Digital Talent Scholarship 2022

Multivariate Calculus 1

Lead a sprint through the Machine Learning Track

Agenda

- Introduction to Calculus
- Functions
- Gradients and derivatives
- Time saving rules

Objektif Pembelajaran

- Memahami definisi differentiation
- Menggunakan differentiation terhadap fungsi
- Menggunakan sum, product and chain rules
- Mengetahui differentiation bisa digunakan untuk beberapa variabel
- Menggunakan multivariate calculus pada sebuah ekuasi
- Mengetahui kegunaan vector/matrix dalam multivariate calculus
- Menganalisis sebuah masalah dua dimensi menggunakan the Jacobian

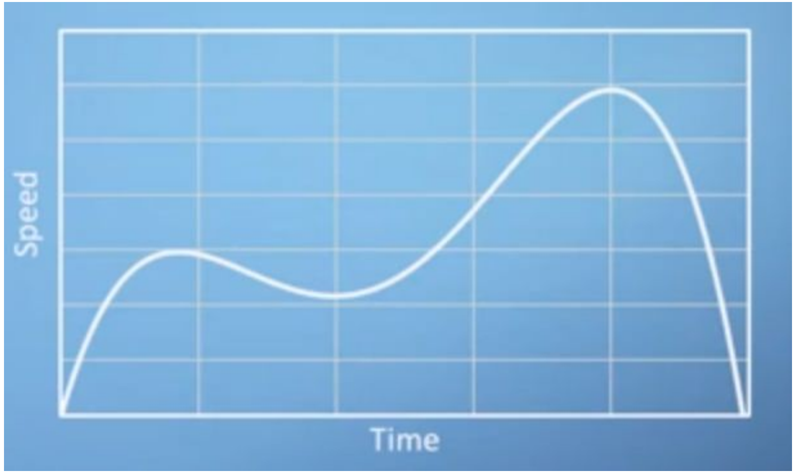
Are your students ML-ready?

Recap

Apa itu Kalkulus

Kalkulus adalah matematika gerak dan perubahan. Dimana ada gerak atau pertumbuhan. Matematika berkaitan dengan menggambarkan cara yang tepat di mana perubahan dalam satu variabel berhubungan dengan perubahan yang lain.

Contoh: Menghitung percepatan sesaat mobil dengan mencari gradien titik tertentu.



Apa itu Fungsi

Fungsi, dalam matematika, merupakan ekspresi, aturan, atau hukum yang mendefinisikan hubungan antara satu variabel (variabel bebas) dan variabel lain (variabel terikat).

Fungsi umumnya direpresentasikan sebagai f(x)
Misal , f(x)= x3
Fungsi juga dapat diwakili oleh g(), t(),… dll.

Apa itu Multivariable Function

Multivariable Function hanyalah fungsi yang input dan/atau outputnya terdiri dari beberapa angka. Sebaliknya, fungsi dengan input satu angka dan output satu angka disebut sebagai **single-variable function**.

	Single-number input	**Multiple-number inputs**
Single-number output	$f(x)=x^2$	$f(x,y)=x^2+y^3$
Multiple-number output	$f(t)=(\cos(t),\sin(t))$	$f(u,v)=(u^2-v,v^2+u)$

Multivariable Function

Jika keluaran suatu fungsi terdiri dari beberapa bilangan, maka dapat juga disebut multivariabel, tetapi yang ini juga biasa disebut **vector-valued function**.

$$f\underbrace{(x, y)}_{\substack{\text{Multiple numbers} \\ \text{in the input}}} = x^2 y$$

$$f(x) = \begin{bmatrix} \cos(x) \\ \sin(x) \end{bmatrix} \leftarrow \text{Multiple numbers in output}$$

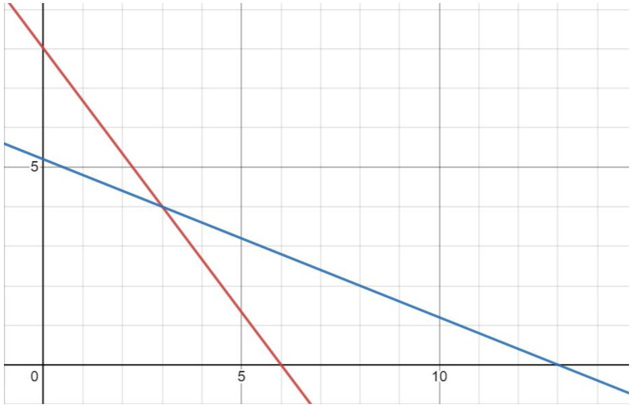
Jenis Fungsi

- Linear Function
- Quadratic Function
- Polynomial Function
- Power Function
- Rational Function
- Exponential Function
- Logarithmic Function

Jenis Function

- **Linear Function**

Fungsi linier adalah fungsi yang grafiknya berupa garis lurus berdimensi n. Ini adalah fungsi dari bentuk y = mx + c



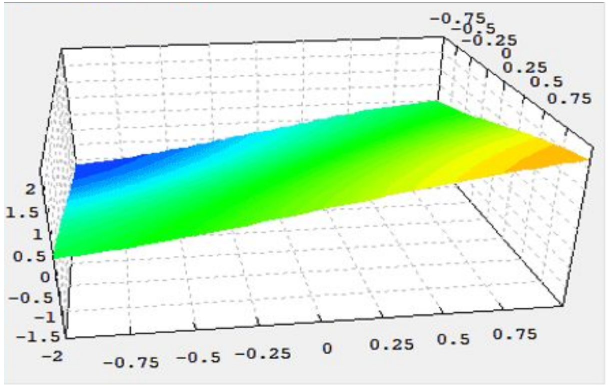
Jenis Function

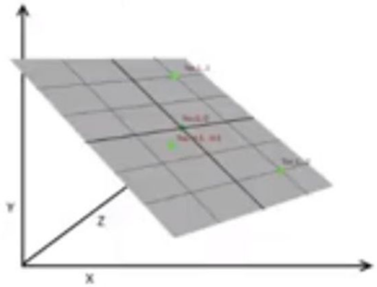
- **Jenis Linear Function**

Satu Dimensi : $y = a$

Dua Dimensi : $y = mx + c$

Lebih dari dua Dimensi : $y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n$





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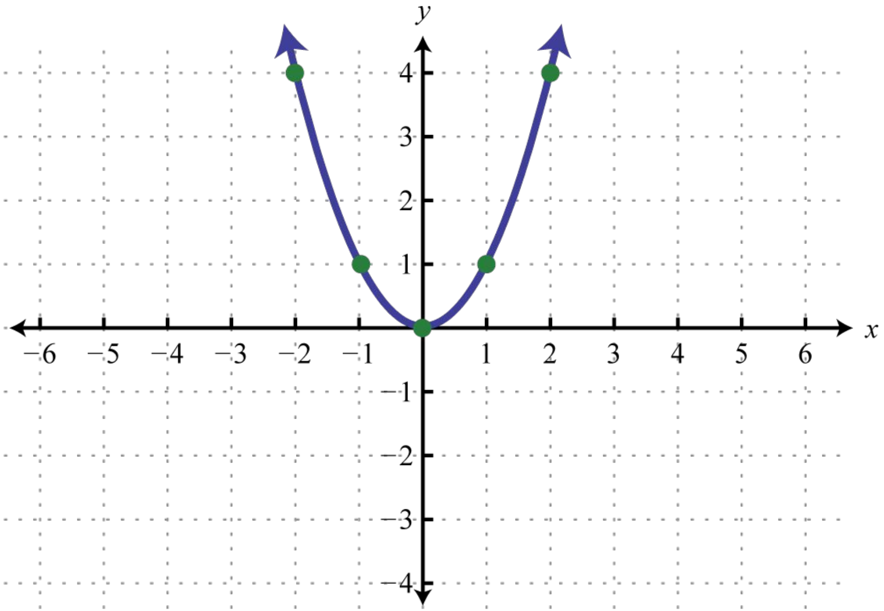
Jenis Function

- **Quadratic Function**

  $y = ax^2 + bx + c$

$$f(x) = x^2$$

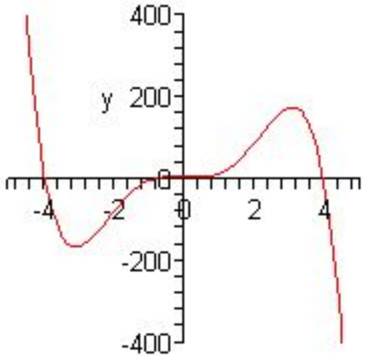
x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

