Reducing Congestion Through Information Design

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***Abstract*— We consider the problem of designing information in games of uncertain congestion, such as traffic networks where road conditions are uncertain. Using the framework of Bayesian persuasion, we show that suitable information structures can mitigate congestion and improve social welfare.**

I. Introduction

When contracting frictions induce inefficiences, less information can lead to better outcomes [1], [2]. Routing games, where agents seek to minimize their own travel time, are a canonical example of a setting where externalities tend to induce Pareto inefficient outcomes [3]. In this paper, we illustrate how providing agents in a routing game with partial information about the state of the network can improve outcomes and even restore efficiency. Real-time traffic apps, such as Google Maps and Waze, provide potential ways of implementing such welfare-improving garbled information.

We first present a very simple example. A continuum of agents choose one of two paths between a single origin and a single destination. Travel time on Path 1 is independent of the number of agents on that path but depends on an uncertain state of the world. Travel time on Path 2 is subject to congestion; the more agents take this path, the longer it takes to traverse it. Under full information, too many agents take the congestion-prone Path 2 when Path 1 happens to be slow. But, by sending a suitable i.i.d. signal about the conditions on Path 1 to each agent, we can reduce the traffic on Path 2 down to its social optimum and achieve the first-best outcome. We also illustrate how partial information can reduce congestion in the Wheatstone Network, which plays a central role in Braess' paradox [4], [5].

We focus our analysis on *i.i.d. signals* that reveal an independent draw of a given signal to each agent. In both of our examples, every *public signal* (that reveals the same signal realization to all agents and thus leads to a common posterior belief) leads to the same social welfare. In other information-design problems, it can be helpful to consider *arbitrary information structures* that allow the designer to correlate agents' signal realizations and to provide more informative signals to some agents than to others, but in the class of games we consider (with a continuum of symmetric agents), no arbitrary information structure can improve on an optimal i.i.d. signal.

Arnott et al provide an early analysis of the impact of information on traffic congestion [6]. Acemoglu et al show that making a subset of agents *aware* of the existence of a route can make those agents worse off; they fully characterize the set of networks where such "informational Braess' paradox" can occur [7].

The closest paper to ours is independent work by Whinston and Liu [8]. They also apply Bayesian persuasion to routing games but consider a setting with finitely many agents and thus focus on optimal correlation of signals; they also examine a dynamic environment where vehicles depart over time.

Kremer, Mansour, and Perry [9] analyze how crowdsourced traffic apps could induce agents to

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explore under-utilized paths so the app would obtain information about the state of the network. Our paper examines a complementary question of how to then deploy this information in a way that minimizes congestion externalities.

We also contribute to the literature on information design in games, i.e., Bayesian persuasion with multiple receivers [10], [11]. Prominent examples of this literature include studies of voting [12], bank runs [13], and auctions [14]. In contrast to the aforementioned papers, we consider an information designer who seeks to maximize welfare of the players,[1] and we establish the possibility of restoring efficiency through information design. We also note that our results could be leveraged for coordinating actions in multiagent teams; congestion games are the standard approach for studying such topics [16].

II. A Simple Example

A. Set-up

A unit measure of agents simultaneously choose one of two paths, P_1 or P_2, between the origin and the destination. The travel time on P_1 is independent of the share of agents that take that path, but depends on the state of the world $\omega \in \{0, 1\}$. The travel time on P_2 is increasing in the share of agents that travel on P_2. The costs (or travel times) on P_1 and P_2 are given by:

$$c(P_1) = \omega$$
$$c(P_2) = \frac{1}{3} + 2s$$

where $s \in [0, 1]$ is the share of agents on P_2. Figure 1 illustrates the network. Agents are risk-neutral and seek to minimize their travel costs. Agents share the prior that the two states are equally likely.

One interpretation of these costs is that P_1 is a high-capacity but construction-prone highway where ω indicates presence of construction, while P_2 is safe from construction but sensitive to congestion. Denoting the share of agents who take P_2 in state ω by $s(\omega)$, the aggregate travel cost in each state is

$$(1 - s(\omega))\,\omega + s(\omega)\left(\frac{1}{3} + 2s(\omega)\right).$$

We seek to minimize expected aggregate travel costs. We consider a number of benchmarks.

