Project 2: SATPlan and the Tower of Hanoi

CSCI 561

Fall 2018

The *Tower of Hanoi* is a classic puzzle consisting of round disks stacked on pegs. One must move all disks to the final peg, subject to the following constraints:

- 1. Only one disk can be moved at a time.
- 2. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack.
- 3. No disk may be placed on top of a smaller disk.

Answer the following questions:

- 1. (15 pts) Create PDDL domains (operators and facts) for the following Tower of Hanoi instances (it is possible that the PDDL operators will be the same):
 - (a) Three pegs and two disks.

Answer:

Operators:

Listing 1: Getting labels

```
1 (define (domain hanoi)
     (: predicates
       (clear ?x)
       (smaller ?x ?y)
      (on ?x ?y))
     (:action move-disk
      :parameters (?disk ?from ?to)
8
      :precondition (and (smaller ?disk ?to)
9
                           (on ?disk ?from)
10
                                    (clear ?disk)
11
                                    (clear ?to))
12
      : effect
                (and (clear ?from)
                      (on ?disk ?to)
14
                      (not (on ?disk ?from))
15
                              (not (clear ?to)))))
16
  Facts:
1 (define (problem towers-of-hanoi-three-pegs-two-disks)
     (:domain hanoi)
     (:objects disk1 disk2 peg1 peg2 peg3)
     (:init)
       (on disk1 disk2)
       (on disk2 peg1)
       (clear disk1)
       (clear peg2)
       (clear peg3)
       (smaller disk1 disk2)
```

```
\begin{array}{lll} (smaller \ disk1 \ peg1) \ (smaller \ disk1 \ peg2) \ (smaller \ disk1 \ peg3) \\ (smaller \ disk2 \ peg1) \ (smaller \ disk2 \ peg2) \ (smaller \ disk2 \ peg3)) \end{array}
 11
 12
       (:goal (and (on disk2 peg3)
 13
                     (on disk1 disk2))))
(b) Three pegs and four disks. Answer: Operators:
    (define (domain hanoi)
       (: predicates
         (clear ?x)
         (smaller ?x ?y)
         (on ?x ?y))
  6
       (:action move-disk
  7
         :parameters (?disk ?from ?to)
         :precondition (and (smaller ?disk ?to)
  9
 10
                                (on ?disk ?from)
                                          (clear ?disk)
 11
                                          (clear ?to))
 12
         : effect
                   (and (clear ?from)
 13
                          (on ?disk ?to)
 14
                          (\mathbf{not} \ (\mathbf{on} \ ? \mathbf{disk} \ ? \mathbf{from}))
 15
                                    (not (clear ?to)))))
 16
    Facts:
  1 (define (problem towers-of-hanoi-three-pegs-four-disks)
       (:domain hanoi)
       (:objects disk1 disk2 disk3 disk4 peg1 peg2 peg3)
       (:init)
         (on disk1 disk2)
         (on disk2 disk3)
         (on disk3 disk4)
         (on disk4 peg1)
  8
         (clear disk1)
  9
         (clear peg2)
 10
          clear peg3)
 11
          smaller disk1 disk2)
 12
          smaller disk1 disk3)
 13
          smaller disk1 disk4)
 14
          smaller disk2 disk3)
 15
          smaller disk2 disk4)
 16
          smaller disk3 disk4)
 17
          smaller disk1 peg1) (smaller disk1 peg2) (smaller disk1 peg3)
 18
         (smaller disk2 peg1) (smaller disk2 peg2) (smaller disk2 peg3)
 19
         (smaller disk3 peg1) (smaller disk3 peg2) (smaller disk3 peg3)
 20
 21
         (smaller disk4 peg1) (smaller disk4 peg2) (smaller disk4 peg3))
       (:goal (and (on disk1 disk2)
 22
 23
                     (on disk2 disk3)
                     (on disk3 disk4)
 24
                     (on disk4 peg3))))
```

- 2. (10 pts) Download one or more of the following planners and use them to produce plans for your PDDL domains:
 - Blackbox: https://www.cs.rochester.edu/u/kautz/satplan/blackbox/
 - Madagascar: http://research.ics.aalto.fi/software/sat/madagascar/
 - TMKit: http://tmkit.dyalab.org/

What plans are produced by each planner for each instance (two and four disks)?

