MA 323 : MONTE CARLO SIMULATION

LAB 12

NAME : MOHAMMAD HUMAM KHAN

ROLL NUMBER : 180123057

# Van-der-Corput Sequence and Halton Sequence

Question-1 : Part-A

Using The **Radical_Inverse_Function()** implemented in code, the values of Van-der-Corput Sequence is calculated.

★ The first 25 terms in Van-der-Corput Sequence are as follows:

0, 0.5, 0.25, 0.75, 0.125, 0.625, 0.375, 0.875, 0.0625, 0.5625, 0.3125, 0.8125, 0.1875, 0.6875, 0.4375, 0.9375, 0.03125, 0.53125, 0.28125, 0.78125, 0.15625, 0.65625, 0.40625, 0.90625, 0.09375.

★ The plot of (x[i], x[i+1]) as obtained is:
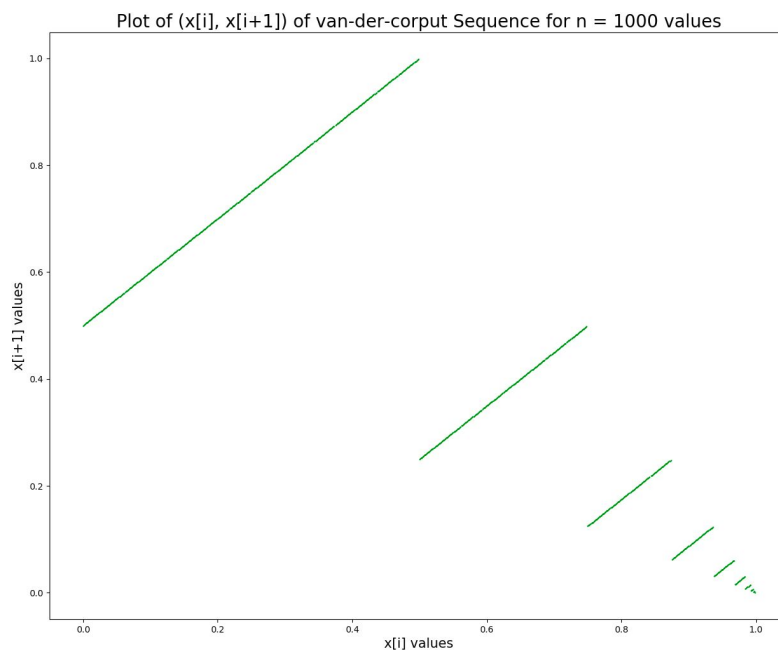
The following observations can be drawn from the plot:

1) The plot is not completely random and a specific pattern can be observed from it.
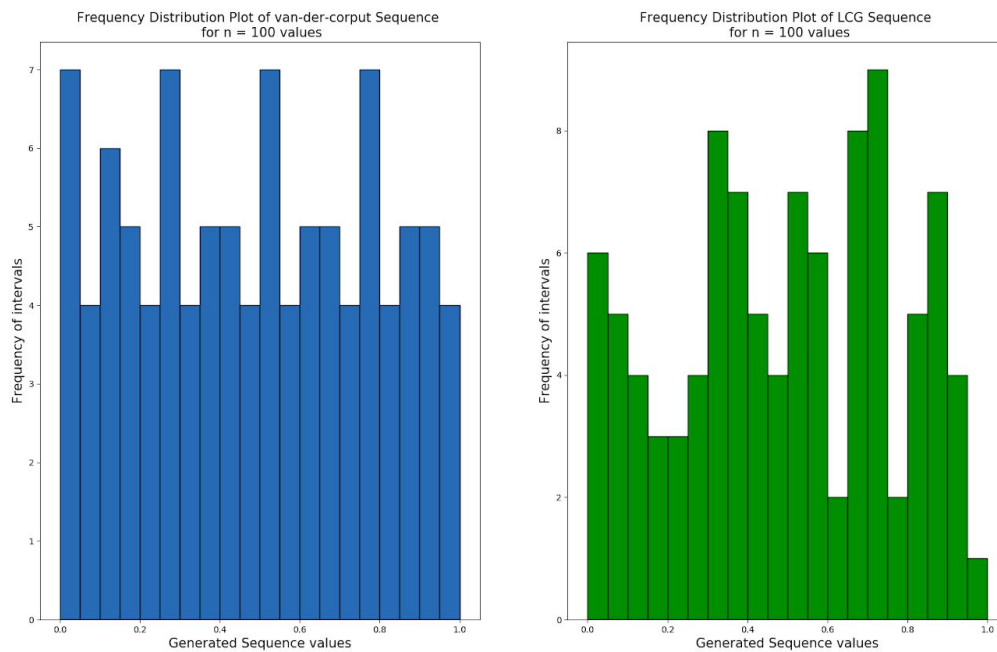
Question-1 : Part-B
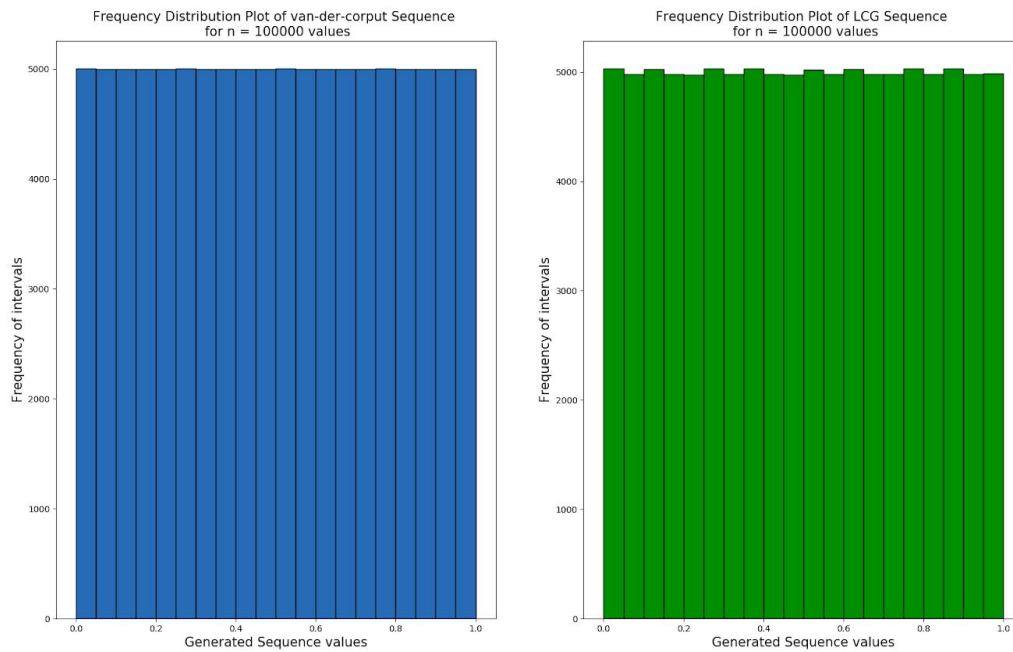
The LCG used is given by the parameters:

a = 1229, b = 1, m = 2048, x_0 = 213.

$$X_{n+1} = (a * X_n + b) \% m$$

$$u_{n+1} = X_{n+1} / m$$

The frequency distribution histograms are plotted side-by-side for comparison.

