

MA 323 : MONTE CARLO SIMULATION LAB 4

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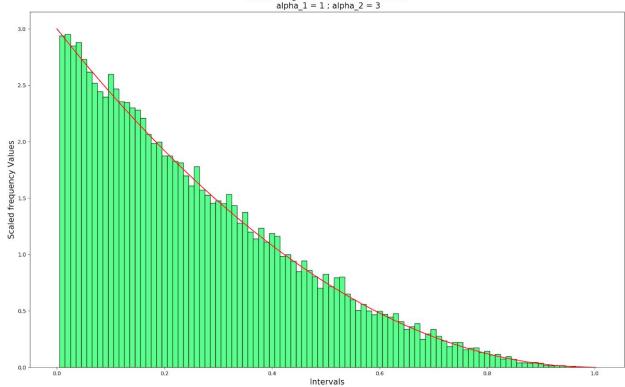
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BETA DISTRIBUTION

CASE-I

1. ALPHA_1 = 1, ALPHA_2 = 3

Histogram showing Frequencies of Generated Numbers Value of c = 3.0 :::: Avg number of times steps 1 repeated = 2.98329 Average value of Generated Numbers = 0.2501828633651551 Number of generated numbers = 33520 alpha $_1$ = 1; alpha $_2$ = 3



2.
$$x^* = (alpha_1 - 1)/(alpha_1 + alpha_2 - 2) = (1-1)/(1+3-2) = 0$$

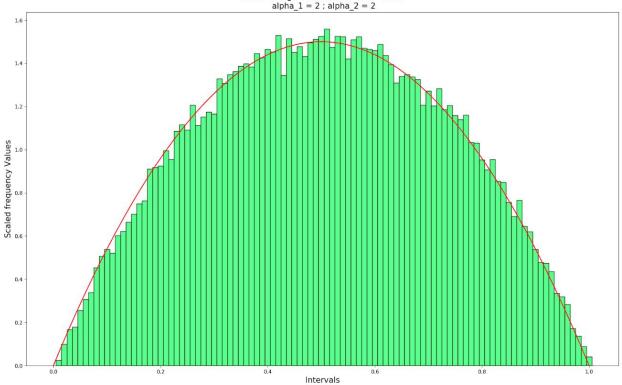
3.
$$1/B(1,3) = Gamma(1+3)/(Gamma(1)*Gamma(3)) = 3$$

4.
$$c = f(x^*) = 3 * (1-x^*)^2 = 3 * 1 = 3$$

CASE-II

1. ALPHA_1 = 2, ALPHA_2 = 2

Histogram showing Frequencies of Generated Numbers Value of c = 1.5:::: Avg number of times steps 1 repeated = 1.50092 Average value of Generated Numbers = 0.5004405254705371 Number of generated numbers = 66626 alpha $_1$ = 2; alpha $_2$ = 2



2.
$$x^* = (alpha_1 - 1)/(alpha_1 + alpha_2 - 2) = (2-1)/(2+2-2) = 0.5$$

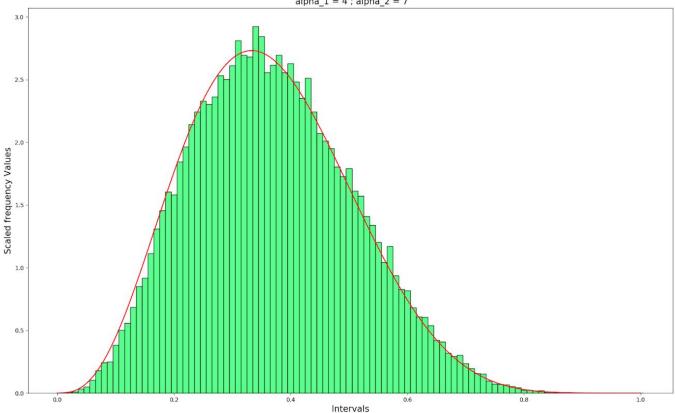
3.
$$1/B(2,2) = Gamma(1+3)/(Gamma(1)*Gamma(3)) = 6$$

4.
$$c = f(x^*) = 6 * (x^*) * (1-x^*) = 6 * 0.5 * 0.5 = 1.5$$

CASE-III

1. ALPHA_1 = 4, ALPHA_2 = 7

Histogram showing Frequencies of Generated Numbers Value of c = 2.731290961743638 :::: Avg number of times steps 1 repeated = 2.72094 Average value of Generated Numbers = 0.3642567775903352 Number of generated numbers = 36752 alpha_1 = 4; alpha_2 = 7