(a) Three pegs and two disks

Answer:

```
1 (move-disk disk1 disk2 peg2)
2 (move-disk disk2 peg1 peg3)
3 (move-disk disk1 peg2 disk2)
```

(b) Three pegs and four disks **Answer:**

```
(move-disk disk1 disk2 peg2)
  (move-disk disk2 disk3 peg3)
  (move-disk disk1 peg2 disk2)
   move-disk disk3 disk4 peg2)
   move-disk disk1 disk2 disk4)
5
   move-disk disk2 peg3 disk3)
   (move-disk disk1 disk4 disk2)
   (move-disk disk4 peg1 peg3)
   move-disk disk1 disk2 disk4)
   move-disk disk2 disk3 peg1)
10
   move-disk disk1 disk4 disk2)
  (move-disk disk3 peg2 disk4)
12
  (move-disk disk1 disk2 peg2)
13
  (move-disk disk2 peg1 disk3)
  (move-disk disk1 peg2 disk2)
```

3. (15 pts) Encode the two-disk instance as a Boolean formula using the SATPlan method. Answer:

```
( or (NOT noop_clear_peg2-1)
                                            clear_peg2-0 )
        (NOT noop_clear_peg3-1)
                                     clear_peg3-0 )
    \mathbf{or}
        (NOT noop_smaller_disk1_disk2-1)
                                                smaller_disk1_disk2-0)
    \mathbf{or}
3
        (NOT move-disk_disk1_disk2_peg2-1)
                                                   clear_peg2-0)
    \mathbf{or}
        (NOT move-disk_disk1_disk2_peg2-1)
                                                   clear_disk1-0)
    \mathbf{or}
5
        (NOT move-disk_disk1_disk2_peg2-1)
                                                   on_disk1_disk2-0)
        (NOT move-disk_disk1_disk2_peg2-1)
                                                  smaller_disk1_peg2-0)
    or
        (NOT move-disk_disk1_disk2_peg3-1)
                                                  clear_peg3-0)
    \mathbf{or}
        (NOT move-disk_disk1_disk2_peg3-1)
                                                  clear_disk1-0)
    \mathbf{or}
9
        (NOT move-disk_disk1_disk2_peg3-1)
                                                 on_disk1_disk2-0
10
    or
        (NOT move-disk_disk1_disk2_peg3-1)
                                                 smaller_disk1_peg3-0)
11
        (NOT noop_smaller_disk1_peg2-1)
                                               smaller_disk1_peg2-0)
12
    or
        (NOT noop_smaller_disk1_peg3-1)
                                               smaller_disk1_peg3-0)
14
    or
        (NOT noop_on_disk1_disk2-1)
                                           on_disk1_disk2-0)
        (NOT noop_on_disk2_peg1-1)
                                          on_disk2_peg1-0)
    \mathbf{or}
15
16
    or
        (NOT noop_clear_disk1-1)
                                       clear_disk1-0)
        (NOT noop\_smaller\_disk2\_peg2-1)
                                               smaller_disk2_peg2-0)
17
    or
    \mathbf{or}
        (NOT noop_smaller_disk2_peg3-1)
                                               smaller_disk2_peg3-0)
        (NOT noop\_clear\_peg3-2)
                                      clear_peg3-1 )
19
    or
20
    or
             noop_smaller_disk1_disk2-2)
                                                smaller_disk1_disk2-1)
        (NOT move-disk_disk1_peg3_disk2-2)
                                                 clear_disk2-1)
21
    or
        (NOT move-disk_disk1_peg3_disk2-2)
                                                  clear_disk1-1)
    or
22
        (NOT move-disk_disk1_peg3_disk2-2)
                                                 on_disk1_peg3-1
                                                 smaller_disk1_disk2-1)
        (NOT move-disk_disk1_peg3_disk2-2
24
    \mathbf{or}
             move-disk\_disk1\_disk2\_peg2-2
                                                  clear_peg2-1)
25
    \mathbf{or}
        (NOT
        (NOT move-disk_disk1_disk2_peg2-2)
                                                  clear_disk1-1)
26
    or
        (NOT move-disk_disk_1_disk_2_peg_2-2)
                                                   on_disk1_disk2-1)
27
    or
        (NOT move-disk_disk1_disk2_peg2-2
                                                  smaller_disk1_peg2-1)
28
    \mathbf{or}
                                                 clear_peg3-1)
        (NOT move-disk_disk1_disk2_peg3-2
29
    \mathbf{or}
             move-disk_disk1_disk2_peg3-2
                                                  clear_disk1-1
30
        (NOT
        (NOT move-disk_disk1_disk2_peg3-2
31
    or
                                                 on_disk1_disk2-1)
        (NOT move-disk_disk1_disk2_peg3-2
                                                 smaller_disk1_peg3-1)
    or
32
33
    or
        (NOT move-disk_disk_2-peg_1-peg_2-2)
                                                 clear_peg2-1)
    or
        (NOT move-disk_disk_2_peg1_peg2-2)
                                                  clear_disk2-1
34
35
    \mathbf{or}
        (NOT move-disk_disk2_peg1_peg2-2
                                                 on_disk2_peg1-1)
        (NOT\ move-disk_disk2\_peg1\_peg2-2
                                                 smaller_disk2_peg2-1)
36
    or
        (NOT move-disk_disk1_peg3_peg2-2
                                                  clear_peg2-1 )
37
    or
                                                 clear_disk1-1)
    \mathbf{or}
        (NOT move-disk_disk1_peg3_peg2-2)
38
39
    \mathbf{or}
        (NOT move-disk_disk1_peg3_peg2-2
                                                 on_disk1_peg3-1
        (NOT move-disk_disk1_peg3_peg2-2
