MA 323 : MONTE CARLO SIMULATION

LAB 6

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MULTIVARIATE NORMAL DISTRIBUTION

To simulate points of form (x1, x2) from given bivariate normal, Cholesky Factorization method was used. Initially Z1~N(0,1) and Z2~N(0,1) were generated which were then transformed to X1 and X2 using Cholesky factorization method.

$$X = (X1, X2) = \mu + A*Z$$

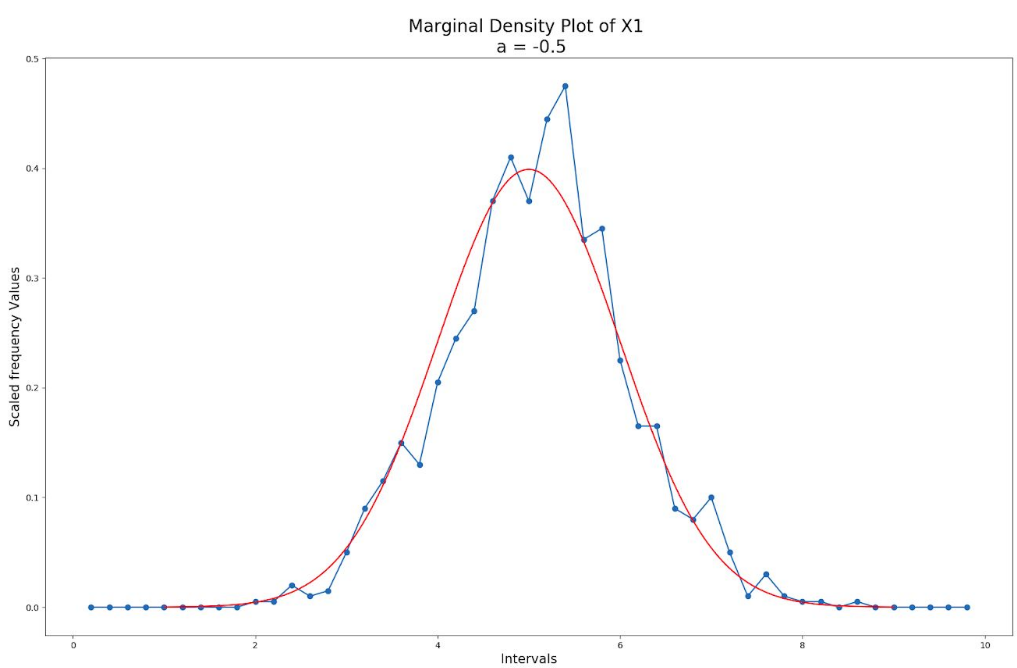
$$X1 = \mu1 + \sigma1*Z1$$

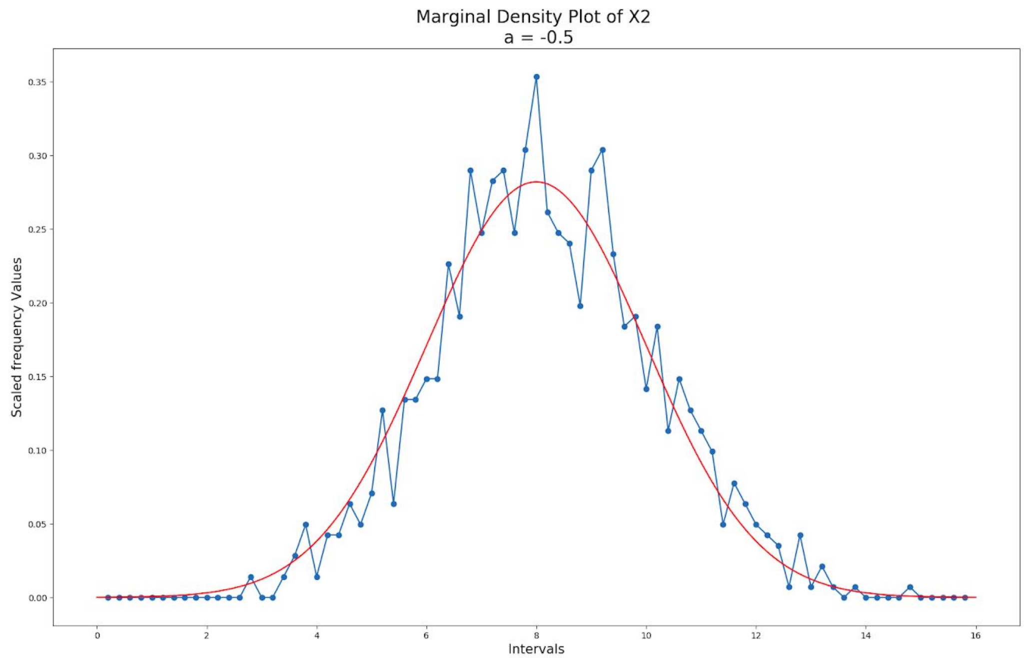
$$X2 = \mu2 + \rho*\sigma2*Z1 + \mathrm{sqrt}(1 - \rho^2)*\sigma2*Z2$$

