Problem Statement:
Use the following Monte Carlo estimator to approximate the expected value, $I = E\left[\exp\left(\sqrt{U}\right)\right]$, where, $U \sim \mathcal{U}[0,1]$:

$$I_M = \frac{1}{M}\sum_{i=1}^{M} Y_i, \text{ where } Y_i = \exp\left(\sqrt{U_i}\right), \text{ with } U_i \sim \mathcal{U}[0,1].$$

Repeat the problem, using antithetic variates via the following estimator:

$$\widehat{I}_M = \frac{1}{M}\sum_{i=1}^{M} \widehat{Y}_i, \text{ where } \widehat{Y}_i = \frac{\exp\left(\sqrt{U_i}\right) + \exp\left(\sqrt{1-U_i}\right)}{2}, \text{ with } U_i \sim \mathcal{U}[0,1].$$

Taking the values of M to be $10^2, 10^3, 10^4$ and 10^5, determine the 95% confidence interval for I_M and $\widehat{I}_M$, for all these four values of M, that you have taken, and present the results that you have obtained above in a tabular form. Your table must consist of the values of I_M, $\widehat{I}_M$, 95% confidence intervals for I_M, 95% confidence intervals for $\widehat{I}_M$, and the ratio of lengths of both the intervals.

Submission Deadline: 18th November 2020, 11:59 PM