

$$2H_1. \quad P(A) = P(B) = 0.5$$

$$P(\text{twins} \mid A) = 0.1, \quad P(\text{single} \mid A) = 0.9$$

$$P(\text{twins} \mid B) = 0.2, \quad P(\text{single} \mid B) = 0.8$$

$$P(\text{twins } 2 \mid \text{twins } 1) = ?$$

$$\begin{aligned} P(\text{twins } 1) &= P(\text{twins } 1 \mid A) \overbrace{P(A)}^{\text{prior}} + P(\text{twins } 1 \mid B) \overbrace{P(B)}^{\text{prior}} \\ &= 0.1(0.5) + 0.2(0.5) \\ &= 0.15 \end{aligned}$$

$$\overbrace{P(A \mid \text{twins } 1)}^{\text{posterior}} = \frac{P(\text{twins } 1 \mid A) P(A)}{P(\text{twins } 1)} = \frac{0.1(0.5)}{0.15} = \frac{1}{3}$$

$$\overbrace{P(B \mid \text{twins } 1)}^{\text{posterior}} = \frac{P(\text{twins } 1 \mid B) P(B)}{P(\text{twins } 1)} = \frac{0.2(0.5)}{0.15} = \frac{2}{3}$$

$$\begin{aligned} P(\text{twins } 2 \mid \text{twins } 1) &= P(\text{twins } 2 \mid A) \overbrace{P(A \mid \text{twins } 1)}^{\text{new prior from previous posterior}} + P(\text{twins } 2 \mid B) \overbrace{P(B \mid \text{twins } 1)}^{\text{new prior from previous posterior}} \\ &= 0.1\left(\frac{1}{3}\right) + 0.2\left(\frac{2}{3}\right) \\ &= \frac{1}{6} \\ &= 0.1667 \end{aligned}$$

$$2H_2. P(A | \text{twins}) = \frac{1}{3} = 0.333$$

Probabilitas A turun dari 0.5 ke 0.333 karena B memiliki peluang kelahiran twins lebih besar

$$2H_3. P(A | \text{twins, single}) = ? \quad \text{new prior from previous posterior}$$

$$P(A | \text{twins, single}) = \frac{P(\text{single} | A) P(A | \text{twins})}{P(\text{single} | \text{twins})}$$

$$= \frac{P(\text{single} | A) P(A | \text{twins})}{P(\text{single} | A) P(A | \text{twins}) + P(\text{single} | B) P(B | \text{twins})}$$

$$= \frac{0.9 \left(\frac{1}{3}\right)}{0.9 \left(\frac{1}{3}\right) + 0.8 \left(\frac{2}{3}\right)}$$

$$= \frac{9}{25}$$

$$= 0.36$$

Probabilitas A naik dari 0.333 ke 0.36 karena A memiliki peluang kelahiran single lebih besar

$$\begin{aligned}
 214. \quad & P(\text{test A} | A) = 0.8 \\
 & P(\text{test B} | B) = 0.65 \\
 & P(\text{test A} | B) = 1 - 0.65 = 0.35
 \end{aligned}$$

$$\begin{aligned}
 P(A | \text{test A}) &= \frac{P(\text{test A} | A) \overset{\text{prior}}{P(A)}}{P(\text{test A})} \\
 &= \frac{P(\text{test A} | A) P(A)}{P(\text{test A} | A) P(A) + P(\text{test A} | B) P(B)} \\
 &= \frac{0.8 (0.5)}{0.8 (0.5) + 0.35 (0.5)} \\
 &= \frac{16}{23} \\
 &= 0.695
 \end{aligned}$$

hasil tes meningkatkan probabilitas A dari 0.5 ke 0.695

$$\begin{aligned}
 P(A | \text{test A, twins, single}) &= \frac{P(\text{test A} | A) \overset{\text{new prior from previous posterior}}{P(A | \text{twins, single})}}{P(\text{test A} | \text{twins, single})} \\
 &= \frac{P(\text{test A} | A) P(A | \text{twins, single})}{P(\text{test A} | A) P(A | \text{twins, single}) + P(\text{test A} | B) P(B | \text{twins, single})} \\
 &= \frac{0.8 \left(\frac{9}{25}\right)}{0.8 \left(\frac{9}{25}\right) + 0.35 \left(\frac{16}{25}\right)} \\
 &= \frac{9}{16} \\
 &= 0.5625
 \end{aligned}$$

hasil tes meningkatkan probabilitas A dari 0.36 ke 0.5625