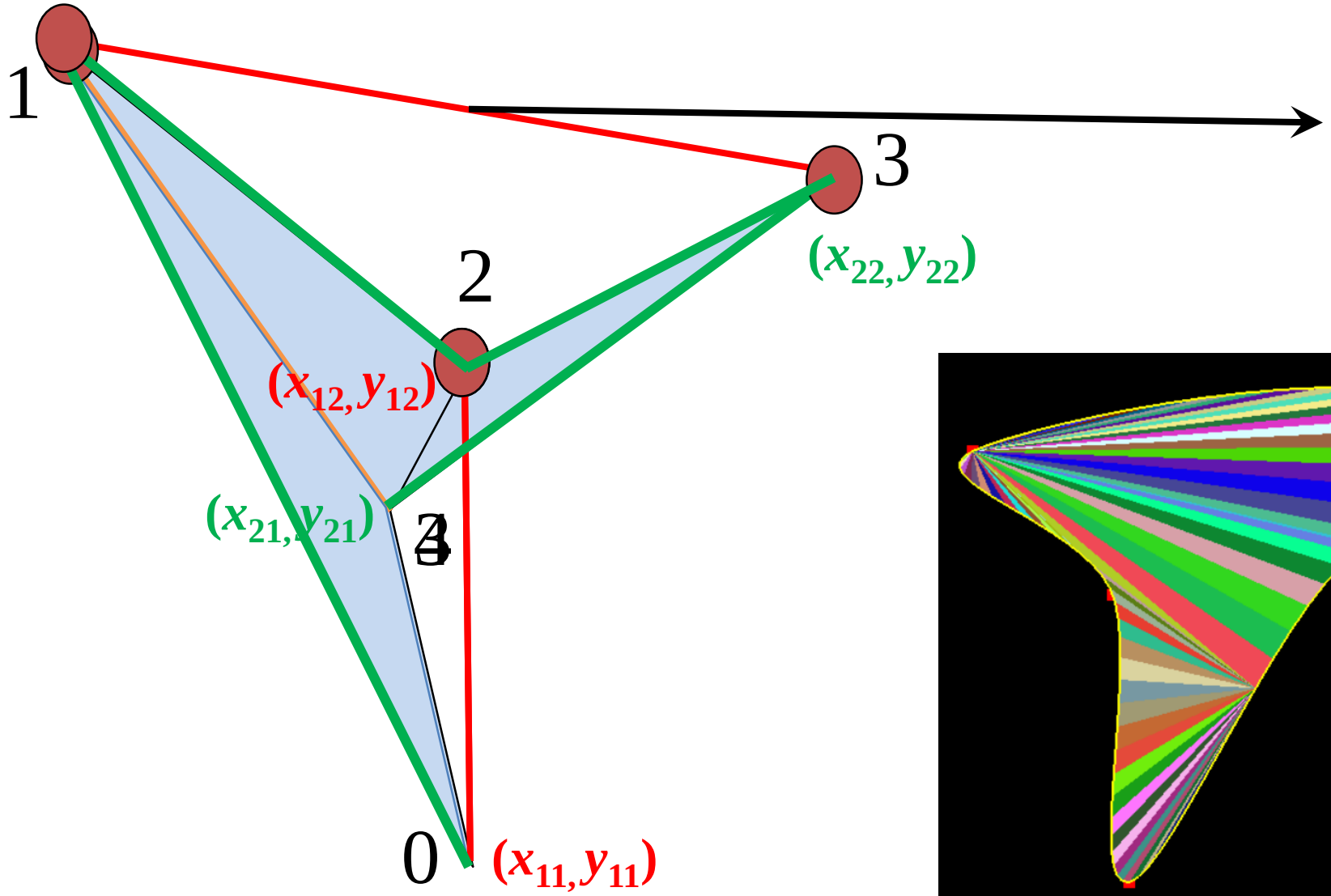
- p_i fül, ha $p_{i-1} \leftrightarrow p_{i+1}$ diagonál
- Fül levágható!
- **Fülvágás**: keress fület és nyissz!
- $O(n^3)$

Két fül tétel: Minden legalább 4 csúcsú egyszerű sokszögnek van legalább 2 füle.



„Minden fának van legalább két levele.”

Fülvágó algoritmus: $O(n^3)$



Szakasz-szakasz metszés

Algebrai megoldás:

$x_1(t_1) = x_{11}t_1 + x_{12}(1 - t_1)$

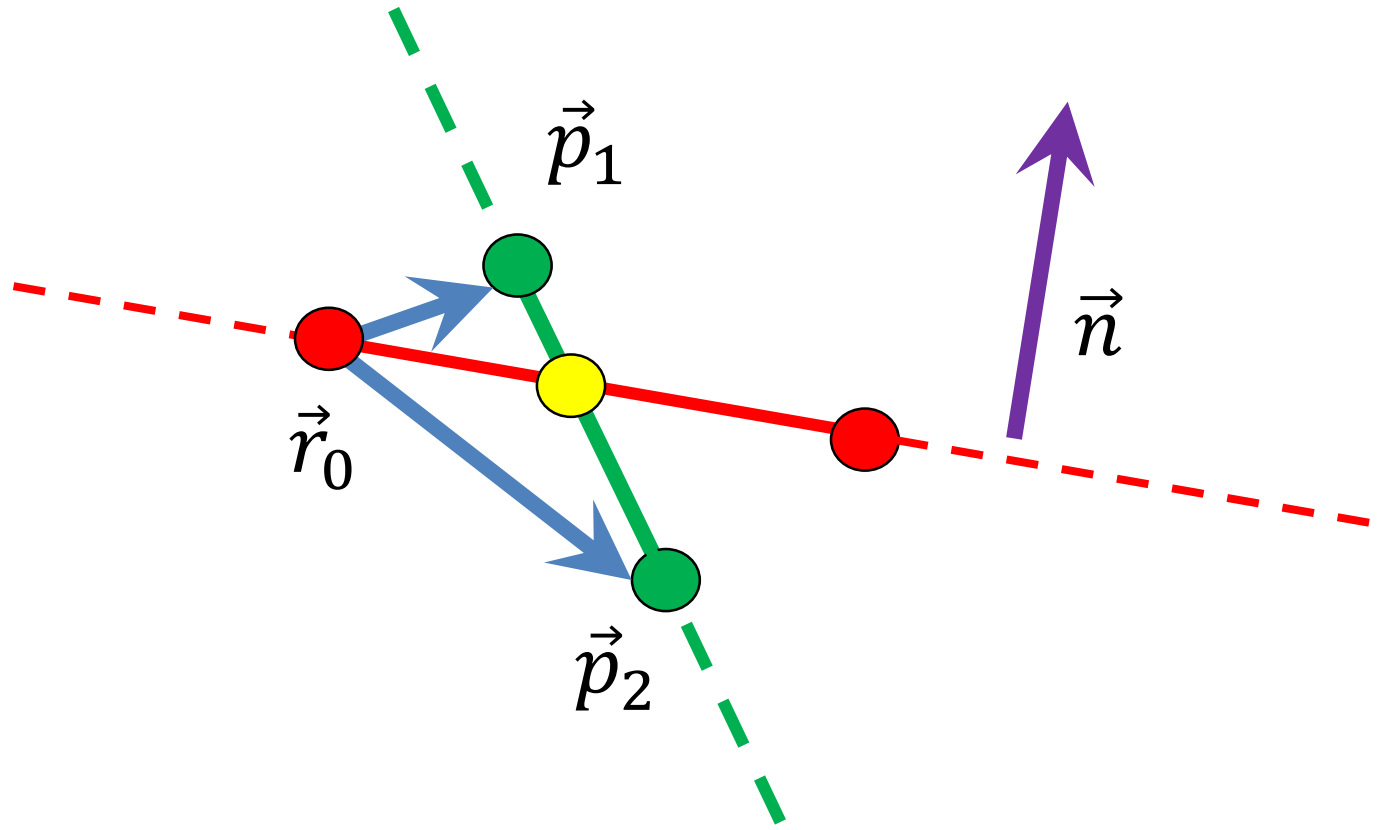
$y_1(t_1) = y_{11}t_1 + y_{12}(1 - t_1), t_1 \in (0,1)$

$x_2(t_2) = x_{21}t_2 + x_{22}(1 - t_2)$

$y_2(t_2) = y_{21}t_2 + y_{22}(1 - t_2), t_2 \in (0,1)$

$x_1(t_1) = x_2(t_2)$

$y_1(t_1) = y_2(t_2)$? $t_1, t_2 \in (0,1)$



$$(\vec{n} \cdot (\vec{p}_1 - \vec{r}_0))\ (\vec{n} \cdot (\vec{p}_2 - \vec{r}_0)) < 0$$

“For geometry, you know, is the gate of science, and the gate is so low and small that we can only enter it as a little child.”