                                                 smaller_disk1_peg2-1)
40
    \mathbf{or}
                                                 clear_peg3-1 )
        (NOT move-disk_disk2_peg1_peg3-2
41
    or
        (NOT move-disk_disk2_peg1_peg3-2
                                                  clear_disk2-1)
        (NOT\ move-disk_disk2_peg1_peg3-2)
                                                 on_disk2_peg1-1)
43
    or
        (NOT move-disk_disk2_peg1_peg3-2)
                                                 smaller_disk2_peg3-1)
    \mathbf{or}
44
    or
        (NOT noop_on_disk1_disk2-2)
                                           on_di
                                                 sk1_disk2-1)
45
        (NOT move-disk_disk1_peg2_disk2-2)
                                                   clear_disk2-1
46
    or
        (NOT move-disk_disk1_peg2_disk2-2)
                                                   clear_disk1-1)
    \mathbf{or} (NOT move-disk-disk1-peg2-disk2-2)
                                                   on_disk1_peg2-1)
```

```
or (NOT move-disk_disk1_peg2_disk2-2)
                                                                               smaller_disk1_disk2-1)
 49
              (NOT noop_on_disk2_peg1-2)
                                                                  on_disk2_peg1-1)
        \mathbf{or}
              (NOT noop_clear_disk1-2)
                                                              clear_disk1-1)
 51
        or
              (NOT noop_clear_disk2-2)
                                                              clear_disk2-1)
 53
        \mathbf{or}
              (NOT move-disk_disk_{1-peg_{2-peg_{3}-2}})
                                                                              clear_peg3-1)
              (NOT move-disk_disk1_peg2_peg3-2)
                                                                              clear_disk1-1
        or
 54
              (NOT move-disk_disk1_peg2_peg3-2)
                                                                              on_disk1_peg2-1)
 55
        \mathbf{or}
              (NOT\ move-disk\_disk1\_peg2\_peg3-2)
                                                                              smaller_disk1_peg3-1)
 56
        or
              (NOT noop_on_disk1_peg2-2)
                                                                  on_disk1_peg2-1)
              (NOT noop_on_disk1_peg3-2)
                                                                  on_disk1_peg3-1)
        or
 58
              (NOT noop_smaller_disk2_peg3-2)
                                                                          smaller_disk2_peg3-1)
 59
        or
              (NOT move-disk_disk2_peg2_peg3-3)
        or
                                                                              clear_peg3-2)
 60
              (NOT move-disk_disk2_peg2_peg3-3)
                                                                              clear_disk2-2 )
        \mathbf{or}
 61
              (NOT move-disk_disk2_peg2_peg3-3)
                                                                              on_disk2_peg2-2 )
 62
              (NOT move-disk_disk2_peg2_peg3-3)
                                                                              smaller_disk2_peg3-2 )
 63
        \mathbf{or}
              (NOT move-disk_disk1_peg3_disk2-3)
                                                                                clear_disk2-2
 64
        or
              (\texttt{NOT move-disk\_disk1\_peg3\_disk2-3})
 65
        or
                                                                                clear_disk1-2)
              (NOT move-disk_disk1_peg3_disk2-3)
                                                                                on_disk1_peg3-2)
        \mathbf{or}
 66
              (NOT move-disk_disk1_peg3_disk2-3)
                                                                                smaller_disk1_disk2-2)
        or
                                                                              clear_peg3-2 )
              (NOT move-disk_disk2_peg1_peg3-3)
 68
        \mathbf{or}
 69
        \mathbf{or}
              (NOT move-disk_disk2_peg1_peg3-3)
                                                                              \operatorname{clear\_disk} 2 - 2
              (NOT move-disk_disk2_peg1_peg3-3)
                                                                              on_disk2_peg1-2 )
        or
 70
              (NOT move-disk_disk2_peg1_peg3-3)
                                                                              smaller_disk2_peg3-2)
 71
        or
              (NOT noop_on_disk1_disk2-3)
                                                                   on_disk1_disk2-2)
 72
        or
              (NOT move-disk_disk1_peg2_disk2-3)
                                                                                clear_disk2-2
        \mathbf{or}
 73
              (NOT move-disk_disk1_peg2_disk2-3
                                                                                clear_disk1-2
 74
              (NOT\ move-disk\_disk1\_peg2\_disk2-3\ )
                                                                                on\_disk1\_peg2-2
 75
        or
              (NOT move-disk_disk1_peg2_disk2-3)
                                                                                smaller_disk1_disk2-2)
 76
        \mathbf{or}
                                                                  on_disk2_peg3-2 )
 77
        \mathbf{or}
              (NOT noop_on_disk2_peg3-3)
              (NOT clear_peg3-1)
                                                     noop\_clear\_peg3-1 )
 78
        \mathbf{or}
              (NOT smaller_disk2_peg2-1)
                                                                   noop_smaller_disk2_peg2-1
 79
              NOT smaller_disk2_peg3-1)
                                                                   noop\_smaller\_disk2\_peg3-1
 80
        or
              (NOT on_disk1_peg2-1)
                                                           move-disk_disk_1_disk_2_peg_2-1 )
 81
 82
        \mathbf{or}
              (NOT on_disk1_peg3-1)
                                                           move-disk_disk1_disk2_peg3-1
              (NOT clear_disk1-1)
                                                       noop_clear_disk1-1
        or
 83
              (NOT clear_disk2-1)
        or
                                                       move-disk\_disk1\_disk2\_peg2-1 move-disk\_disk1\_disk2\_peg3-1)
                                                                     noop_smaller_disk1_disk2-1
              (NOT smaller_disk1_disk2-1)
 85
        or
              (NOT smaller_disk1_peg2-1)
                                                                    noop\_smaller\_disk1\_peg2-1 )
                                                                   noop\_smaller\_disk1\_peg3-1
              (NOT smaller_disk1_peg3-1)
 87
        or
        or
              (NOT on_disk1_disk2-1)
                                                            noop\_on\_disk1\_disk2-1 )
 88
              (NOT on_disk2_peg1-1)
        \mathbf{or}
                                                           noop\_on\_disk2\_peg1-1 )
 89
              (NOT clear_peg2-1)
                                                      noop\_clear\_peg2-1 )
        \mathbf{or}
 90
              (NOT noop\_clear\_peg2-1)
                                                             (NOT move-disk_disk1_disk2_peg2-1 ))
 91
              (NOT noop\_clear\_peg3-1)
                                                             (NOT move-disk_disk1_disk2_peg3-1))
 92
        \mathbf{or}
              (NOT move-disk_disk_1_disk_2_peg_2-1)
                                                                                (NOT move-disk_disk1_disk2_peg3-1 ))
        \mathbf{or}
 93
              (NOT\ move-disk_disk1_disk2_peg2-1)
                                                                                (NOT\ noop\_on\_disk1\_disk2-1\ ))
 94
        or
              (NOT move-disk_disk1_disk2_peg3-1)
                                                                                (NOT noop_on_disk1_disk2-1)
 95
        or
              (NOT on_disk2_peg3-2)
                                                         move-disk_disk_2-peg_1-peg_3-2
             (NOT clear_peg3-2)
                                                      noop\_clear\_peg3-2 \quad move-disk\_disk1\_peg3\_disk2-2 \quad move-disk\_disk1\_peg3\_peg2
 97
        \mathbf{or}
 98
        or
             (NOT smaller_disk2_peg3-2)
                                                                   noop\_smaller\_disk2\_peg3-2 )
                                                           noop\_on\_disk1\_peg2-2 move\_disk\_disk1\_peg3\_peg2-2 move\_disk\_disk1\_disk1\_disk1
              (NOT on_disk1_peg2-2)
        \mathbf{or}
99
100
        \mathbf{or}
              (NOT on_disk1_peg3-2)
                                                           noop\_on\_disk1\_peg3-2
                                                                                                move-disk\_disk1\_peg2\_peg3-2
                                                                                                                                                   move-disk_disk1_disk
              (NOT clear_disk1-2)
                                                       noop\_clear\_disk1-2 )
        or
101
102
        \mathbf{or}
              (NOT smaller_disk1_disk2
                                                       -2)
                                                                     noop_smaller_disk1_disk2-2 )
             (NOT clear_disk2-2)
                                                       noop\_clear\_disk2-2 \\ move-disk\_disk1\_disk2\_peg2-2 \\ move-disk\_disk2\_peg2-2 \\ move-disk\_disk2\_peg2-2 \\ move-disk\_disk2\_peg2-2 \\ move-disk\_disk2\_peg2-2 \\ move-disk\_disk2\_peg2-2 \\ move-disk\_disk2\_peg2-2 \\ move-disk2\_peg2-2 \\ mo
103
        or
        or (NOT on_disk1_disk2-2)
                                                             noop\_on\_disk1\_disk2-2 move-disk\_disk1\_peg2\_disk2-2 move-disk\_disk1\_peg2\_disk2-2
104
             (NOT on_disk2_peg1-2)
                                                           noop\_on\_disk2\_peg1-2 )
105
              (NOT on_disk2_peg2-2)
                                                           move-disk_disk_2-peg_1-peg_2-2 )
106
        \mathbf{or}
              (NOT noop\_clear\_peg3-2)
                                                             (NOT move-disk_disk_1-peg_2-peg_3-2))
107
              (NOT noop_clear_peg3-2)
                                                             (NOT move-disk_disk2_peg1_peg3-2))
108
        \mathbf{or}
