MA 323 : MONTE CARLO SIMULATION

LAB 3

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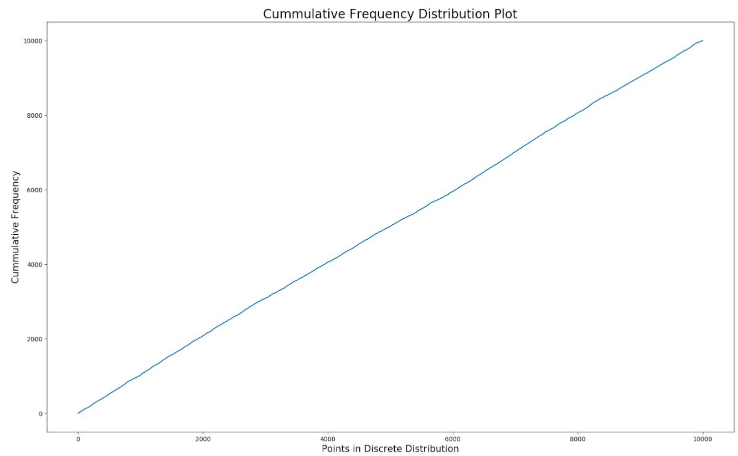
ACCEPTANCE-REJECTION METHOD

QUESTION 1: OBSERVATIONS

10000 Uniform Discrete Random Variables were generated from the given discrete distribution. Since we have to generate uniform discrete random variables, the value of p_i associated with each of values is 1/5000.

On running the attached code, 10000 numbers will be saved in an output .txt file. The plot for cumulative frequency is plotted to verify that numbers are sampled from uniform distribution.



QUESTION 2: OBSERVATIONS

a) Using **U[0,1]** as known density function g, we get g(x)=1 for 0<=x<=1. Then **f(x) <= c*g(x)** will become **f(x) <= c**. Now to get the minimum value of c we have to find the global maximum value attained by f(x). The calculations for finding the min value of c is attached below. On calculating we get the min value of **c = 135/64 = 2.109375**.

Calculation of Minimum Value of c

We have here $g(x) = 1 \quad 0 \leq x \leq 1$

Thus $f(x) \leq c\, g(x) \Rightarrow f(x) \leq c$.

i.e. we have to find min value of c s.t.

$$f(x) = 20x(1-x)^3 \leq c.$$

To find this, we have to find maximum value of $f(x)$ in $(0,1)$.

Differentiating w.r.t x, we get.

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(20x(1-x)^3\right). = 20\left[(1-x)^3 - 3x(1-x)^2\right]$$

Setting this equal to 0, we get.

$$20(1-x)^2(1-x-3x) = 0 \Rightarrow \boxed{x=1} \text{ OR } \boxed{x = 1/4}$$

At $x=1$, $f(1) = 0$

At $x = 1/4$, $f(1/4) = 20 \cdot \frac{1}{4}\left(1-\frac{1}{4}\right)^3 = \frac{135}{64}$.

