Cooling Atomic Gases With Disorder

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Cold atomic gases have proven capable of emulating a number of fundamental condensed matter phenomena including Bose-Einstein condensation, the Mott transition, Fulde-Ferrell-Larkin-Ovchinnikov pairing and the quantum Hall effect. Cooling to a low enough temperature to explore magnetism and exotic superconductivity in lattices of fermionic atoms remains a challenge. We propose a method to produce a low temperature gas by preparing it in a disordered potential and following a constant entropy trajectory to deliver the gas into a non-disordered state which exhibits these incompletely understood phases. We show, using quantum Monte Carlo simulations, that we can approach the Neél temperature of the three-dimensional Hubbard model for experimentally achievable parameters. Recent experimental estimates suggest the randomness required lies in a regime where atom transport and equilibration are still robust.

Introduction: The interplay of disorder and interactions is a central problem in condensed matter physics, both from the viewpoint of materials like the heavy fermions [1, 2], high-temperature superconductors [3], and manganites [4], and also because of intriguing theoretical issues such as the fate of Anderson localization in the presence of interactions, especially in two dimensions [5, 6]. Ultracold atomic gases offer the opportunity to emulate these fundamental issues using optical speckle [7, 8], impurities [9], or a quasiperiodic optical lattice [10, 11] to introduce randomness. In the bosonic case, the competition between strong interactions and strong disorder has been studied in the context of the elusive Bose glass phase [7, 9, 11], while for fermions, a recent experiment has explored disorder-induced localization in the three-dimensional (3D) Hubbard model of strongly-interacting fermions [12].

In this paper, we explore the thermodynamics of interacting, disordered systems and suggest that, in addition to studies of the many-body phenomena noted above, preparing a gas in a random potential might be exploited to cool the atoms. Specifically, we show using an unbiased numerical method that one can lower the temperature and access the regime with long-range magnetic order by adiabatically decreasing the randomness in the chemical potential or hopping energies of the Hubbard Hamiltonian. Results for the double occupancy and antiferromagnetic structure factor lend physical insight into this effect. We also present arguments, partially based on recent experiments, that our suggestion is achievable in practice.

We consider the disordered Hubbard Hamiltonian,

$$H = -\sum_{\langle ij\rangle\sigma} t_{ij}(c^\dagger_{i\sigma}c_{j\sigma} + c^\dagger_{j\sigma}c_{i\sigma}) + U\sum_i (n_{i\uparrow}-\frac{1}{2})(n_{i\downarrow}-\frac{1}{2}) - \sum_i \mu_i\,(n_{i\uparrow}+n_{i\downarrow}) \quad (1)$$

whose emulation [13, 14] with optical lattices is possible using two hyperfine species of fermionic atoms. Here $c^\dagger_{i\sigma}(c_{i\sigma})$ is the creation (destruction) operator for a fermion at spatial site i and spin (or hyperfine state) σ. We consider a cubic lattice of N sites, and hopping t_{ij} between near neighbors $\langle ij\rangle$. The hopping, and the on-site repulsion U, can be tuned with the lattice depth and the Feshbach resonance [13], allowing for the successful exploration of the Mott transition [15–17].

Disorder is introduced via a spatially random chemical potential μ_i or hopping t_{ij}. We choose uniform distributions $\mu_0 - \Delta_\mu < \mu_i < \mu_0 + \Delta_\mu$ or $t_0 - \Delta_t < t_{ij} < t_0 + \Delta_t$, and set the mean of the hopping energy $t_0 = 1$ as the energy scale. For most of this paper we choose $\mu_0 = 0$, which makes the lattice half-filled (average density $n = 1$). However, we also gain insight into the effects of a confining potential, in which the chemical potential increases as one moves spatially away from the trap center, by presenting data for different densities.

Our computational method, determinant quantum Monte Carlo (QMC) [18, 19], treats disorders and interactions on an equal, exact footing, and provides a solution to the Hubbard Hamiltonian on lattices of finite spatial size. We focus on the disorder dependence of the entropy $S(T)$, obtained via a thermodynamic integration of the energy [20] down from $T = \infty$. We also report results for the (site-averaged) double occupancy $D = 1/N\sum_i \langle n_{i\uparrow}n_{i\downarrow}\rangle$, and the structure factor