William Kingdon Clifford

2D képszintézis
3. Transzformációk és vágás

Szirmay-Kalos László

Modellezési transzformáció

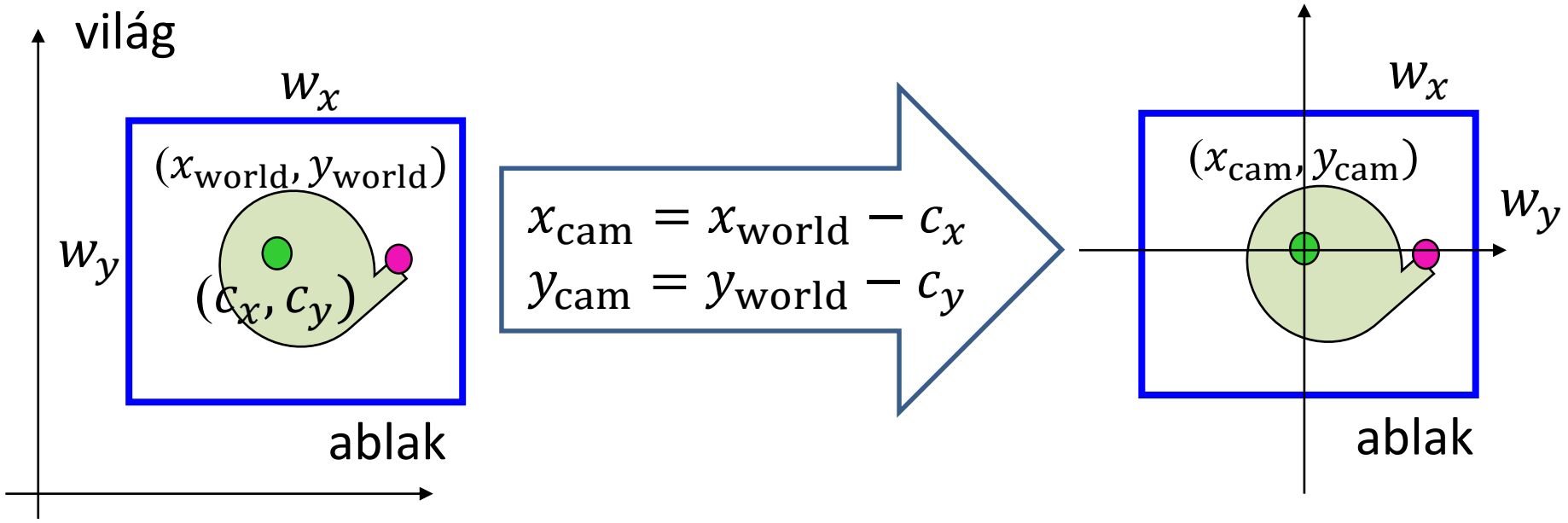
- Mátrixokat a CPU-n számítjuk, a transzformációt a GPU hajtja végre
- Homogén lineáris transzformáció:

$$\begin{bmatrix} x_{\text{world}} \\ y_{\text{world}} \\ z_{\text{world}} \\ 1 \end{bmatrix} = \boldsymbol{T}_{4\times4} \cdot \begin{bmatrix} x_{\text{model}} \\ y_{\text{model}} \\ z_{\text{model}} \\ 1 \end{bmatrix}$$

- Példa: skálázás, forgatás, eltolás:

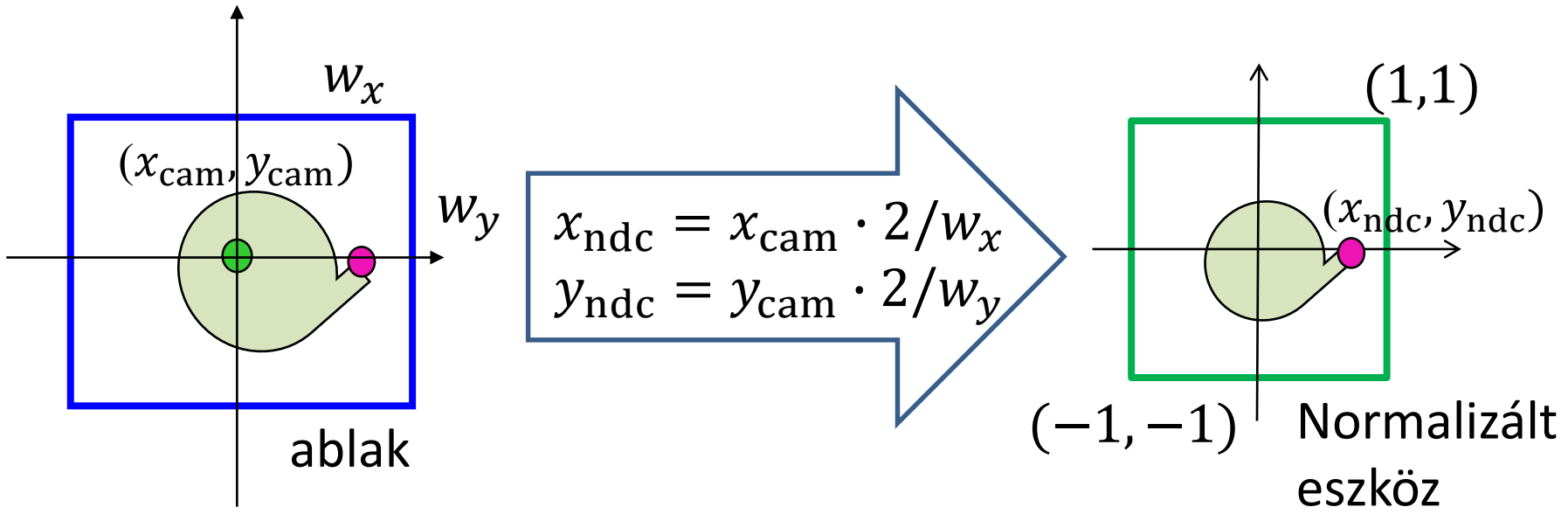
$$\boldsymbol{T}_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

View transzformáció: V()
Kameraablak közepe az origóba



$$\begin{bmatrix} x_{\text{cam}} \\ y_{\text{cam}} \\ z_{\text{cam}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -c_x \\ 0 & 1 & 0 & -c_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{world}} \\ y_{\text{world}} \\ z_{\text{world}} \\ 1 \end{bmatrix}$$

Projekció: P()
Kameraablak a (-1, -1)-(1, 1) négyzetbe



$$\begin{bmatrix} x_{\text{ndc}} \\ y_{\text{ndc}} \\ z_{\text{ndc}} \\ 1 \end{bmatrix} = \begin{bmatrix} 2/w_x & 0 & 0 & 0 \\ 0 & 2/w_y & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{cam}} \\ y_{\text{cam}} \\ z_{\text{cam}} \\ 1 \end{bmatrix}$$

2D kamera

```
class Camera2D {
   vec2 wCenter; // center in world coords
   vec2 wSize;   // width and height in world coords

public:
   mat4 V() { return translate(vec3(-wCenter.x, -wCenter.y, 0)); }

   mat4 P() { // projection matrix
      return scale(vec3(2/wSize.x, 2/wSize.y, 1));
   }
   mat4 Vinv() { // inverse view matrix
       return translate(vec3(wCenter.x, wCenter.y, 0));
   }
   mat4 Pinv() { // inverse projection matrix
      return scale(vec3(wSize.x/2, wSize.y/2, 1));
   }
   void Zoom(float s) { wSize = wSize * s; }
   void Pan(vec2 t) { wCenter = wCenter + t; }
};
```

Geometry

```
template<class T> class Geometry {
   unsigned int vao, vbo; // GPU
   vector<T> vtx;         // CPU
public:
   Geometry() { glGenVertexArrays(1, &vao); glBindVertexArray(vao); glGenBuffers(1, &vbo); }
   void updateGPU() {
      glBindVertexArray(vao); glBindBuffer(GL_ARRAY_BUFFER, vbo);
      glBufferData(GL_ARRAY_BUFFER, vtx.size() * sizeof(T), &vtx[0], GL_DYNAMIC_DRAW);
      glEnableVertexAttribArray(0);
      int nf = sizeof(T)/sizeof(float);
      if (nf <= 4) glVertexAttribPointer(0, nf, GL_FLOAT, GL_FALSE, 0, NULL);
   }
   void Bind() { glBindVertexArray(vao); }
   vector<T>& Vtx() { return vtx; }
   void Draw(GPUProgram* gpuProgram, int type, vec3 color) {
      if (vtx.size() > 0) {
         gpuProgram->setUniform(color, "color");
         Bind(); glDrawArrays(type, 0, (int)vtx.size());
      }
   }
   virtual ~Geometry() { glDeleteBuffers(1, &vbo); glDeleteVertexArrays(1, &vao); }
};
```

2D Object

```
class Object : public Geometry<vec2> {
protected:
   vec2  scaling = vec2(1, 1), pos = vec2(0, 0);
   float phi = 0;          // rotation
public:
   // vectorization, ear clipping, etc.
   virtual vector<vec2> GenVertexData() = 0;
   void update() {
      Vtx() = GenVertexData();
      updateGPU();
   }
   void Draw(GPUProgram* gpuProgram, int type, vec3 color, Camera& camera) {
      mat4 M = translate(pos) * rotate(phi, vec3(0, 0, 1)) * scale(scaling);
      mat4 MVP = camera.P() * camera.V() * M;
      gpuProgram->setUniform(MVP, "MVP");
      Draw(gpuProgram, type, color);
   }
};
```

Csúcspont és pixel árnyalók

Vertex shader:

```
uniform mat4 MVP;
layout(location = 0) in vec2 vertexPosition;

void main() {
    gl_Position = MVP * vec4(vertexPosition, 0, 1);
}
```