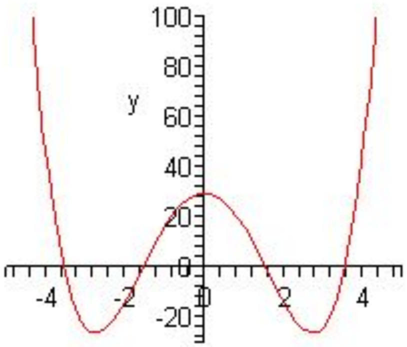


Jenis Function

- **Polynomial Function**

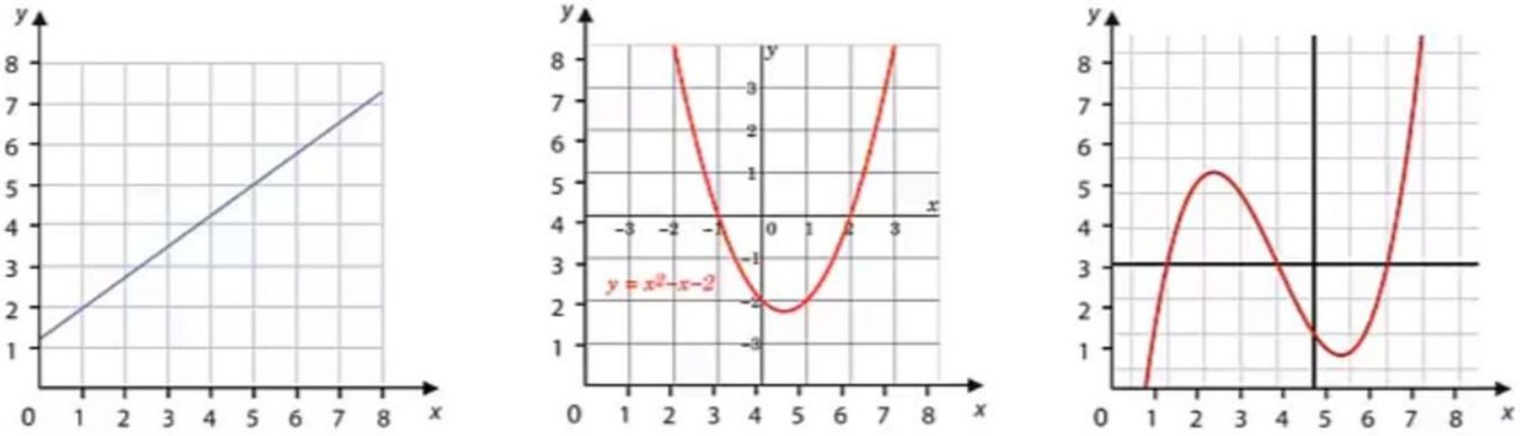
  $y = a_n*x^n + a_{n-1}*x^{n-1} + \dots + a_2*x^2 + a_1*x + a_0$





2 Dimensi

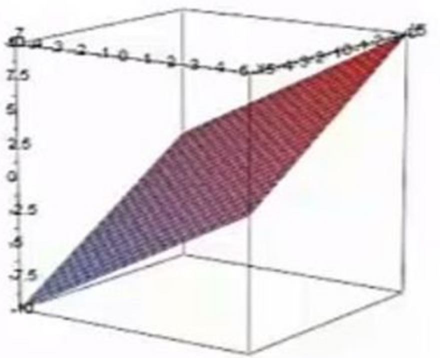
Linear Function	Quadratic Function	Polynomial Function
$y = \theta_0 + \theta_1 x$	$y = \theta_o + \theta_1 x + \theta_2 x^2$	$y = \theta_o + \theta_1 x + \theta_2 x^2 + \theta_3 x^3$



3 Dimensi

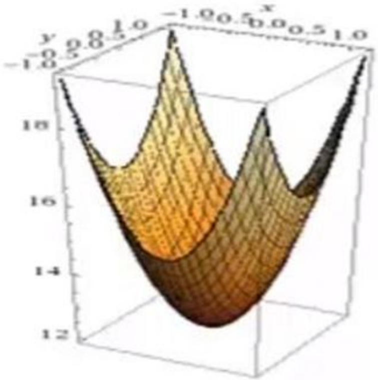
Linear Function

$z = \theta_0 + \theta_1 x + \theta_2 y$



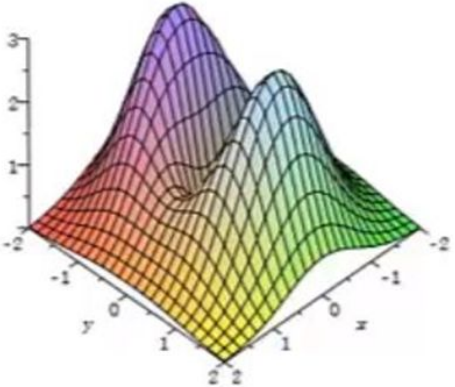
Quadratic Function

$z = \theta_o + \theta_1 x^2 + \theta_2 y^2$



Polynomial Function

$z = \theta_o + \theta_1 x + \theta_2 y^2 + \theta_3 x^3$



Definisi Derivative

Dalam kalkulus, kemiringan garis singgung kurva pada titik tertentu pada kurva. Karena kurva mewakili suatu fungsi, turunannya juga dapat dianggap sebagai **laju perubahan fungsi** yang bersesuaian pada titik tertentu.

Ini adalah fungsi untuk mendapatkan **kemiringan.**

Rise over run, perubahan tinggi dibagi dengan waktu

Notasi Derivative sebuah Fungsi

$f'(x)$	"*f* prime of *x*"
y'	"*y* prime"
$\frac{dy}{dx}$	"the derivative of *y* with respect to *x*"

Derivative Rules

- **The Constant Rule**

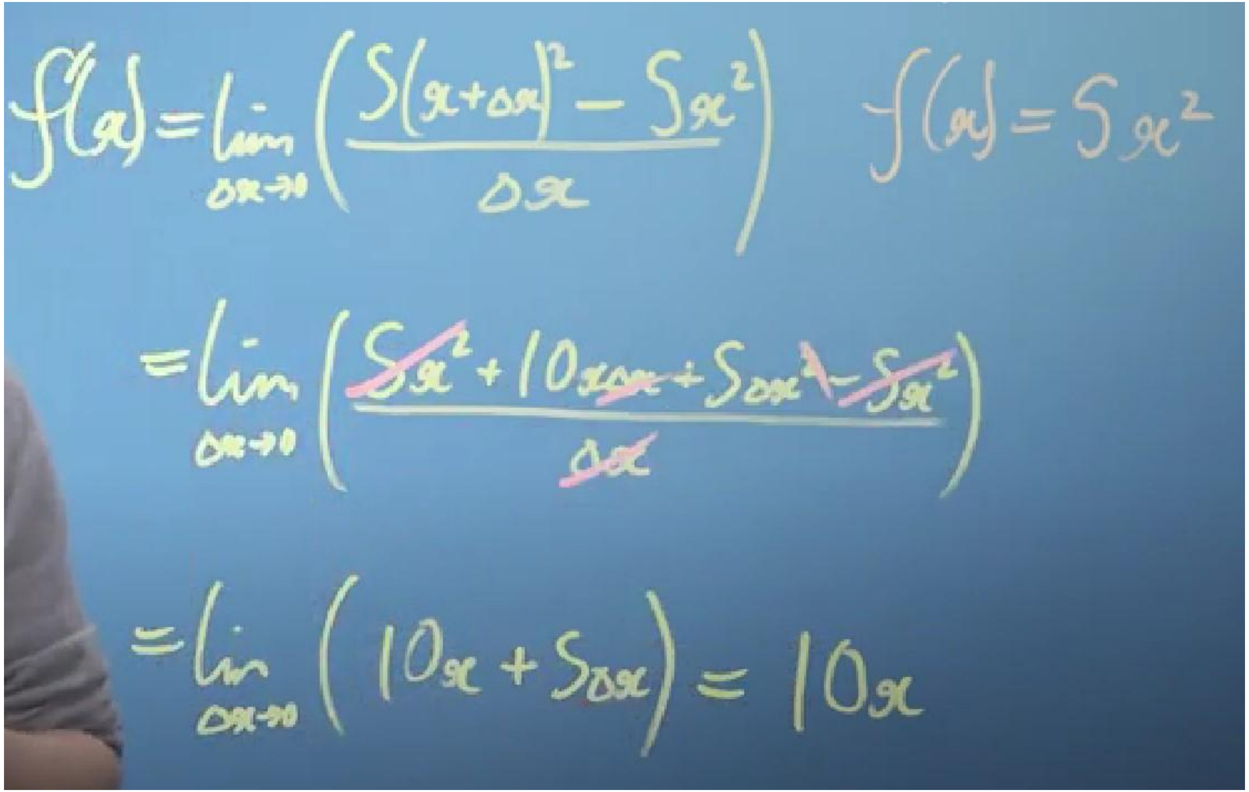
$$\frac{d}{dx}[c] = 0, \qquad c \text{ is a constant}$$

- **Examples**

$$f(x) = 7$$
$$f'(x) = 0$$

$$y = -3$$
$$\frac{dy}{dx} = 0 \quad \text{or} \quad y' = 0$$

Power Rule



$f(x) = \lim_{\Delta x \to 0} \left(\frac{5(x+\Delta x)^2 - 5x^2}{\Delta x} \right)$
$f(x) = 5x^2$
$= \lim_{\Delta x \to 0} \left(\frac{\cancel{5x^2} + 10x\cancel{\Delta x} + 5\Delta x^{\cancel{2}} - \cancel{5x^2}}{\cancel{\Delta x}} \right)$
$= \lim_{\Delta x \to 0} \left(10x + 5\Delta x \right) = 10x$

