Deception For The Greater Good: Minimizing Traffic Congestion With Information Design

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Abstract

We consider the problem of misleading agents to increase social welfare in traffic networks. As agents are naturally selfish when interacting in a traffic system, we investigate information design techniques that the total decrease congestion.

Introduction

A fundamental problem in road networks is the question of how to reduce traffic congestion which leads to an increased social welfare for all agents interacting with it. In this paper we want to investigate techniques that can be used to show specific information to agents that influence their roadpathing that reduces congestion or a global minimum of traffic. We base the main ideas touched in this article from Das et al. (2017).

We start by defining a simple road network, discovered by (Pigou, 1920), that touches on the first problems of total cost of a controlled system against a system with only selfish agents. This is still a very active topic with papers such as (Roughgarden and Tardos, 2002) or (Roughgarden, 2005) exploring the impact of selfish routing in traffic systems.

Various information techniques are then investigated, (Arnott et al., 1991) and (Acemoglu et al., 2016), to see the effect of information design on systems with selfish routing. We explain linear programming models necessary to obtain the optimal network variables to be able to provide the optimal information to agents.

After looking into these basic definitions we advance to a more complicated network discovered by (Braess, 1968) where a seemingly helpful link in a network can have a negative effect on the total cost.

We finish this paper by explaining some of the technical implementation details of the software we build in order to simulate the networks.

Methods

Pigou's example (Pigou, 1920)

Let us consider a first basic example in which agents wish to travel from state s to state t by the means of choosing path P_1 or path P_2 as displayed in Figure 1 with $x, \omega_i \in [0,1]$ and $v \geq 0$. Each edge is labeled with a cost function $c(\cdot)$ which represents the travel time of that route produced by agents taking that route. The upper edge P_1 has a constant cost function $c(x) = \omega_1$ and can be interpreted as a road immune to congestion. The cost of the lower edge $P_2 : c(x) = \omega_2 + vx$ additionally depends upon the number of agents choosing it (x) and factor v setting the congestion increase per traffic share which increases the travel time by the share of agents taking P_2.

By denoting s_i as the share of agents taking route R_i, the combined travel cost is

$$\begin{aligned} &s_1\omega_1 + s_2(\omega_2 + vx) \\ =&(1 - s_2)\omega_1 + s_2(\omega_2 + vx) \end{aligned}$$

or in this specific example

$$(1 - x)\omega_1 + x(\omega_2 + vx)$$

Our main goal is to increase social welfare by decreasing congestion, so we wish to minimize this formula.

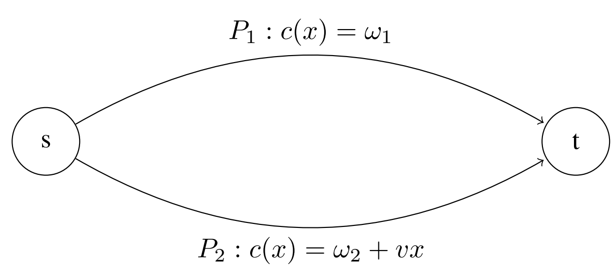


Figure 1: Pigou's example with an additional constant ω_2 (Pigou, 1920)

Suppose we set $\omega_1 = 1, \omega_2 = 0, v = 1$ and that one unit of traffic defines a large amount of vehicles. If users with knowledge of the total state were to chose freely their path which minimizes their time it takes to travel from destination s to t, they would all choose path P_2 and they would all receive the travel cost of 1. This case of *Full information* for the agents and without cooperation is also known as the *Wardrop Equilibrium* (WARDROP, 1952). Setting $\omega_1 = 0.5$ would lead to agents choosing route P_2 until $c(P_1) = c(P_2) = 0.5$, followed by choosing route P_1.

If we could, on the other hand, direct users to take a specific route, we would be able to decrease the total average travel cost while not worsening anyone's situation. Directing half the traffic to P_1 and the other half to P_2 would not worsen the situation for half the users who would continue to have a cost of 1. The other half would experience a cost of $1/2$ and the total average cost would be decreased to 3/4.