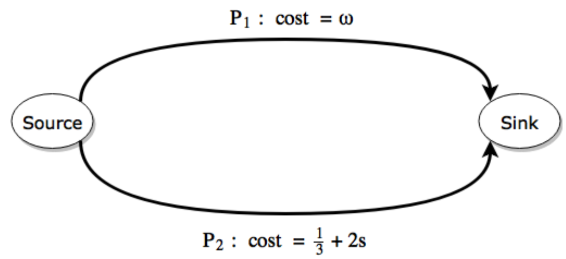

Fig. 1: ω is the unknown state, and s is the share of agents using P_2.

B. First-best

Suppose that a central planner can mandate which route each agent will take and acts to maximize aggregate (utilitarian) social welfare. In state $\omega = 0$, she clearly sends all agents to P_1 and incurs zero costs. In state $\omega = 1$, she minimizes aggregate total cost by sending $\frac{1}{6}$ of agents to P_2 and incurs aggregate travel cost of $\frac{17}{18}$. The first-best expected aggregate travel cost is thus $\frac{17}{36}$.

C. Full information

Now suppose all the agents know the state and play non-cooperatively (i.e., play Wardrop equilibrium [17]). In state $\omega = 0$, all agents will still go to P_1 and incur no cost. In $\omega = 1$, agents will crowd P_2 until the costs of the two paths are equalized, i.e., $\frac{1}{3} + 2s = 1$, or $s = \frac{1}{3}$, which leads to aggregate travel cost of 1. Thus, under full information expected aggregate travel cost is $\frac{1}{2}$.[2]

[1]Bergemann and Morris derive the information structure that maximizes firms' profits in an oligopoly setting [15].

[2]The "price of anarchy" here is around 5.8%. Roughgarden and Tardos prove that under linear congestion costs, price of anarchy is never more than a third [18].

D. Public signal

If agents observe a public signal about ω, following each signal realization they share some common belief $\mu = \Pr(\omega = 1)$. At each such belief, traffic will equilibrate so the share of agents on P_2 is $s(\mu) = \max\left\{0, \frac{3\mu-1}{6}\right\}$. While it might not be obvious at first glance, the expected aggregate total cost,

$$C(\mu) = (1 - s(\mu))\mu + s(\mu)\left(\frac{1}{3} + 2s(\mu)\right)$$

simplifies to $C(\mu) = \mu$. The reason for this is simple; when $\mu \leq \frac{1}{3}$, $s = 0$ and no one is on P_2, so $C(\mu)$ is just the cost of P_1, namely μ. When $\mu > \frac{1}{3}$, some agents take P_1 and some take P_2, which means that the cost of P_1 and P_2 must be the same; hence, their convex combination (e.g., $C(\mu)$) is also equal to the cost of P_1, namely μ.

The fact that $C(\mu) = \mu$ is linear implies that all public signals generate the exact same expected aggregate cost: providing no information, partial information, or full information all yield expected aggregate travel cost of $\frac{1}{2}$.

E. Optimal information structure

Consider the following signal. There are two signal realizations, r_1 and r_2. We will interpret these two realizations as recommendations to take P_1 or P_2, respectively. Suppose

$$\Pr(r_1|\omega = 0) = 1 \qquad \Pr(r_2|\omega = 0) = 0$$
$$\Pr(r_1|\omega = 1) = \frac{5}{6} \qquad \Pr(r_2|\omega = 1) = \frac{1}{6}$$

First note that if agents observe independent draws of this signal and follow the implied recommendations, we will achieve the first-best outcome: when $\omega = 0$, everyone will go on P_1 and when $\omega = 1$, $\frac{1}{6}$ of the agents will go on P_2.

Now we need to show that following these recommendations constitutes a Bayes Nash equilibrium.

If an agent observes r_1, her expectation of the cost of P_1 is

$$\Pr(\omega = 1|r_1) = \frac{\frac{5}{6}}{\frac{5}{6} + 1} = \frac{5}{11}$$

whereas her expectation of the cost of P_2 is

$$\frac{1}{3} + 2 \times \left(\Pr(\omega = 0|r_1) \times 0 + \Pr(\omega = 1|r_1) \times \frac{1}{6}\right)$$
$$= \frac{1}{3} + 2 \times \frac{5}{11} \times \frac{1}{6} = \frac{16}{33} > \frac{5}{11}.$$

Hence, following the recommendation and taking P_1 after observing r_1 is individually optimal.