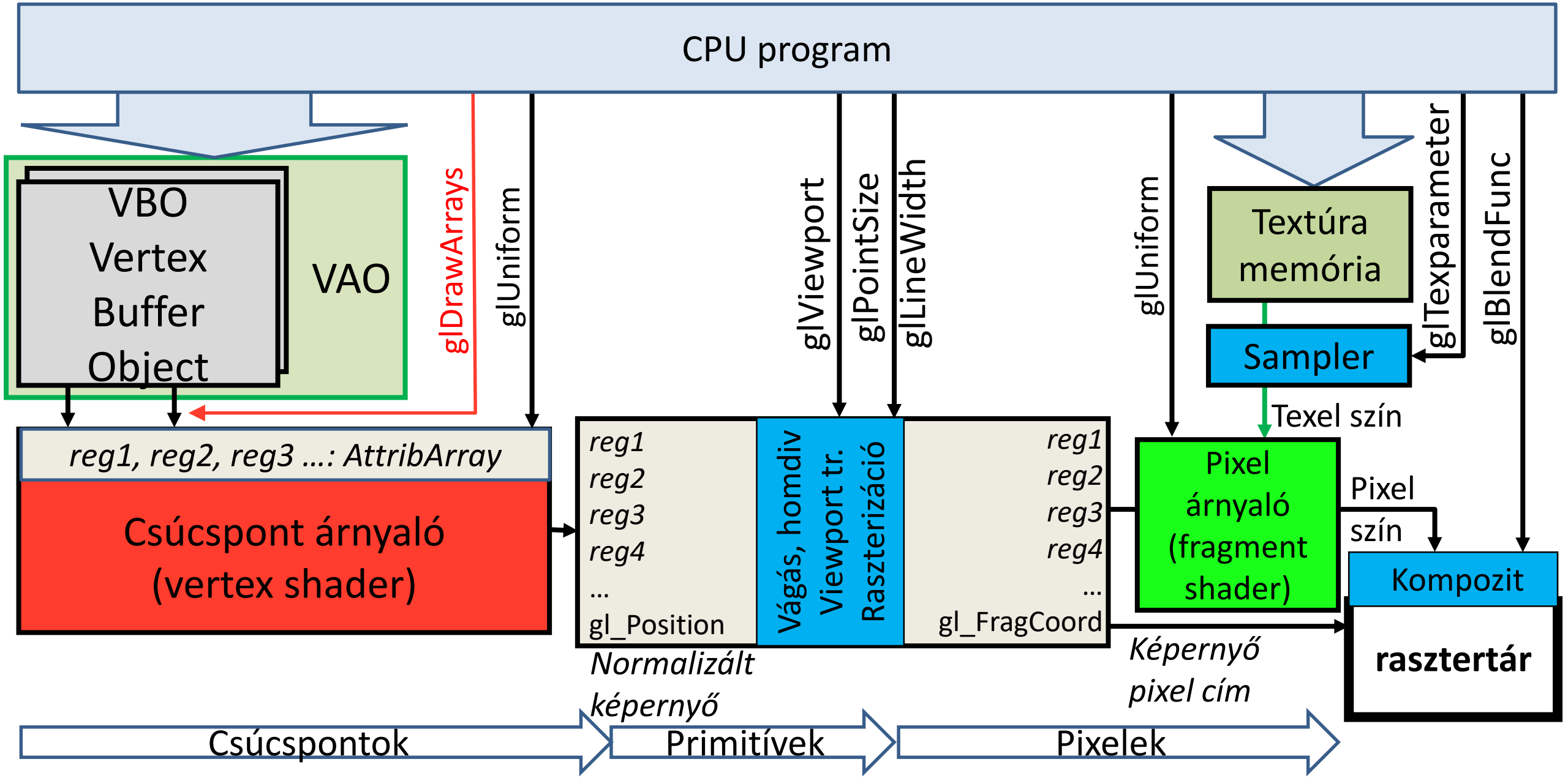
Fragment shader:

```
uniform vec3 color;
out vec4 fragmentColor;

void main() {
    fragmentColor = vec4(color, 1);
}
```

Példa: Bézier görbe

```
const int nTessVertices = 100;

class BezierCurve : public Object {
   vector<vec2> cps;   // control pts
   float B(int i, float t) {
        float choose = 1;
        for(int j = 1; j <= i; j++) choose *= (float)(cps.size()-j)/j;
        return choose * pow(t, i) * pow(1-t, cps.size()-1-i);
   }
public:
   void AddControlPoint(vec2 cp) { cps.push_back(cp); update(); }
   vec2 r(float t) {
        vec2 rt(0, 0);
        for(int i = 0; i < cps.size(); i++) rt += cps[i] * B(i,t);
        return rt;
   }
   vector<vec2> GenVertexData() { // vectorization
        vector<vec2> vertices;
        for(int i = 0; i <= nTessVertices; ++i) {
           float t = (float)i / nTessVertices;
           vertices.push_back(r(t));
        }
        return vertices;
   }
};
```

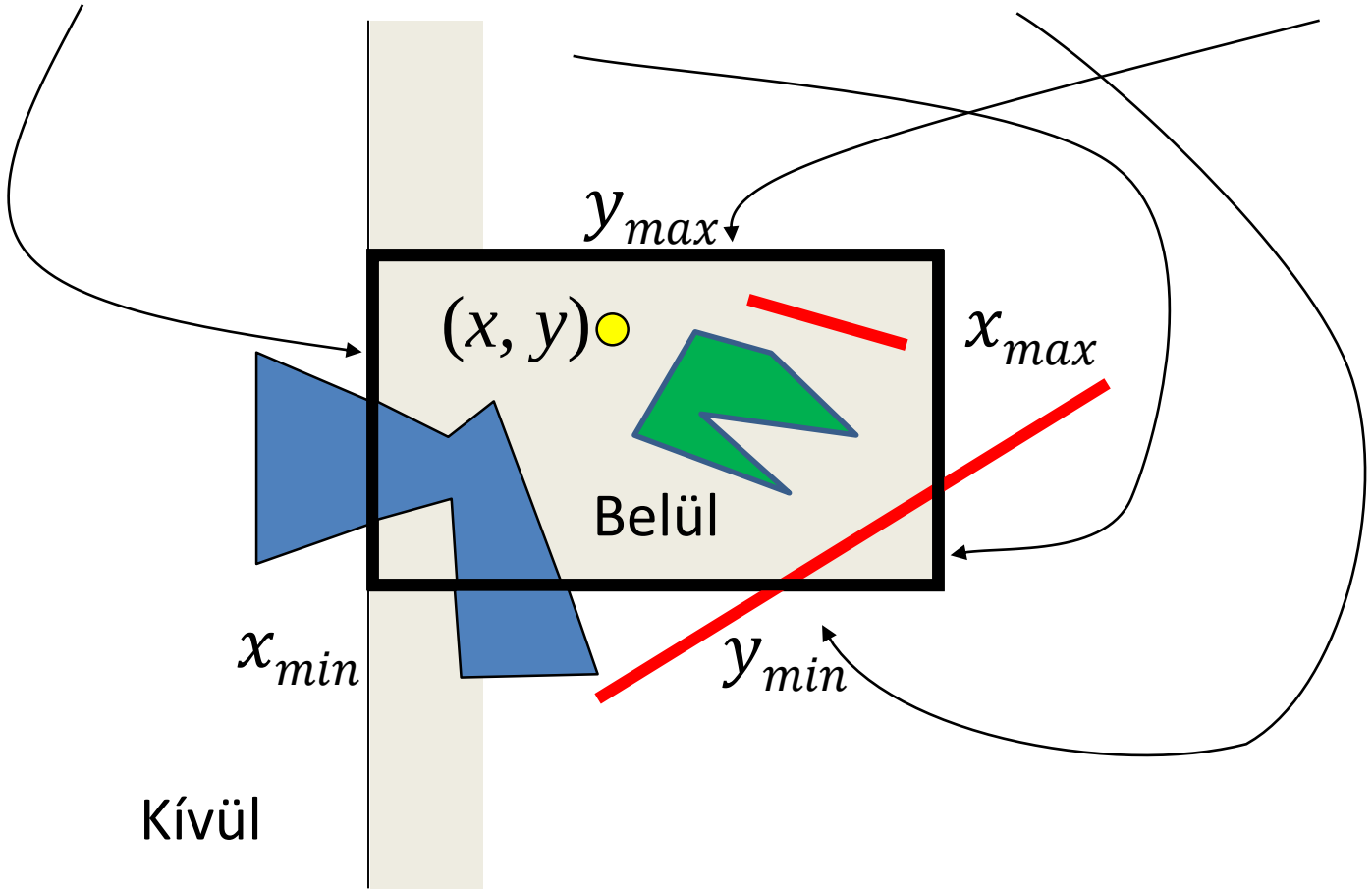
OpenGL 3.3 … 4.6 (Modern OpenGL)