With this example Pigou showed that *selfish routing* - or users deciding the path while having full information of the network - does not necessarily find the minimal global cost because of the selfishness of agents.

Finding The Optimal Agent Distribution

The problem of finding the optimal way to distribute the agents in a system can be be solved by minimizing a linear programming problem. Especially in more complicated systems, brute forcing the problem by trying each possibility is a unfeasible method.

Let us consider a system of m complete paths, noted as CP (cycles excluded), and n single paths. As defined previously, each complete path consists of at least one single path P_i and has has a specific cost $c(P_i)$. Note that a single path can be part of several complete paths at the same time [1]. We formulate the problem as the following:

$$\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{n} \Big(P_i \big(\sum_{j=1}^{m} s_j \times l_{ij} \big) \Big) \\
\text{subject to} \quad & \sum_{j=1}^{m} s_j = 1 && (1) \\
& 0 \leq s_j \leq 1, \qquad j = 1, ..., m && (2)
\end{aligned}$$

with several additional variables: s_j denotes the share of agents assigned to the complete path j. As each complete path CP_j consists of at least one path P_i, we let the binary variable $l_{ij} \in \{0, 1\}$ denote if path i is included in the complete path j. The condition (1) forces one full unit of traffic through the system. (2) says that the non-integer flow in each complete path to be at most one unit of traffic and at least zero. The single path cost function is of the following definition: $P_i(x) = \omega_i + v_i \times x$. The objective function iterates each path i and inputs into the single path cost function (P_i) the sum of all agents taking path i. This is done by the expression $\sum_{j=1}^{m} s_j \times l_{ij}$, as only the agent shares of complete paths that include path i with l_{ij} are summarized for the single path function.

[1] See Figure 3b, where the path $s \to v$ is entailed in two complete paths: $s \to v \to t$ and $s \to v \to w \to t$

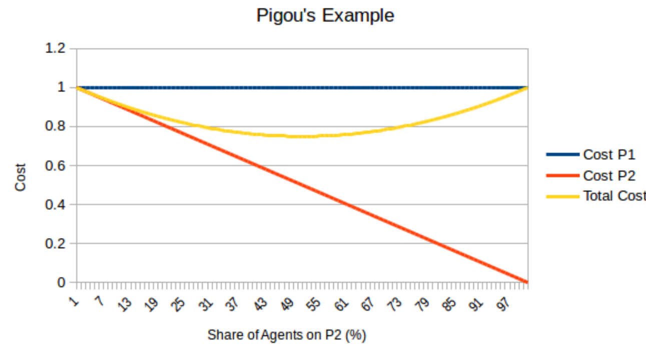


Figure 2: Demonstrating the minimization problem using Pigou's example

Example using Pigou's model In Pigou's simple model we have two single paths and two identical complete paths. We need to minimize the sum two of their cost multiplied by their share of agents. This is graphically displayed in Figure 2. We can observe the cost for each complete path and the resulting complete path. The Total Cost is visually minimum with 50% of agents on either path, resulting in agents on P2 having a cost of 0.5 and the agents on P1 having a cost of 1. The global minimum average is at 0.75. Here $l_{1,1} = 1$, $l_{1,2} = 0$, $l_{2,1} = 0$ and $l_{2,2} = 1$

Public Signal

A method to influence the choice of agents is by letting agents only observe public signals about variables ω_i, denoted by μ_i, that is they all share a common belief about the fixed costs of the system. With these beliefs, the number of agents taking each route will equilibrate to perceived equal cost for all agents. The share of agents on P_2 will for example be

$$s_2(\mu_1, \mu_2) = min\Big(1, max\Big(0, \frac{1}{v}(\mu_1 - \mu_2)\Big)\Big)$$

with an expected aggregated cost of

$$s_1\mu_1 + s_2\left(\mu_2 + vs_2\right)$$

Optimal Information Structure

A different way of thinking about this problem is with signal realizations r_i which act as recommendations to either take route R_1 or R_2 in this example by providing μ_i values. It should be noted that we are only changing the perception of the fixed path cost ω with information design. Let us consider the system configuration used in (Das et al., 2017) to introduce the notion of information structures, which is Pigou's example (Pigou, 1920) with $\omega_2 = \frac{1}{3}$ as a public constant, regardless of the used information structure, and $v = 2$.

