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Fuzzy Sliding Mode Control for a Quadrotor UAV

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Abstract: In this paper, a fuzzy sliding mode control (FSMC) method is presented for the altitude and attitude tracking problem of a quadortor unmanned aerial vehicle (UAV) system. The nonlinear dynamic model of the quadrotor UAV is first established based on the Euler-Lagrange method. Then, a sliding mode controller (SMC) is designed based on the nonlinear model. Further, to address the severe chattering phenomenon associated with the pure SMC, additional fuzzy control is adopted to adjust the switching gain. Finally, the proposed controller is tested via Matlab/Simulink. The results show that the control performance of FSMC is superior to PID control and fuzzy PID control in the sense of tracking accuracy and suppression of switching chattering.

Key Words: Quadrotor, Unmanned aerial vehicle, Altitude and attitude tracking, Fuzzy sliding mode control

1 Introduction

In recent years, the quadrotor UAV has gradually become a research hotspot in the aviation field. Quadrotor UAV has unique advantages in many aspects, including vertical take-Off and landing (VTOL) as well as hovering. At present, UAV has a wide range of applications in all kinds of important occasions, e.g., collection of photogrammetry images[1], inspection missions of railways[2], road detection and tracking[3], traffic management[4], and monitoring power lines [5]. In particular, they can perform tasks efficiently in dangerous conditions without any aid of human beings.

Study on UAV is many-sided and comprehensive, and many control schemes have been proposed for path planning[6], formation flying[7, 8], etc. Among them, proportional-integral-derivative (PID) control is the most commonly used approach due to its simple structure and acceptable control performance. In other words, PID control is still the preferred method in actual control for quadrotor systems[9, 10]. Nevertheless, in principle, the gain tuning for PID control is difficult and highly depends on the experience of the debugger, by noticing that the model of UAV is nonlinear, strongly coupled and underactuated.

Meanwhile, as another well-established control framework, sliding mode control (SMC) has also been introduced for position and attitude control of UAV[11–13]. More specifically, second-order SMC is presented in [14, 15] for the stable control of UAV in order to overcome the chattering phenomena, while preserving the invariance property of sliding mode. In[16], three nonlinear control techniques including SMC, model reference adaptive control (MRAC) and adaptive SMC are applied for quadrotor system, respectively, where numerical and experimental results reveal that the adaptive SMC is the most promising one when tracking precision, robustness, and computational time are of concern. In addition, SMC is combined with linear quadratic regulator in [17], with backstepping design in [18, 19] for UAV control, achieving remarkable tracking performance.

To address more practical implementation issues, fault tolerant control (FTC) of UAV has become more and more popular. In [20], a method of FTC for a quadrotor has been introduced so as to realize a good tolerance to faults without requiring explicit detection. To enhance the adaptive capability of FTC, [21] considers a new FTC scheme based on nonlinear adaptive observer, which is developed to compensate for the partial loss of actuators effectiveness. Further, in[22], small time local controllability of the attitude dynamics of UAV is analyzed using the nonlinear controllability theory with unilateral control inputs, by assuming that one or several actuators' fault occurs.

The main contribution of the paper is to develop a fuzzy sliding mode control (FSMC) approach for a quadrotor UAV system, which can successfully reduce chattering in implementation. Traditional SMC has an outstanding control performance for nonlinear dynamics models, which however tends to produce extremely detrimental chattering to the actual system. Thorough analysis of the structure of SMC renders to that the generation of the sliding mode chattering is closely related to the switching gain. Therefore, fuzzy control is used to achieve intelligent adjustment for the switching gain in our work, which can not only maintain the remarkable performance of traditional SMC, but also reduce chattering opportunely.

This paper is organized as follows. Section 2 presents the mathematical model of UAV. The details of controller design for the UAV are described in Section 3. In Section 4, the proposed controller is tested via simulation. Section 5 concludes the work.