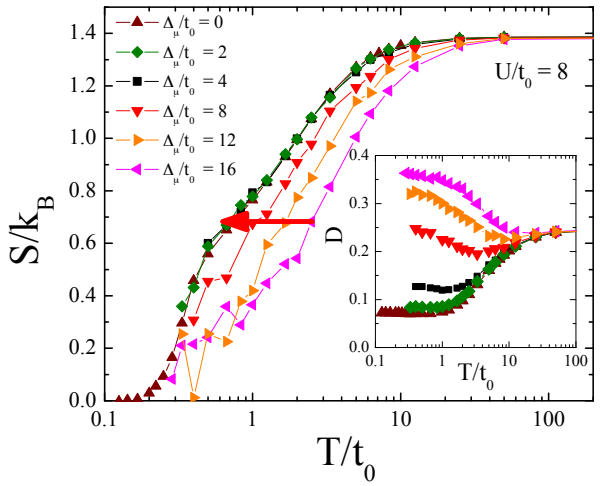

FIG. 1: Entropy S as a function of temperature T for different site disorder strengths Δ_μ at $U/t_0 = 8$. S is largely independent of disorder strength for $\Delta_\mu/t_0 = 2, 4 \lesssim U/t_0 = 8$. For larger randomness, $S(T)$ decreases with Δ_μ so that if disorder is turned off adiabatically, the temperature T decreases, as indicated by the horizontal arrow at $S/k_B = \ln 2$. The inset shows the double occupancy $D(T)$. Large disorder Δ_μ changes the sign of the slope dD/dT from mostly positive, to mostly negative. Here, and in all subsequent figures, unless otherwise indicated, the lattice size is 6^3, the density $n = 1$ is at half filling, and the Trotter discretization is $\Delta\tau = 1/(20t_0)$. Up to 300 disorder realizations are used in the disorder averages.

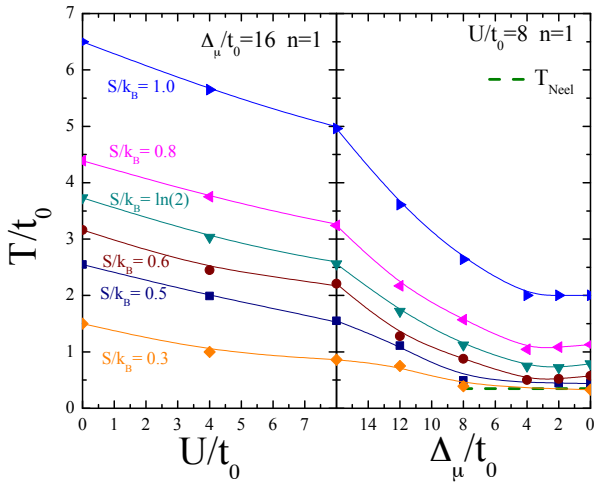

FIG. 2: Adiabats of the disordered 3D Hubbard model along a path which combines an increase of the interaction strength from $U/t_0 = 0$ to $U/t_0 = 8$ at fixed $\Delta_\mu/t_0 = 16$ followed by a reduction of the site disorder. T/t_0 decreases along both trajectories, and, in particular, by about a factor of three at fixed $S/k_B = \ln 2$ along the second path. For $S/k_B = 0.5$, the same reduction brings T down to near T_N.

$S_{\mathbf{q}} = \sum_r e^{i\mathbf{q}\cdot\mathbf{r}} c(\mathbf{r})$ at $\mathbf{q} = (\pi, \pi, \pi)$; S_π, where $c(\mathbf{r}) = \langle c^\dagger_{i+\mathbf{r}\downarrow} c_{i+\mathbf{r}\uparrow} c^\dagger_{i\uparrow} c_{i\downarrow} \rangle$ are spin-spin correlation functions.

Results: The effect of site disorder on $S(T)$ is shown in Fig. 1 at $U/t_0 = 8$, where the Neél transition temperature (T_N) in the homogeneous 3D Hubbard model attains its maximal value [21]. $S(T)$ is largely unaffected by disordered site energies until Δ_μ becomes comparable to U. This is a consequence of the fact that for temperatures less than the repulsion U, the Hubbard model has the character of a Mott insulator in which U blocks transport of Fermions away from singly occupied sites. Such a Mott state is immune to the effects of small disorder $\Delta_\mu/U \lesssim 1$. Our calculated entropy $S(T)$ and double occupancy $D(T)$ (see the inset of Fig. 1) confirm this picture. However, when $\Delta_\mu/U \gtrsim 1$ the entropy curves shift systematically to higher T, reflecting a disorder-driven decrease in S at constant T. The reduction in S can be viewed as the transfer of weight in the specific heat $C(T)$ to a higher temperature: Disorder suppressing the peak in $C(T)$ associated with local magnetic ordering at the exchange energy scale $T \sim J = 4t_0^2/U$, and increasing $C(T)$ at a higher T that scales like Δ_μ due to excitations arising from the transfer of charge between sites of different local μ_i [22]. It is expected that at very low temperatures, the disorder increases the degeneracy of the low lying states, and hence the entropy. However, our results indicate that in the temperature range of interest, $T \gtrsim T_N$ (the Neél temperature), disorder reduces S.

The family of $S(T)$ curves in Fig. 1 indicates that if Δ_μ is switched to zero at constant entropy, the temperature T decreases, in analogy to Pomeranchuk cooling which occurs in a non-disordered lattice when the ratio of repulsion to hopping U/t_0 is increased adiabatically [35]. For the case of site disorder, the double occupancy shows a negative slope $dD/dT < 0$ as seen in the inset of Fig. 1. At high enough temperatures $T \gtrsim t_0, U, \Delta_\mu$, up and down spin fermions are uncorrelated, and D factorizes, $D = \langle n_{i\uparrow} n_{i\downarrow} \rangle \to \langle n_{i\uparrow} \rangle \langle n_{i\downarrow} \rangle$ ($= 1/4$ at half-filling). In the clean limit, as T is lowered, the on-site repulsion eliminates double occupancy, and D falls. Disordered site energies reduce the penalty for double occupancy from U to $U_{\rm eff} = U - |\mu_i - \mu_j|$ so that as Δ_μ grows, $U_{\rm eff}$ becomes negative. The low T phase consists predominantly of doubly occupied and empty sites so that in the limit $\Delta_\mu/t_0 \gg 1$, D approaches $\frac{1}{2}$.

From Fig. 1 we can infer the behavior of T as Δ_μ/t_0 is lowered adiabatically at *fixed* U/t_0. Optical lattice experiments, however, typically involve an increase of U/t_0 from zero to its final value. Figure 2 presents the adiabatic curves of a combined protocol in which the interaction is increased from $U/t_0 = 0$ to $U/t_0 = 8$ in the presence of fixed disorder $\Delta_\mu/t_0 = 16$, followed by the suppression of the disorder to $\Delta_\mu/t_0 = 0$. Data are shown for different values of the starting entropy S/k_B. Figure 2 contains the central observation of our paper: a significant decrease in temperature results from following these adiabats. The substantial cooling in the second part of the path, at fixed U/t_0, is implicit in Fig. 1. A

reduction in T/t_0 also occurs in the initial turning on of the interaction, more so in the presence of disorder than occurs in the clean system [36, 37]. Our QMC results indicate that beginning at temperatures $T/t_0 \lesssim 2.5$ at $\Delta_\mu = 16t_0$ would be sufficient to reach T_N by the time the clean limit is reached. However, an important question arises: Can the trapped system in the presence of disorder be cooled down to an initial temperature $T/t_0 \sim 1.5$, or possibly even lower, close to what is initially needed for the clean system to reach the Neél phase ($T_N/t_0 \sim 0.35$) [37]. Current cooling capabilities have achieved a final temperature of $T/t_0 = 0.5$ ($1.4T_N$) for $U/t_0 \sim 11$ at the trap center [38]. We provide several suggestions concerning its feasibility in our concluding remarks.

Since random μ_i and t_{ij} occur together with optical speckles [39, 40], we also explore the case of bond disorder. Figure 3 shows $S(T)$ for nonzero Δ_t (and $\Delta_\mu = 0$). Significant disorder-induced cooling occurs. It is notable that $\Delta_t/t_0 \sim 1$ is sufficient to produce an effect on the entropy, whereas the scale of random site energies required to change S is much larger (Fig. 1). This is a consequence of the fact that random hopping immediately leads to a range of exchange energies $J_{ij} \sim 4t_{ij}^2/U$ which reduces the moment ordering. Random μ_i also smear J_{ij} but, since they are added to U in the energy denominator, initially have only a small effect. Random hopping thus offers cooling at lower temperature (entropy) scales for $\Delta_t \sim t_0$ than does random chemical potential, without requiring a 'threshold value', $\Delta_\mu > U$. Unlike for the chemical potential disorder, the basic structure of $D(T)$ remains unaltered for the clean system [22].

To provide some insight into possible effects of the inhomogeneous densities resulting from a confining potential, we show the entropy as a function of density for the clean system and for chemical potential disorder $\Delta_\mu/t_0 = 16$ and hopping disorder $\Delta_t/t_0 = 4$ in the inset of Fig. 3. Although there is some structure to the curves, entropy is systematically lowered for all densities as disorder is introduced. Thus disorder cooling is not a special feature of half-filling, but likely occurs for a broad range of densities.

We note that there are important questions of principle which would arise in a full treatment of a trap [36, 37]. QMC calculations for clean systems employed a set of homogeneous simulations, combined with the local density approximation (LDA), to understand how the density, double occupancy, and entropy are inhomogeneously distributed in a system with smoothly varying chemical potential. This is a considerably harder task in the presence of disorder, because the implementation, and indeed even the validity, of the LDA is much less straightforward with a rapidly varying μ_i or t_{ij}. In fact, the LDA has the curious feature that thermodynamic properties are insensitive to the specific geometric organization of the sites with the different chemical potentials: The local entropy s_{μ_i} is unaltered for any two systems with the same collection $\{\mu_i\}$ whether they are randomly distributed or ordered spatially in some pattern, a patently unphysical result.