Derivative Rules

- **The Power Rule**

$$\frac{d}{dx}[x^N] = Nx^{N-1},\ N \text{ is any real number}$$

$$\frac{d}{dx}[x] = 1$$

Derivative Rules

- **The Power Rule Examples**

$$g(x) = x^{100}$$
$$g'(x) = 100x^{99}$$

$$f(x) = x^{3}$$
$$f'(x) = 3x^{2}$$

$$y = x^{9}$$
$$\frac{dy}{dx} = 9x^{8}$$

Derivative Rules

- **The Constant Multiple Rule**

$$\frac{d}{dx}[c(f(x))] = c(f'(x)), \quad c \text{ is a constant}$$

Derivative Rules

- **The Constant Multiple Rule Example**

$$f(x) = \frac{4x^2}{5} = \frac{4}{5}x^2$$

$$f'(x) = \tfrac{4}{5}(2x)$$

$$f'(x) = \frac{8}{5}x$$

$$g(x) = 5x^7$$

$$g'(x) = 35x^6$$

$$y = 2x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = 2\left(\tfrac{1}{3}x^{-\frac{2}{3}} \right)$$

$$\frac{dy}{dx} = \frac{2}{3x^{\frac{2}{3}}}$$

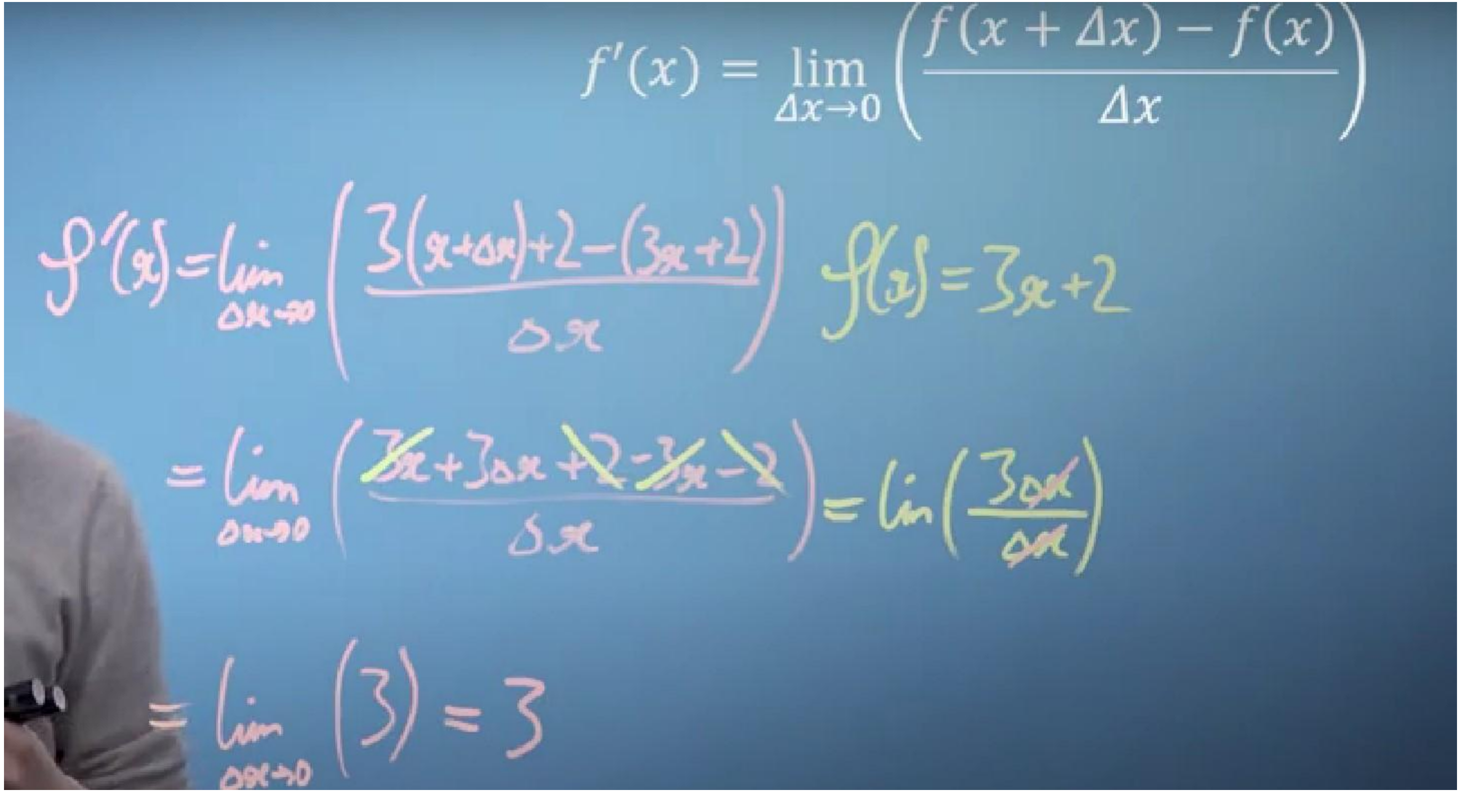
Derivative Rules

- **The Sum and Difference Rule**

$$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$$

$$\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$$

Sum Rule


$f'(x) = \lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$
$f'(x) = \lim_{\Delta x \to 0} \left(\frac{3(x+\Delta x)+2-(3x+2)}{\Delta x} \right)$
$f(x) = 3x+2$
$= \lim_{\Delta x \to 0} \left(\frac{\cancel{3x}+3\Delta x+\cancel{2}-\cancel{3x}-\cancel{2}}{\Delta x} \right) = \lim \left(\frac{3\cancel{\Delta x}}{\cancel{\Delta x}} \right)$
$= \lim_{\Delta x \to 0} (3) = 3$

Derivative Rules

- **The Sum and Difference Rule Example**

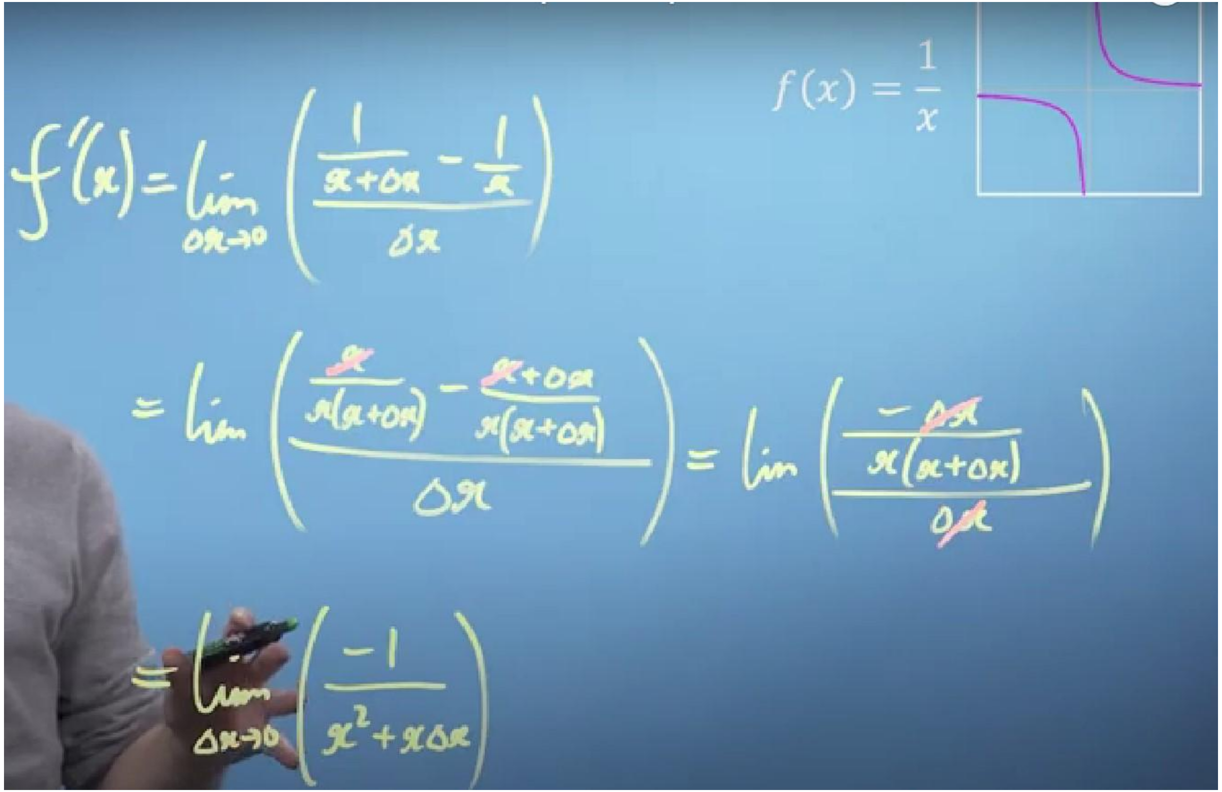
$$f(x) = 5x^2 + 7x - 6$$

$$f'(x) = 10x + 7$$

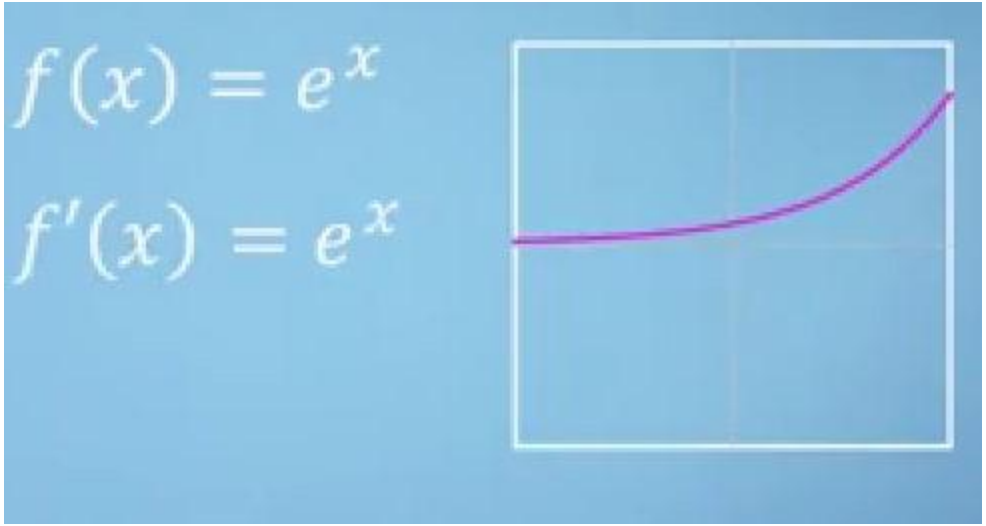
$$g(x) = 4x^6 - 3x^5 - 10x^2 + 5x + 16$$