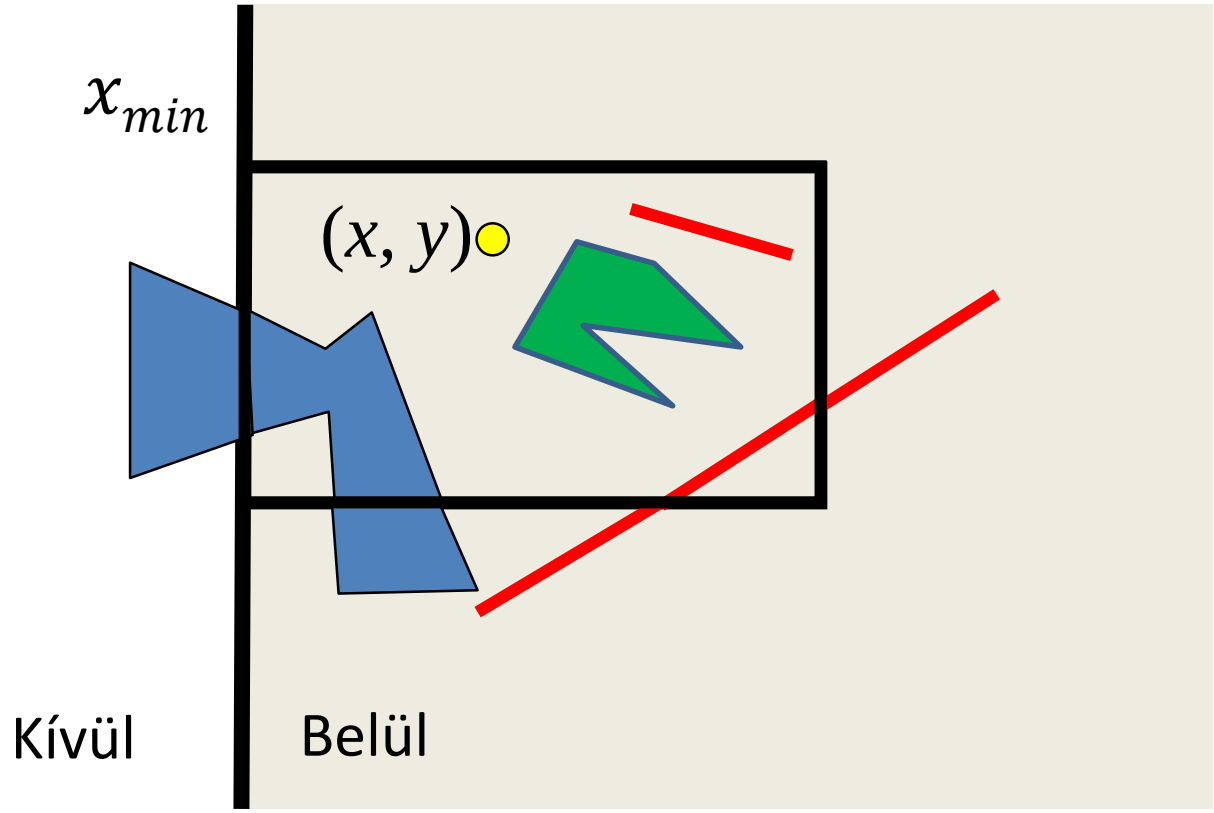
2D vágás

Pont vágás:

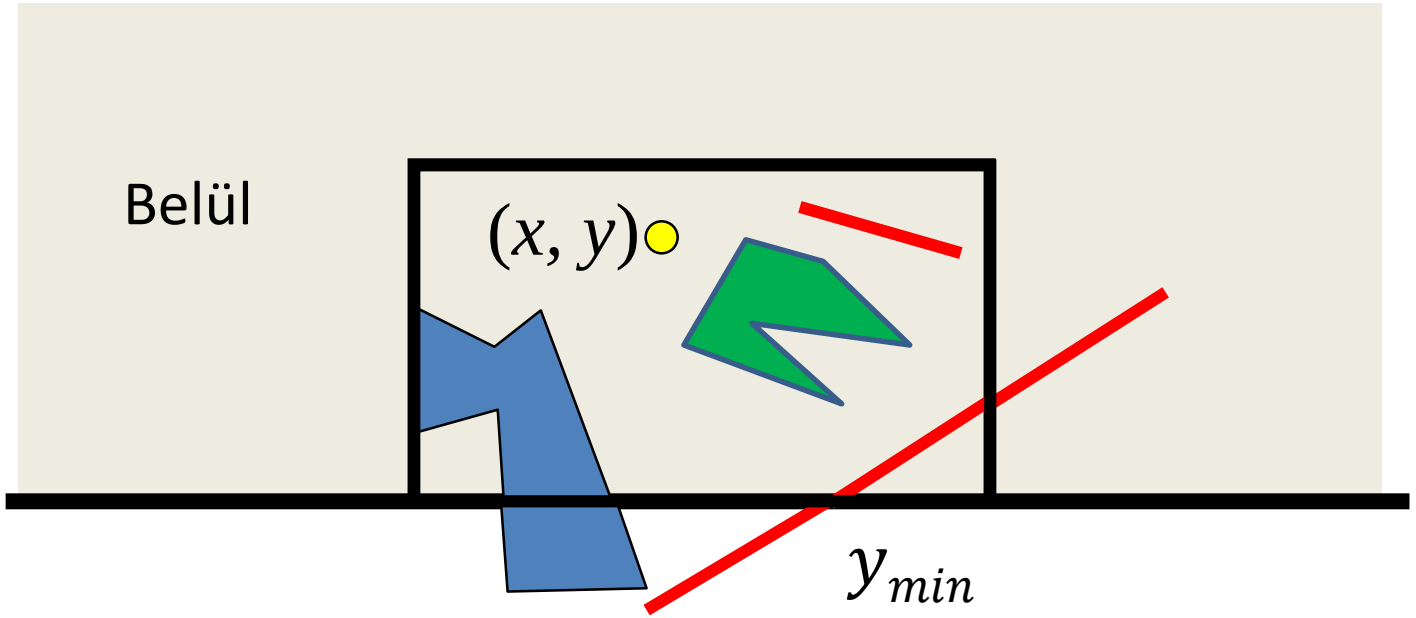
$$x > x_{min} = -1,\ x < x_{max} = +1,\quad y > y_{min} = -1,\ y < y_{max} = +1$$