If an agent observes r_2, her expectation of the cost of P_1 is

$$\Pr(\omega = 1|r_2) = 1$$

whereas her expectation of the cost of P_2 is

$$\frac{1}{3} + 2 \times \left(\Pr(\omega = 0|r_2) \times 0 + \Pr(\omega = 1|r_2) \times \frac{1}{6}\right)$$
$$= \frac{1}{3} + 2 \times \frac{1}{6} = \frac{2}{3} < 1.$$

Hence, following the recommendation and taking P_2 after observing r_2 is individually optimal.

The structure of the optimal signal is intuitive. When $\omega = 0$, the equilibrium behavior that guides everyone to P_1 suits the social planner. Thus, setting $\Pr(r_1|\omega = 0) = 1$ is clearly a good idea. (Reducing $\Pr(r_1|\omega = 0)$ would not only increase costs when $\omega = 0$ but also make it more difficult to reduce the traffic on P_2 when $\omega = 1$.) On the other hand, when $\omega = 1$, with full information too many people take P_2. The social planner wishes to "persuade" some of them to take P_1. It is not possible (nor desirable) to send too many agents to P_1 – if $\Pr(r_1|\omega = 1)$ gets too high, it will no longer be privately optimal to take P_1 when it is recommended. That said, when P_2 is recommended to the socially optimal share of agents, following the recommendations happens to be incentive compatible under the assumed parameter values.

III. Wheatstone network

The example above is meant as the simplest, highly distilled illustration of the power of information design to reduce congestion. In this section we consider information design in the case of the Wheatstone network that induces Braess' paradox [5].

A. Set-up

A unit measure of agents simultaneously choose one of three paths shown in Figure 2, denoted P_{up}, P_{down}, and P_{bridge}, where the third of these is the path that uses the zero-cost bridge. Let s_{up} be the share of agents choosing P_{up}, s_{down} the share of agents choosing P_{down}, and $s = 1 - s_{\text{up}} - s_{\text{down}}$ the share of agents choosing P_{bridge}. Note that $\nu_1 = s_{\text{up}} + s$ and $\nu_2 = s_{\text{down}} + s$.

The extent of the externality on the congestion-prone edges is determined by an uncertain state of the world $\omega \in \{1, 1.75\}$. The costs of the paths can be written as :

$$c(P_{\text{up}}) = 1 + \omega(s_{\text{up}} + s)$$
$$c(P_{\text{down}}) = 1 + \omega(s_{\text{down}} + s)$$
$$c(P_{\text{bridge}}) = \omega(1 + s)$$

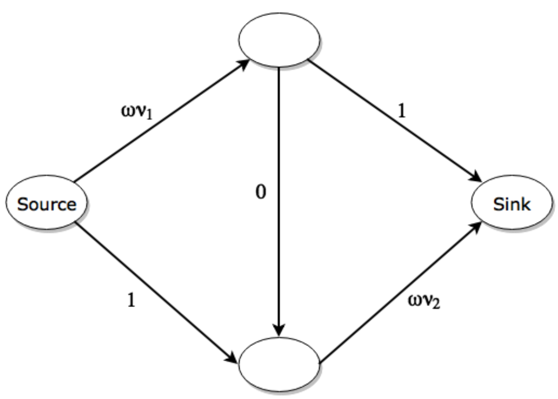

Fig. 2: A version of the classic Braess' paradox network where edges are labeled with their costs. 1 is a fixed cost paid along the edges so labeled, ω is the unknown congestion state, and ν_1 and ν_2 are the share of agents using those edges. Agents can take one of three paths from source to sink. We call the path using the top two edges P_{up}, the path using the bottom two edges P_{down} and the internal path (which uses the 0-cost edge) P_{bridge}.

As before, agents are risk-neutral and share the prior that the two states are equally likely.