              (NOT noop_clear_peg3-2)
                                                             (NOT move-disk_disk1_disk2_peg3-2))
        \mathbf{or}
109
        or
              (NOT noop\_clear\_peg3-2)
                                                             (NOT move-disk_disk_1-peg_3-peg_2-2))
110
             (NOT noop\_clear\_peg3-2)
                                                             (NOT move-disk_disk1_peg3_disk2-2))
111
        \mathbf{or}
             (NOT noop\_clear\_peg3-2)
                                                             (NOT noop_on_disk1_peg3-2))
113 ( or (NOT move-disk_disk1_peg3_disk2-2)
                                                                               (NOT move-disk_disk1_peg2_peg3-2))
```

```
(NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT move-disk_disk1_peg3_peg2-2))
     \mathbf{or}
114
        (NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT noop_on_disk1_peg3-2))
115
     \mathbf{or}
        (NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT move-disk_disk1_disk2_peg3-2))
116
     or
117
        (NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT move-disk_disk1_disk2_peg2-2 ))
        (NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT move-disk_disk2_peg1_peg2-2))
118
     or
         (NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT noop_clear_disk2-2)
     or
119
                                                  (NOT\ move-disk_disk1_peg2_disk2-2))
     or
        (NOT move-disk_disk1_peg3_disk2-2)
120
        (NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT noop_on_disk1_peg2-2)
     \mathbf{or}
121
        (NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT move-disk_disk2_peg1_peg3-2))
        (NOT move-disk_disk1_peg3_disk2-2)
                                                  (NOT noop_on_disk1_disk2-2 ))
123
     or
             move-disk_disk_1_disk_2_peg_2-2)
                                                  (NOT move-disk_disk1_disk2_peg3-2))
124
     or
        (NOT move-disk_disk1_disk2_peg2-2)
                                                  (NOT noop_on_disk1_disk2-2)
     or
125
        (NOT move-disk_disk1_disk2_peg2-2)
                                                  (NOT move-disk_disk1_peg2_peg3-2))
     or
126
        (NOT move-disk_disk1_disk2_peg2-2)
                                                  (NOT move-disk_disk_1-peg_3-peg_2-2))
127
                                                  (NOT noop_on_disk1_peg3-2 ))
        (NOT move-disk_disk1_disk2_peg2-2)
128
     or
        (NOT move-disk_disk1_disk2_peg2-2)
                                                  (NOT move-disk_disk_2-peg_1-peg_3-2))
     \mathbf{or}
129
        (NOT move-disk_disk1_disk2_peg2-2)
                                                  (NOT move-disk_disk2_peg1_peg2-2))
130
     or
        (NOT move-disk_disk1_disk2_peg2-2)
                                                  (NOT noop_clear_disk2-2)
     or
131
                                                  (NOT move-disk_disk1_peg2_disk2-2))
         (NOT move-disk_disk1_disk2_peg2-2)
132
     or
         (NOT move-disk_disk1_disk2_peg2-2)
                                                  (NOT noop_on_disk1_peg2-2 ))
     or
133
134
         (NOT move-disk_disk1_disk2_peg3-2)
                                                  (NOT noop_on_disk1_disk2-2)
                                                  (NOT move-disk_disk1_peg2_peg3-2 ))
        (NOT move-disk_disk1_disk2_peg3-2)
135
     or
        (NOT move-disk_disk1_disk2_peg3-2)
                                                  (NOT noop_on_disk1_peg2-2))
     or
136
                                                  (NOT move-disk_disk1_peg2_disk2-2))
         (NOT move-disk_disk1_disk2_peg3-2)
137
     or
     or
        (NOT move-disk_disk1_disk2_peg3-2)
                                                 (NOT move-disk_disk_2-peg_1-peg_3-2))
138
         (NOT move-disk_disk1_disk2_peg3-2)
                                                 (NOT move-disk_disk_2-peg_1-peg_2-2))
139
        NOT move-disk_disk1_disk2_peg3-2)
                                                  (NOT noop\_clear\_disk2-2))
140
     or
        (NOT move-disk_disk1_disk2_peg3-2)
                                                  (NOT move-disk_disk1_peg3_peg2-2))
     \mathbf{or}
141
        (NOT move-disk_disk1_disk2_peg3-2)
                                                  (NOT noop_on_disk1_peg3-2))
142
     or
         (NOT move-disk_disk2_peg1_peg2-2)
                                                 (NOT move-disk_disk2_peg1_peg3-2))
143
     or
        (NOT move-disk_disk_2-peg_1-peg_2-2)
                                                 (NOT noop_on_disk2_peg1-2)
144
        (NOT move-disk_disk2_peg1_peg2-2)
                                                 (NOT move-disk_disk1_peg2_peg3-2 ))
145
     or
        (NOT move-disk_disk2_peg1_peg2-2)
                                                 (NOT move-disk_disk1_peg2_disk2-2))
146
        (NOT\ move-disk_disk2_peg1_peg2-2)
                                                 (NOT move-disk_disk_1-peg_3-peg_2-2))
147
     or
        (NOT move-disk_disk2_peg1_peg2-2)
                                                 (NOT noop_on_disk1_disk2-2)
     \mathbf{or}
148
        NOT move-disk_disk2_peg1_peg2-2)
                                                 (NOT noop_on_disk1_peg2-2 ))
149
     or
        (NOT move-disk_disk1_peg3_peg2-2)