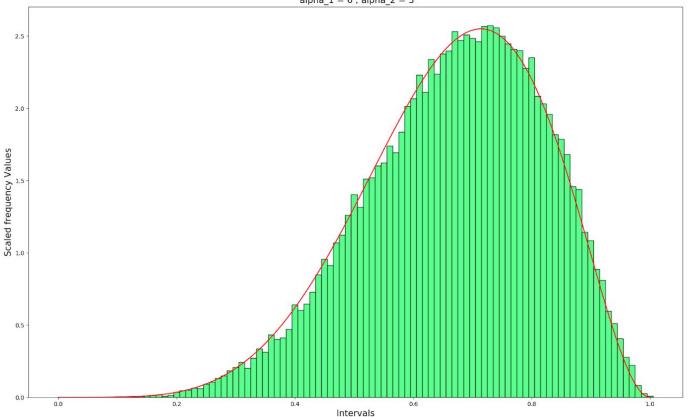


- 2. $x^* = (alpha_1 1)/(alpha_1 + alpha_2 2) = (4-1)/(4+7-2) = 0.33333333...$
- 3. 1/B(4,7) = Gamma(4+7)/(Gamma(4)*Gamma(7)) = 840
- 4. $c = f(x^*) = 840 * (x^*)^3 * (1-x^*)^6 = 2.731290961743638$

CASE-IV

1. ALPHA_1 = 6, ALPHA_2 = 3

Histogram showing Frequencies of Generated Numbers Value of c = 2.549957925694226:::: Avg number of times steps 1 repeated = 2.53824 Average value of Generated Numbers = 0.6669460060410691 Number of generated numbers = 39397 alpha_1 = 6; alpha_2 = 3

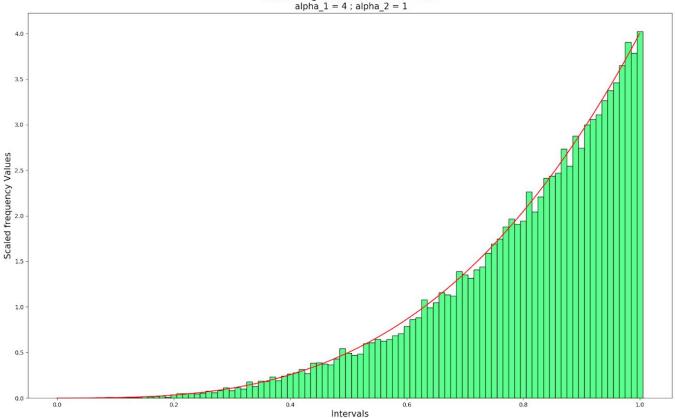


- 2. $x^* = (alpha_1 1)/(alpha_1 + alpha_2 2) = (6-1)/(6+3-2) = 0.7142857142857143$
- 3. 1/B(6,3) = Gamma(6+3)/(Gamma(6)*Gamma(3)) = 168
- 4. $f(x^*) = 168 * (x^*)^5 * (1-x^*)^2 = 2.549957925694226$

CASE-V

1. ALPHA_1 = 4, ALPHA_2 = 1

Histogram showing Frequencies of Generated Numbers
Value of c = 4.0 :::: Avg number of times steps 1 repeated = 4.01962
Average value of Generated Numbers = 0.8009021629552214
Number of generated numbers = 24878



2.
$$x^* = (alpha_1 - 1)/(alpha_1 + alpha_2 - 2) = (4-1)/(4+1-2) = 1$$

3.
$$1/B(4,1) = Gamma(4+1)/(Gamma(4)*Gamma(1)) = 4$$

4.
$$c = f(x^*) = 4 * (x^*) * (1-x^*) = 4$$

CONCLUSION

- 1. When alpha_1 < alpha_2, average of numbers generated is less than 0.5 and also peak of graph is achieved at $x^* < 0.5$
- 2. When alpha_1 > alpha_2, average of numbers generated is greater than 0.5 and also peak of graph is achieved at $x^* > 0.5$.
- 3. When alpha_1 = alpha_2, the average of numbers generated is approximately 0.5 and also the peak of the graph is achieved at $x^* = 0.5$.