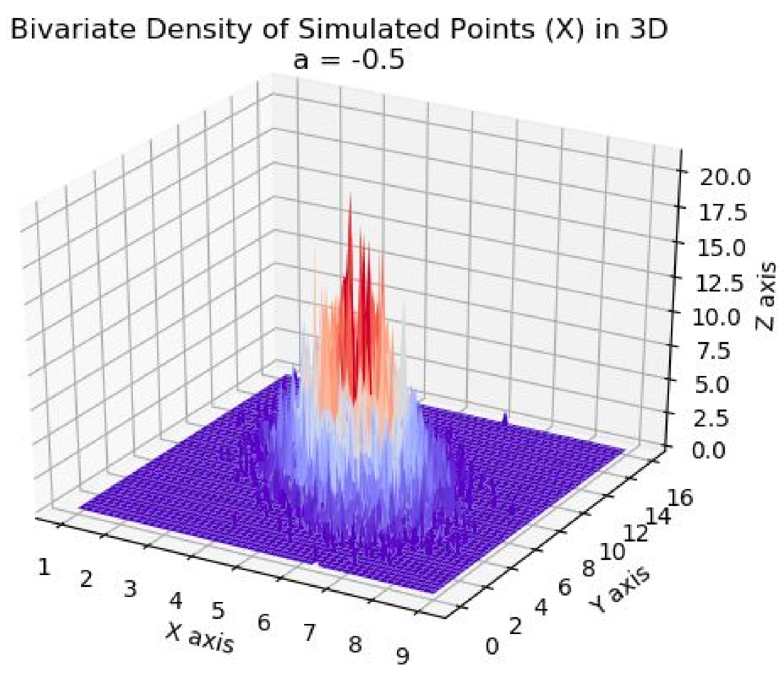
NOTE: The simulation is run for 1000 rounds only. However as the number of rounds in simulation increase, the simulated density converges to actual density.

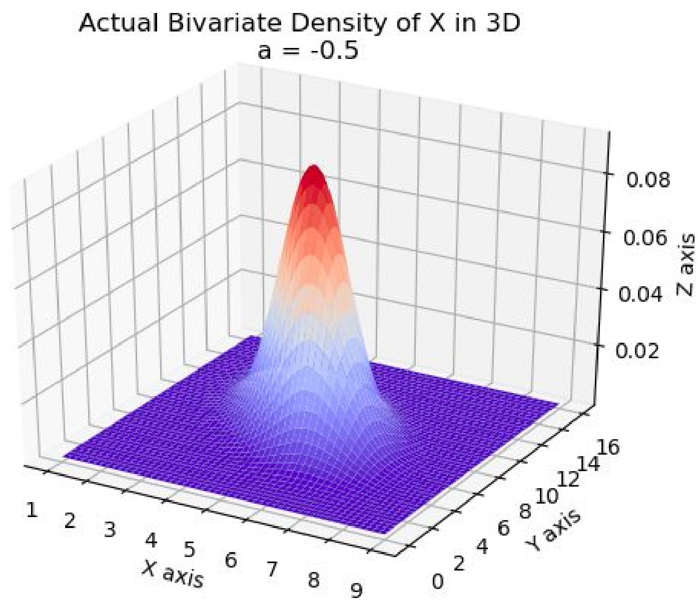
The plots obtained for different values of **a** are as follows:

PART-1: a = -0.5

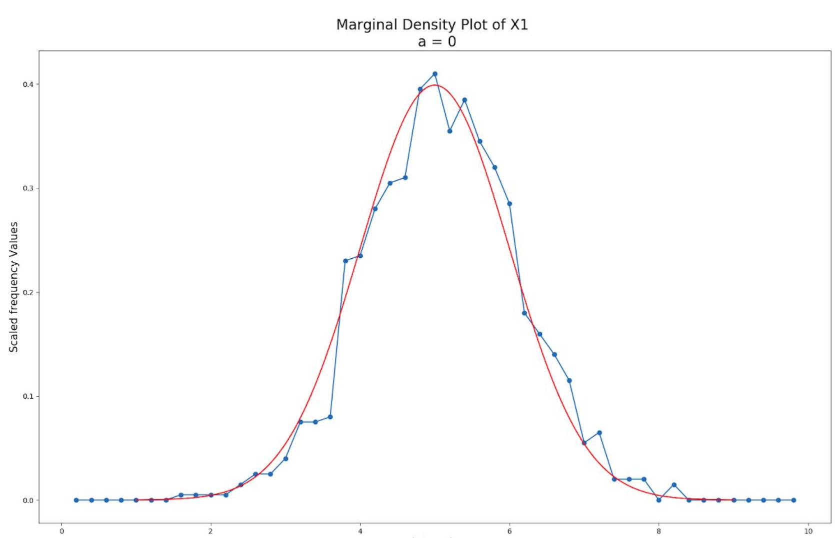


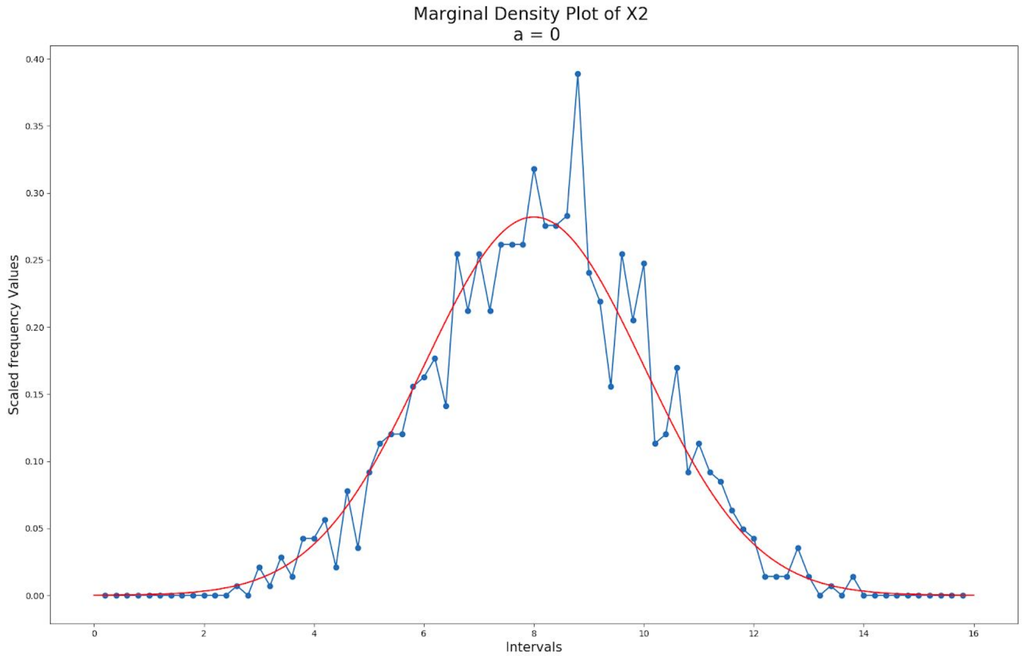

Marginal Density Plot of X2
a = -0.5
Scaled frequency Values
Intervals

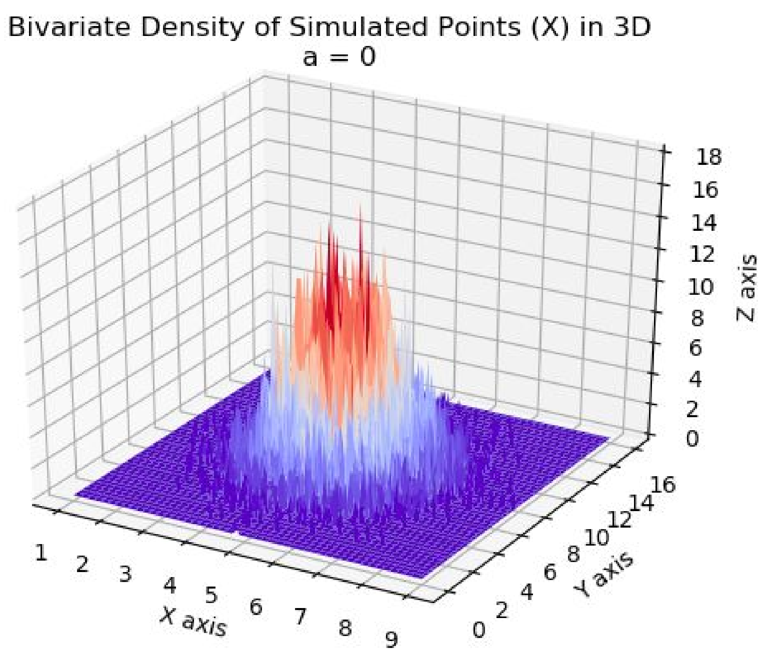