2 Mathmatical Modelling for the UAV

In this section, the mathematical model and control principle of the quadrotor UAV is introduced. The schematic diagram of the UAV is shown in Fig.1, where E and B represent the earth-frame and body-frame, respectively. In flight experiment of quadrotor UAV, Rotor 1 and Rotor 3 are controlled simultaneously, while Rotor 2 and Rotor 4 are controlled together. It is worth noticing that the left or right motion of the quadrotor UAV will create a roll angle by increasing (decreasing) the speeds of Rotors 1 and 3, and decreasing (increasing) the speeds of Rotors 2 and 4. Meanwhile, in order to obtain the pitch angle around the x-axis, the UAV can be controlled by increasing (decreasing) the speeds of Rotors 1 and 2, and decreasing (increasing) the speeds of Rotors 3 and 4. Further, to rotate the quadrotor around the z-axis, it is suggested to increase (decrease) the speeds of Rotors 1

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and 4, and decrease (increase) the speeds of Rotors 2 and 3. In addition, taking-off/landing task can be accomplished by increasing or decreasing all the rotors' speed uniformly.

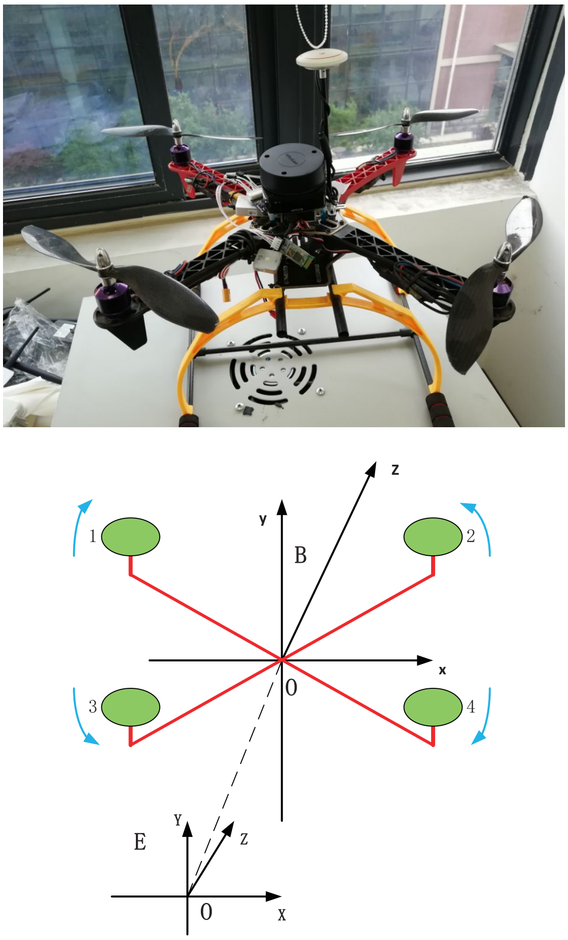

Fig. 1: Diagram of quadrotor UAV

As usual, the following hypothesises are given to make the consequent analysis and controller design rigorous.

- The structure of UAV is rigid, and there are no internal forces or deformations.
- The structure of UAV is symmetrical.
- The center of mass and the body frame origin are assumed to be coincided.
- The earth frame is the inertial frame.

The six degrees of freedom of the quadrotor UAV are defined as:

$$V = [x, y, z, \phi, \theta, \varphi] \in \mathcal{R}^6, \tag{1}$$

where $[x, y, z]$ is the position vector relative to the inertial frame E, and the three Euler angles of the quadrotor UAV system, $[\phi, \theta, \varphi]$, denote the roll angle around the y-axis, the pitch angle around the x-axis as well as the yaw angle around the z-axis, respectively.

The dynamical model of the quadrotor UAV can be described by the following two sets of equations.

$$\begin{cases} \ddot{x} = \dfrac{U_1(\sin\theta\cos\phi\cos\varphi + \sin\phi\sin\varphi) - k_x\dot{x}}{m}, & \text{(2a)} \\ \ddot{y} = \dfrac{U_1(\sin\theta\cos\phi\sin\varphi - \sin\phi\cos\varphi) - k_y\dot{y}}{m}, & \text{(2b)} \\ \ddot{z} = \dfrac{U_1(\cos\phi\cos\theta) - k_z\dot{z} - mg}{m}, & \text{(2c)} \end{cases}$$

$$\begin{cases} \ddot{\phi} = \dfrac{(J_y - J_z)\dot{\theta}\dot{\varphi} - J_r\dot{\theta}\Omega_{11} + LU_2}{J_x}, & \text{(3a)} \\ \ddot{\theta} = \dfrac{(J_z - J_x)\dot{\phi}\dot{\varphi} - J_r\dot{\phi}\Omega_{11} + LU_3}{J_y}, & \text{(3b)} \\ \ddot{\varphi} = \dfrac{(J_x - J_y)\dot{\phi}\dot{\theta} + fU_4}{J_z}. & \text{(3c)} \end{cases}$$