$$g'(x) = 24x^5 - 15x^4 - 20x + 5$$

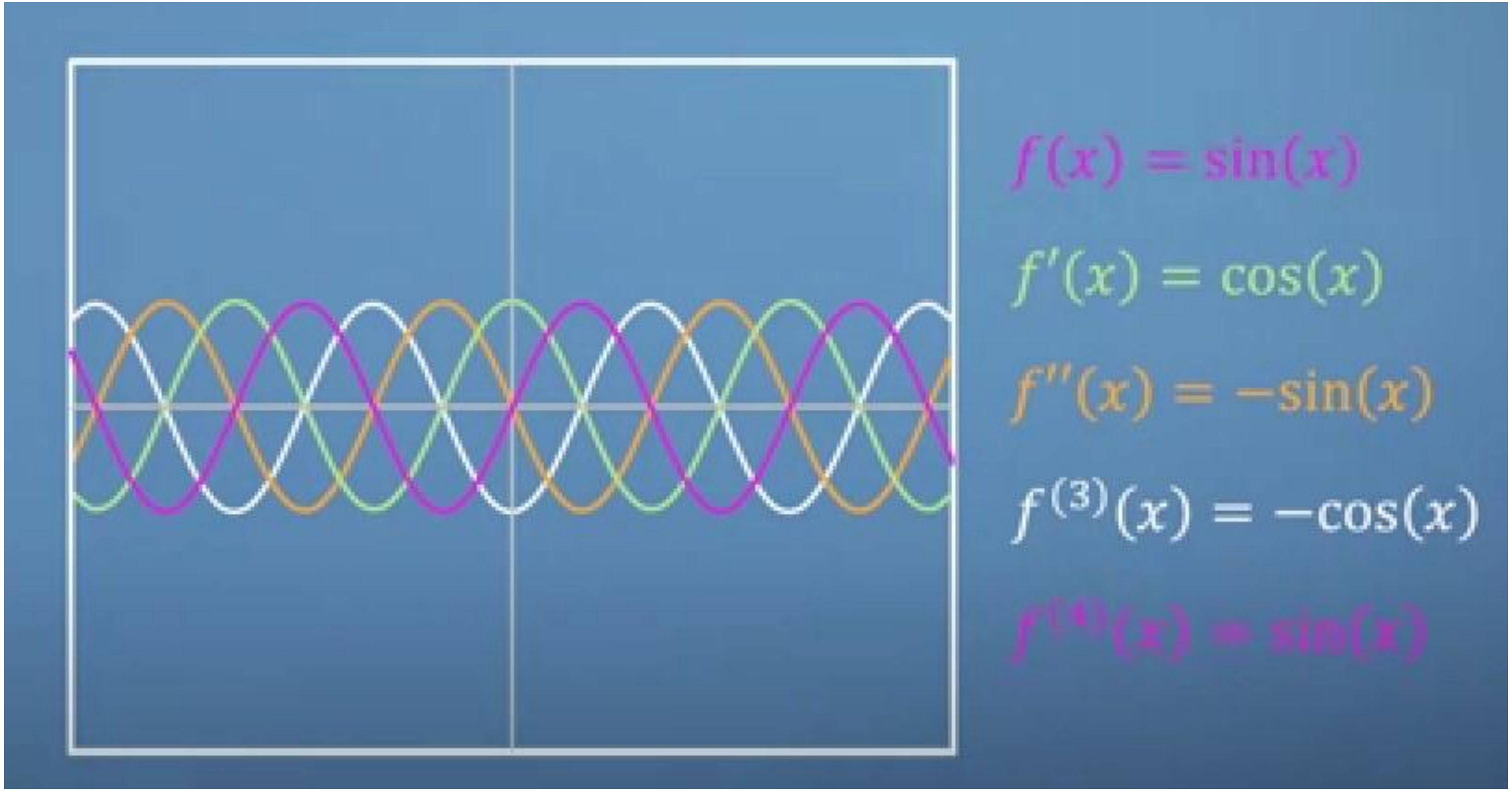
Contoh Derivative dan special cases



Contoh Derivative dan special cases



Contoh Derivative dan special cases



Contoh Derivative dan special cases

Use the identities

$$sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

$$cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

Product Rule

If $A(x) = f(x)g(x)$

then $A'(x) = f(x)g'(x) + g(x)f'(x)$

Chain Rule

If $h = h(p)$ and $p = p(m)$

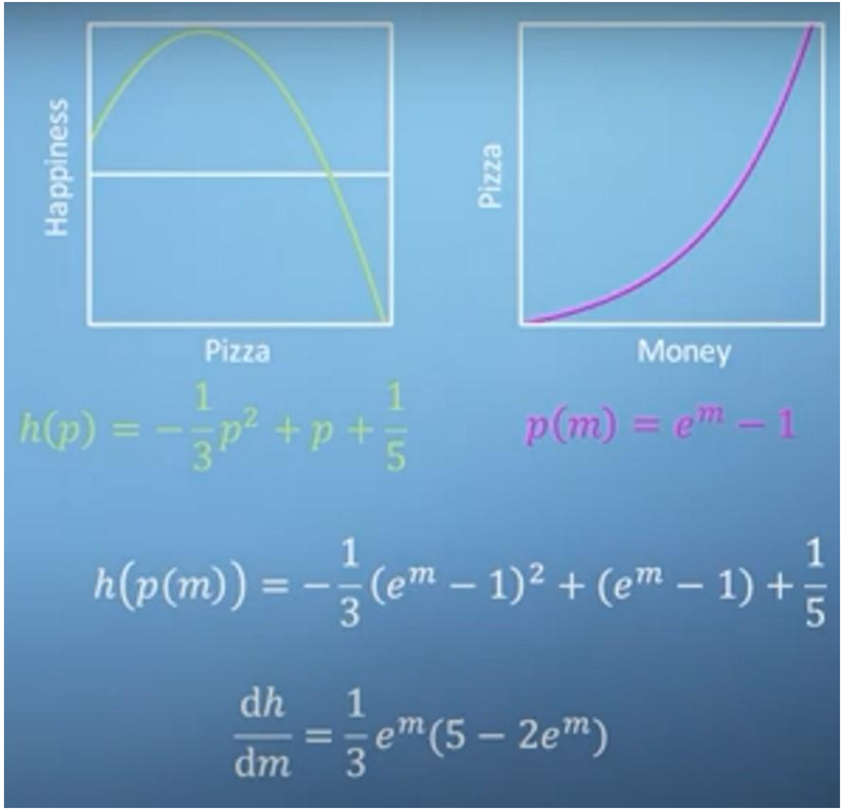
then $$\frac{dh}{dm} = \frac{dh}{dp} \times \frac{dp}{dm}$$

Contoh Chain Rule

Ibarat h(p(m))

h berarti happiness
p berarti pizza
m berarti money

Dan kita sedang melihat hubungan dari ketiga variabel tersebut.



Contoh Chain Rule

$$h(p) = -\frac{1}{3}p^2 + p + \frac{1}{5}$$

$$\frac{dh}{dp} = 1 - \frac{2}{3}p$$

$$p(m) = e^m - 1$$

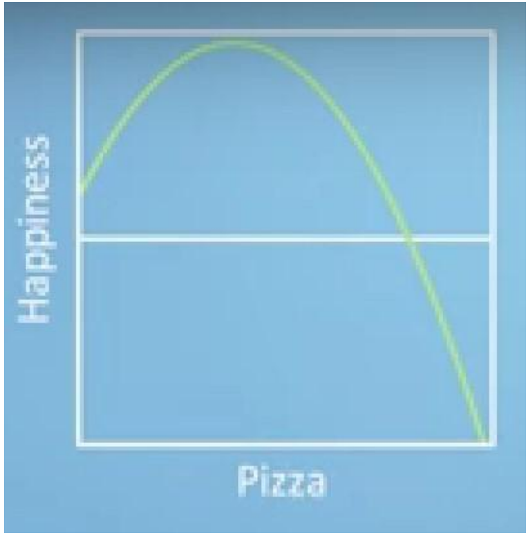
$$\frac{dp}{dm} = e^m$$

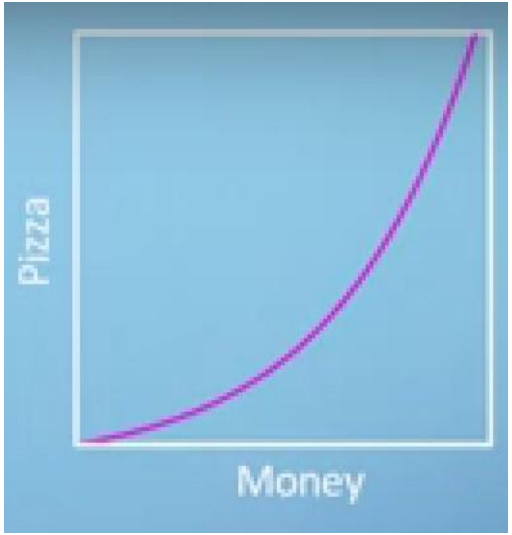
$$\frac{dh}{dp} \times \frac{dp}{dm} = \left(1 - \frac{2}{3}p\right)e^m$$

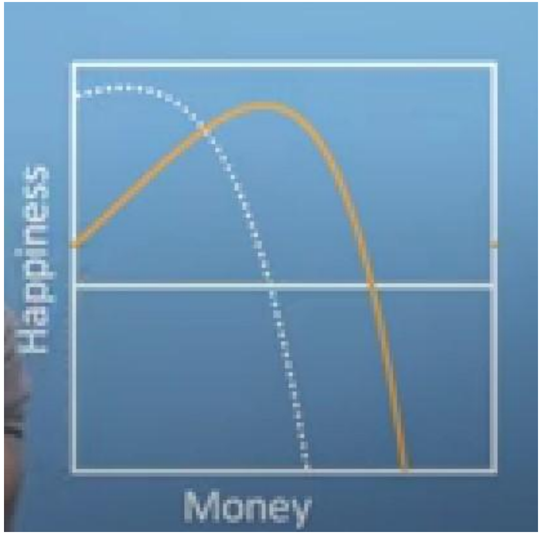
$$= \left(1 - \frac{2}{3}(e^m - 1)\right)e^m$$

$$\frac{dh}{dm} = \frac{1}{3}e^m(5 - 2e^m)$$

Contoh Chain Rule







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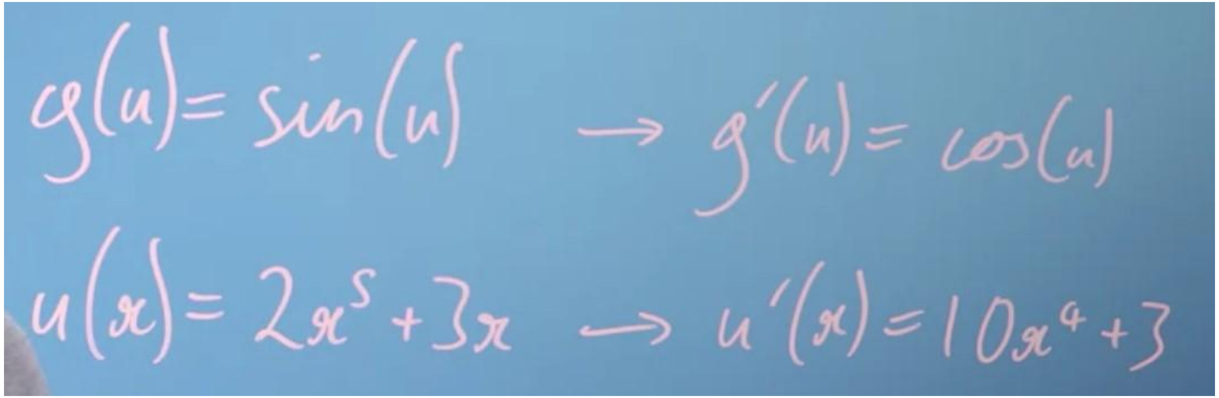
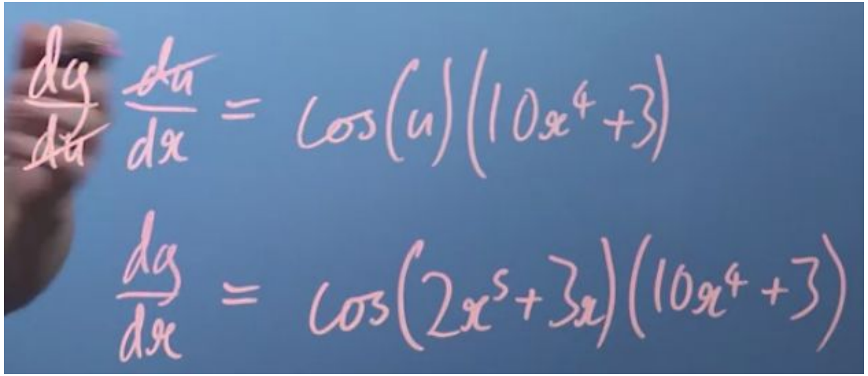
Challenge!

A. $g(x) = \sin(2x^5 + 3x)$ $g(x)' = ?$

B. $h(x) = 6e^{7x}$ $h(x)' = ?$

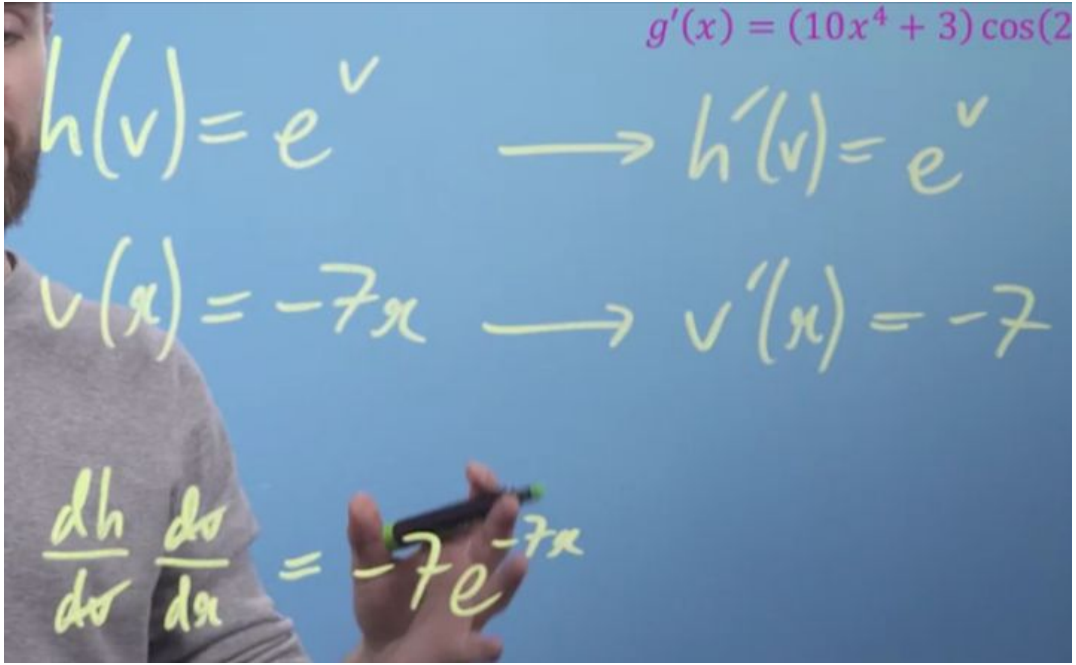
Challenge Solution A





$\cos(2x^5+3x)(10x^4+3)$

Challenge Solution B



$-7e^{-7x} * 6 = -42e^{-7x}$

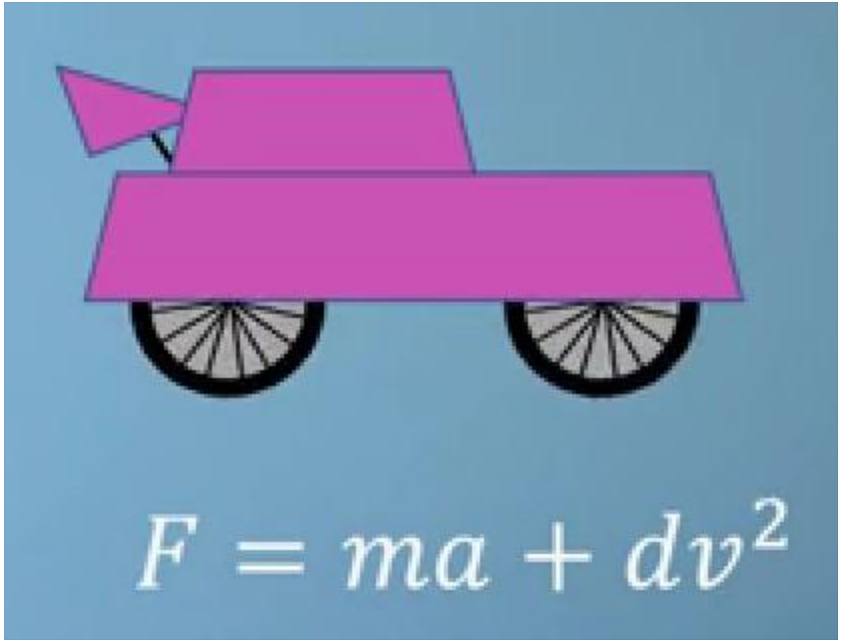
Apa itu Multivariable Calculus

Dalam kalkulus, **kalkulus multivariabel** adalah perluasan dari kalkulus satu dimensi biasa ke lebih dari satu dimensi. Sebagian besar konsep dari kalkulus, seperti continuity dan chain rule, masih berfungsi di lebih dari satu dimensi.