Bivariate Density of Simulated Points (X) in 3D
a = -0.5
Z axis
X axis
Y axis

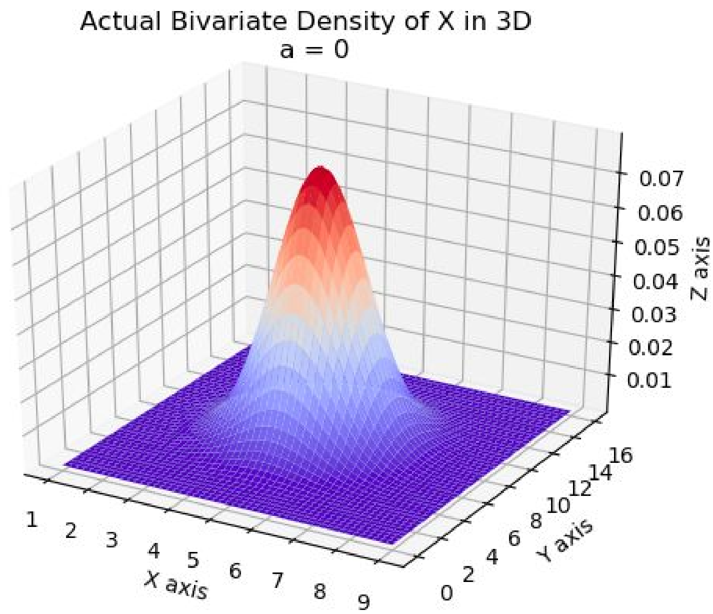

Actual Bivariate Density of X in 3D
a = -0.5
0.08
0.06
0.04
0.02
Z axis
1 2 3 4 5 6 7 8 9
X axis
0 2 4 6 8 10 12 14 16
Y axis

