MA 323 : MONTE CARLO SIMULATION

LAB 2

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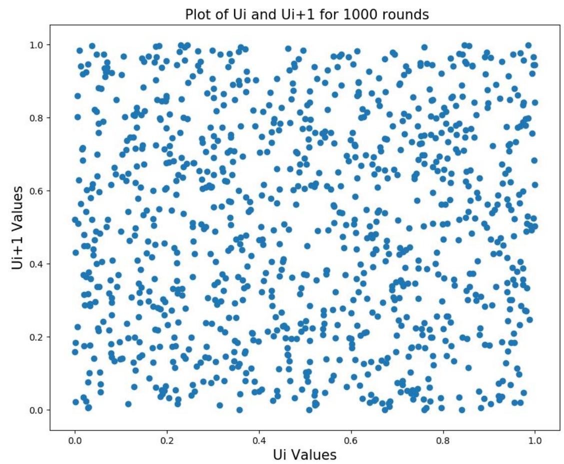
LAGGED FIBONACCI GENERATOR AND INVERSE TRANSFORMATION SAMPLING

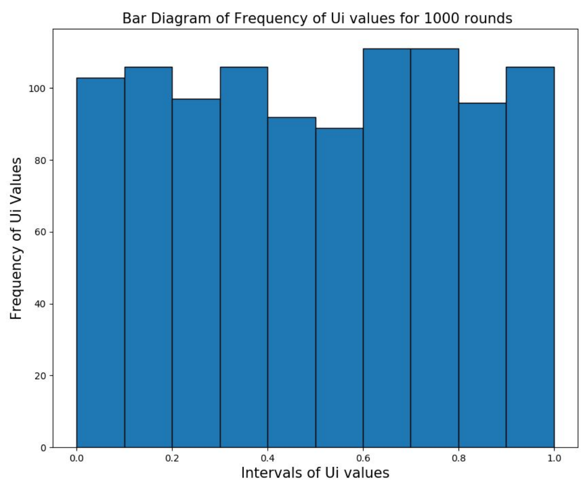
QUESTION 1: OBSERVATIONS

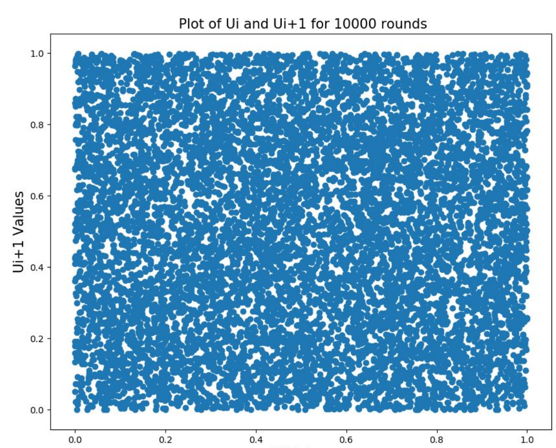
The code attached with submission generates two plots each for 1000, 10000, and 100000 rounds of simulation. The values used for generating the first 17 numbers using Linear Congruential Generators are : a = 1229, b = 1, m = 2048, x0 = 295. Then we use the Lagged Fibonacci Generator to generate further sequences of numbers.

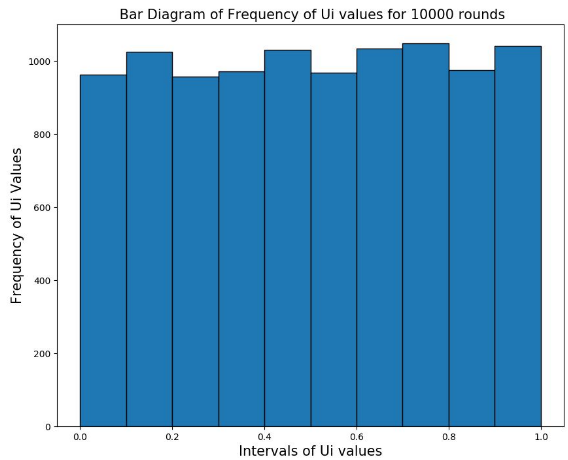
The following observations can be drawn from the plot of (Ui, Ui+1) and respective bar diagrams :