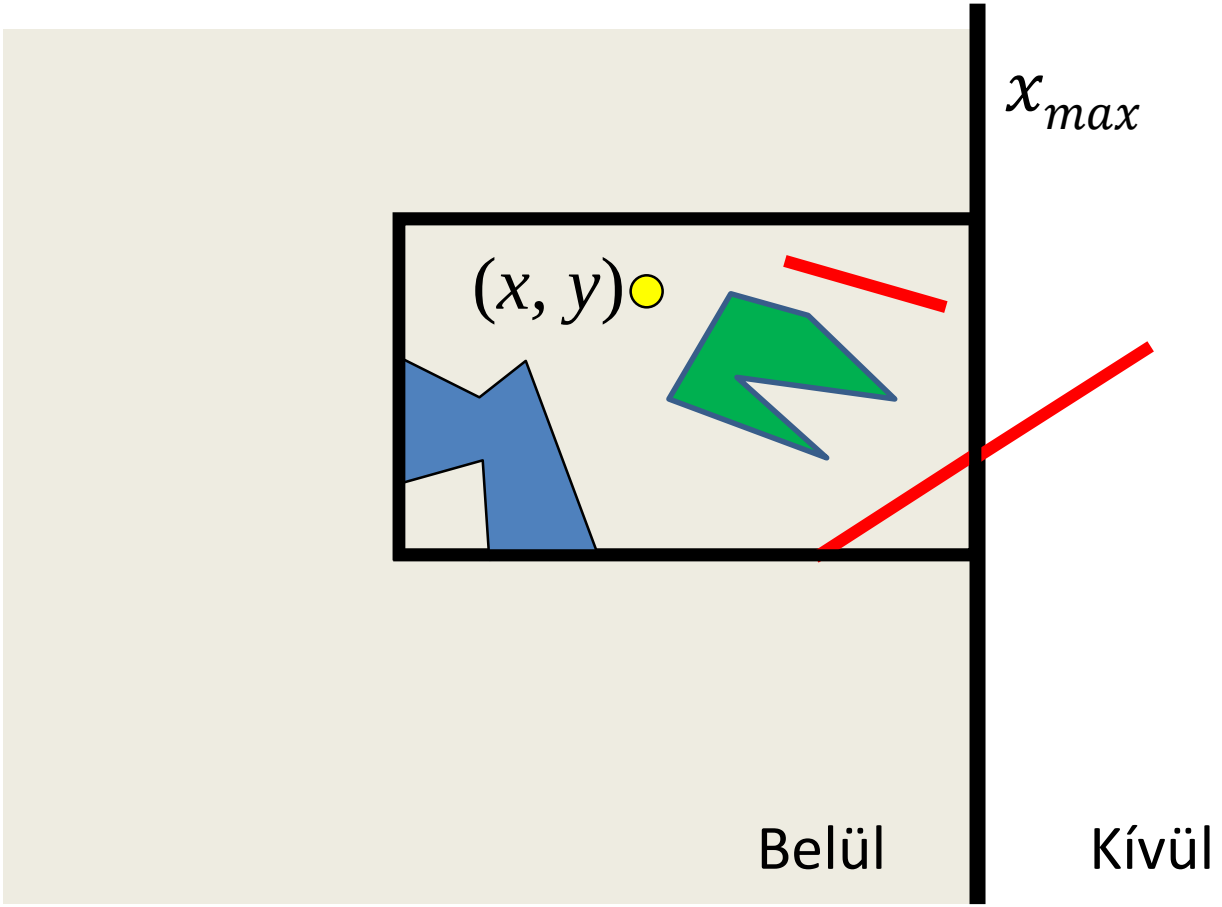
Vágás



Vágás

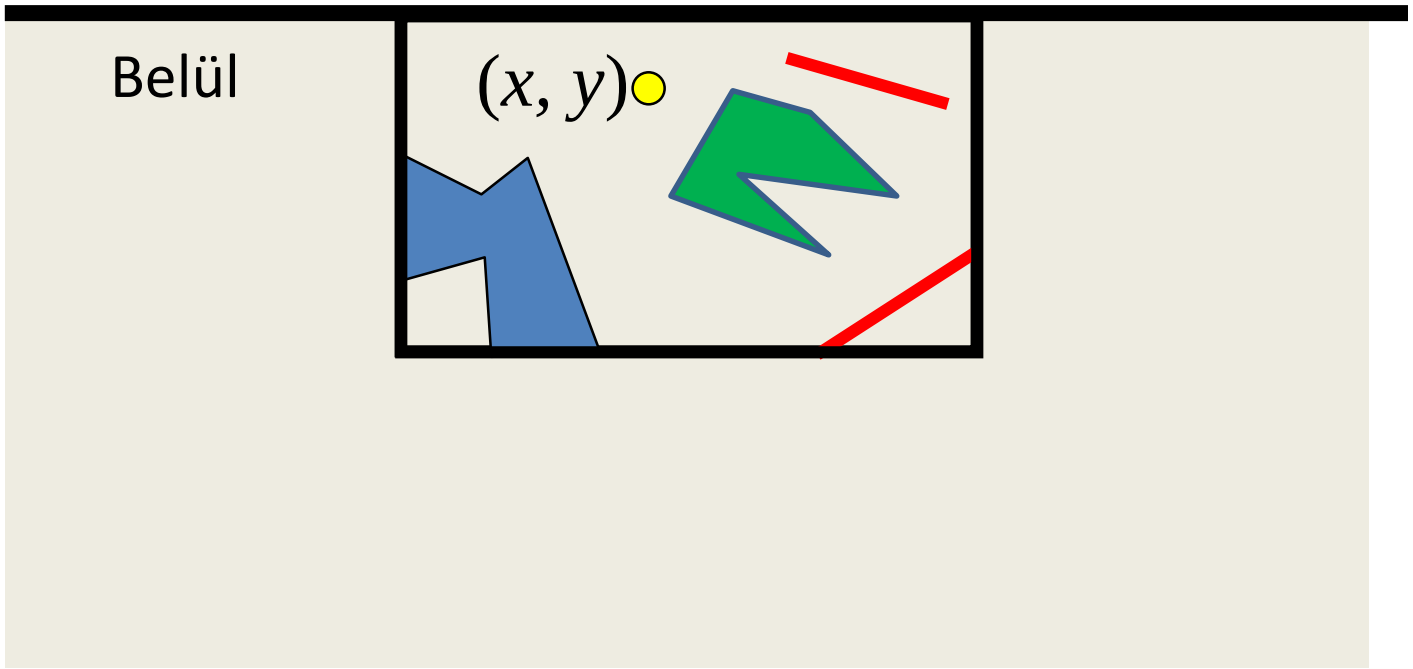


Vágás

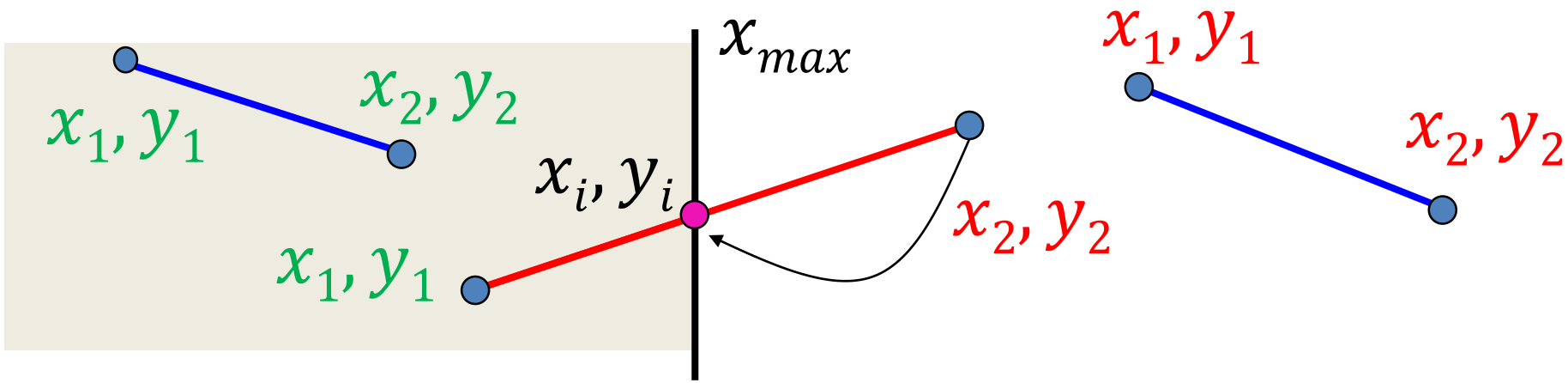


Vágás

Kívül



2D szakasz vágás: $x < x_{max}$

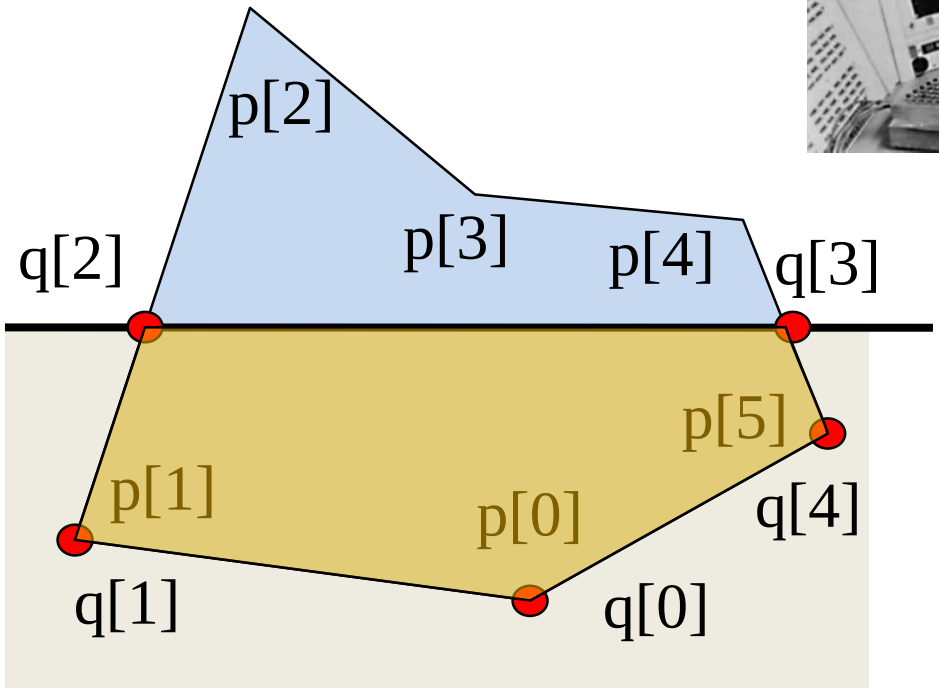


$$x(t) = x_1 + (x_2 - x_1)t, \qquad y(t) = y_1 + (y_2 - y_1)t$$
$$x = x_{max}$$

Metszés: $x_{max} = x_1 + (x_2 - x_1)t \Rightarrow t = (x_{max} - x_1)/(x_2 - x_1)$

$$x_i = x_{max} \qquad y_i = y_1 + (y_2 - y_1)\,(x_{max} - x_1)/(x_2 - x_1)$$

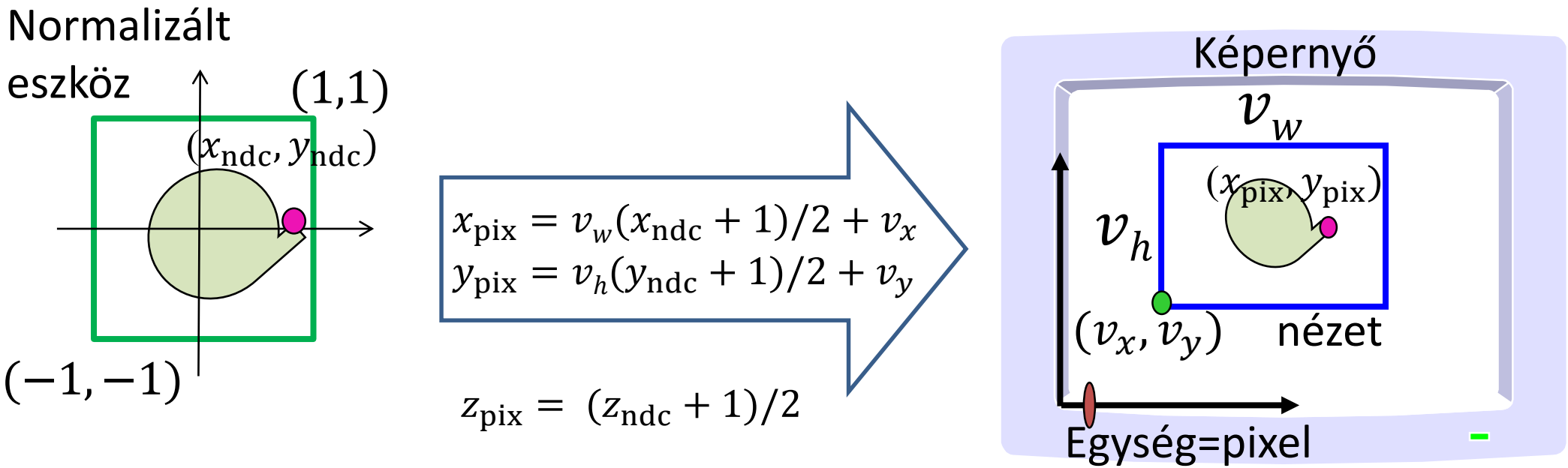
(Ivan) Sutherland-(Gary) Hodgman poligonvágás




```
PolygonClip(p[n] ⇨ q[m])
    m = 0;
    for(i=0; i < n; i++) {
        if (p[i] belső) {
            q[m++] = p[i];
            if (p[i+1] külső)
                q[m++] = Intersect(p[i], p[i+1], vágóegyenes);
        } else {
            if  (p[i+1] belső)
                q[m++] = Intersect(p[i], p[i+1], vágóegyenes);
        }
    }
}
```