The model has two complete paths from s to t which results in two route signal realizations, r_1 and r_2. As we set ω_2 to a public constant, agents only perceive different ω_1 values in this particular simple example. Before advancing to the probabilities of each signal realization, we have to define what values ω_1 can take. This undocumented part, denoted by *state of the world* in (Das et al., 2017), assigns the constant path cost ω the following two values: The first value is the cost of the complete path without any agent share. The second value is the cost of the complete path with selfish routing. This results in $\omega_1 \in \{0, 1\}$. Complete paths with several non-public fixed costs are not touched upon [2].

By finding the optimal agent share on each path, we solve the system like previously and find that when $\omega_1 = 0$, all agents take path P_1 with zero cost and when $\omega_2 = 1$, $\frac{5}{6}$ of agents take path P_1 with a total cost of $\frac{17}{36}$.

This leads to the two following signal realization which act as a recommendation to take either P_1 or P_2.

$$Pr(r_1|\omega_1 = 1) = \frac{5}{6} \qquad Pr(r_1|\omega_1 = 0) = 1$$
$$Pr(r_2|\omega_1 = 1) = \frac{1}{6} \qquad Pr(r_2|\omega_1 = 0) = 0$$

The interpretation of these probabilities is the following: When the actual value of $\omega_1 = 0$, all agents will receive the recommendation to take path P_1. Assuming $\omega_1 = 1$ results in $\frac{5}{6}$ of agents receiving the recommendation to take path P_1.

To show that the agents (who are acting in a selfish manner and choose their shortest perceived path) will take the recommended path, we need calculate their expected cost of P_1 and P_2. Only if that costs is lower than the opposite cost the hypothesis holds. An agent observing r_1 will have the following expected cost of P_1:

$$Pr(\omega_1 = 1|r_1) = \frac{\frac{5}{6}}{\frac{5}{6} + 1} = \frac{5}{11}$$

and a expectation of P_2:

$$\frac{1}{3} + 2 \times \left(Pr(\omega_1 = 1|r_1) \times \frac{1}{6} \right) = \frac{16}{33} > \frac{5}{11}$$

which is larger than the expectation for P_1 meaning that the signal realization holds for r_1: agents receiving the signal choose recommended. We now need to check for the opposite signal. Agents observing r_2 believe the cost of path P_1 to be:

$$Pr(\omega_1 = 1|r_2) = 1$$

and the cost of path P_2 to be:

$$\frac{1}{3} + 2 \times \left(Pr(\omega_1 = 1|r_2) \times \frac{1}{6} \right) = \frac{2}{3} < 1$$

which also holds: agents receiving the signal r_2 believe path P_2 is optimal. This *deception* of selfish agents wishing to choose the shortest path has reached the previously calculated global minimum for this system. This also constitutes a Nash Equilibrium: individual agents changing their decision will be worse of.

Assuming agents are able to receive all signal realizations would result in the full information assignment with a lower global average cost.

Braess's Paradox (Braess, 1968)

Let us now consider a network with two identical costs from s to t, as displayed in Figure 3a with $v_1 = v_2 = 1$. Assuming a *selfish routing*, half the users will take the upper route while the other half will take the bottom route, resulting in a average of 3/2 for every user.

We now add a intuitively helpful link in the network as displayed in Figure 3b. Clearly the link $s - v - w$ with cost x is better than the link $s - w$ with cost 1, resulting in all users to choose the path $s - v$. From the node v, the best path to the target t is $v - w - t$ with cost x instead of $v - t$ with cost 1. This results in all users choosing the path $s - v - w - t$ with cost $2x = 2$ which is worse than the cost of the initial network $3/2$.