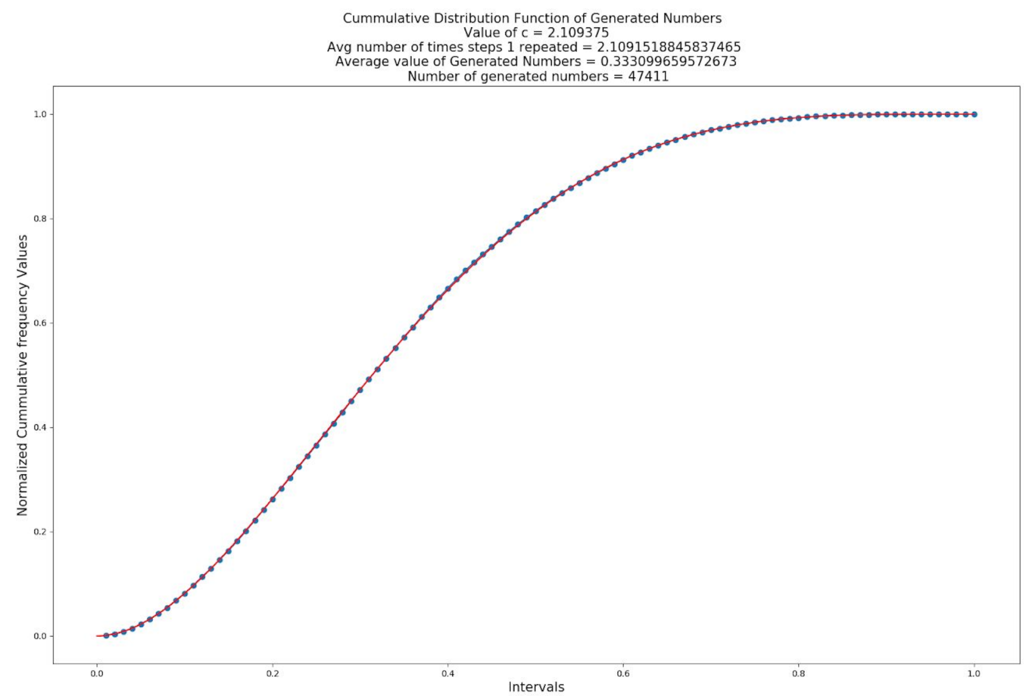
which is max^m value of $f(x)$ attained.

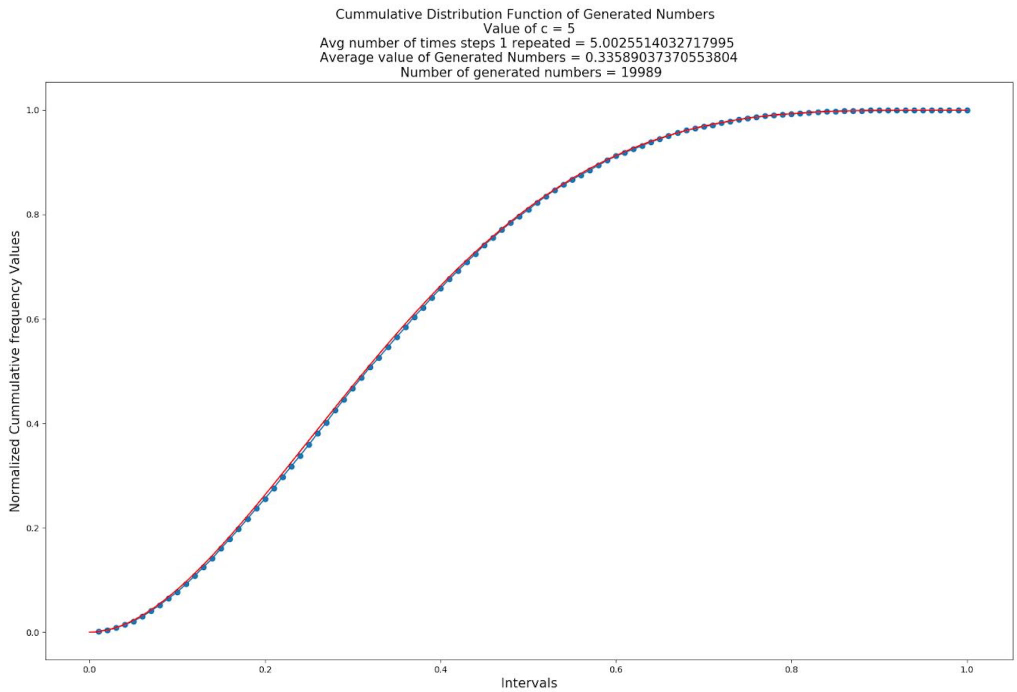
Therefore min value of c is $\boxed{c = \frac{135}{64} = 2.109375}$