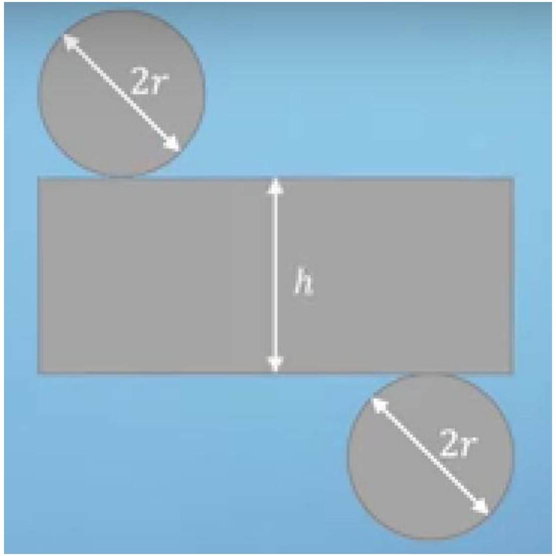
Kalkulus multivariabel dapat digunakan untuk apa?
Kalkulus multivariat dapat digunakan dalam analisis regresi untuk mendapatkan rumus untuk memperkirakan hubungan antara berbagai set data.

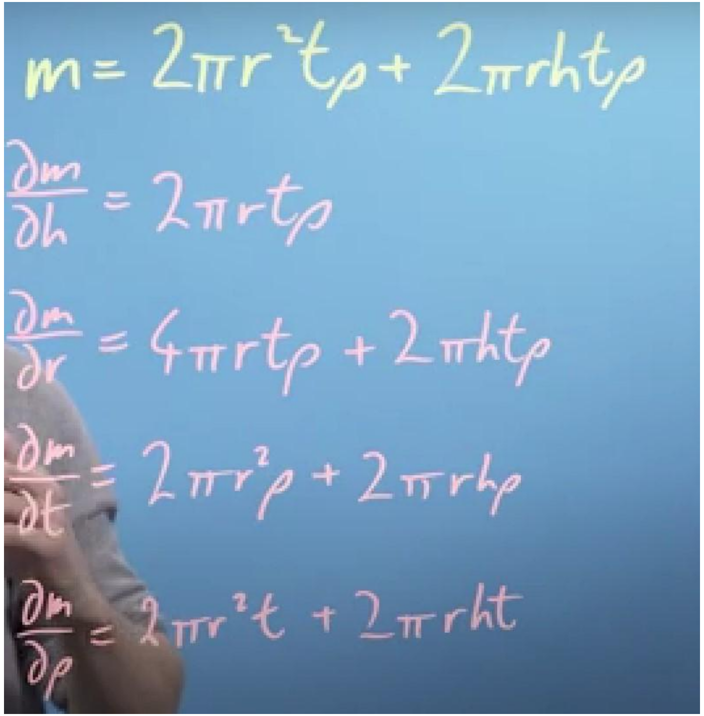
Variables, constants & context

Perbedaan context dapat mengubah variabel dan konstan.