PART-2: a = 0

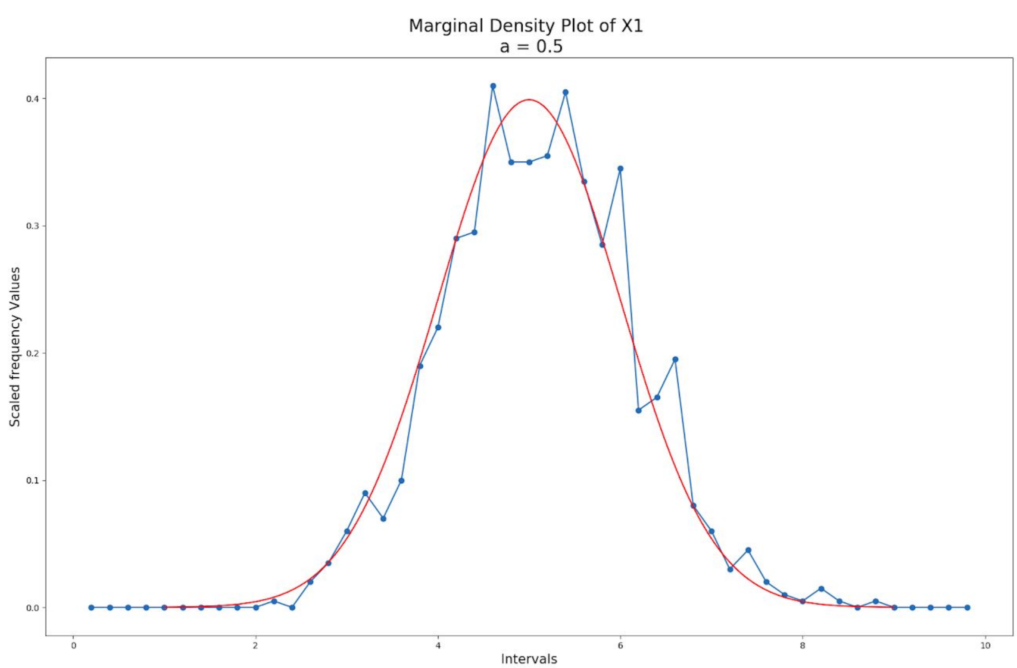

Marginal Density Plot of X1
a = 0
Scaled frequency Values
Intervals

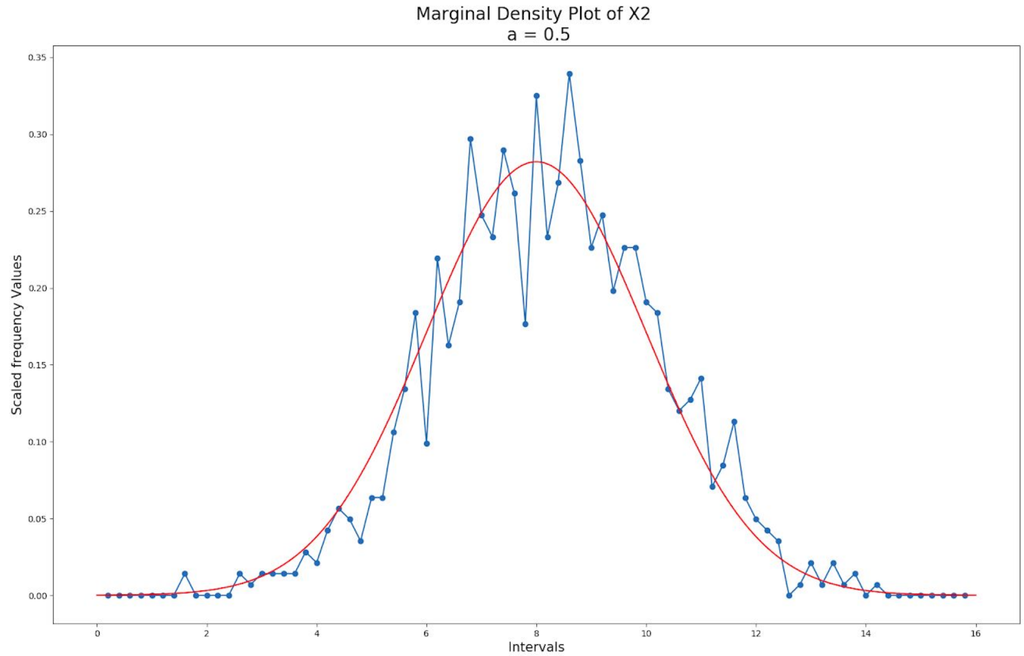

Marginal Density Plot of X2
a = 0
Scaled frequency Values
Intervals
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10
12
14
16

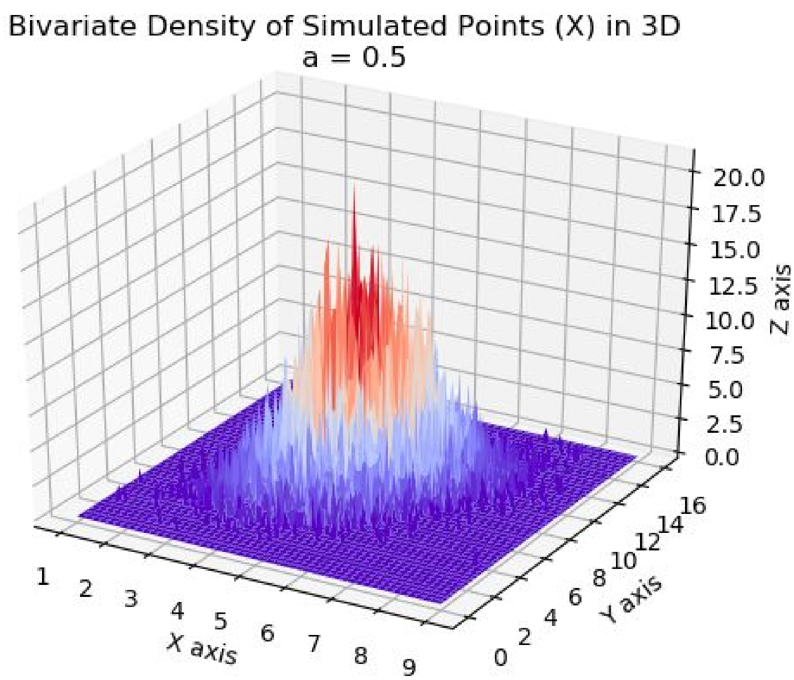

Bivariate Density of Simulated Points (X) in 3D
a = 0
X axis
Y axis
Z axis
1
2
3
4
5
6
7
8
9
0
2
4
6
8
10
12
14
16
18