B. First-best

It is easy to see that the socially optimal policy is to set $s = 0$ and $s_{\text{up}} = s_{\text{down}} = 0.5$ in both states, sending all agents on the external paths, as is usually the case in formulations of Braess' paradox. When $\omega = 1$, this achieves a cost of 1.5, whereas when $\omega = 1.75$ this achieves a cost of 1.875. Therefore the aggregate cost of the first-best policy is 1.6875.

C. Full information and public signals

If agents were fully aware of the underlying state, the equilibrium when $\omega = 1$ is that all agents take P_{bridge} at cost 2. Meanwhile, when $\omega = 1.75$, $\frac{1}{7}$ of agents take P_{bridge}, and $\frac{3}{7}$ take each of P_{up} and P_{down}; the costs of all paths are equal and again happen to be 2. Therefore, the aggregate cost under full information is 2.

A computation analogous to the one in Section II-D shows that every public signal leads to the same aggregate cost. Thus, in both of our examples, public signals have no impact on agents' welfare.

D. Optimal information structure

As mentioned in the introduction, to identify the optimal information structure we can focus our attention on i.i.d. signals. Moreover, since there are no multiple equilibria, we can restrict our attention to signals that recommend specific paths [15]. Further, since it is both an equilibrium and socially optimal for an equal share of agents to take P_{up} and P_{down}, we can restrict our attention to signals that generate one of two realizations, r_{o} or r_{b}. Realization r_{b} will be interpreted as a recommendation to take P_{bridge} while r_{o} can be thought of as a recommendation to randomize equiprobably between P_{up} and P_{down}.

Therefore, we consider signals of the form:

$$\Pr(r_{\text{o}}|\omega = 1) = \lambda \qquad \Pr(r_{\text{b}}|\omega = 1) = 1 - \lambda$$
$$\Pr(r_{\text{o}}|\omega = 1.75) = 1 - \rho \quad \Pr(r_{\text{b}}|\omega = 1.75) = \rho$$

An optimal λ, ρ pair can be found by solving a quadratic optimization problem with quadratic constraints. Total social cost is increasing in:

$$4\lambda^2 - 8\lambda + 7\rho^2 + 6\rho$$

which we seek to minimize subject to two obedience constraints

$$4\lambda^2 + 7\rho^2 - 8\rho + 1 \leq 0$$
$$4\lambda^2 - 4\lambda + 7\rho^2 - \rho \leq 0.$$

The first constraint ensures that when an agent receives r_{o}, the expected cost of P_{up} or P_{down} is less than the expected cost of P_{bridge} while the second ensures that when an agent receives r_{b}, the expected cost of P_{bridge} is less than the expected cost of P_{up} or P_{down}.

Numerical optimization reveals that the optimal signal sets $\lambda = 0.3419, \rho = 0.2295$, yielding an aggregate cost of 1.9049. Thus, under these parameter values the optimal signal closes about 30% of the gap between providing full information (or any public signal) and the first-best outcome.

IV. Practical issues

Our analysis is relevant for situations where an informationally-advantaged social planner cannot dictate behavior but can influence it by providing information. One example is route recommendation by traffic apps like Google Maps or Waze. Our results suggest that a traffic app could benefit its customers by giving each of them garbled information about the current state of the roads. Several practical issues, however, need to be considered in terms of implementing such recommendation systems in the specific case of traffic apps.

A. Competition

Suppose two traffic apps compete for customers and one of them offers socially optimal imperfect information about traffic. The other app will then have an incentive to provide more information. Even though all customers would be hurt should all get this additional information, any individual customer would benefit by switching to the deviating firm. This could lead the whole industry to unravel to full information. One hope is that repeated interaction between a stable set of firms could allow for an equilibrium that discourages such myopic deviations. Economies of scale inherent in crowd-sourced traffic apps could generate barriers to entry needed for such an equilibrium.

B. Multiple handsets

I.i.d. signals would give each agent an incentive to gather multiple draws of the signal, say by asking other individuals in the car to check for the optimal route. The scale of this problem is unlikely to be significant given that (according to the US National Household Travel Survey) a majority of passenger-miles on the road are single occupant. Further, this could also be amenable to technological solutions: because of the precision of GPS, it may be possible to ensure that users in the same vehicle all receive the same signal.

C. Fairness

When signals are public, there are no fairness concerns. With an i.i.d. signal, all agents have the same ex ante travel cost, but the idiosyncratic signal realizations induce ex post inequality. In our simple example, the optimal i.i.d. signal is a Pareto improvement over full information in ex post outcomes,[3] but in our Wheatstone Network example, some agents in some states are ex post worse off than they would be under full information.

V. Future research

We are interested in characterizing how much benefit optimal signal structures can bring in reducing the price of anarchy in different types of traffic networks. We are also interested in modeling more realistic networks with many source and sink nodes, stochastic arrival of vehicles at source nodes, and stochastic road

[3] When $\omega = 0$, all agents pay zero costs under both full information and the optimal signal; when $\omega = 1$, all agents incur cost of 1 under full information while the optimal signal imposes a cost of 1 to some agents and a cost of $\frac{1}{6}$ to others.

conditions (accidents, construction, etc.). Such network structures might not be amenable to an analytic derivation of an optimal information structure, but the basic idea of reducing congestion through information design could lead to the development of dynamic information-provision-algorithms that reduce time wasted in traffic.

References

[1] E. Kamenica and M. Gentzkow, "Bayesian persuasion," *The American Economic Review*, vol. 101, no. 6, pp. 2590–2615, 2011.

[2] S. Das and E. Kamenica, "Representations of information structures," in *Proceedings of the 53rd Allerton Conference on Communication, Control, and Computing*, 2015, pp. 737–743.

[3] T. Roughgarden, *Selfish Routing and the Price of Anarchy*. Cambridge, MA: MIT Press, 2005.

[4] D. Braess, "Über ein Paradoxon aus der Verkehrsplanung," *Unternehmensforschung*, vol. 12, no. 1, pp. 258–268, 1968.

[5] I. Milchtaich, "Network topology and the efficiency of equilibrium," *Games and Economic Behavior*, vol. 57, no. 2, pp. 321–346, 2006.

[6] R. Arnott, A. De Palma, and R. Lindsey, "Does providing information to drivers reduce traffic congestion?" *Transportation Research Part A: General*, vol. 25, no. 5, pp. 309–318, 1991.

[7] D. Acemoglu, A. Makhdoumi, A. Malekian, and A. Ozdaglar, "Informational Braess' paradox: The effect of information on traffic congestion," 2017, Working paper.

[8] A. Whinston and Y. Li, "Efficient real-time routing for autonomous vehicles through Bayes correlated equilibrium: An information design framework," 2017, Working paper.

[9] I. Kremer, Y. Mansour, and M. Perry, "Implementing the "wisdom of the crowd"," *Journal of Political Economy*, vol. 122, no. 5, pp. 988–1012, 2014.

[10] D. Bergemann and S. Morris, "Bayes correlated equilibrium and the comparison of information structures in games," *Theoretical Economics*, vol. 11, no. 2, pp. 487–522, 2016.

[11] L. Mathevet, J. Perego, and I. Taneva, "On information design in games," 2017, Working paper.

[12] R. Alonso and O. Câmara, "Persuading voters," *The American Economic Review*, vol. 106, no. 11, pp. 3590–3605, 2016.

[13] J. C. Ely, "Beeps," *The American Economic Review*, vol. 107, no. 1, pp. 31–53, 2017.

[14] D. Bergemann, B. Brooks, and S. Morris, "First-price auctions with general information structures: Implications for bidding and revenue," *Econometrica*, vol. 85, no. 1, pp. 107–143, 2017.

[15] D. Bergemann and S. Morris, "Robust predictions in games with incomplete information," *Econometrica*, vol. 81, no. 4, pp. 1251–1308, 2013.

[16] K. Tumer and S. Proper, "Coordinating actions in congestion games: Impact of top-down and bottom-up utilities," *Autonomous Agents and Multi-Agent Systems*, vol. 27, no. 3, pp. 419–443, 2013.

[17] J. G. Wardrop, "Some theoretical aspects of road traffic research," *Proceedings of the Institution of Civil Engineers, Part II*, pp. 325–378, 1952.

[18] T. Roughgarden and É. Tardos, "How bad is selfish routing?" *Journal of the ACM*, vol. 49, no. 2, pp. 236–259, 2002.