In (2) and (3), $U_i, i = 1, \cdots, 4$ are the manipulated inputs of the system, defined as

$$\begin{cases} U_1 = (F_1 + F_2 + F_3 + F_4), & \text{(4a)} \\ U_2 = (F_1 + F_3 - F_2 - F_4), & \text{(4b)} \\ U_3 = (F_3 + F_4 - F_1 - F_2), & \text{(4c)} \\ U_4 = (F_1 + F_4 - F_2 - F_3), & \text{(4d)} \end{cases}$$

where $F_i, i = 1, \cdots, 4$ are the lifting forces generated by the 4 rotors, m the mass of quadrotor, L the arm length of quadrotor, k_x, k_y and k_z the air resistance coefficients in all the three directions, J_x, J_y, J_z the inertias of the quadrotor, and J_r the inertia of the propeller. Moreover, $\Omega_{11} \equiv \Omega_2 + \Omega_3 - \Omega_1 - \Omega_4$, where $\Omega_i, i = 1, \cdots, 4$ stand for the angular speeds of the 4 propellers, and f is the scaling factor from force to moment. Compared with the brushless motor, the propeller is very light, therefore, it is reasonable to ignore the moment of inertia caused by the propellers here [11]. In addition, air resistance is also ignored during the simulation process in the paper.

3 Controller Design

SMC approach is recognized as an effective tool to design robust control law for complex nonlinear system in the presence of uncertainties. One of the main benefits of SMC is insensitive to parameter variations. Compared with other nonlinear control methods, SMC needs to change the structure of the controller during the control process. Although the controlled system can follow the specified trajectory to perform sliding mode motion and converge to the equilibrium point, chattering might exist when the system state is switched at the sliding mode switching surface. As such, the control performance could be degraded. This motivates us to tune the switching gains of SMC via some fuzzy logic rules. Therefore, the proposed control scheme belongs to the category of Fuzzy SMC.

Let

$$\mathbf{x} = [x_1, \cdots, x_{12}]^T, \tag{5}$$

where $x_1 = x$, $x_2 = \dot{x}$, $x_3 = y$, $x_4 = \dot{y}$, $x_5 = z$, $x_6 = \dot{z}$, $x_7 = \phi$, $x_8 = \dot{\phi}$, $x_9 = \theta$, $x_{10} = \dot{\theta}$, $x_{11} = \varphi$, $x_{12} = \dot{\varphi}$. Then, the dynamic system of the quadrotor UAV, (2) and (3), can be represented by the following state space.

$$\dot{\mathbf{x}} =$$

$$
\begin{cases}
x_2, \\
\dfrac{U_1(\sin x_9 \cos x_7 \cos x_{11} + \sin x_7 \sin x_{11}) - k_x x_2}{m}, \\
x_4, \\
\dfrac{U_1(\sin x_9 \cos x_7 \sin x_{11} - \sin x_7 \cos x_{11}) - k_y x_4}{m}, \\
x_6, \\
\dfrac{U_1 \cos x_7 \cos x_9 - k_z x_6 - mg}{m}, \\
x_8, \\
\dfrac{(J_y - J_z)x_{10}x_{12} + LU_2}{J_x}, \\
x_{10}, \\
\dfrac{(J_z - J_x)x_8 x_{12} + LU_3}{J_y}, \\
x_{12}, \\
\dfrac{(J_x - J_y)x_8 x_{10} + fU_4}{J_z}.
\end{cases} \tag{6}
$$

where the altitude controller U_1, and the roll, pitch and yaw controllers $U_i, i = 2, \cdots, 4$ will be designed under the framework of SMC.