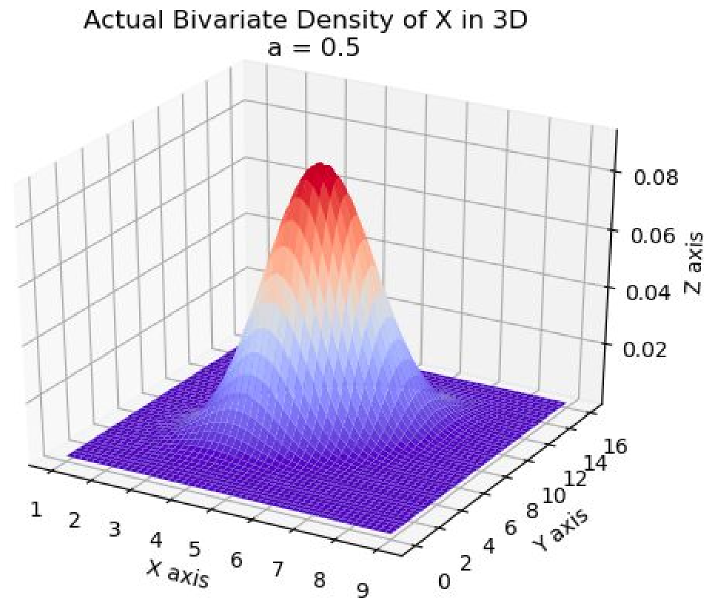

Actual Bivariate Density of X in 3D
a = 0
0.07
0.06
0.05
0.04
0.03
0.02
0.01
Z axis
1 2 3 4 5 6 7 8 9
X axis
0 2 4 6 8 10 12 14 16
Y axis