                                                 (NOT noop_on_disk1_peg3-2))
150
     or
151
     \mathbf{or}
        (NOT move-disk_disk1_peg3_peg2-2)
                                                 (NOT move-disk_disk1_peg2_peg3-2))
        (NOT move-disk_disk1_peg3_peg2-2)
                                                 (NOT move-disk_disk2_peg1_peg3-2 ))
152
     or
153
     or
             move-disk_disk_{1-peg_{3-peg_{2}-2}}
                                                 (NOT noop_on_disk1_disk2-2)
                                                 (NOT move-disk_disk1_peg2_disk2-2 ))
        (NOT\ move-disk_disk1_peg3_peg2-2)
     or
154
        (NOT move-disk_disk1_peg3_peg2-2)
                                                 (NOT noop_on_disk1_peg2-2)
     or
155
        (NOT move-disk_disk2_peg1_peg3-2)
                                                 (NOT move-disk_disk_1-peg_2-disk_2-2))
156
        (NOT move-disk_disk2_peg1_peg3-2)
                                                 (NOT noop_on_disk2_peg1-2))
157
     or
             move-disk_disk2_peg1_peg3-2)
                                                 (NOT move-disk_disk1_peg2_peg3-2))
     \mathbf{or}
         (NOT
158
                                                 (NOT noop_on_disk1_disk2-2 ))
        (NOT move-disk_disk2_peg1_peg3-2)
159
     or
        (NOT move-disk_disk2_peg1_peg3-2)
                                                 (NOT noop_on_disk1_peg3-2))
160
     or
         (NOT noop_on_disk1_disk2-2)
                                                noop\_on\_disk1\_peg3-2 ))
161
     \mathbf{or}
         (NOT noop_on_disk1_disk2-2)
                                          (NOT noop_on_disk1_peg2-2 ))
162
     or
         (NOT)
             noop_on_disk1_disk2-2)
                                          (NOT move-disk_disk1_peg2_peg3-2))
163
        (NOT noop_on_disk1_disk2-2)
                                          (NOT noop_clear_disk2-2)
164
     or
                                          (NOT move-disk_disk1_peg2_disk2-2))
        (NOT noop_on_disk1_disk2-2)
     or
165
         (NOT move-disk_disk1_peg2_disk2-2)
                                                  (NOT move-disk_disk1_peg2_peg3-2))
166
     or
                                                  NOT noop_on_disk1_peg2-2 ))
     or
        (NOT move-disk_disk1_peg2_disk2-2)
167
                                                  (NOT noop_clear_disk2-2 ))
168
     \mathbf{or}
        (NOT move-disk_disk1_peg2_disk2-2)
        (NOT move-disk_disk1_peg2_disk2-2)
                                                  (NOT noop_on_disk1_peg3-2))
169
     or
        (NOT move-disk_disk1_peg2_peg3-2)
                                                 (NOT noop_on_disk1_peg2-2)
170
        (NOT move-disk_disk1_peg2_peg3-2)
                                                 (NOT noop_on_disk1_peg3-2))
171
     or
                                         (NOT noop_on_disk1_peg3-2))
172
        (NOT noop_on_disk1_peg2-2)
        noop\_on\_disk2\_peg3-3 \quad move-disk\_disk2\_peg1\_peg3-3 \quad move-disk\_disk2\_peg2\_peg3-3
173
        noop\_on\_disk1\_disk2-3 move-disk\_disk1\_peg2\_disk2-3 move-disk\_disk1\_peg3\_disk2-3
174
     or
        (NOT\ move-disk\_disk2\_peg2\_peg3-3
                                                 (NOT move-disk_disk1_peg2_disk2-3))
175
     or
        (NOT move-disk_disk2_peg2_peg3-3
                                                 (NOT move-disk_disk1_peg3_disk2-3 ))
     \mathbf{or}
176
        NOT move-disk_disk2_peg2_peg3-3
                                                 (NOT move-disk_disk2_peg1_peg3-3 ))
     or
177
        (NOT move-disk_disk2_peg2_peg3-3
                                                 (NOT noop_on_disk1_disk2-3)
178
     \mathbf{or}
        (NOT move-disk_disk2_peg2_peg3-3
                                                 (NOT noop_on_disk2_peg3-3)
        (NOT move-disk_disk1_peg3_disk2-3)
                                                  (NOT noop_on_disk2_peg3-3)
```

```
or (NOT move-disk_disk1_peg3_disk2-3)
                                                   (NOT move-disk_disk2_peg1_peg3-3))
181
         (NOT move-disk_disk1_peg3_disk2-3 )
                                                   (NOT move-disk_disk1_peg2_disk2-3))
     \mathbf{or}
         (NOT move-disk_disk1_peg3_disk2-3)
                                                   (NOT noop_on_disk1_disk2-3))
183
     or
         (NOT move-disk_disk2_peg1_peg3-3)
                                                  (NOT move-disk_disk1_peg2_disk2-3))
184
         (NOT move-disk_disk2_peg1_peg3-3)
                                                  (NOT noop_on_disk1_disk2-3)
185
     \mathbf{or}
         (NOT move-disk_disk2_peg1_peg3-3)
                                                  (NOT noop_on_disk2_peg3-3)
     \mathbf{or}
186
         (NOT noop_on_disk1_disk2-3)
                                            (NOT noop_on_disk2_peg3-3 ))
     \mathbf{or}
187
   ( or (NOT noop_on_disk1_disk2-3)
                                           (NOT move-disk_disk1_peg2_disk2-3))
188
    clear_peg3-0
189
190
    smaller\_disk2\_peg2-0
    smaller_disk2_peg3-0
191
    clear_disk1-0
192
    smaller_disk1_disk2-0
193
    smaller\_disk1\_peg2-0
    \verb|smaller_disk1_peg3-0|
195
    on_disk1_disk2-0
196
197
    on\_disk2\_peg1-0
    clear_peg2-0
198
    on_disk2_peg3-3
    on_disk1_disk2-3
200
```