First address U_1. Define the altitude error as

$$e_z = x_{5d} - x_5, \tag{7}$$

where x_{5d} is the desired altitude. Further set the sliding surface as

$$s_1 = c_1 e_z + \dot{e}_z, \quad c_1 > 0. \tag{8}$$

The method of reaching law is applied to design SMC in the paper [23]. Consider the following exponential reaching law:

$$\dot{s}_1 = -\eta_1 sgn(s_1) - k_1 s_1 \tag{9}$$

where η_1 and k_1 are positive constants and $sgn(\cdot)$ denotes the signum function.

Differentiating (8) yields

$$\dot{s}_1 = c_1 \dot{e}_z + \ddot{e}_z. \tag{10}$$

Then, we have from (9) and (10) that

$$c_1 \dot{e}_z + \ddot{e}_z = -\eta_1 sgn(s_1) - k_1 s_1. \tag{11}$$

By substituting the dynamics of $\dot{x}_6$ in (6), it follows from (11) that

$$U_1 = \frac{m(\eta_1 sgn(s_1) + k_1 s_1 + c_1 \dot{e}_z + \ddot{z}_d + g) + k_z x_6}{\cos x_7 \cos x_9}. \tag{12}$$

Now, we are in the position of designing the roll, pitch and yaw angle controllers. Define

$$
\begin{aligned}
e_\phi &= x_{7d} - x_7, \\
e_\theta &= x_{9d} - x_9, \\
e_\varphi &= x_{11d} - x_{11},
\end{aligned} \tag{13}
$$

and three corresponding sliding surfaces

$$
\begin{aligned}
s_2 &= c_2 e_\phi + \dot{e}_\phi, \\
s_3 &= c_3 e_\theta + \dot{e}_\theta, \\
s_4 &= c_4 e_\varphi + \dot{e}_\varphi,
\end{aligned} \tag{14}
$$

where $c_i > 0, i = 2, \cdots, 4$, and x_{7d}, x_{9d}, x_{11d} are the desired roll, pitch and yaw angles, respectively. Similar to (9), the corresponding exponential reaching laws are defined as

$$\dot{s}_i = -\eta_i sgn(s_i) - k_i s_i, \tag{15}$$

with positive η_i and k_i, $i = 2, \cdots, 4$. Hence, by virtue of the state-space model of the UAV, (6), $U_i, i = 2, \cdots, 4$ can be designed in the following way.

$$U_2 = \frac{J_x}{L}(\eta_2 sgn(s_2) + k_2 s_2 + c_2 \dot{e}_\phi + \ddot{\phi}_d) - \frac{J_y - J_z}{L} x_{10} x_{12} \tag{16}$$

$$U_3 = \frac{J_y}{L}(\eta_3 sgn(s_3) + k_3 s_3 + c_3 \dot{e}_\theta + \ddot{\theta}_d) - \frac{J_z - J_x}{L} x_8 x_{12} \tag{17}$$

$$U_4 = \frac{J_z}{f}(\eta_4 sgn(s_4) + k_4 s_4 + c_4 \dot{e}_\varphi + \ddot{\varphi}_d) - \frac{J_x - J_y}{f} x_8 x_{10} \tag{18}$$

Theorem 1 *Consider the system (6) with the controller (12) and (16)-(18). The output tracking errors of the closed-loop system are guaranteed to converge to zero asymptotically, i.e.,* $e_z, e_\phi, e_\theta, e_\varphi \to 0$ *as* $t \to \infty$.

Proof 1 *Consider the following Lyapunov function candidate*

$$V = \frac{1}{2}\sum_{i=1}^{4} s_i^2. \tag{19}$$

The time derivative of V *is*

$$\dot{V} = \sum_{i=1}^{4} s_i \dot{s}_i. \tag{20}$$

By substituting (9) and (15) into (20), we have

$$
\begin{aligned}
\dot{V} &= \sum_{i=1}^{4} s_i(-\eta_i sgn(s_i) - k_i s_i) \\
&= -\sum_{i=1}^{4} s_i \eta_i sgn(s_i) - \sum_{i=1}^{4} k_i s_i^2 \\
&= -\sum_{i=1}^{4} \eta_i |s_i| - \sum_{i=1}^{4} k_i s_i^2 \\
&\le -\sum_{i=1}^{4} k_i s_i^2 \le 0,
\end{aligned} \tag{21}
$$

implying that all s_i *will converge to zero asymptotically. Observing the detailed expressions of the sliding surfaces (8) and (14), the asymptotical convergence of* $s_i, i = 1, \cdots, 4$ *directly give that of* $e_z, e_\phi, e_\theta, e_\varphi$.