PART-3: a = 0.5

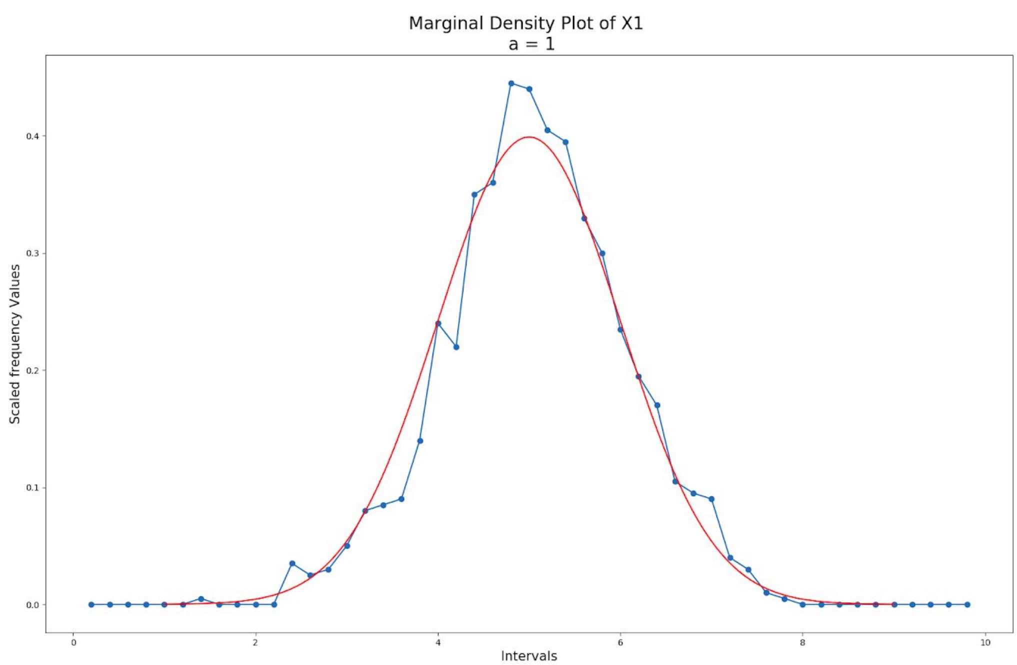

Marginal Density Plot of X1
a = 0.5
Scaled frequency Values
Intervals

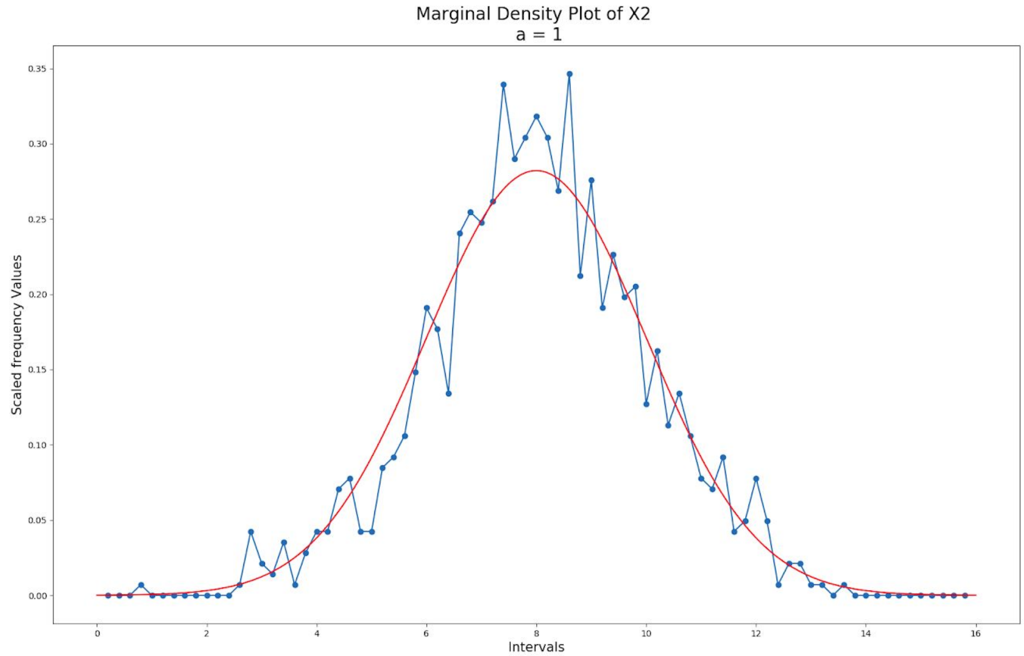

Marginal Density Plot of X2
a = 0.5
Scaled frequency Values
Intervals


Bivariate Density of Simulated Points (X) in 3D
a = 0.5
Z axis
X axis
Y axis


Actual Bivariate Density of X in 3D
a = 0.5
0.08
0.06
0.04
0.02
Z axis
1 2 3 4 5 6 7 8 9
X axis
0 2 4 6 8 10 12 14 16
Y axis

PART-4: a = 1

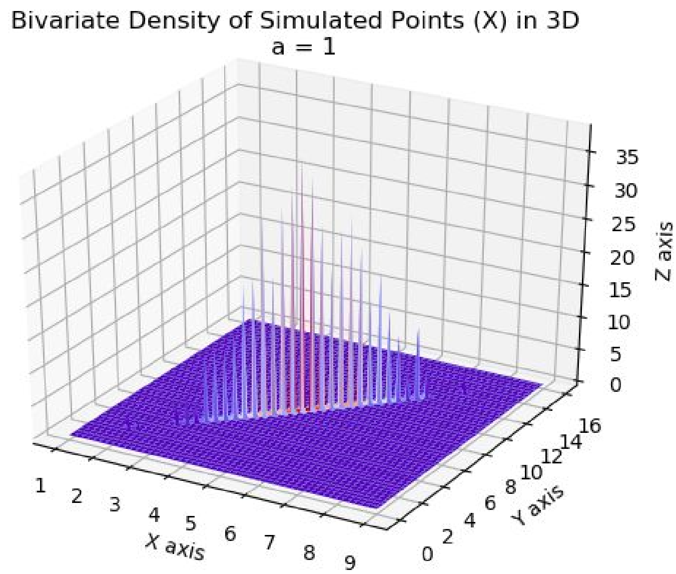

Marginal Density Plot of X1
a = 1
Scaled frequency Values
0.0
0.1
0.2
0.3
0.4
0 2 4 6 8 10
Intervals



In this case the covariance matrix becomes sigma = [[1,2], [2,4]]. Thus the **determinant of the covariance matrix becomes 0**. So in this case the Multivariate Normal Distribution does not exist. For this case the Normal distribution is defined by

$$\mathbf{X = \mu + A*Z, \text{ where } AA^T = Sigma \text{ and } Z\sim N(0,1)}$$

Hence the plot obtained on simulating for 1000 rounds is as follows:



Also to approximate how Multivariate density approaches this density as a->1, I plotted Multivariate density for a = 0.99999. The plot is as follows: