Solution 1

(a) Set $p = [X \ \ Y \ \ Z]^T$ as the point of the object. $\bar{p}_1$ is the projection of the first camera. $\bar{p}_2$ is the projection of the second camera.

$$\bar{p}_1 \sim K[I|\bar{0}]p = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} * p \tag{1}$$

$$\bar{p}_2 \sim K[I|t]p = \begin{bmatrix} f & 0 & 0 & -f * t_x \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} * p \tag{2}$$

Then we can compute X and X'.

$$x = \frac{fX}{Z} \tag{3}$$

$$y = \frac{fY}{Z} \tag{4}$$

$$x' = \frac{fX - ft_x}{Z} \tag{5}$$

$$y' = \frac{fY}{Z} \tag{6}$$

Then the disparity is

$$d = x - x' = \frac{ft_x}{Z} \tag{7}$$

(b) On the left sensor:

$$\begin{aligned} X =& Zx/f \\ Y =& Zy/f \end{aligned}$$

With plugging in the equation $X\alpha + Y\beta + Z\gamma = k$, we can know:

$$Z = \frac{k}{x\frac{\alpha}{f} + y\frac{\beta}{f} + \gamma} \tag{8}$$

then replace Z with ft_x/d from part a

$$\begin{aligned} d =& \frac{t_x\alpha}{k}x + \frac{t_x\beta}{k}y + \frac{t_x f\gamma}{k} && (9) \\ =& ax + bx + c && (10) \end{aligned}$$

(c) Since the translation vector now is $t = [0, 0, -t_z]^T$, we can get the corresponding x' and y':

$$x' = \frac{fX}{Z - t_z}$$
$$y' = \frac{fY}{Z - t_z}$$

Replace f with Zx/X and Zy/y separately into the equation above:

$$x' = \frac{Zx}{Z - t_z}$$
$$y' = \frac{Zy}{Z - t_z}$$

If we know the value of Z, then we can know the relation between (x', y') and (x, y).