Első pontot még egyszer
a tömb végére

Viewport transzformáció: Normalizáltból képernyő koordinátákba (GPU)



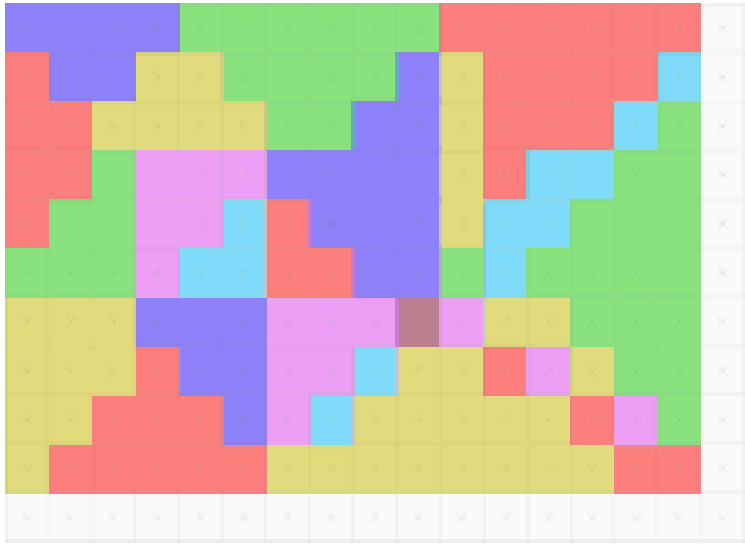

```
glViewport(vx, vy, vw, vh)
```

“The computer was born to solve problems that did not exist before.”

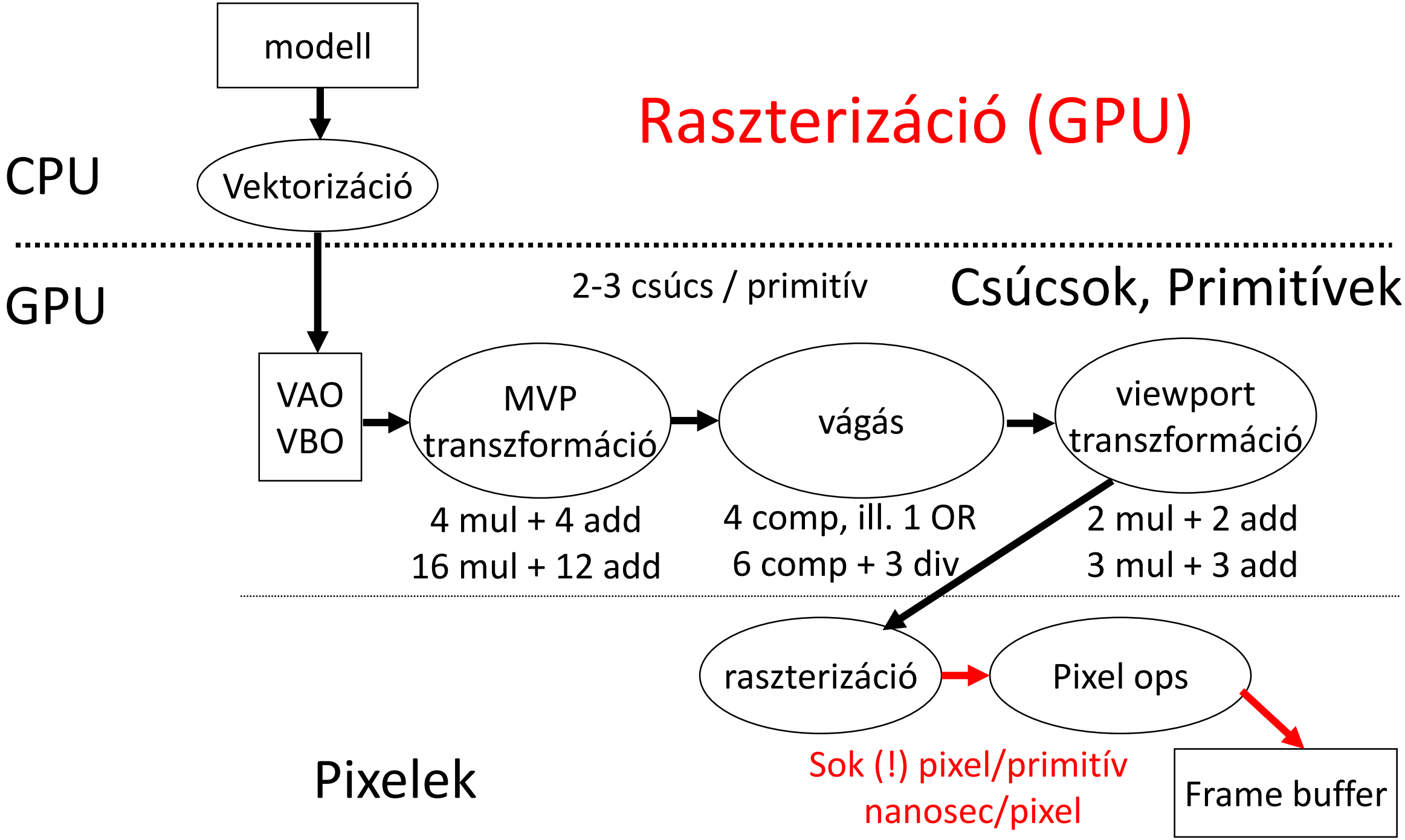
Bill Gates

2D képszintézis
4. Raszterizáció

Szirmay-Kalos László



Rasterizáció (GPU)

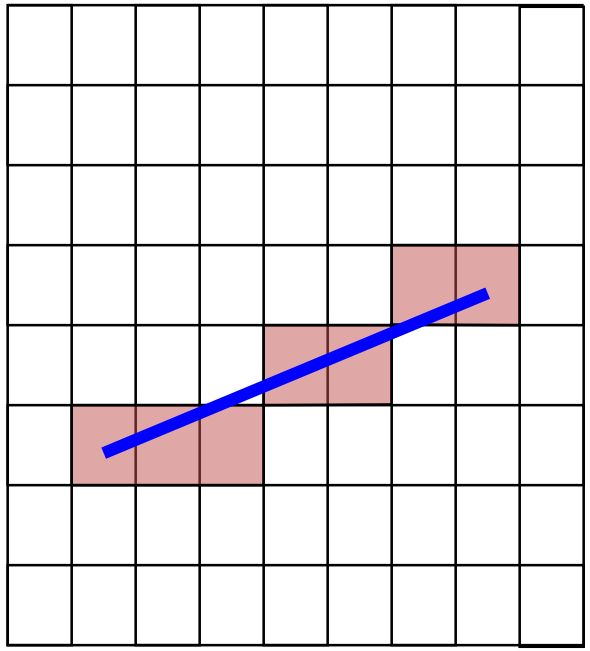


Pont rajzolás

A pont „**kicsi**” és van jellemző helye:

- A kiszínezett tartomány is legyen kicsi
- A legkisebb dolog, amit át lehet színezni, a pixel
- Színezzünk ki egy (vagy néhány) pixelt, amely legközelebb van a ponthoz
- Pixelkoordináták egészek
- **Legközelebbi pixel** = koordináták kerekítése

Szakasz rajzolás



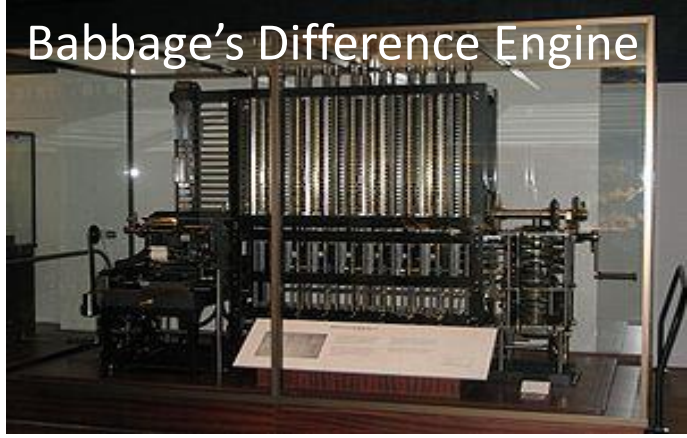
Egyenes „**vékony**” és **összefüggő**.
Pontjai kielégítik az egyenletét:

$$y = mx + b$$

x2 > x1, |x2-x1| >= |y2-y1|
típusú szakasz rajzolása:

```
float m = (float)(y2-y1)/(x2-x1);
for(int x = x1; x <= x2; x++) {
      float y = m*x + b;
      int Y = round( y );
      write( x, Y );
}
```