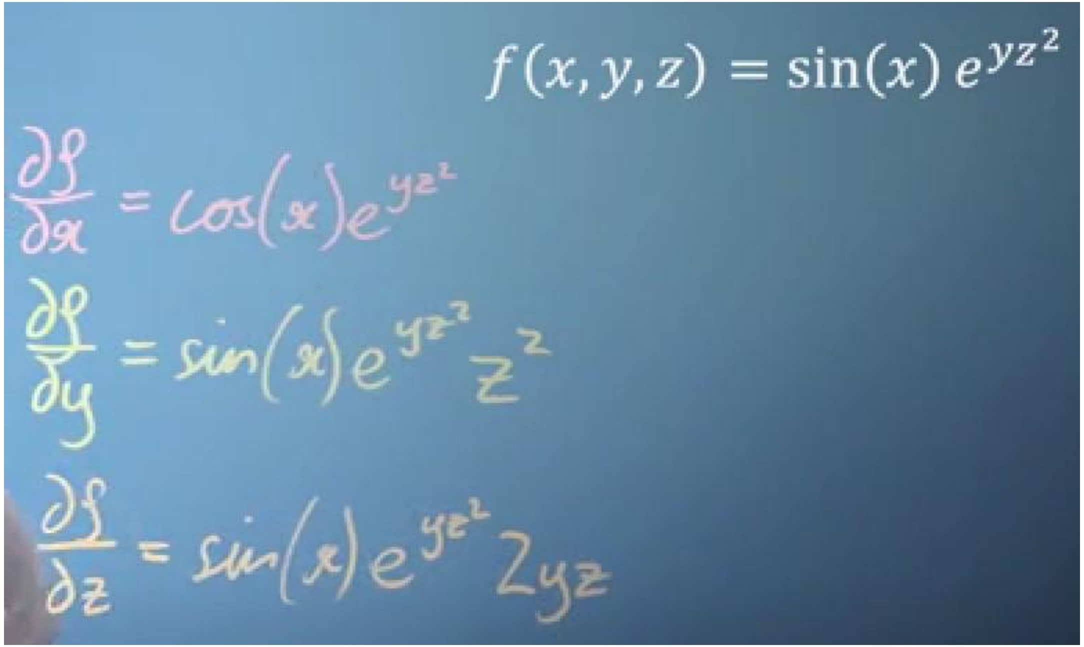
Contoh Variables, constants & context





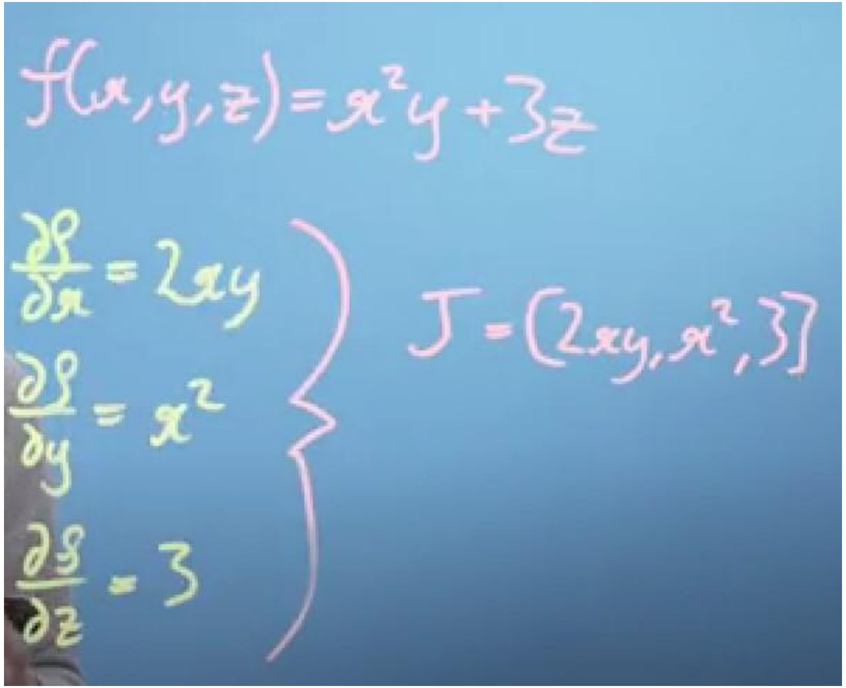
Differentiate with respect to anything

Total Derivation



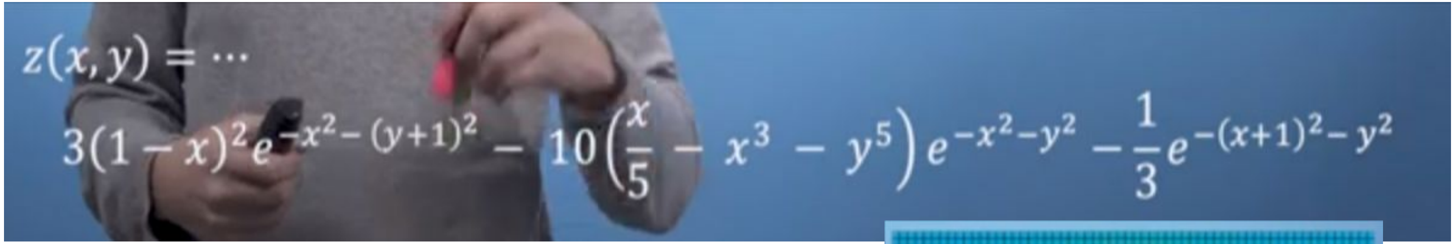
The Jacobian

Vector yang akan menunjuk ke tempat tertinggi

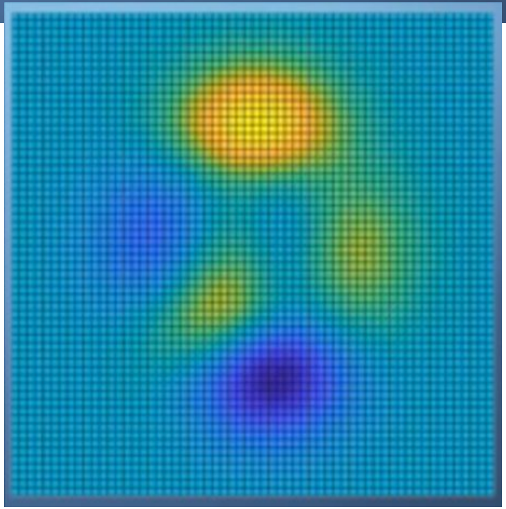


Contoh The Jacobian

1. Ekuasi

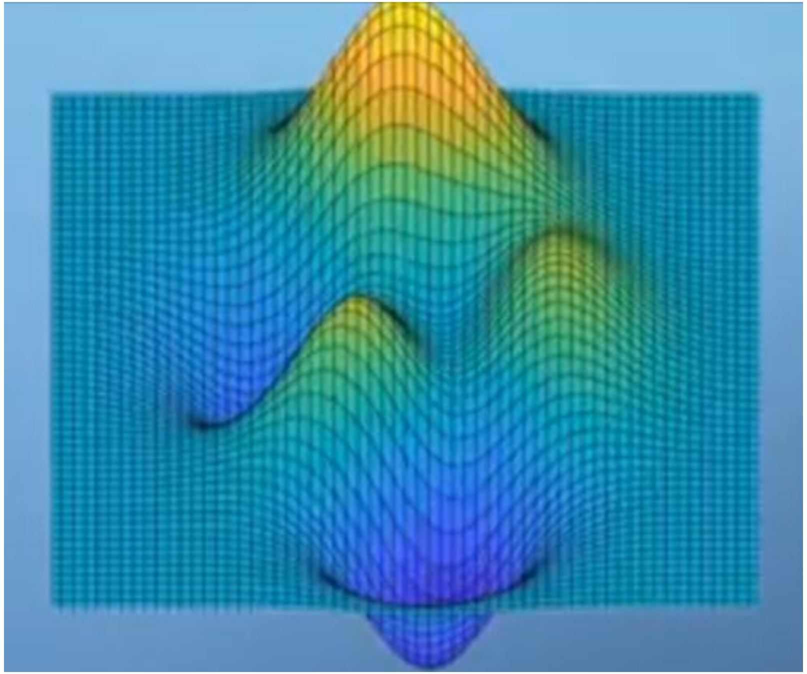


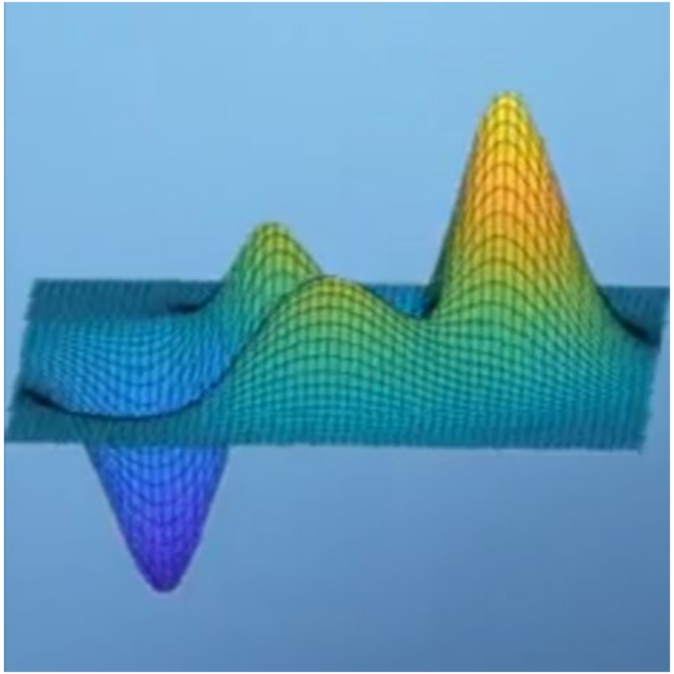
2. Mapping semua gradient di map



Contoh The Jacobian

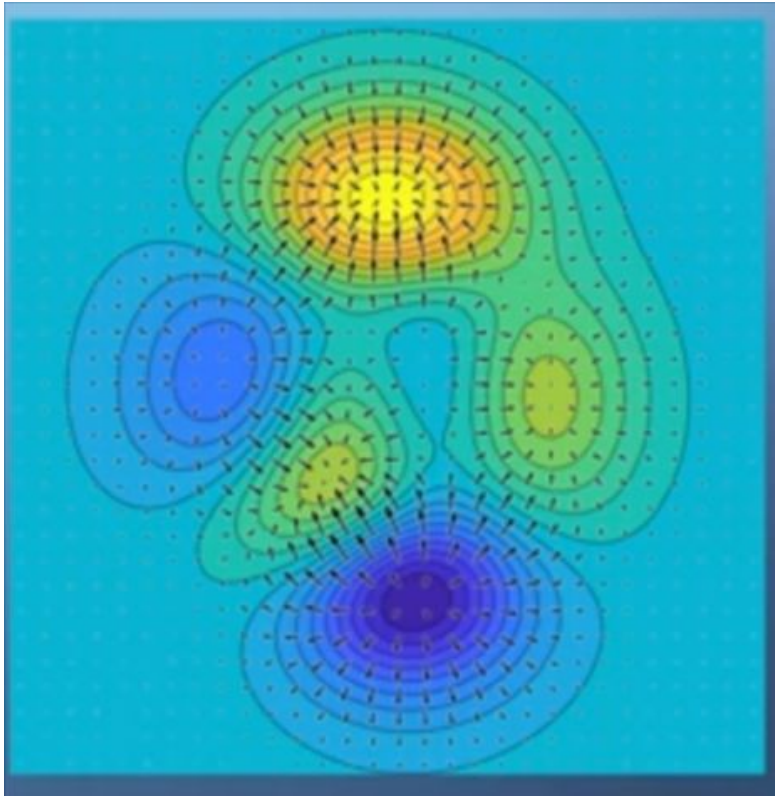
3. Visualisasi



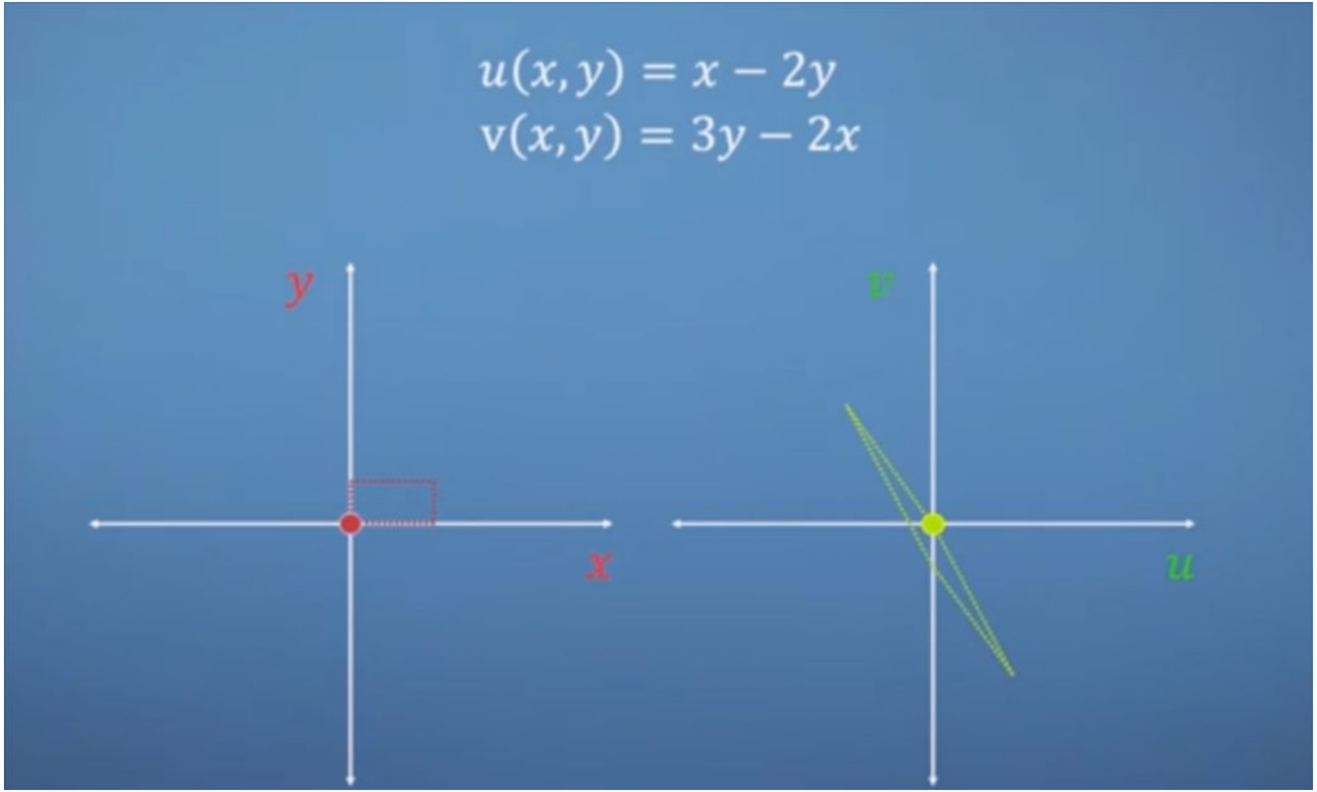


Contoh The Jacobian

4. Jacob vector field



Jacobian Applied



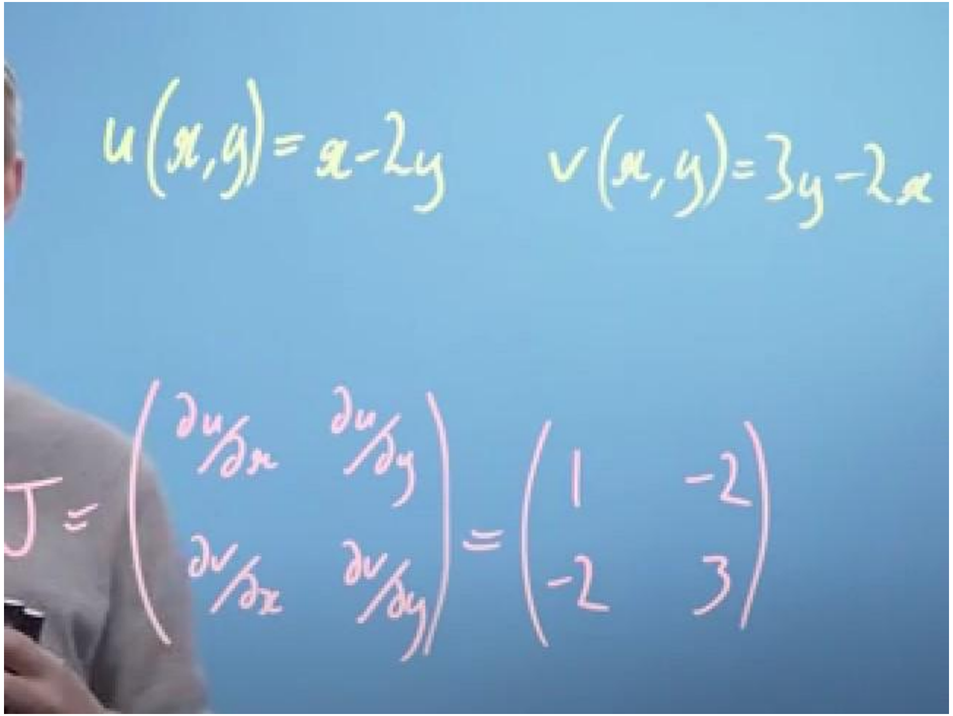
Jacobian Applied

$$u(x,y) = x - 2y$$
$$v(x,y) = 3y - 2x$$

$$J_u = \left[\partial u / \partial x \quad \partial u / \partial y \right]$$

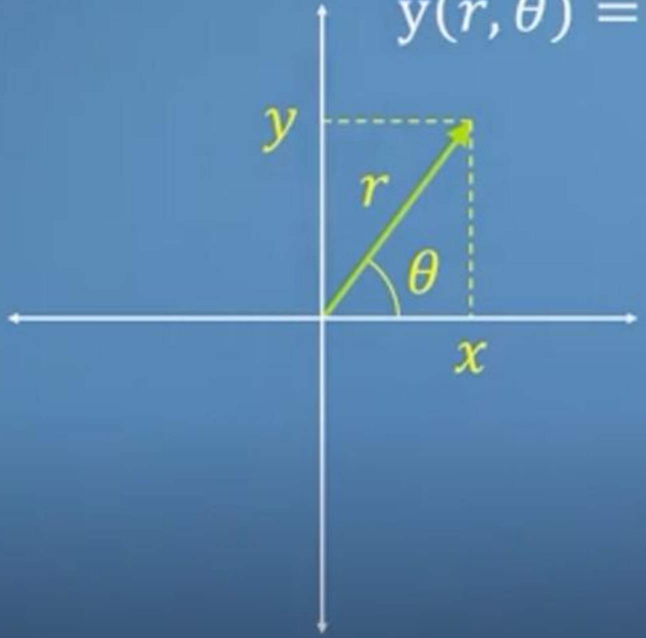
$$J_v = \left[\partial v / \partial x \quad \partial v / \partial y \right]$$

$$J = \begin{bmatrix} \partial u / \partial x & \partial u / \partial y \\ \partial v / \partial x & \partial v / \partial y \end{bmatrix}$$



Jacobian Applied

$$x(r,\theta) = r\cos(\theta)$$
$$y(r,\theta) = r\sin(\theta)$$



$$J = \begin{bmatrix} \partial x/\partial r & \partial x/\partial \theta \\ \partial y/\partial r & \partial y/\partial \theta \end{bmatrix}$$

$$J = \begin{bmatrix} \cos(\theta) & -r\sin(\theta) \\ \sin(\theta) & r\cos(\theta) \end{bmatrix}$$

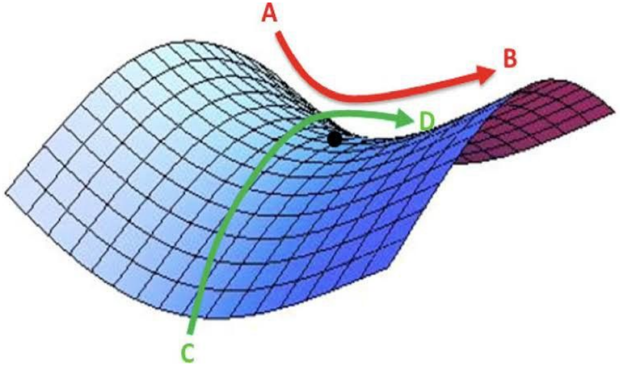
$$|J| = r(\cos^2(\theta) + \sin^2(\theta))$$

$$|J| = r$$

The Hessian

Dalam matematika, matriks Hesse/Hessian adalah matriks persegi dari turunan parsial orde kedua dengan fungsi bernilai skalar, atau medan skalar. Matriks ini mendeskripsikan **kelengkungan** lokal dari fungsi banyak peubah. Dapat membantu menentukan saddle point, dan minimum maximum point sebuah fungsi.

$$\mathbf{H} = \begin{bmatrix} \partial_{x_1 x_1} f & \partial_{x_1 x_2} f & \cdots & \partial_{x_1 x_n} f \\ \partial_{x_2 x_1} f & \partial_{x_2 x_2} f & \cdots & \partial_{x_2 x_n} f \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{x_n x_1} f & \partial_{x_n x_2} f & \cdots & \partial_{x_n x_n} f \end{bmatrix}$$



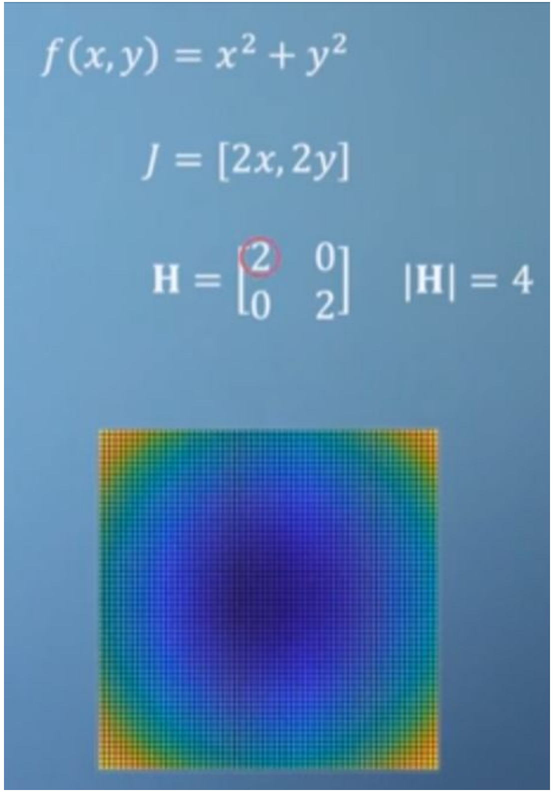
The Hessian

$$f(x,y,z) = x^2 y z$$

$$J = [2xyz,\ x^2 z,\ x^2 y]$$

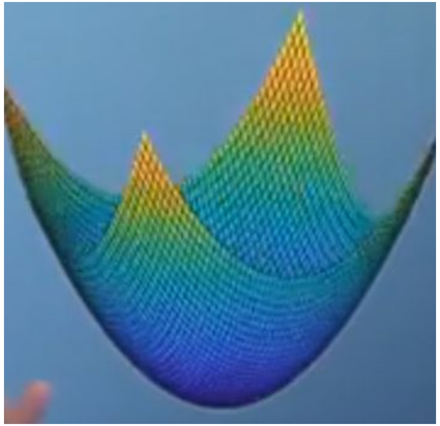
$$H = \begin{bmatrix} 2yz & 2xz & 2xy \\ 2xz & 0 & x^2 \\ 2xy & x^2 & 0 \end{bmatrix}$$

The Hessian



Kalau atas kiri **positif** dan determinan **positif**, **minimal**.

Kalau atas kiri **negatif** dan determinan **positif**, **maximal**.



Q & A

Thank You