However, since the signum function is involved in the controller, chattering phenomenon could be serious when the proposed SMC law (12) and (16)-(18) is applied in UAV system. To address this, an intelligent fuzzy controller is further proposed to adjust the switching gains to reduce chattering as much as possible. The fuzzy rules among s_i, $\dot{s}_i$ and η_i can

Table 1: Fuzzy rules of $\eta_i, i = 1, \cdots, 4$

$s_i \backslash \dot{s}_i$	NB	ZO	PB
NB	NB	NB	ZO
ZO	NB	ZO	ZO
PB	ZO	ZO	PB

be described in Table 1. In Table 1, three fuzzy sets, namely negative big (NB), zero (ZO), positive big (PB) are chosen for s_i, $\dot{s}_i$, $\eta_i (i = 1, \cdots, 4)$ simultaneously, all with triangle membership functions.

In such way, the control objective of the resultant fuzzy system is changed from the normal tracking error to the sliding mode function. As long as the synthesis control is applied to make the sliding mode function zero, the tracking error of altitude and attitude will converge to zero in consequence. In general, the significance of fuzzy controller is that it softens the control signal and reduces the chattering of traditional SMC.

4 Simulation and Discussion

This section presents simulation results regarding PID, fuzzy PID, SMC and FSMC algorithms for the quadortor UAV. The initial altitude and angle values of the quadrotor UAV are set as 0 m and $[0, 0, 0]$ rad. The desired/reference position and angle values are assumed to be $z_d = 6$ m, $\phi_d = \theta_d = \varphi_d = 0.1$ rad. The parameters of four sliding mode controllers are listed in Table 2.

Table 2: Controller parameters

Variable	Value	Variable	Value
c_1	10	c_3	15
k_1	10	k_3	15
η_1	1	η_3	1
c_2	15	c_4	15
k_2	15	k_4	15
η_2	1	η_4	1

Time histories of the sliding surfaces are illustrated in Fig.2. It means that the sliding surfaces are performed well and converge to zero in finite time.

The control performances of SMC and FSMC are demonstrated in Figs. 3 and 4, respectively. It can be seen that the amplitude of chattering of SMC is much greater than that of FSMC in all the four channels. In other words, fuzzy controller does have good performance for reducing the chattering, which is conducive to the robust control of quadrotor UAV system.

Simulation result of altitude control for the quadrotor UAV is shown in Fig. 5, where the PID and the fuzzy PID controllers induce greater overshoot than SMC and FSMC controllers. In terms of stable tracking time, PID controller is greater than 8 s, while the fuzzy PID achieves stable tracking of the reference within 6.5 s. Besides, compared with PID and fuzzy PID, SMC and FSMC possess a better tracking performance for the quadortor. It is worth noting that SMC and FSMC are provided with nearly equivalent tracking ability. Nevertheless, FSMC can greatly weaken the chattering.

In addition, the tracking results of roll angle, pitch angle and yaw angle are shown in Figs. 6-8, respectively. It can be