Frequency Distribution Plot of van-der-corput Sequence for n = 100000 values / Frequency Distribution Plot of LCG Sequence for n = 100000 values
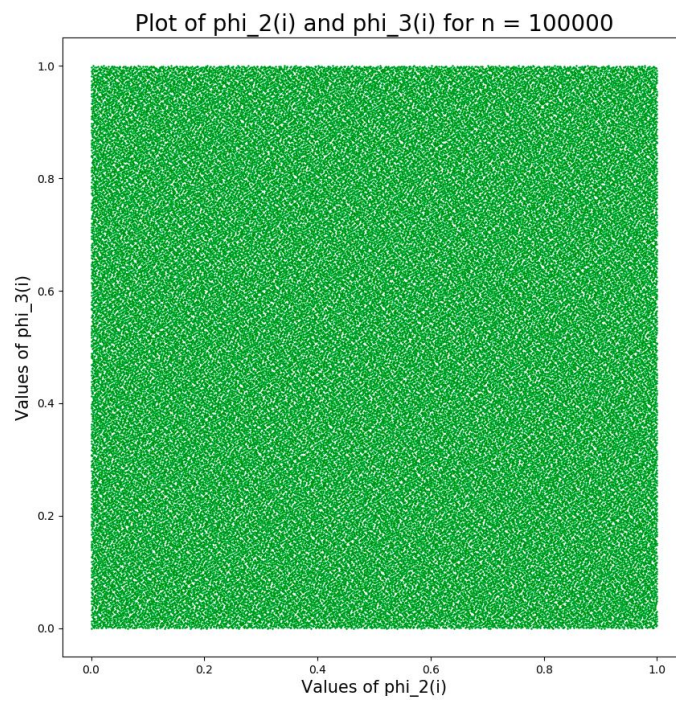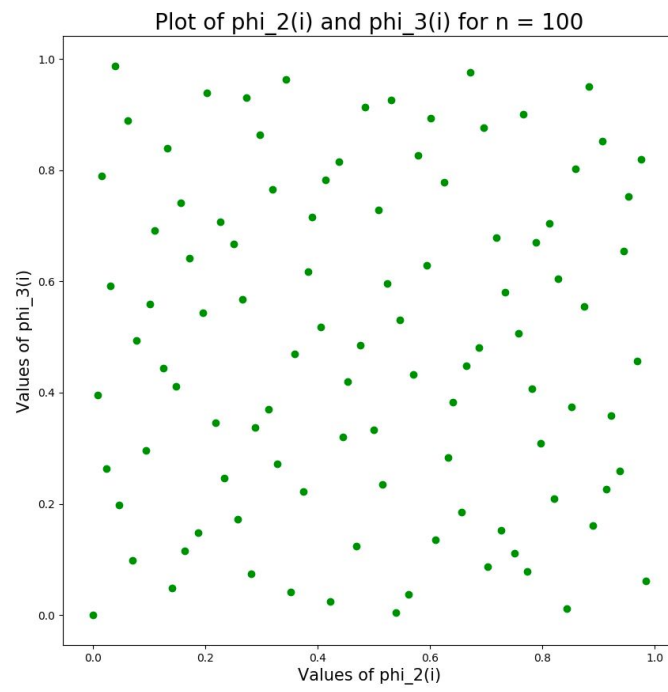
The following observations can be drawn from above two plots:

1. For n = 100, both methods i.e. Van-der-Corput and LCG poorly mimic Standard uniform distribution i.e. there is large deviation from standard uniform distribution.
2. As the number of terms in sequence is increased i.e. n = 100000, it can be observed that Van-der-Corput Sequence mimics standard uniform distribution better than LCG sequence.
3. For large values of n, Van-der-Corput Sequence deviates less from standard uniform distribution.

Question-2

The points of Halton Sequence in $\mathbb{R}^2$ are calculated using Radical_Inverse_Function with base values as **2 and 3.** The plots obtained for n = 100 and n = 100000 are shown below.

Plot of phi_2(i) and phi_3(i) for n = 100

Plot of phi_2(i) and phi_3(i) for n = 100000

The following observations can be made from above plots:

1) Both the plots fill the whole hypercube in $\mathcal{R}^2$ which is the unit square. This is because we chose 2 and 3 as bases which are relatively prime to each other.
2) For n = 100, the plot is less dense but shows no noticeable pattern.
3) For n = 100000, the unit square is completely filled.
4) In both the plots the density of points appear to be uniform.
5) Thus Halton Sequence mimics Uniform Distribution and it is a low-discrepancy sequence.