QFGB 8933 Homework #2 Part I

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1 GARCH Maximum Likelihood Function

For GARCH(1,1) model, the process is

$$\begin{cases} y_t = \sigma_t \epsilon_t \\ \sigma_t^2 = \alpha_0 + \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \end{cases} \tag{1}$$

If ϵ_t has a standard Gaussian distribution, then the log-likelihood function is

$$LLF = \frac{N}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{N}\log\sigma_t^2 - \frac{1}{2}\sum_{t=1}^{N}\frac{Y_t^2}{\sigma_t^2}$$

In MATLAB program, we only contains the part useful to calculate extreme value.

```
function y = garchFunction(x, data)
numData = size(data,1);
sigma = zeros(numData,1);
uSq = data.^2;
sigma(2) = sqrt( uSq(2) );
likelihood = 0;
for i=3:numData
    sigma(i) = sqrt( x(1) +  x(2) *uSq(i-1) + x(3)*sigma(i-1)^2 );
    likelihood = likelihood + (-log(sigma(i)^2) - uSq(i)/sigma(i)^2);
end
y = -likelihood;
```

2 Example

We choose the particular GARCH(1,1) model with parameter $\theta_0 = (\alpha_0, \alpha_1, \beta_1) = (0.01, 0.25, 0.7)$. We simulate the data and get the estimates. First we list the function we use.

```
% Calculate estimates, CI, p-value for a given GARCH model
function [x, fval, ci, p_value] = GarchMle(Yn, x0)

```

```
[x, fval, ¬, ¬, ¬, h]=fminunc(@(x)garchFunction(x, Yn),[0.1,0.5,0.5]);

h_inv = inv(h);
sigma_hat_m = sqrt(diag(h_inv));

ci = CI(x, sigma_hat_m);

p_value = PValue(x, x0, sigma_hat_m);

end
```

```
% Calculate CI
function [ci] = CI(thetam, sigma_hat_m)

conf_level = 0.05;
[m, n] = size(thetam);
l_m = norminv(1-conf_level/2,0, sigma_hat_m);

for i = 1:1:n
    ci(i,:) = [thetam(i) - l_m(i), thetam(i) + l_m(i)];
end

end
```

```
% Calculate p-value
function [p_value] = PValue(thetam, theta0, sigma_hat_m)

[m, n] = size(thetam);
for i = 1:1:n
    p_value(i) = 2 * (1-normcdf(abs(thetam(i)-theta0(i)), 0, ...
        sigma_hat_m(i)));
end

end
```

```
% Simulate a GARCH series with 1000 data
numData = 1000;

x0 = [0.01, 0.25, 0.7];

Mdl = garch('Constant', x0(1),'ARCH',x0(2),'GARCH',x0(3));
rng default; % For reproducibility
[Vn,Yn] = simulate(Mdl,numData,'NumPaths', 1);

% Estimate
[x, fval, ci, p_value] = GarchMle(Yn(:,1), x0);
x

x =
```

```
    0.0114    0.2662    0.6713

ci =
    0.0076    0.0151
    0.2105    0.3219
    0.6130    0.7296

p_value =

    0.4798    0.5677    0.3344
```

The result of estimation is $\theta = (0.0114, 0.2662, 0.6713)$. According to the p-value, we do not reject the null hypothesis.

We repeat the process 10000 times and record the result that we reject the null hypothesis.

```
numData = 1000;

x0 = [0.01, 0.25, 0.7];

Mdl = garch('Constant', x0(1),'ARCH',x0(2),'GARCH',x0(3));
rng default; % For reproducibility
path = 10000
[Vn,Yn] = simulate(Mdl,numData,'NumPaths', path);

count = 0
for i=1:path
    [x, fval, ci, p_value] = GarchMle(Yn(:,i), x0);
    if sum(p_value>0.05)<1
        count = count + 1;
    end
end

count

count =

   381
```

We get 381 rejection results out of 10,000 simulation. This is because of the confidence level we set.

Then we examine the data generated by an alternative model ARCH(2). The percentage of rejection is $9981/10000 = 99.81\%$.

```
% Alternative model ARCH(2)
numData = 1000;

x0 = [0.01, 0.25, 0.7];

```

```
Mdl = garch('Constant', x0(1),'ARCH',[x0(2),x0(3)]);
rng default; % For reproducibility
path = 10000
[Vn,Yn] = simulate(Mdl,numData,'NumPaths', path);

count = 0
for i=1:path
    [x, fval, ci, p_value] = GarchMle(Yn(:,i), x0);
    if sum(p_value>0.05)<1
        count = count + 1;
    end
end

count

count =
        9918
```