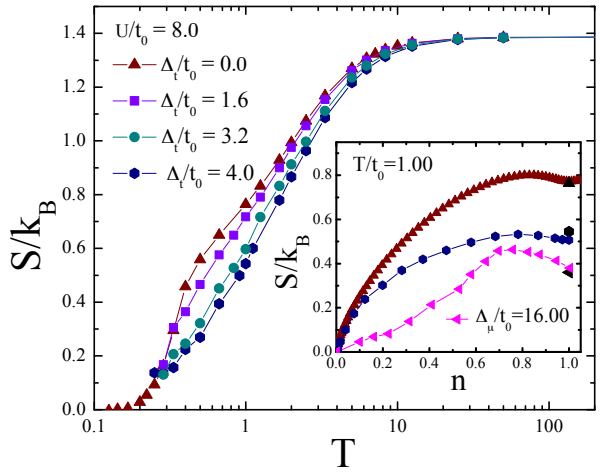

FIG. 3: Entropy versus temperature for hopping disorder. Here, disorder cooling is strongest at lower entropies $S \sim 0.5$. The inset shows the entropy as a function of density of the clean system for $\Delta_\mu/t_0 = 16$ and $\Delta_t/t_0 = 4$ at fixed $U/t_0 = 8$ and $T/t_0 = 1$. Here, the entropy is obtained using $S(\mu,T) = \int_{-\infty}^{\mu} d\mu \frac{\partial n}{\partial T}|_\mu$ [41], except for the three data points in black (darker shade) at $n = 1$, which are obtained via integrating over β.

Further Analysis: Observing the onset of long-range antiferromagnetic (AF) correlations is a central goal of the field. To see the development of these correlations as the disorder is turned off, we show in Fig. 4 the structure factor S_π as a function of T for different site (top panel) and bond (bottom panel) disorder strengths. $\Delta_\mu > U$ completely destroys the sharp rise in S_π, which occurs here on a 6^3 lattice at a value close to the bulk $T_N/t_0 \sim 0.35$ for $U/t_0 = 8$. The suppression of magnetic order is a consequence of the destruction of the local moments $m^2 = \langle (n_{i\uparrow} - n_{i\downarrow})^2 \rangle = 1 - 2D$ at half-filling (see the inset of Fig. 1). S_π is also suppressed by Δ_t despite the fact that it has only a small effect on m^2 [22]. The likely mechanism for the destruction of AF order in this case is the introduction of fluctuations in the near-neighbor exchange $J_{ij} \sim 4t_{ij}^2/U$. As a consequence, of this anisotropy, singlets can form on the bonds with large J_{ij}. When many pairs of sites are effectively removed from the lattice, order is lost. Although both bond and site disorder reduce S_π, it is important to emphasize that low T is reached by turning the disorder *off*, so that the terminal state is the sought after regime of large AF correlations.

Equilibration is crucial to the viability of disorder cooling. Recent experiments by the DeMarco group [12] provide evidence that the requisite Δ_μ lie well below the

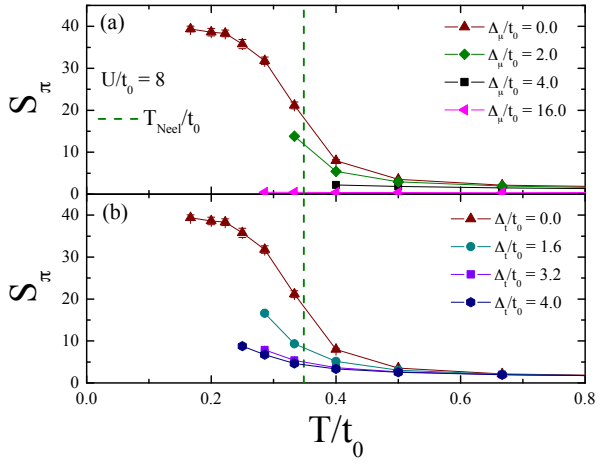

FIG. 4: Antiferromagnetic structure factor S_π as a function of T as the site (top) and bond (bottom) disorder is varied. Site disorder drives S_π to zero for $\Delta_\mu \gtrsim U$ by destroying the magnetic moments $m^2 = 1 - 2D$, whereas singlet formation on bonds with large $J_{ij} \sim t_{ij}^2/U$ is induced by sufficiently large Δ_t and also destroys the AF long-range order.