- 4. (10 pts) Find the satisfying assignments for two-disk boolean formula using your DPLL implementation.
 - (a) What is the satisfying assignment?

```
(NOOP_SMALLER_DISK1_PEG2-1 . T)
         NOOP_SMALLER_DISK1_PEG3-1 . T)
2
         (NOOP_SMALLER_DISK2_PEG2-1 . T)
3
         (NOOP\_SMALLER\_DISK2\_PEG3-2 . T)
         (ON_DISK1_PEG2-1 . T)
         (ON_DISK1_PEG3-2)
         (NOOP_ON_DISK1_PEG2-2 . T)
         (NOOP_CLEAR_DISK2-2 . T)
8
         (CLEAR_PEG3-2)
         (ON_DISK1_DISK2-2)
10
         (ON_DISK1_DISK2-1)
         (ON_DISK2_PEG1-2)
12
13
         (NOOP_ON_DISK1_DISK2-1)
         (ON_DISK2_PEG2-2)
14
         (CLEAR_PEG2-1)
15
         (NOOP_CLEAR_PEG3-2)
16
         (NOOP_CLEAR_PEG2-1)
17
         (MOVE-DISK_DISK1_DISK2_PEG2-2)
         (NOOP\_ON\_DISK2\_PEG1-1 . T)
19
         (MOVE-DISK_DISK1_DISK2_PEG3-2)
20
21
         (MOVE-DISK_DISK1_DISK2_PEG2-1 . T)
         (MOVE-DISK_DISK2_PEG1_PEG2-2)
22
         (NOOP\_CLEAR\_DISK1-1 . T)
23
         (MOVE-DISK_DISK1_PEG2_DISK2-2)
24
         (NOOP_SMALLER_DISK2_PEG3-1 . T)
25
         (NOOP_ON_DISK2_PEG1-2)
26
         (SMALLER_DISK2_PEG3-1 . T)
27
         (MOVE-DISK_DISK1_PEG2_PEG3-2)
         (ON\_DISK2\_PEG1-1 . T)
29
         (NOOP_ON_DISK1_DISK2-2)
30
31
         (CLEAR_DISK2-1 . T)
         (MOVE-DISK\_DISK2\_PEG1\_PEG3-2 . T)
32
         (ON\_DISK2\_PEG3-2 . T)
33
         (NOOP_SMALLER_DISK1_DISK2-1 . T)
34
         (SMALLER_DISK1_DISK2-1 . T)
35
         (NOOP\_SMALLER\_DISK1\_DISK2-2 . T)
36
         (CLEAR_DISK1-1 . T)
37
         (NOOP_ON_DISK2_PEG3-3 T)
38
         (NOOP\_CLEAR\_DISK1-2 . T)
39
         (MOVE-DISK_DISK2_PEG2_PEG3-3)
40
         (SMALLER_DISK1_DISK2-2 . T)
41
         (MOVE-DISK_DISK1_PEG3_DISK2-3)
```

```
(ON_DISK1_PEG2-2 . T)
 43
           (MOVE-DISK_DISK2_PEG1_PEG3-3)
          (CLEAR_DISK1-2 . T)
 45
 46
           (NOOP_ON_DISK1_DISK2-3)
           (CLEAR_DISK2-2 . T)
 47
           (MOVE-DISK_DISK1_PEG2_DISK2-3 . T)
 48
          (MOVE-DISK_DISK1_PEG3_PEG2-2)
 49
          (NOOP_ON_DISK1_PEG3-2)
 50
           (MOVE-DISK_DISK1_PEG3_DISK2-2)
           (ON_DISK1_PEG3-1)
 52
           (MOVE-DISK_DISK1_DISK2_PEG3-1)
 53
          (NOOP_CLEAR_PEG3-1 . T)
 54
          (CLEAR_PEG3-1 . T)
 55
           (SMALLER_DISK1_PEG3-0 . T)
 56
           (SMALLER_DISK1_PEG2-0 . T)
 57
           (ON_DISK1_DISK2-0 . T)
 58
          (SMALLER_DISK1_DISK2-0 . T)
 59
          (ON\_DISK2\_PEG1-0 . T)
 60
          (CLEAR\_DISK1-0 . T)
 61
          (CLEAR_PEG2-0 . T)
 62
          (SMALLER\_DISK2\_PEG3-0 . T)