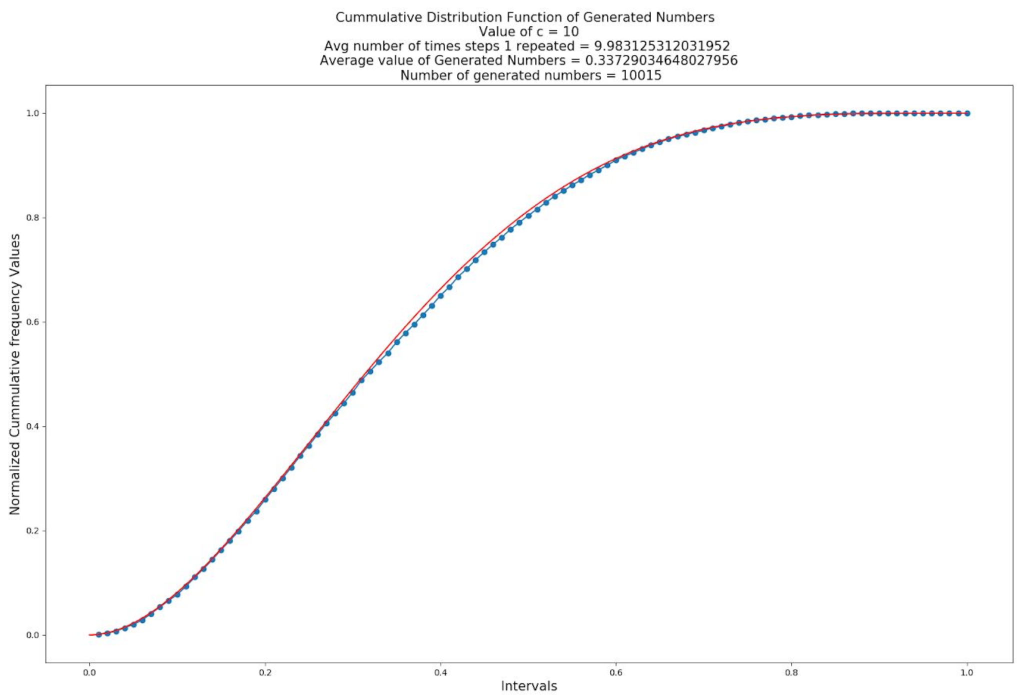
b) The simulation is run for 100000 times to generate numbers using acceptance-rejection method. It is found that the distribution of generated numbers converges to Cumulative Distribution Function of f(x). The distribution function of f(x) can be obtained by integrating f(x) from 0 to x. The distribution function obtained is $\mathbf{10x^2 - 20x^3 + 15x^4 - 4x^5}$**.** This distribution function is plotted in each of plots in red color for comparison. From the plots it is clear that the distribution of generated numbers converges to distribution of f(x).

NOTE: Red line ---> Distribution Function of f(x)

Blue line ---> Distribution of generated numbers




Cummulative Distribution Function of Generated Numbers
Value of c = 5
Avg number of times steps 1 repeated = 5.0025514032717995
Average value of Generated Numbers = 0.33589037370553804
Number of generated numbers = 19989
Normalized Cummulative frequency Values
Intervals


Cummulative Distribution Function of Generated Numbers
Value of c = 10
Avg number of times steps 1 repeated = 9.983125312031952
Average value of Generated Numbers = 0.33729034648027956
Number of generated numbers = 10015
Normalized Cummulative frequency Values
Intervals

c) On calculating the average number of times Step-1 of Algorithm is repeated before the generated number gets accepted it comes out approximately equal to value of c.

The average value of generated numbers is around 0.33

d) I ran the simulation for two more values of c viz. 5 and 10. It is observed that

1. As the value of c increases the acceptance rate gets decreased. In each case the simulation is run for 100000 rounds and it is observed that for c=2.109375, around 48000 numbers are generated; for c = 5, around 20000 numbers are generated; while for c = 10, around 10000 numbers are generated. This is because as value of c increases the quantity **f(x)/(c*(g(x)))** decreases and hence the chance for a number to get accepted decreases.
2. In each case the number of times step-1 of algorithm is repeated increases with increase in value of c i.e. the average number of iterations required to generate a number is equal to value of c and hence increases with value of c.

QUESTION 3: OBSERVATIONS

Here in this case we have the value of g(x) = 1/10 due to uniform discrete distribution. To sample numbers from discrete uniform distribution g(x), the algorithm of q1 of this assignment is used.

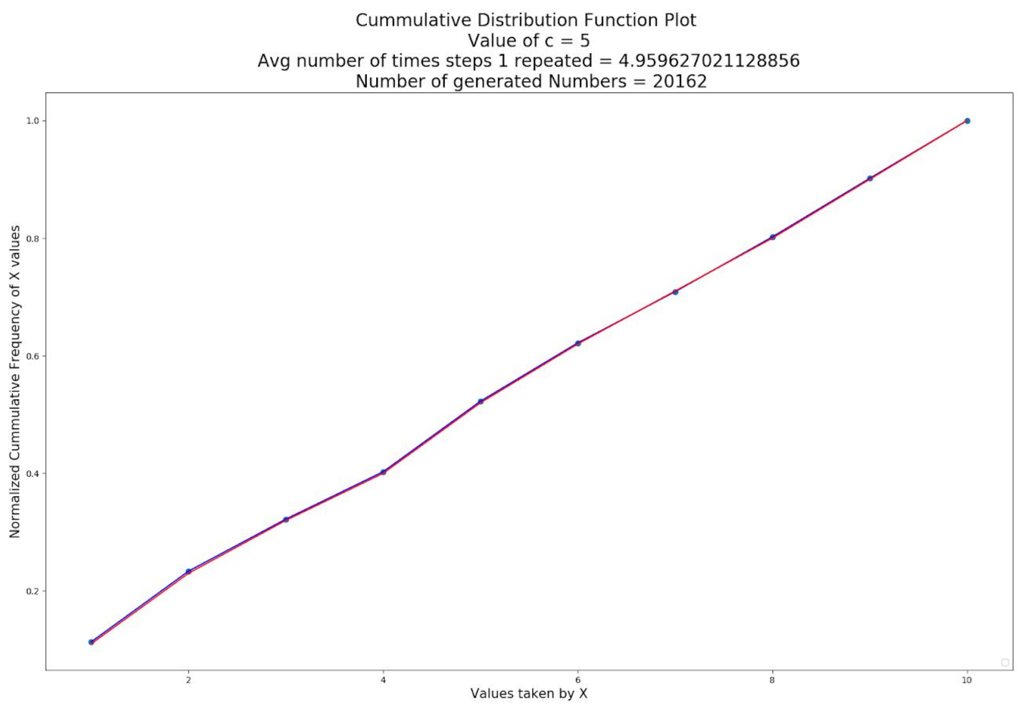
Now we have **f(x)/(c*g(x)) = (10/c)*f(x)**. The simulation is run for three values of c viz. 5, 10, 20. In step-3 of the algorithm we perform the check **u <= f(x)/(c*g(x)) i.e. u <= (10/c)*f(x).** Now as the value of c increases, the chance of the generated number getting accepted gets decreased because the value of **10/c** decreases. Thus it is observed that as the value of c increases, the number of generated numbers decreases.

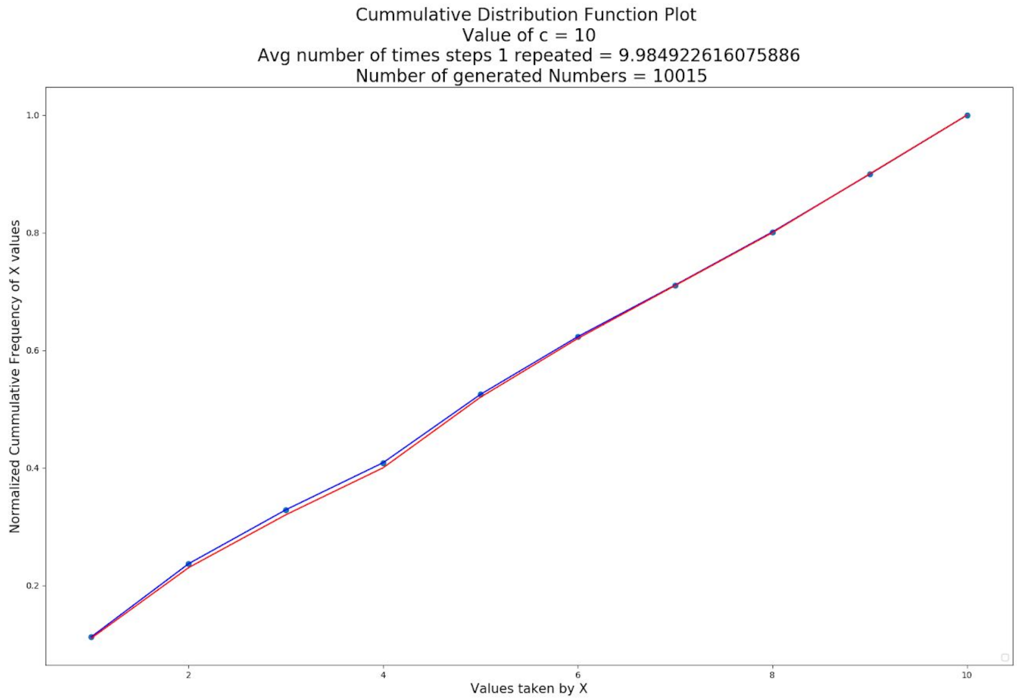
For c = 5 around 20000 get accepted; for c = 10 around 10000 numbers get accepted; while for c = 20 around 6000-7000 numbers get accepted.

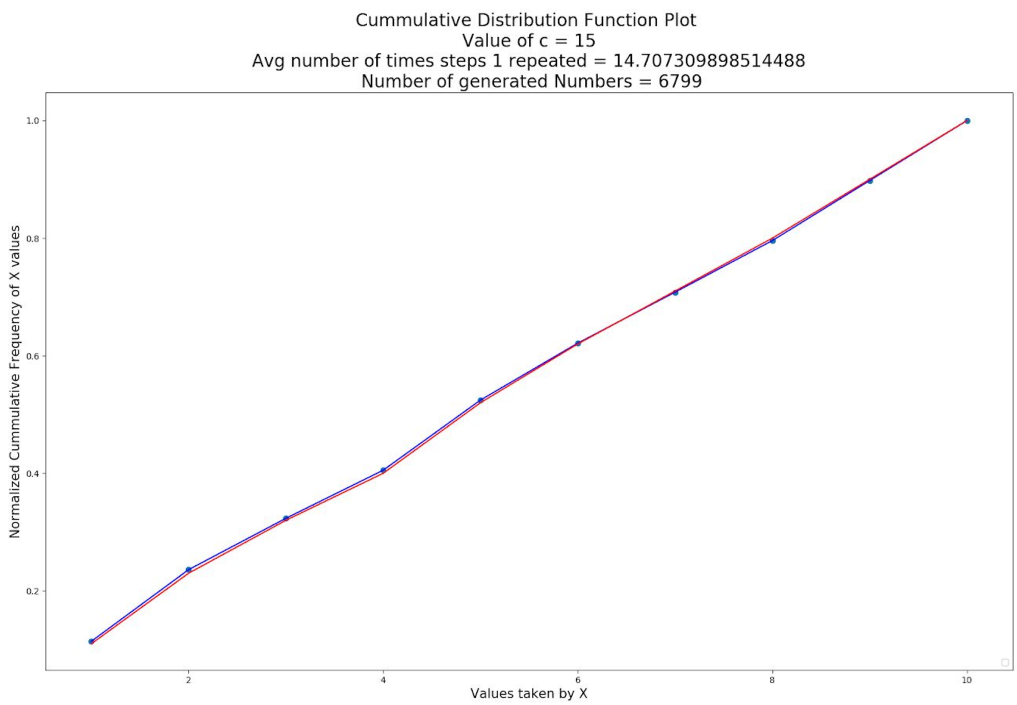
Also in each case the average number of iterations needed to generate a number is approximately equal to value of c.

NOTE: Red line ---> Distribution Function of X

Blue line ---> Distribution of generated numbers




Cummulative Distribution Function Plot
Value of c = 10
Avg number of times steps 1 repeated = 9.984922616075886
Number of generated Numbers = 10015
Normalized Cummulative Frequency of X values
Values taken by X


Cummulative Distribution Function Plot
Value of c = 15
Avg number of times steps 1 repeated = 14.707309898514488
Number of generated Numbers = 6799
Normalized Cummulative Frequency of X values
Values taken by X