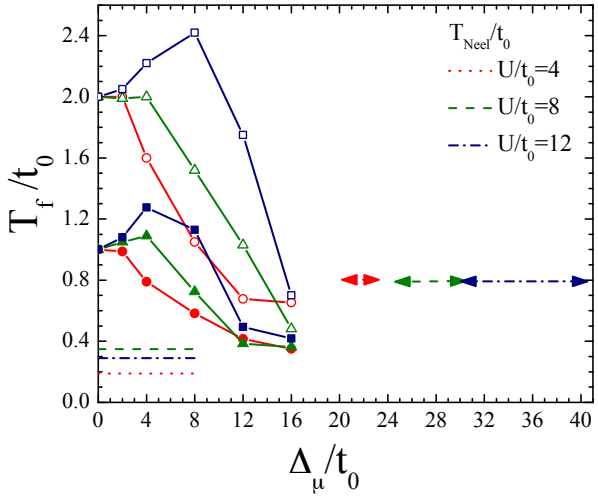

FIG. 5: Final temperature T_f/t_0 resulting from adiabatically turning off disorder in a system with initial disorder strength Δ_μ/t_0, shown on the horizontal axis, and two different initial temperatures $T_i/t_0 = 1,\ 2$. As Δ_μ increases beyond U/t_0, T_f decreases. The Neél temperatures $T_N/t_0 = 0.19, 0.35, 0.29$ for $U/t_0 = 4, 8, 12$, respectively, are shown as dashed horizontal lines. The horizontal arrows are estimates for the onsets of MBL for (from left to right) $U/t_0 = 4.0, 8.0, 12.0$, obtained by linearly interpolating $\Delta_c/12t_0$ vs $U/12t_0$ in Fig. 3 of Ref. [12].

threshold where randomness drives atomic velocities to zero: Measurements of mass transport show that the center-of-mass velocity only vanishes above $\Delta_\mu/t_0 \sim 21.7 \pm 1.6$ for $U/t_0 = 3.8$ and $\Delta_\mu/t_0 \sim 31.7 \pm 4.2$ for $U/t_0 = 9.1$. The implications of these results for disorder cooling are considered in Fig. 5, which shows the final temperature $T_f(T_i, \Delta_\mu)$ which would result from starting at initial temperature T_i and disorder Δ_μ, and turning off randomness adiabatically. Figure 5 complements Fig. 2 and provides another way of analyzing the lowering of T_f starting from states at T_i with Δ_μ beyond U/t_0 and adiabatically following a path to $\Delta_\mu = 0$. The reduction in temperature, $T_i - T_f$, can be as large as $0.65t_0$ for $\Delta_\mu/t_0 = 16$ and $U/t_0 = 4$, starting at $T_i/t_0 = 1$ and $1.35t_0$ for $T_i/t_0 = 2$. The many-body localization (MBL) critical disorder strengths for $U/t_0 = 4.0, 8.0$ and 12.0 (denoted by horizontal arrows in Fig. 5) lie above the range which provides substantial cooling. These comparisons provide considerable support to the likelihood that equilibration will still occur in the regime where disorder-induced cooling is effective.

Implementation and Concluding Remarks: The feasibility of disordered-induced cooling depends on the ability to realize low initial temperatures in the disordered lattice. Since turning on disorder heats the gas, this energy must be removed before attempting to cool more deeply using our method. There have been no direct attempts to cool in a disordered lattice, but several schemes are promising. One such method is sympathetic cooling by another atomic species [42] or spin-state of the same species [43, 44] that by proper choice of lattice wavelength or polarization is unaffected by the lattice. Another approach is to implement a compensated lattice, where the overall confinement created by the infrared lattice beams is compensated by overlapping blue-detuned beams [45]. By tuning the intensity of the blue-detuned beams the threshold for evaporation can be brought near the chemical potential, resulting in very low temperatures [38]. While this scheme has only been implemented in a clean lattice, it seems plausible that it can work in any situation where there is sufficient mobility.

A second approach is to mask the disorder in such a way that it is applied only to a small spatial subregion of the entire gas. Through thermal contact, atoms in this region could be cooled by the larger reservoir region outside the disordered volume. If the clean gas is then discarded, one again has the starting point of a disordered gas at the same initial T as a clean one. Complex optical potentials to perform these roles can be created using phase-imprinting spatial light modulators [46, 47] or micro-mirror devices [48].

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Supplemental Material: Cooling Atomic Gases With Disorder

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In this Supplement, we present further quantum Monte Carlo (QMC) data which (1) analyzes $D(T)$ in case of bond disorder; (2) presents results at $U/t_0 = 4, 12$ to expand upon the $U/t_0 = 8$ data in the main paper; (3) examines the shifts in the specific heat $C(T)$ curve, and its kinetic and potential energy components, as disorder is increased; (4) presents entropy S as a function of disorder Δ_μ at fixed temperature T; (5) connects to previous determinant QMC (DQMC) and dynamical cluster approximation (DCA) studies of the effects of disorder on magnetic transitions and conductivity, and experimental work in 1D; and (6) discusses more general disorder distributions.

1. Double occupancy.

Figure S1 shows the double occupancy $D(T)$ for hopping disorder. The effect of randomness is very different from the case with chemical potential disorder: there is some enhancement of D, but the basic structure remains unaltered from the clean limit. $D(T)$ falls with decreasing T at a charge fluctuation scale associated with U, reaches a minimum at $T/t_0 \sim 1.0$, and then has a region where dD/dT can be slightly negative at low temperatures. The effect of Δ_t is only to shift the values of D upwards (from $D \sim 0.07$ to $D \sim 0.12$ at the largest $\Delta_t = 4.0$), in an almost temperature independent way at low T. These data suggest that disorder-induced cooling is not uniquely linked to a negative dD/dT, as is the case with Pomeranchuk cooling.