Inkrementális elv és fixpontos számítás

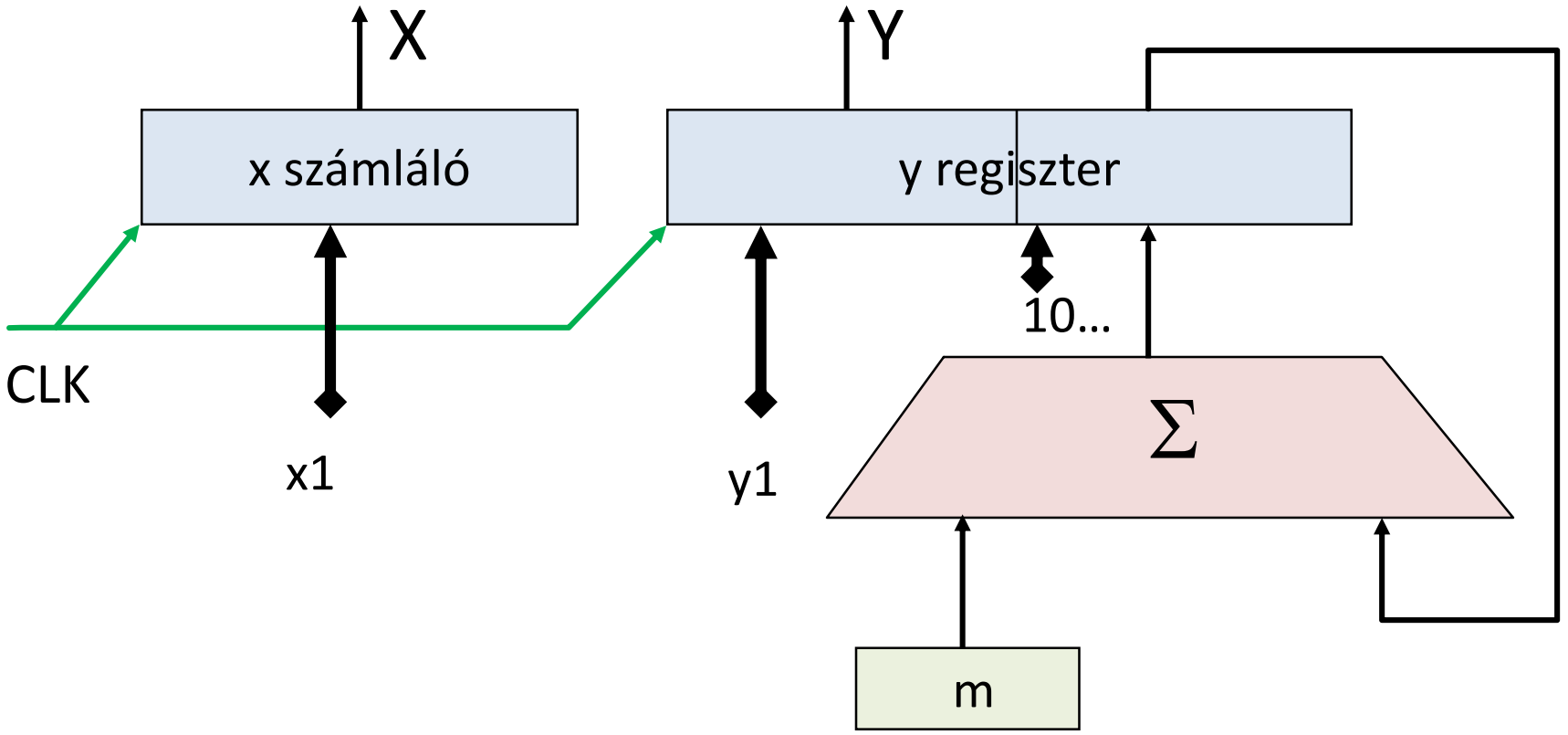


$$y(x) = mx + b = y(x-1) + m$$

```
LineFloat (short x1, short y1,
           short x2, short y2) {
    float m = (float)(y2-y1)/(x2-x1);
    float y = y1;
    for(short x = x1; x <= x2; x++) {
        short Y = round(y);
        write(x, Y, color);
        y = y+m;
    }
}
```

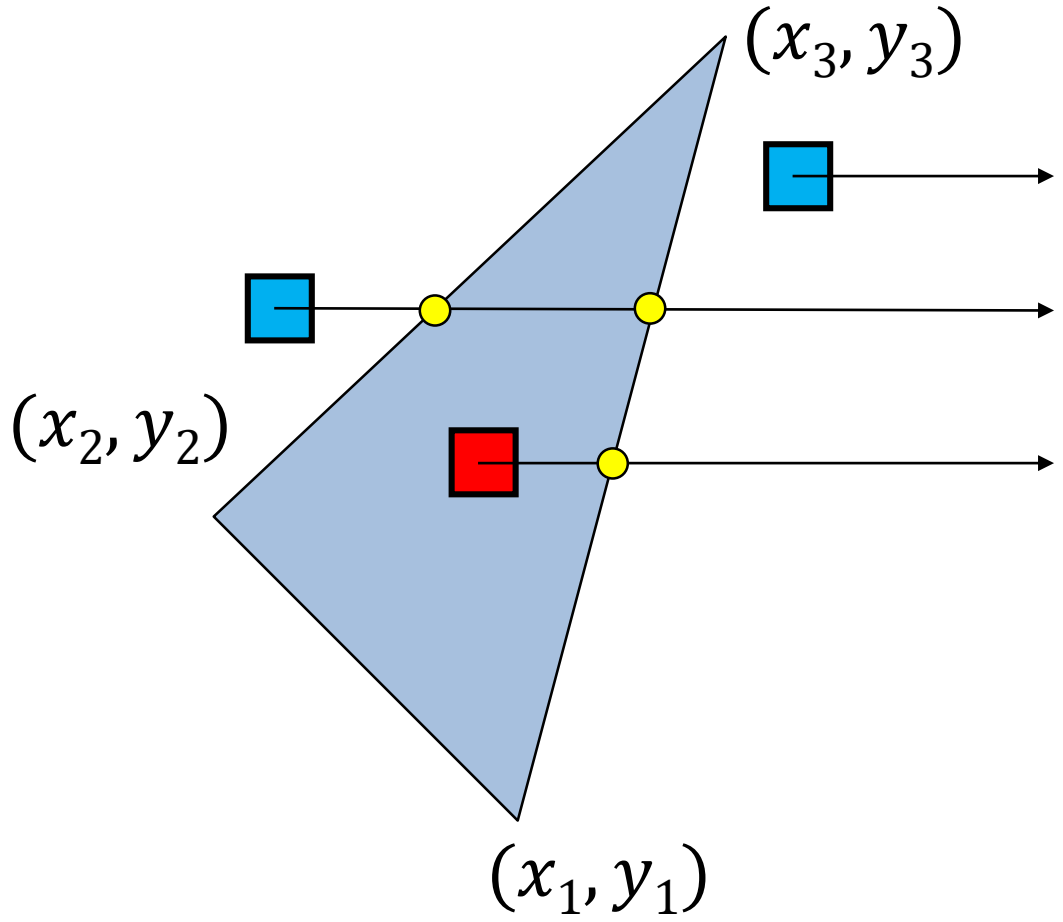
```
const int T=12; // fractional bits

LineFix (short x1, short y1,
         short x2, short y2) {
    int m = ((y2 - y1)<<T)/(x2 - x1);
    int y = (y1<<T)+(1<<(T-1)); // +0.5
    for(short x = x1; x <= x2; x++) {
        short Y = y >> T;          // trunc
        write(x, Y, color);
        y = y+m;
    }
}
```