          (ON\_DISK2\_PEG3-3 . T)
 64
          (SMALLER_DISK2_PEG2-0 . T)
 65
           (ON\_DISK1\_DISK2-3 . T)
 66
          (CLEAR_PEG3-0 . T)
 67
(b) What is the corresponding plan?
          (NOOP_SMALLER_DISK1_PEG2-1 . T)
           (NOOP_SMALLER_DISK1_PEG3-1 . T)
 2
          (NOOP_SMALLER_DISK2_PEG2-1 . T)
 3
          (NOOP_SMALLER_DISK2_PEG3-2 . T)
           (ON_DISK1_PEG2-1 . T)
           (NOOP_ON_DISK1_PEG2-2 . T)
           (NOOP_CLEAR_DISK2-2 . T)
          (NOOP\_ON\_DISK2\_PEG1-1 . T)
          (MOVE-DISK_DISK1_DISK2_PEG2-1 . T)
           (NOOP_CLEAR_DISK1-1 . T)
 10
           (NOOP_SMALLER_DISK2_PEG3-1 . T)
 11
 12
           (SMALLER_DISK2_PEG3-1 . T)
          (ON_DISK2_PEG1-1 . T)
 13
          (CLEAR_DISK2-1 . T)
 14
 15
           (MOVE-DISK_DISK2_PEG1_PEG3-2 . T)
           (ON_DISK2_PEG3-2 . T)
 16
           (NOOP\_SMALLER\_DISK1\_DISK2-1 . T)
          (SMALLER_DISK1_DISK2-1 . T)
 18
          (NOOP_SMALLER_DISK1_DISK2-2 . T)
 19
           (CLEAR_DISK1-1 . T)
 20
           (NOOP_ON_DISK2_PEG3-3 . T)
 21
          (NOOP\_CLEAR\_DISK1-2 . T)
 22
          (SMALLER_DISK1_DISK2-2 . T)
 23
          (ON\_DISK1\_PEG2-2 . T)
 24
          (CLEAR_DISK1-2 . T)
(CLEAR_DISK2-2 . T)
 25
 26
           (MOVE-DISK_DISK1_PEG2_DISK2-3 . T)
          (NOOP_CLEAR_PEG3-1 . T)
 28
           (CLEAR_PEG3-1 . T)
           (SMALLER_DISK1_PEG3-0 . T)
 30
           (SMALLER_DISK1_PEG2-0 . T)
 31
          (ON\_DISK1\_DISK2-0 . T)
 32
          (SMALLER_DISK1_DISK2-0 . T)
 33
          (ON_DISK2_PEG1-0 . T)
 34
          (CLEAR_DISK1-0 . T)
(CLEAR_PEG2-0 . T)
 35
 36
          (SMALLER\_DISK2\_PEG3-0 . T)
 37
          (ON_DISK2_PEG3-3 . T)
 38
           (SMALLER_DISK2_PEG2-0 . T)
 39
           ON_DISK1_DISK2-3 . T)
 40
          (CLEAR\_PEG3-0 . T)
```

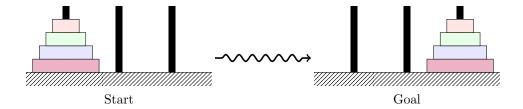


Figure 1: Tower of Hanoi Puzzle with 3 pegs and 4 disks.

- 5. Extra Credit: Compare the performance/scalability of your DPLL implementation with one or more state-of-the-art SMT solvers such as:
 - Z3: https://github.com/Z3Prover/z3
 - CVC4: http://cvc4.cs.stanford.edu/web/

(Note: you might find the Lisp TIME macro and SBCL's statistical profiler (http://www.sbcl.org/manual/#Statistical-Profiler) useful to evaluate performance).

6. Extra Credit: Optimize your DPLL implementation. For example, you could improve the implementation of DPLL-CHOOSE-LITERAL. Discuss the optimizations you implement and characterize the speedup (for example, using TIME or SBCL's statistical profiler).