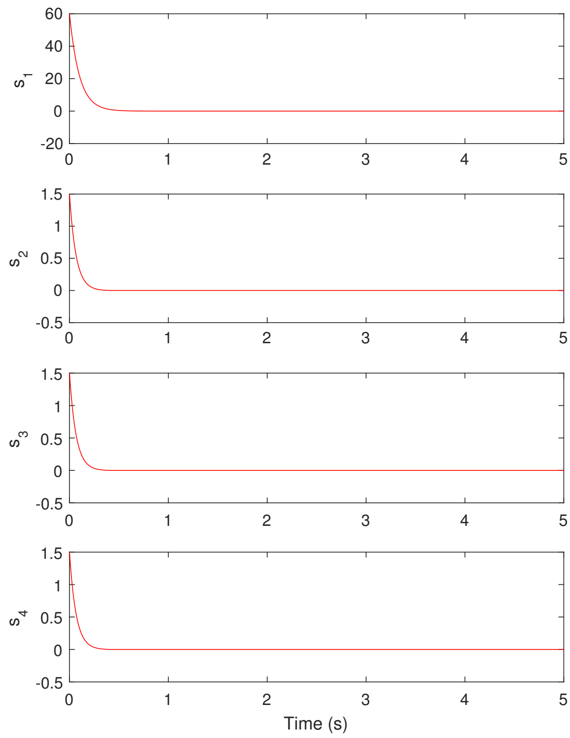

Fig. 2: sliding surfaces

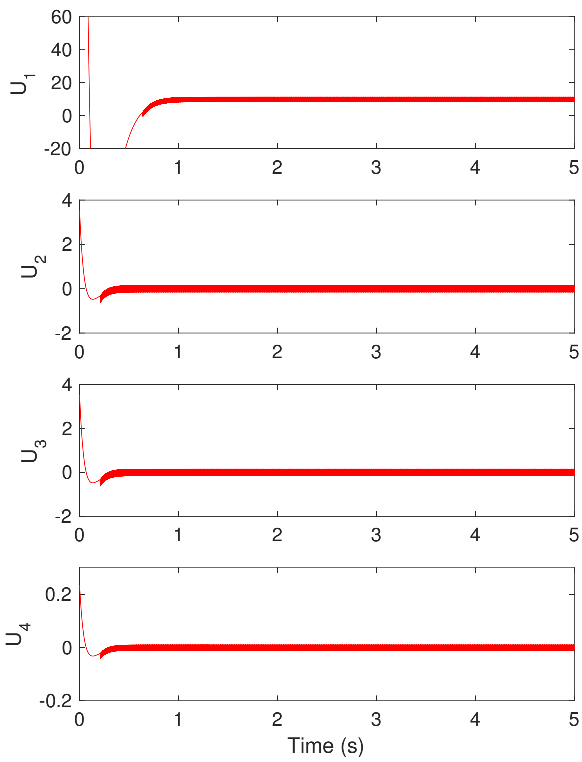

Fig. 3: Output of SMC for the quadrotor UAV system

seen that the steady-state error appears when PID controller is applied, but fuzzy SMC, SMC, fuzzy PID achieve error-

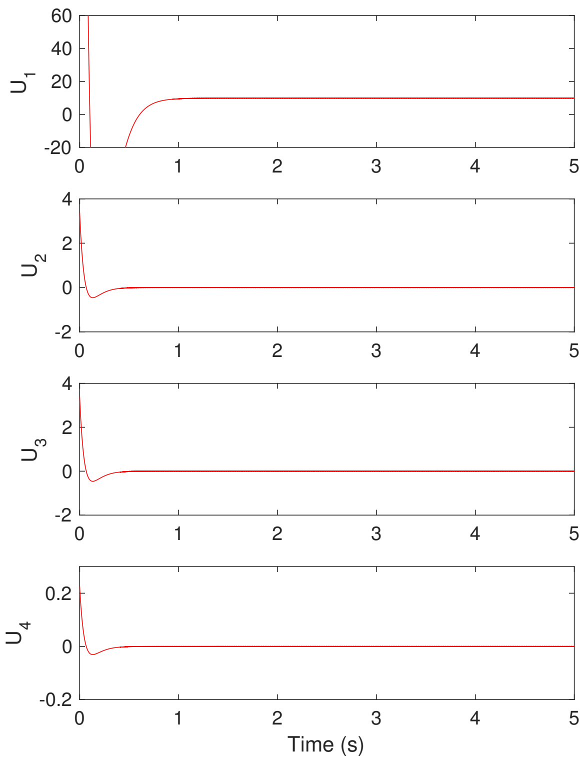

Fig. 4: Output of FSMC for the quadrotor UAV system

free tracking for the given reference. Further, the response time of fuzzy SMC and SMC are less than that of fuzzy PID and PID.

5 Conclusions

This paper studies the problem of altitude and attitude tracking control of a quadrotor UAV. The nonlinear dynamic model of the quadrotor UAV is first built, and then a sliding mode controller with/without fuzzy parametric tuning is presented. The proposed control laws have been tested in simulation. The results show that FSMC has a better control performance than PID or SMC in the sense that the chattering is greatly suppressed through adjusting the switching gain via the proposed fuzzy rule.