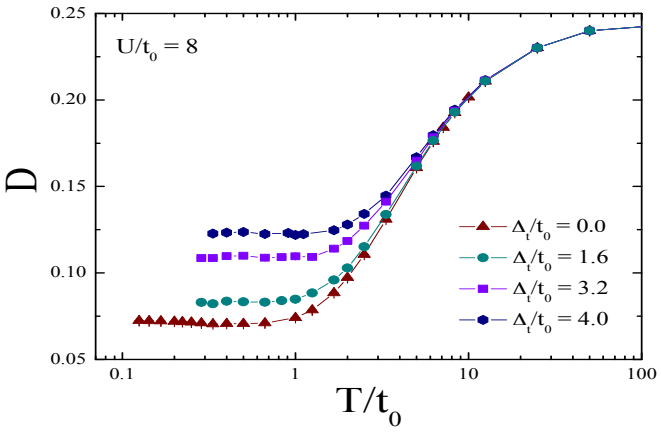

Fig. S1: Double occupancy vs temperature for the clean and disordered systems with bond disorder strength $\Delta_t = 0.0 - 4.0$. Unlike the case of chemical potential disorder, there is no qualitative change in the structure of $D(T)$: double occupancy decreases at the temperature scale $T \sim U$ for all Δ_t.

2. Weak and strong coupling.

Figure 2 of the main text showed the evolution of temperature following a protocol starting with a trapped

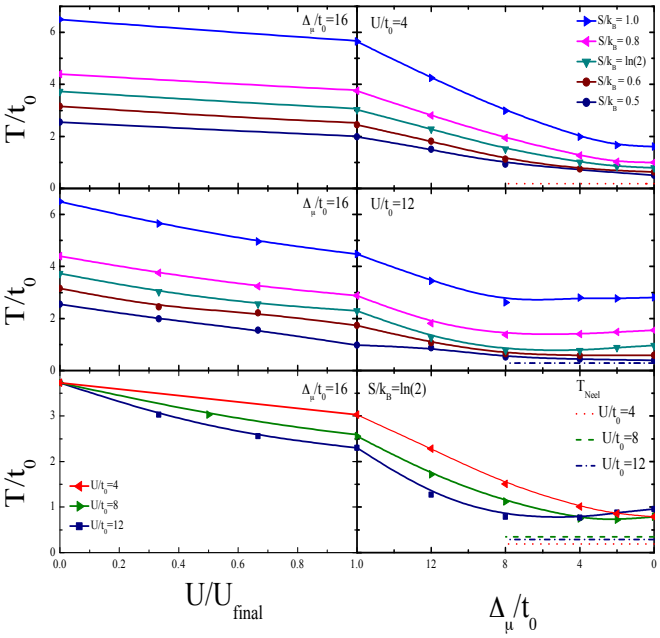

Fig. S2: QMC results for the adiabats of the disordered 3D Hubbard model along paths which combine an increase of U/t_0 from 0 to 4 (top row, left) and to 12 (central row, left), both at fixed $\Delta_\mu/t_0 = 16$. This trajectory is followed by a reduction of the disorder to zero at constant U (top and middle rows, right). These $U/t_0 = 4, 12$ data complement the $U/t_0 = 8$ results in the main text. The bottom row compares the $S/k_B = \ln 2$ adiabats for $U/t_0 = 4, 8, 12$. Note the horizontal axis of left panels is $U/U_{\rm final}$ to allow results for different U/t_0 to be displayed together.

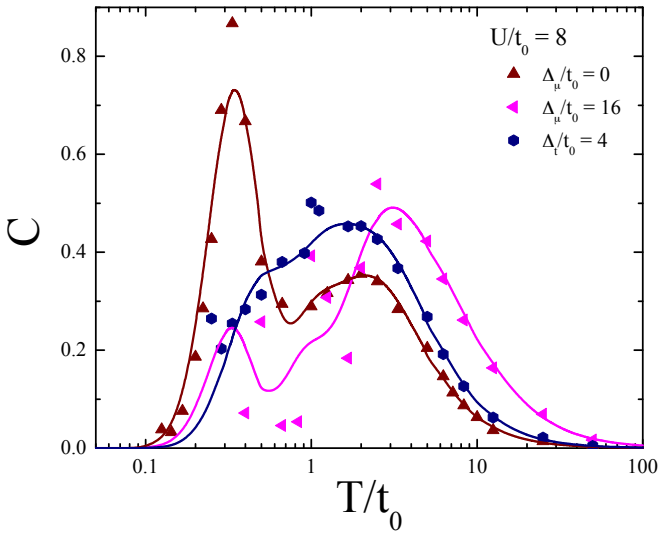

Fig. S3: The specific heat $C(T)$ has two features in the clean limit: a high temperature ($T \sim U$) peak associated with moment formation, and a low temperature peak $T \sim J \sim 4t^2/U$ where moment ordering occurs. As disorder increases, these peaks are smeared, and the low T moment ordering peak especially is disrupted. $C(T)$ structure resulting from excitations linked to occupying sites with large local chemical potential appears at temperatures T which scale with Δ_μ. Symbols represent the numerical derivative of the energy with respect to temperature, whereas lines were obtained from fitting the QMC energy data to a physically motivated function: $E(T) = E(0) + \sum_{l=1}^{M} c_l \exp(-l\Delta/T)$ [1]. We have used $M = 7$ or 8 exponentials to fit the data.

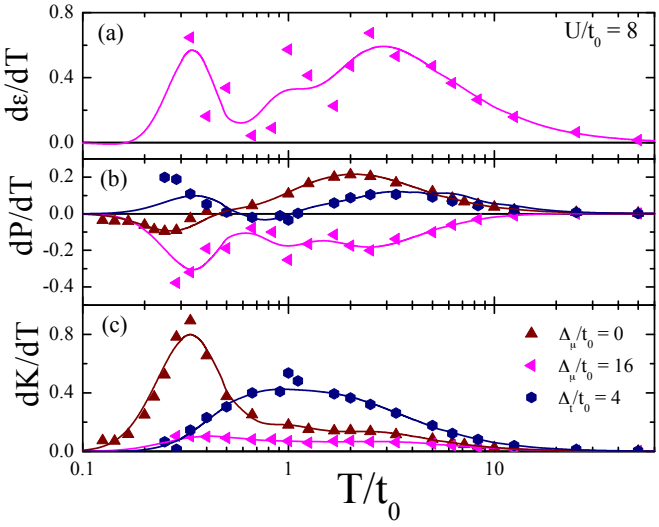

Fig. S4: Temperature derivatives of the different components of the energy. The greatest qualitative changes are in the sign of dP/dT for $\Delta_\mu/t_0 = 0$ versus $\Delta_\mu/t_0 = 16$, and the reduction of the low T peak in dK/dT. For $\Delta_\mu/t_0 = 16$ lines were obtained from fitting the QMC data to $f(T) = f(0) + \sum_{l=1}^{M} c_l \exp(-l\Delta/T)$ [1].

gas in a disordered potential $\Delta_\mu/t_0 = 16$ and with no interaction $U/t_0 = 0$, through a ramp of the lattice to $U/t_0 = 8$, followed by turning off the disorder $\Delta_\mu = 0$. Figure S2 presents similar data at weaker ($U/t_0 = 4$) and stronger ($U/t_0 = 12$) couplings.

The main message is that disorder cooling occurs through all values of U/t_0, an important practical observation because a confining potential not only changes the density across the trap, but also the lattice depth. Figure S2, in combination with Fig. 2 of the main text, indicate some interesting trends with U/t_0. Since cooling takes place during the first phase (left panels) of the protocol, when the interaction is increased in the presence of disorder, the larger U/t_0, the greater the cooling. However, the cooling in the second phase (right panels), when the disorder is turned off, is less at larger U/t_0 so that the final temperature after the full protocol is almost identical ($T/t_0 \sim 0.8$) for $U/t_0 = 4$ and $U/t_0 = 8$ when $S/k_B = \ln 2$, and is only slightly higher ($T/t_0 \sim 1$) for $U/t_0 = 12$.

3. Specific Heat.

The mechanism of entropy reduction at low T is associated with the modification and redistribution of the energy levels. When Δ_μ is sufficiently large, the weight under the specific heat curve C(T), Fig. S3, is shifted to higher temperatures. This occurs both because of the reduction in magnetic ordering, and hence, of the peak in $C(T)$ at $T \sim J$, and also because of the introduction of new, high energy excitation modes associated with the occupation of sites with large μ_i. The shift in the structure of $C(T)$ to higher T signals the reduction of $S(T)$ at low T, and therefore the phenomenon of disorder cooling.

Figure S4 shows the temperature derivatives of the kinetic energy $K = 1/N\langle\sum_{\langle ij\rangle}(c^\dagger_{i\sigma}c_{j\sigma} + c^\dagger_{j\sigma}c_{i\sigma})\rangle$, potential energy $P = U \times D$, and random chemical potential energy $\epsilon = -1/N\langle\sum_i \mu_i (n_{i\uparrow} + n_{i\downarrow})\rangle$. Together these derivatives sum to $C(T)$. The effect of disorder is most dramatic on dP/dT, which is mostly positive for $\Delta_\mu/t_0 = 0$ but negative for $\Delta_\mu/t_0 = 16$: the double occupancy is shrinking as T increases in the highly disordered limit. This is also clearly seen in the inset of Fig. 1 of the main text. A second effect of disorder is to eliminate the low T peak in dK/dT. As site disorder grows, neighboring sites are no longer in resonance, and hopping is diminished. This diminished hopping is also linked to reduced local AF order.

4. Dependence of entropy on disorder

Another way of looking at cooling is to examine the entropy as a function of Δ_μ at fixed temperature T (Fig. S5). This plot can be inferred by taking a vertical cut through the lines in Fig. 1 of the main text. As Δ_μ increases, the entropy is lowered. Thus, if it is possible to cool an atomic gas to a given T regardless of disorder strength, lower initial S_i can be accessed. If the disorder

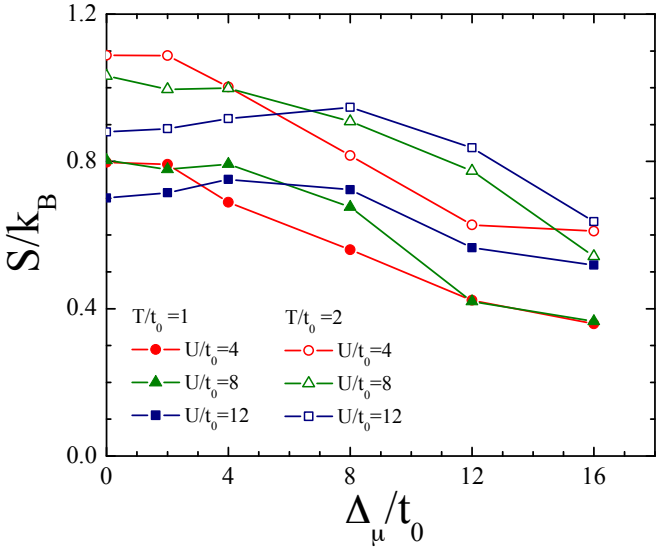

Fig. S5: Entropy S as a function of site disorder strength Δ_μ for several fixed temperatures and interaction strengths. As Δ_μ increases, the entropy is lowered.

is then turned off adiabatically ($S_f = S_i$), a lower final T_f is achieved. Possible approaches to cooling to the same T_i in the presence of disorder, such as putting the disordered gas in thermal contact with a clean gas or a region of the trap with no disorder, were discussed in the Concluding Remarks of the main text.

5. Related DQMC and experimental work.

DQMC results for the effect of site and bond disorder on antiferromagnetism, analogous to Fig. 4 of the main text, have been reported in two dimensions [2, 3] and in the limit of infinite dimensions [4, 5]. In the former case, ordering occurs only at $T = 0$. An interesting effect in the latter case is the initial enhancement of T_N when the chemical potential disorder is small. This occurs as a consequence of an increase in exchange energies. For two neighboring sites with site energy difference Δ_μ the clean system exchange is altered to $J = 2t^2/(U-\Delta_\mu)+2t^2/(U+\Delta_\mu) > 4t^2/U = J_{\rm clean}$ (See main text). The effect of disorder on the conductivity, and the possibility of an Anderson insulator to metal transition, have also been studied [6–8]. An interesting feature of the transport properties of the disordered Hubbard Hamiltonian is the key role of particle-hole symmetry, i.e. whether the disorder is in the site energies, which breaks particle-hole symmetry, or the bond hoppings, which preserves it.

The DCA is a powerful alternate approach to the thermodynamics of the 3D Hubbard Hamiltonian [9]. A recently proposed extension of this method, the typical medium DCA (TMDCA) [10] allows the treatment of disorder, generalizing earlier approaches [11], and accurately captures many features of the Anderson localization transition, including the critical disorder strength, the exponent of the typical density of states at the transition, and reentrant behavior. Given the usefulness of past comparisons of DQMC [12] and DCA [9] in the absence of randomness, it will be useful to incorporate interactions in the TMDCA and study the disorder cooling proposed here.

A recent study of relaxation from an initial charge density wave state in the *one dimensional* Hubbard Hamiltonian with a quasi-periodic potential [13] also provides experimental and exact diagonalization values for the critical disorder strength for MBL. After adjusting for the difference in bandwidth, the size of Δ_μ we propose for disorder cooling lies close to this threshold. Given the strong tendency for localization in $d = 1$, (much more than a simple bandwidth renormalization) these results indicate the Δ_μ reported here are not in a regime where the system will fail to equilibrate. This work [13] also compares relaxation data with and without the presence of a trap and shows they differ only in the vicinity of the MBL transition, suggesting the inclusion of a trap in our 3D calculations will not affect the qualitative results, except perhaps near T_N.

6. More general disorder distributions.

There are a number of possible refinements of the calculations we have reported here. We have assumed a box distribution of the random chemical potentials μ_i and near-neighbor hoppings t_{ij}. In fact, the precise functional forms of these distributions from optical speckle are known [14–16]. Whereas we have studied site and bond randomness separately, the disorder-induced cooling produced by hopping disorder tends to occur at lower T than for chemical potential disorder, making the possible interplay of the two of interest.

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