Solution 2

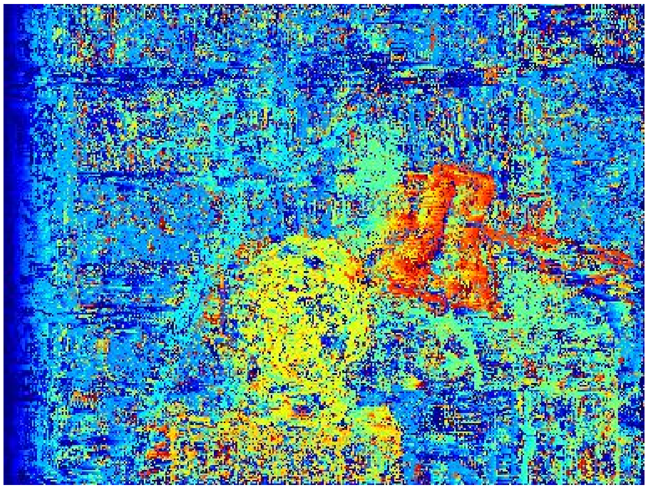

Figure 1: 2a

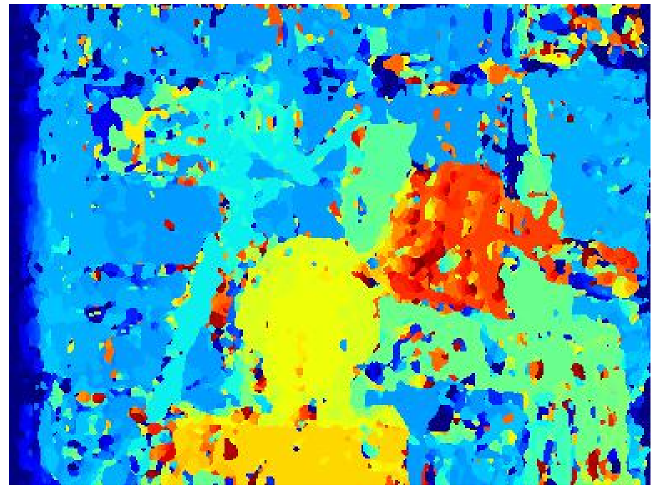

Figure 2: 2b

Solution 3

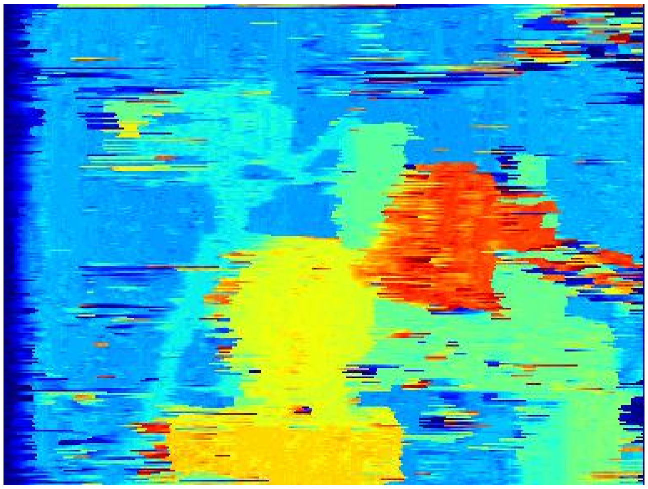

Figure 3: 3a

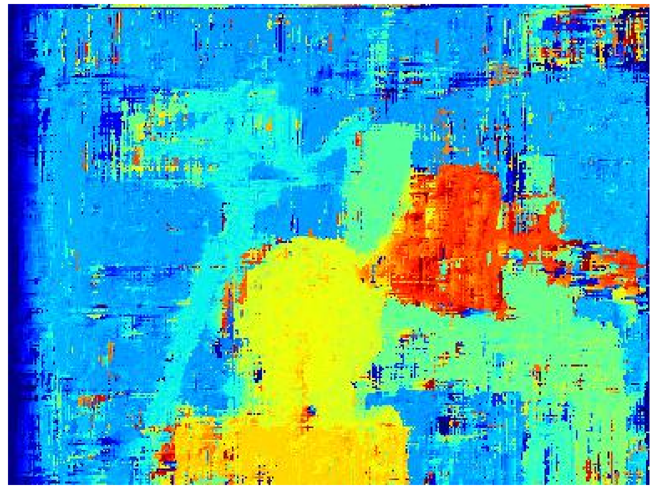

Figure 4: 3b

Solution 4

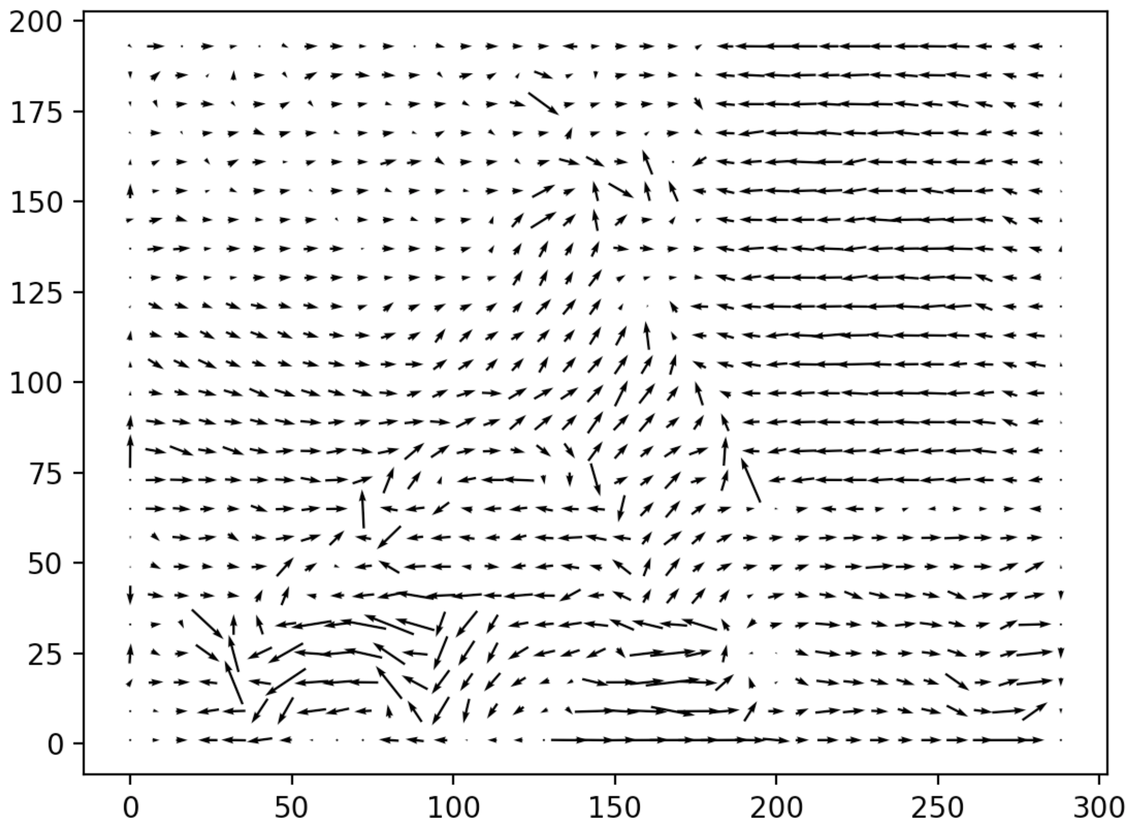

Figure 5: Quiver Plot

Information

This problem set took approximately 20 hours of effort.

I discussed this problem set with:

- Mingyu Cao

I also got hints from the following sources:

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