Section 1.3 Homework

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Example 5

Let $f(n) = 1 + 3 + 5 + ... + (2n - 1)$
Let $p(n)$ be the statement $f(n) = n^2$
When $n = 1$, we can get $1 = 1^2$. So, $p(n)$ is true at $n = 1$
We now assume $\forall k, p(k)$
Our goal is to show $\forall k, p(k) \Rightarrow p(k + 1)$
For $n = k$, we have $f(k) = k^2$
So $f(k + 1) = f(k) + [2 * (k + 1) - 1] = k^2 + 2k + 1 = (k + 1)^2$
We have shown that $\forall k, p(k) \Rightarrow p(k + 1)$.
Therefore, we have shown that $\forall n, p(n)$ ■

Example 7

If x and y are positive odd integers, then we can express x and y as $x = 2m + 1, y = 2n + 1$ where m and n are nonnegative integers. Then, we obtain

$$
\begin{aligned}
xy &= (2m + 1)(2n + 1) && \text{(by definition of } x \text{ and } y\text{)} \\
&= 4mn + 2m + 2n + 1 && \text{(expand the expression)} \\
&= 2(2mn + m + n) + 1 && \text{(2 is a common factor of } 2mn, m \text{ and } n\text{)} \\
&= 2k + 1 && (k = 2mn + m + n \text{ is also a positive integer}) \\
&= \textit{Old Positive Integer} && \text{(by definition of a positive odd integer)}
\end{aligned}
$$

■

Example 9

p	q	$q \Rightarrow p$	$p \Rightarrow q$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	T	T

Conclusion: $q \Rightarrow p$ **is not logically equivalent to** $p \Rightarrow q$

Example 10

p	q	$\sim p$	$\sim q$	$\sim p \Rightarrow \sim q$	$p \Rightarrow q$
T	T	F	F	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Conclusion: $\sim p \Rightarrow \sim q$ **is not logically equivalent to** $p \Rightarrow q$

Example 11

(a) **If it is snowing, then the temperature outside is less than one-hundred degrees Fahrenheit.**

Contrapositive: If the temperature outside is not less than one-hundred degrees Fahrenheit, then it not snowing. (T)

Converse: If the temperature outside is less than one-hundred degrees Fahrenheit, then it is snowing. (F)

Inverse: If it is not snowing, then the temperature outside is not less than one-hundred degrees Fahrenheit. (F)

(b) **If an animal has feet, then it can walk.**

Contrapositive: If an animal can not walk, then it does not have feet. (F)

Converse: If an animal can walk, then it has feet. (T)

Inverse: If an animal does not have feet, then it can not walk. (T)