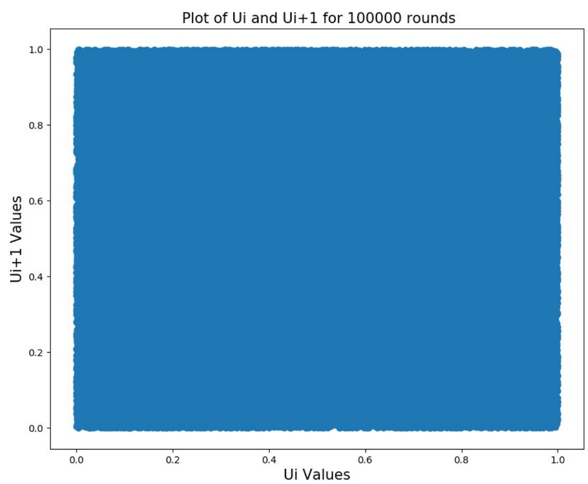
1. In the plot of (Ui, Ui+1), we can observe that there is no particular pattern as was obtained with LCG. This means that the sequence of numbers generated by Lagged Fibonacci Generator is more random than LCG i.e. we can obtain random numbers with longer period length.
2. From the bar diagrams, we can conclude that the numbers generated by LFG are not as uniform in [0,1] as the numbers that were generated by LCG. However. as the number of rounds in simulation increases, the frequency of numbers in all the intervals becomes approximately the same i.e. uniformity increases with the number of rounds in simulation.
3. Also, the plots get denser when the number of rounds in the simulation is increased which implies that with an increase in the number of rounds in simulation, the uniformity of LFG increases.

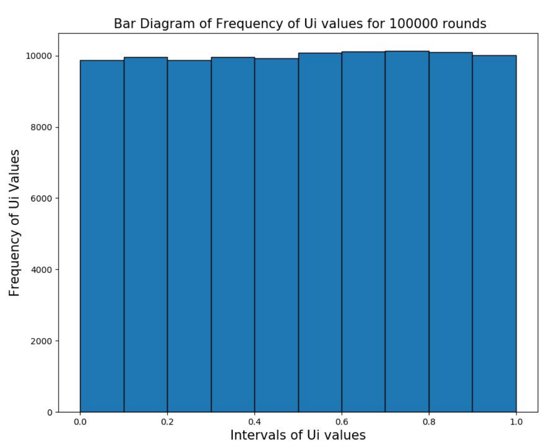

Plot of Ui and Ui+1 for 1000 rounds
Ui+1 Values
Ui Values
0.0
0.2
0.4
0.6
0.8
1.0


Bar Diagram of Frequency of Ui values for 1000 rounds
Frequency of Ui Values
Intervals of Ui values
0
20
40
60
80
100
0.0
0.2
0.4
0.6
0.8
1.0


Plot of Ui and Ui+1 for 10000 rounds
Ui+1 Values
Ui Values
0.0
0.2
0.4
0.6
0.8
1.0


Bar Diagram of Frequency of Ui values for 10000 rounds
Frequency of Ui Values
Intervals of Ui values


Plot of Ui and Ui+1 for 100000 rounds
Ui+1 Values
Ui Values


Bar Diagram of Frequency of Ui values for 100000 rounds
Frequency of Ui Values
Intervals of Ui values

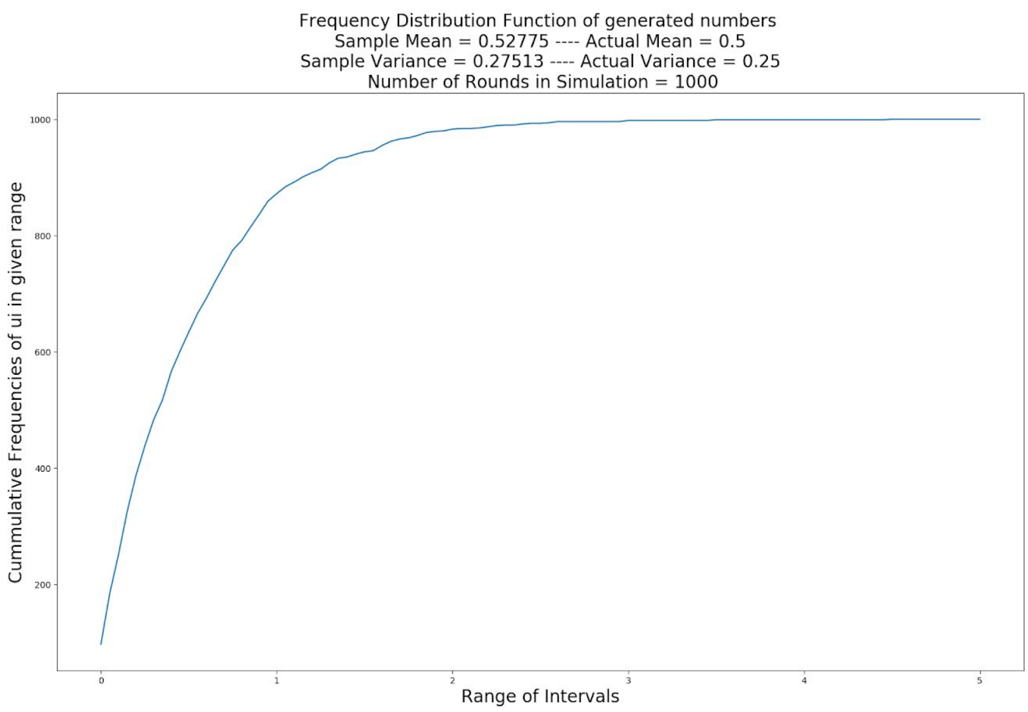
QUESTION 2: OBSERVATIONS

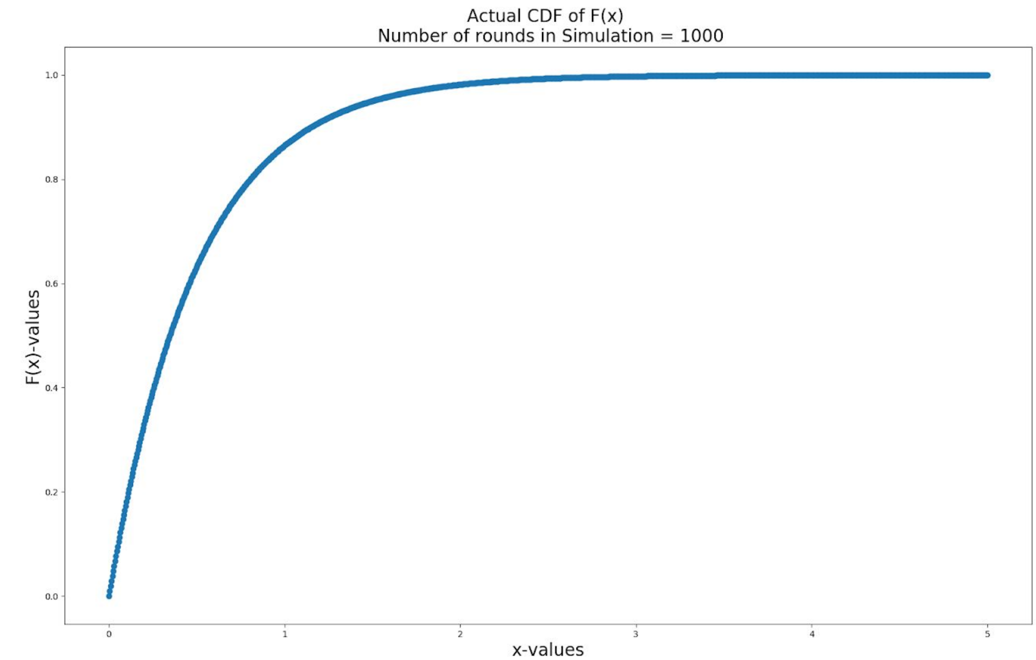
The value of theta taken is : theta = 0.5. Since F(x) is CDF of the exponential distribution, we have **mean = 0.5** and **variance = $(0.5)^2$ = 0.25.**

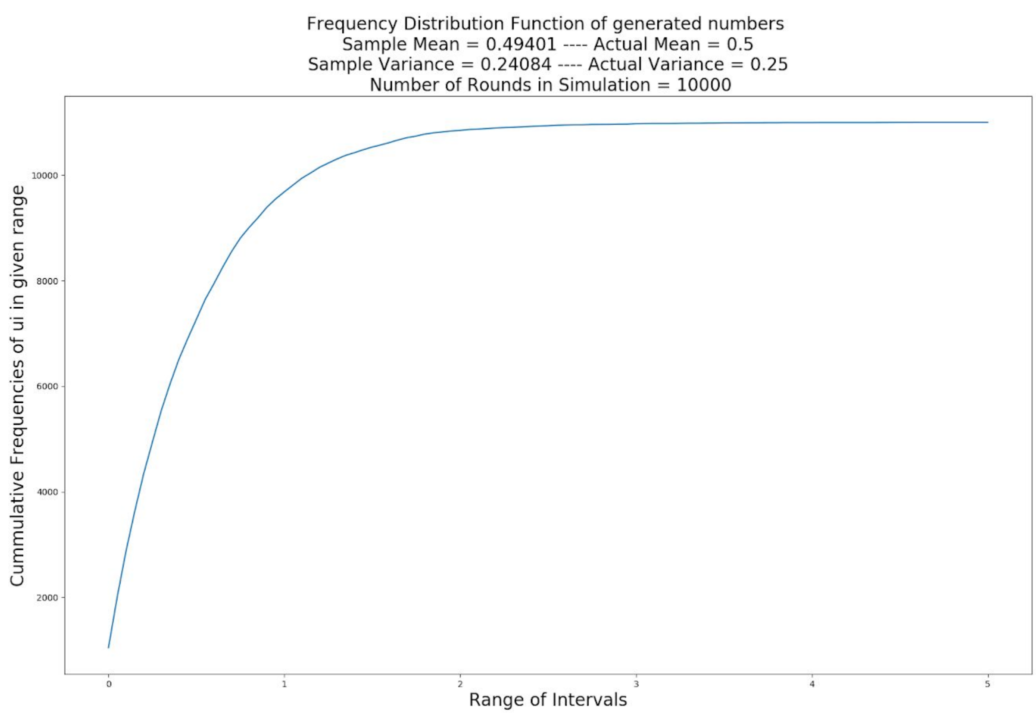
The U ~ **U[0,1]** is generated using inbuilt function of python **random.uniform()** and then the sequence of numbers is generated using given formula of **X = -theta * log(1-u).** Consequently, two plots are obtained for each of 1000, 10000, 100000 rounds of simulation viz.

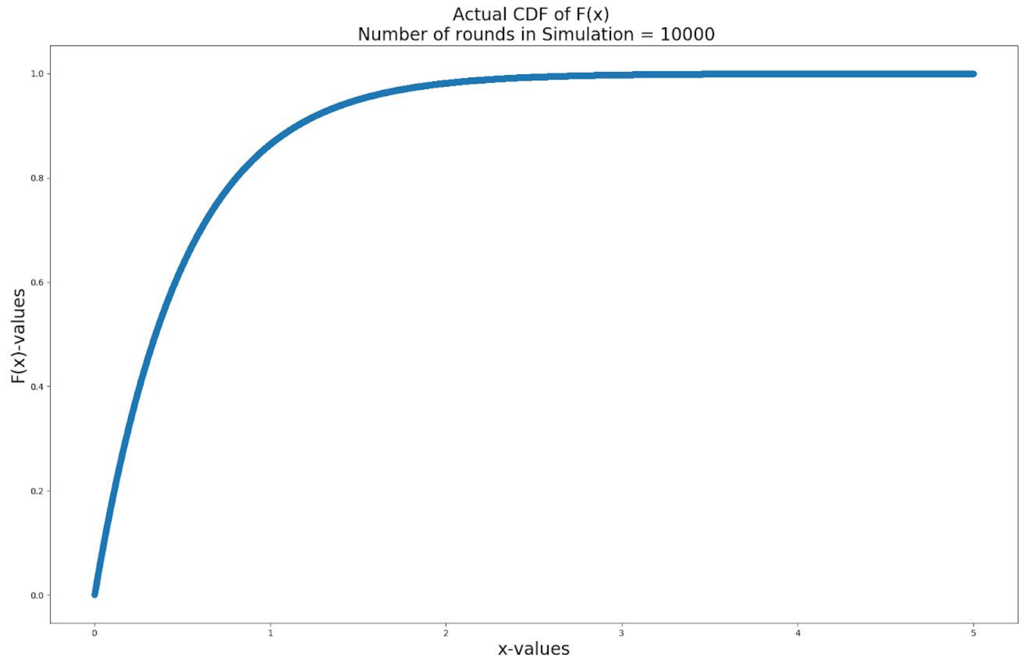
1. **Cumulative Frequency F(x) vs x in [0,5]** with interval length of 0.05. The graph was plotted with the cumulative frequency of intervals against the mid-point of interval.
2. The **Actual CDF** of exponential distribution i.e. **F(x) = 1 - $e^{(-x/theta)}$.**

Both the plots in each case resemble each other. As the number of rounds of simulation increases the sample mean and sample variance converges to actual mean and actual variance respectively.

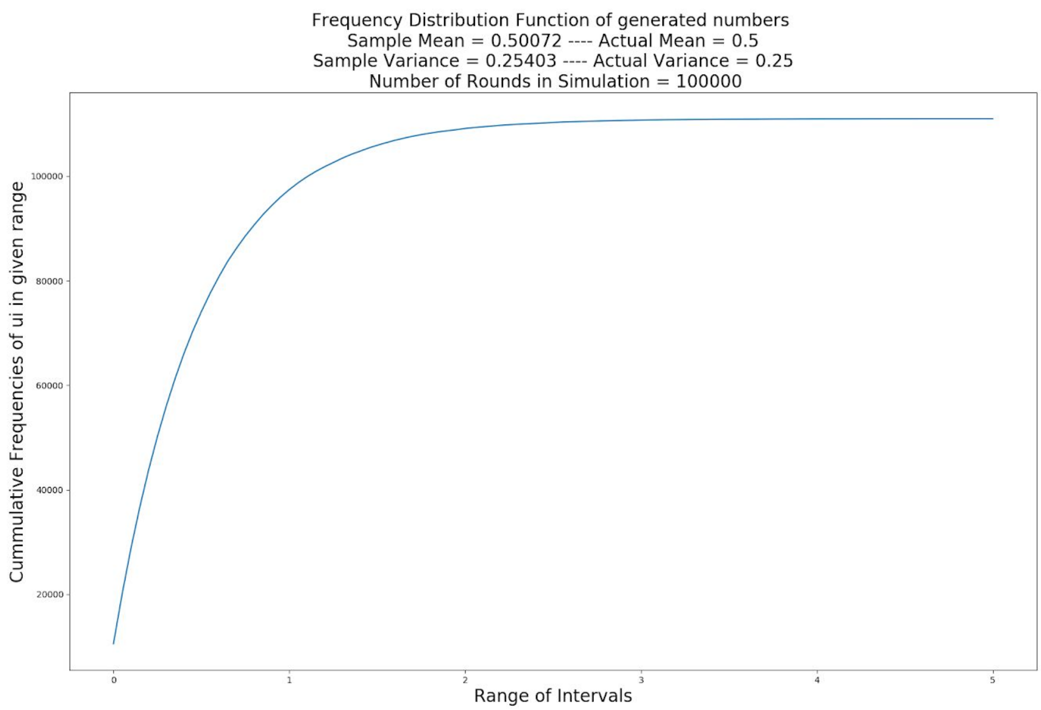



Actual CDF of F(x)
Number of rounds in Simulation = 1000
F(x)-values
x-values
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5

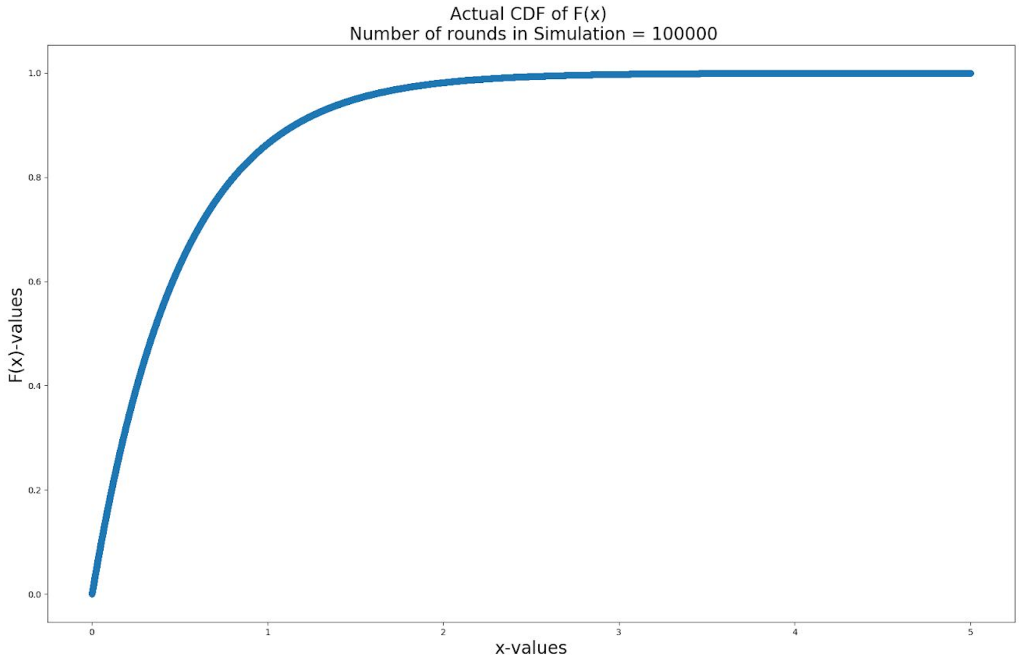

Frequency Distribution Function of generated numbers
Sample Mean = 0.49401 ---- Actual Mean = 0.5
Sample Variance = 0.24084 ---- Actual Variance = 0.25
Number of Rounds in Simulation = 10000
Cummulative Frequencies of ui in given range
Range of Intervals
2000
4000
6000
8000
10000
0
1
2
3
4
5



Actual CDF of F(x)
Number of rounds in Simulation = 10000
F(x)-values
x-values
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5



Frequency Distribution Function of generated numbers
Sample Mean = 0.50072 ---- Actual Mean = 0.5
Sample Variance = 0.25403 ---- Actual Variance = 0.25
Number of Rounds in Simulation = 100000
Cummulative Frequencies of ui in given range
Range of Intervals
20000
40000
60000
80000
100000
0
1
2
3
4
5



QUESTION 3: OBSERVATIONS

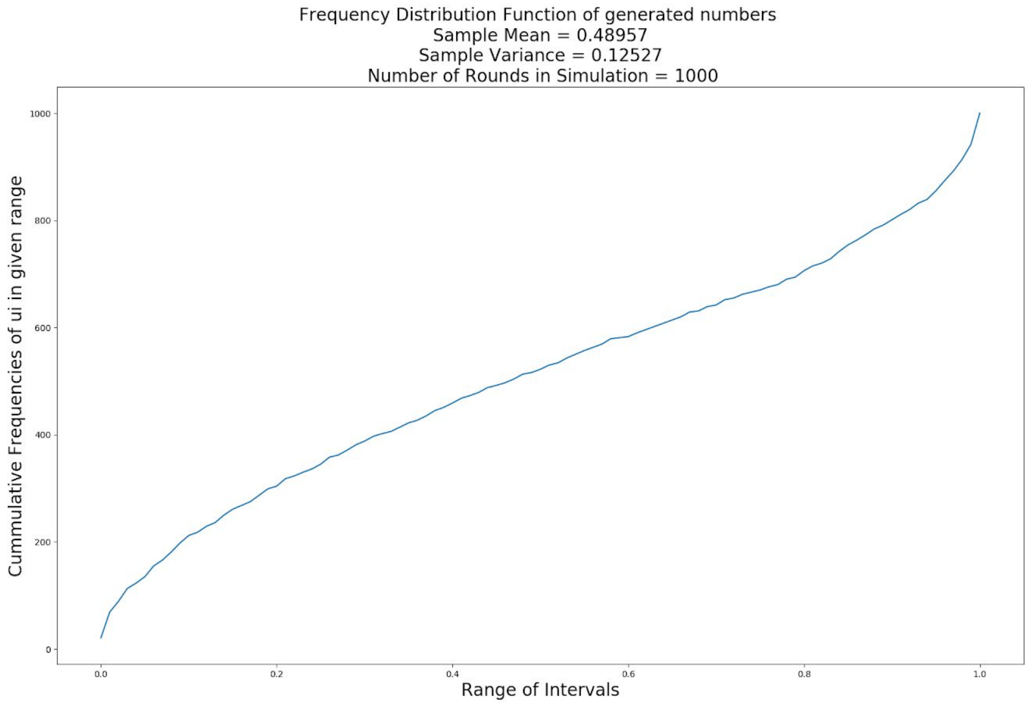
The U ~ **U[0,1]** is generated using inbuilt function of python **random.uniform()** and then the sequence of numbers is generated using given formula of

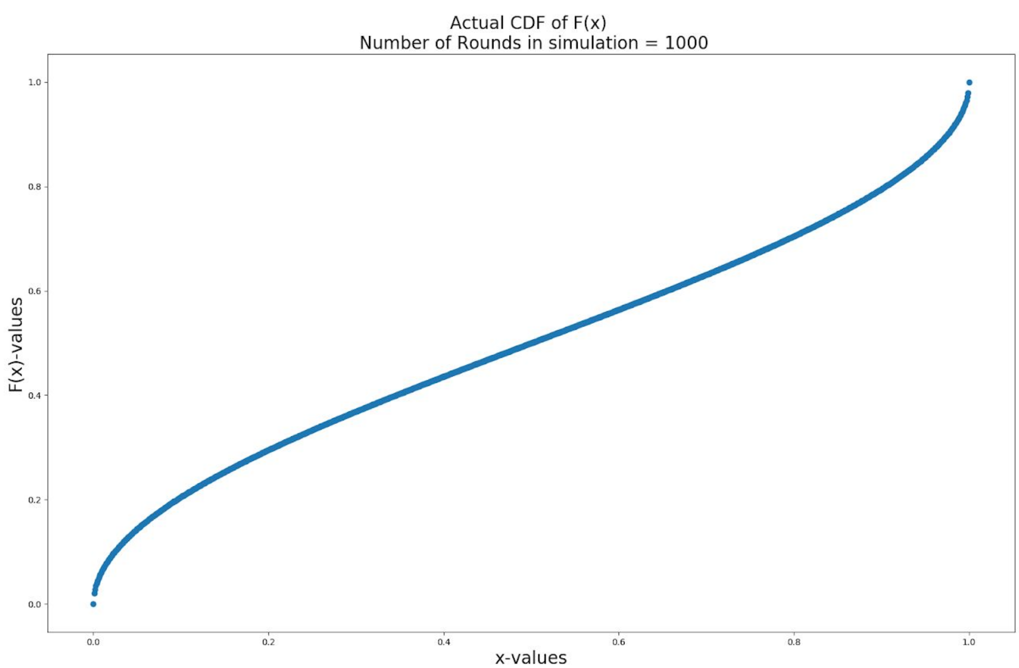
X = $\sin^2(U*pi/2)$ = ½(1 - cos(U*pi)).

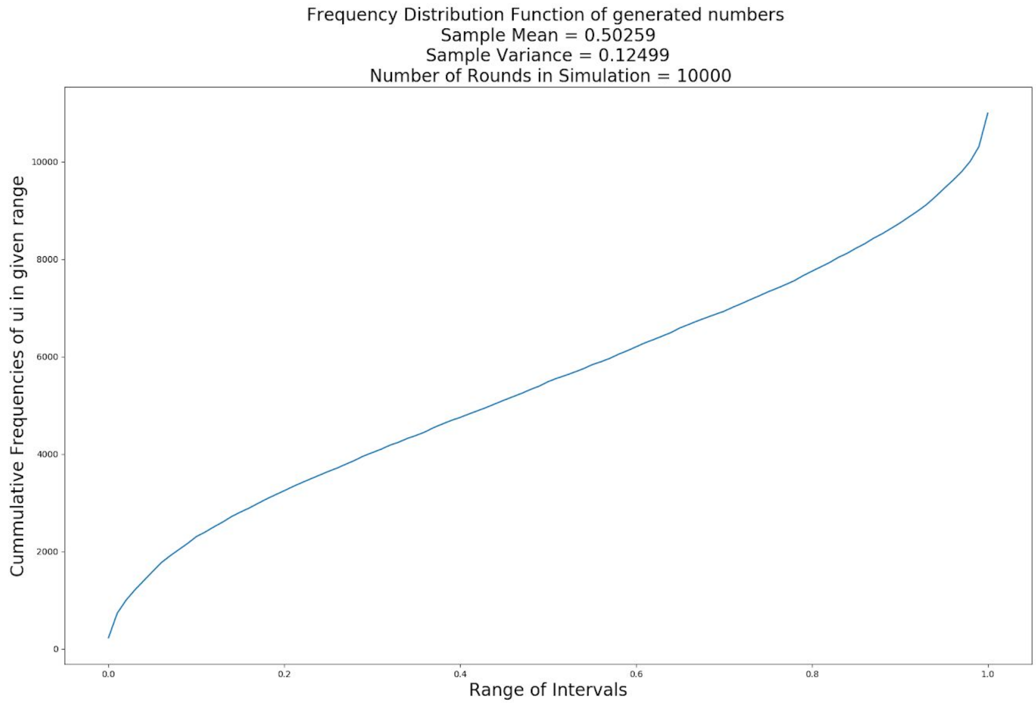
Consequently, two plots are obtained for each of 1000, 10000, 100000 rounds of simulation viz.

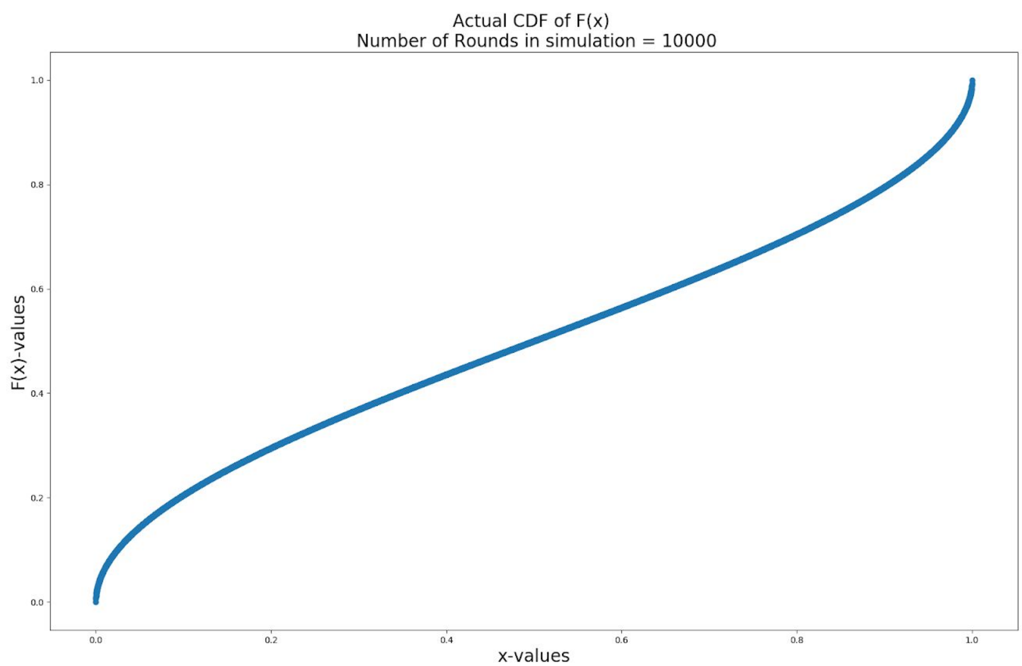
1. **Cumulative Frequency F(x) vs x in [0,5]** with interval length of 0.05. The graph was plotted with the cumulative frequency of intervals against the mid-point of interval.
2. The **Actual CDF** of exponential distribution i.e. **F(x) = 2/pi * arc sin(sqrt(x)).**