DDA szakaszrajzoló hardver

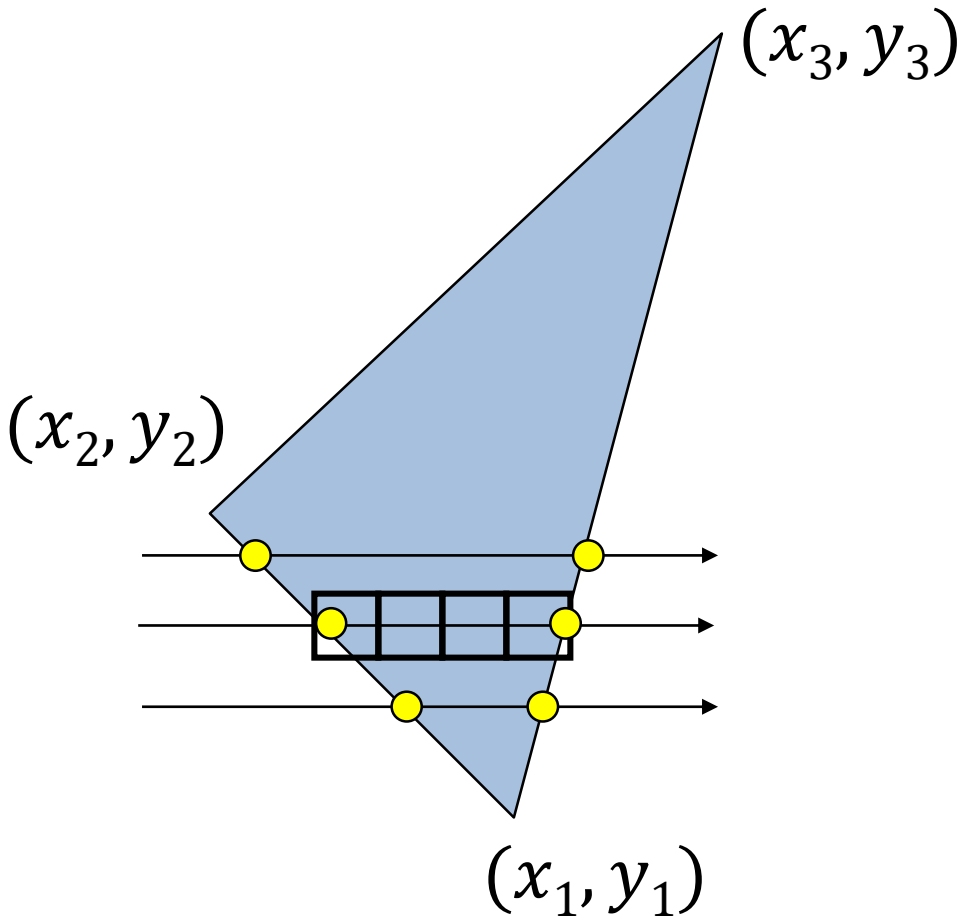


Algoritmikus fotográfia

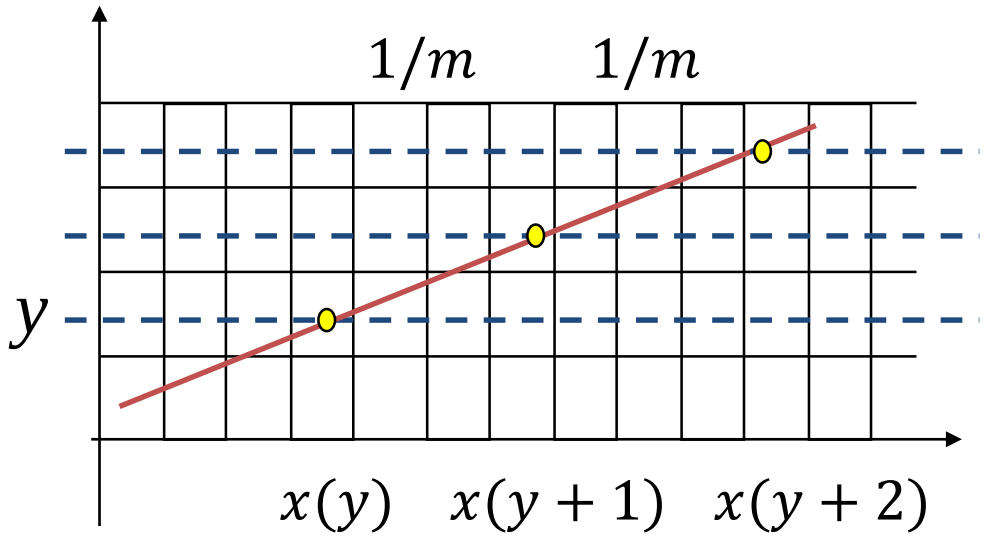
Naív háromszög kitöltés



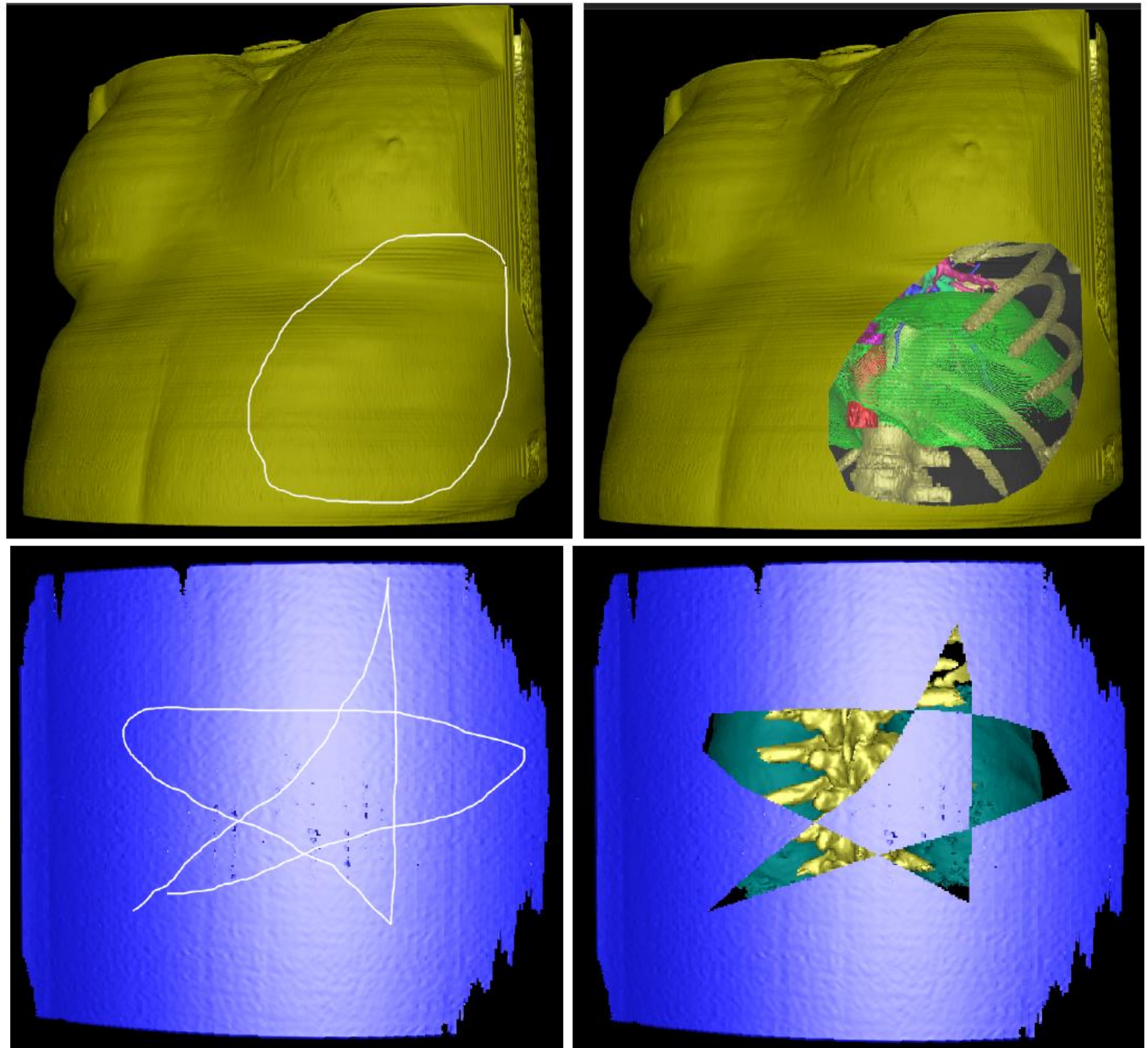
Koherencia és inkrementális elv



$$y = mx + b$$

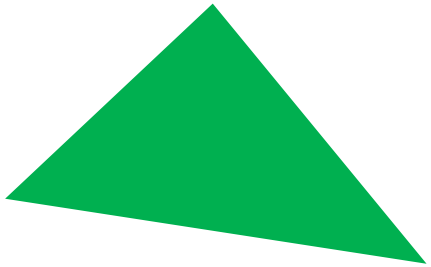


$$x(y+1) = x(y) + 1/m$$

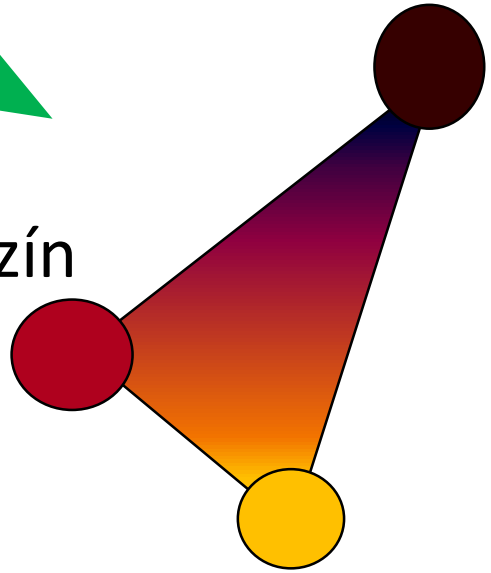


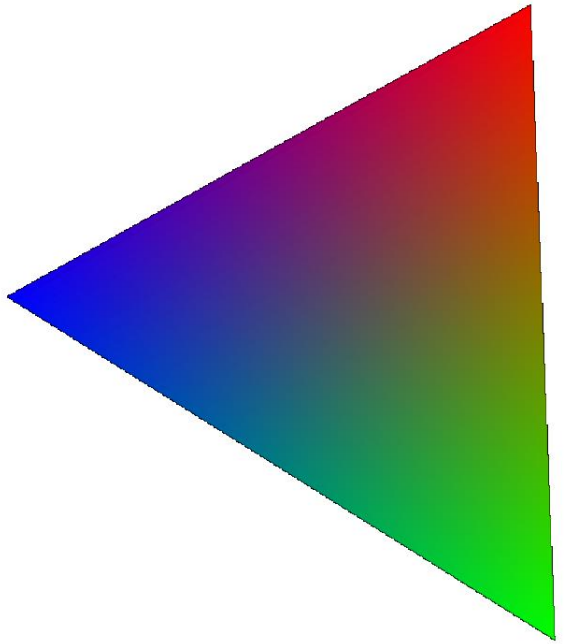
Milyen színű legyen a pixel?

- Uniform az egész objektum



- Csúcspont tulajdonságokból interpolált szín





- Pl. Textúrázás (2D): Csúcspont textúrakoordináták interpolációja a pixelekre, majd textúra kiolvasás.