1. Governing system for the benchmark problem

$$\frac{\partial p}{\partial t} - \nabla \cdot (\boldsymbol{K}\nabla p) = q, \qquad (\boldsymbol{x}, t) \in \Omega \times (0, T]$$

$$\begin{aligned} p &= g_D(\boldsymbol{x}, t) \quad \text{for} \quad \boldsymbol{x} \in \Gamma_D, t \in (0, T] \quad \Gamma_D = \{\boldsymbol{x} | \boldsymbol{x} \in (\cdot, 0) \cup (1, \cdot)\} \\ \boldsymbol{u} \cdot \boldsymbol{n} &= g_N(\boldsymbol{x}, t) \quad \text{for} \quad \boldsymbol{x} \in \Gamma_N, t \in (0, T] \quad \Gamma_N = \{\boldsymbol{x} | \boldsymbol{x} \in (\cdot, 1) \cup (0, \cdot)\} \end{aligned}$$

$$\begin{aligned} g_D &= cos(x_1 + t) \quad \text{for} \quad x \in (\cdot, 0) \\ g_D &= cos(1 - x_2 + t) \quad \text{for} \quad x \in (1, \cdot) \\ g_N &= -x_1 sin(x_1 - 1 + t) \quad \text{for} \quad x \in (\cdot, 1) \\ g_N &= -x_2 sin(-x_2 + t) \quad \text{for} \quad x \in (0, \cdot) \end{aligned}$$

$$\boldsymbol{u} = -\boldsymbol{K}\nabla p \qquad \boldsymbol{K} = \begin{bmatrix} x_2 & 0 \\ 0 & x_1 \end{bmatrix}$$

$$p(\boldsymbol{x}, 0) = p_0(\boldsymbol{x}) \quad \text{for} \quad \boldsymbol{x} \in \Omega \qquad p = cos(x_1 - x_2 + t)$$

q = sin(x - y + t) + (x + y)* cos(x - y + t)

2. Figures:

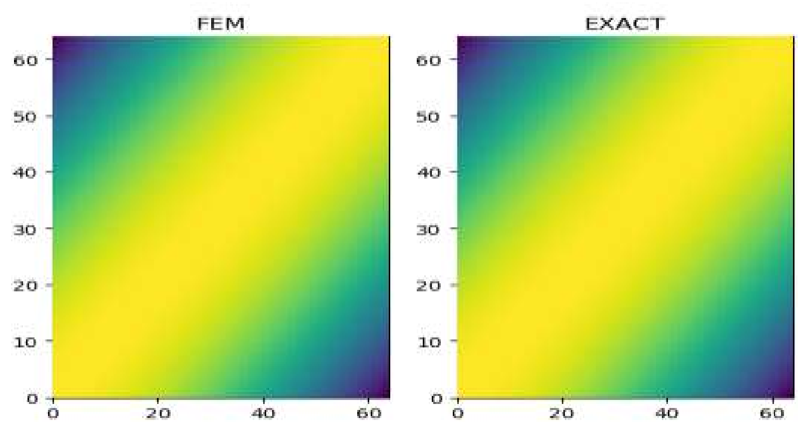

Figure 1. Benchmark t=0

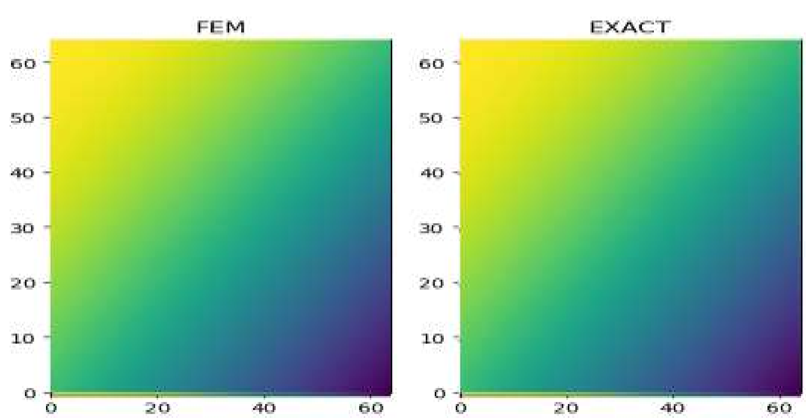

Figure 2. Benchmark t=1

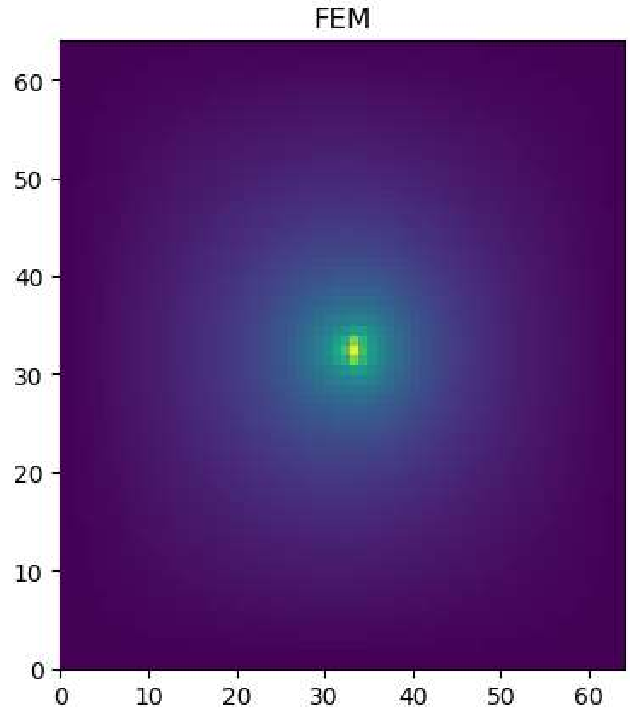

Figure 3. application t=0

3. Error Analysis

Table 1: Spatial convergence rate study

Cycle	Time step/s	Mesh size	#cells	Dofs	L2	convergence
1	0.01	4	16	4	0.0173	
2	0.01	8	64	4	0.0043	4
3	0.01	16	256	4	0.0011	4
4	0.01	32	1024	4	0.0011	1

Table 2: Temporal convergence rate study

Cycle	Time step/s	Mesh size	#cells	Dofs	L2	convergence
1	0.1	32	1024	4	0.0125	
2	0.01	32	1024	4	0.0011	10
3	0.001	32	1024	4	0.000265	4
4	0.0005	32	1024	4	0.00029	1