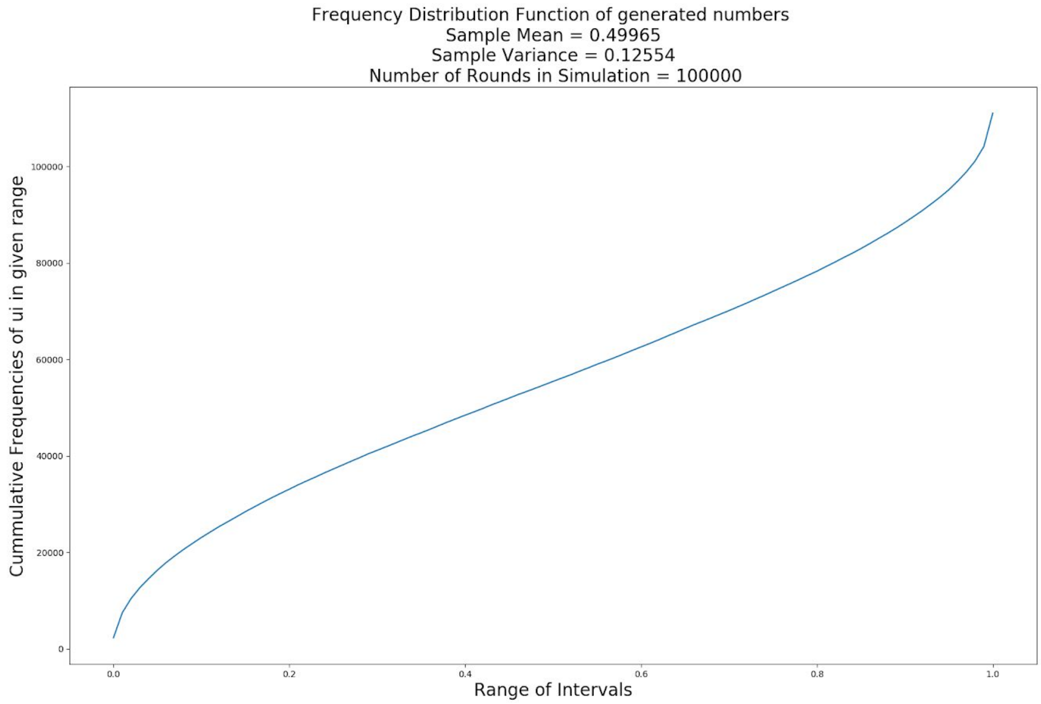
Both the plots in each case resemble each other. As the number of rounds of simulation increases, the plot generated tends towards the actual CDF Plot.

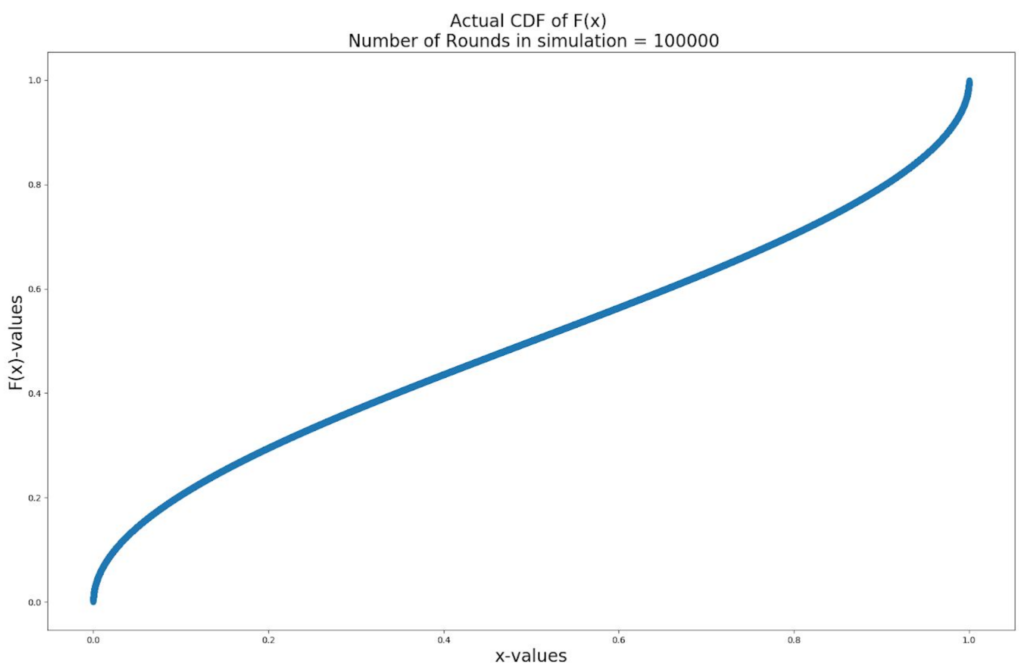

Frequency Distribution Function of generated numbers
Sample Mean = 0.48957
Sample Variance = 0.12527
Number of Rounds in Simulation = 1000
Cummulative Frequencies of ui in given range
Range of Intervals
0
200
400
600
800
1000
0.0
0.2
0.4
0.6
0.8
1.0


Actual CDF of F(x)
Number of Rounds in simulation = 1000
F(x)-values
x-values
0.0
0.2
0.4
0.6
0.8
1.0


Frequency Distribution Function of generated numbers
Sample Mean = 0.50259
Sample Variance = 0.12499
Number of Rounds in Simulation = 10000
Cummulative Frequencies of ui in given range
Range of Intervals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0


Actual CDF of F(x)
Number of Rounds in simulation = 10000
F(x)-values
x-values
0.0
0.2
0.4
0.6
0.8
1.0


Frequency Distribution Function of generated numbers
Sample Mean = 0.49965
Sample Variance = 0.12554
Number of Rounds in Simulation = 100000
Cummulative Frequencies of ui in given range
Range of Intervals
0
20000
40000
60000
80000
100000
0.0
0.2
0.4
0.6
0.8
1.0


Actual CDF of F(x)
Number of Rounds in simulation = 100000
F(x)-values
x-values
0.0
0.2
0.4
0.6
0.8
1.0