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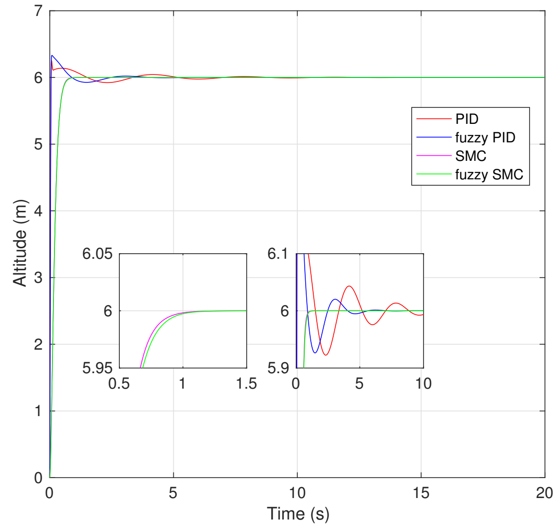

Fig. 5: Altitude tracking for the quadrotor UAV

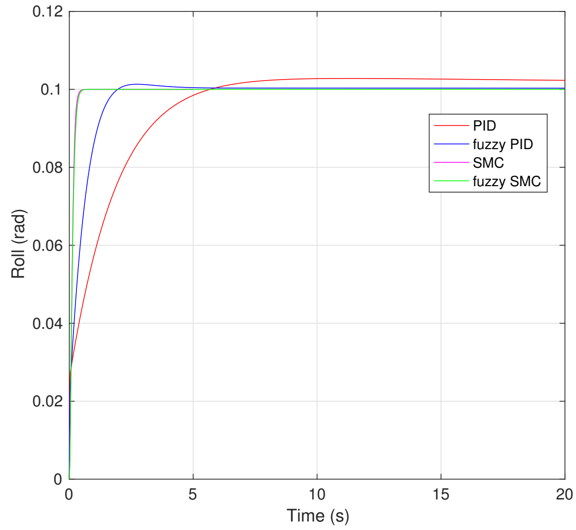

Fig. 6: Roll tracking for the quadrotor UAV

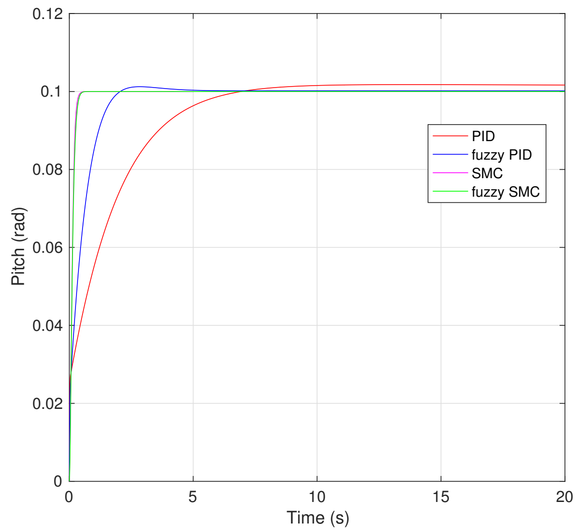

Fig. 7: Pitch tracking for the quadrotor UAV

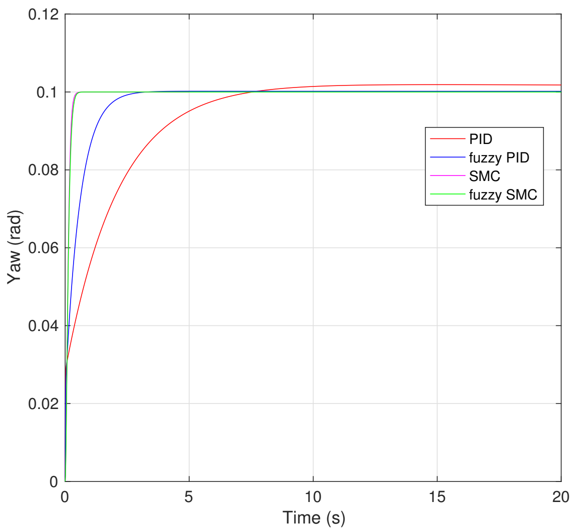

Fig. 8: Yaw tracking for the quadrotor UAV

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