This is known as the Braess's Paradox (Braess, 1968) who observed that a deterioration of a traffic situation can lead to a global improvement.

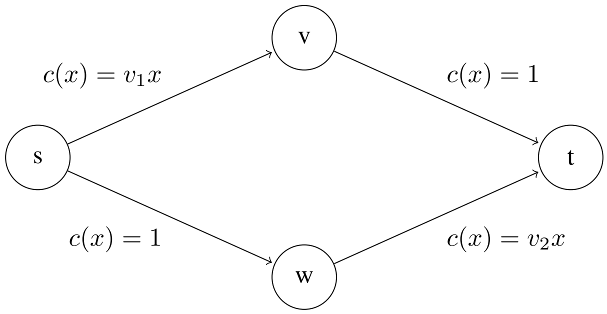
Different from the previous model, here we are changing the perception of the variable cost and not the fixed cost of a path. Using the same method as in the previous model to find out about the *uncertain state of the world* variables, we find $v_i \in \{1, 1.75\}$. The total cost cost with controlled agents and $v_i = 1.75$ is 1.875.

Full Information Assuming agents posses full information of the system then we have the two cases - assuming every v_i is the same: $v_i = 1$ results in all agents taking the added path which results in a total cost of 2. Setting $v_i = 1.75$ also gives a total cost of 2.

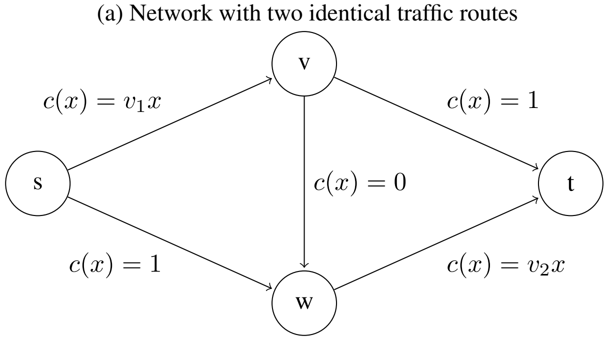
Optimal Information Structure The finding of a minimum here is a bit more challenging as we do not know (yet) how to distribute the agents in a way to obtain a global minimum. Since we have three possible paths, we have three signal realizations.

$$Pr(r_1|v_i = 1) = \lambda \qquad Pr(r_1|v_i = 1.75) = 1 - \lambda$$
$$Pr(r_2|v_i = 1) = \rho \qquad Pr(r_2|v_i = 1.75) = 1 - \rho$$
$$Pr(r_3|v_i = 1) = \psi \qquad Pr(r_3|v_i = 1.75) = 1 - \psi$$

[2] The later shown example of (Braess, 1968) has a *state of the world* of $\omega \in \{1, 1.75\}$ with the same reasoning



(a) Network with two identical traffic routes



(b) Added link with zero cost

Figure 3: Braess's Paradox - The intuitively helpful increased capacity of the network can reduce the flow

The optimal values have to be found by solving a quadratic optimization problem.

As we have seen from these examples, users that posses a global knowledge of the network can result in a increased cost for every user in the network. We will investigate methods of information design to reduce congestion in networks and can lead to an increased social welfare.

Simulator

The policies discussed in the previous sub-sections required us to verify whether similar results could be generated using the same parameters. For this purpose, the implementation of a simulator was a logical step. As mentioned previously, the agents on the network could classified in two categories.

Agents Agents of the first category are referred to as *controlled agents*. These agents follow the instructions that are mandated by a central planner. Consequently, agents of the first category are distributed amongst all possible paths following the configuration that yields the minimal total cost. In order to find the optimal configuration, the central planner uses the First-Best method.

Agents of the second category are referred to as *selfish agents*. Indeed, these agents act non-cooperatively with as main objective to minimize their own travel-cost. Hence, their decisions are disjoint from actions that lead to optimal welfare (Roughgarden and Tardos, 2002). Selfish agents depend on information about the network to decide what path they will choose. The simulator is designed to either provide them with full information about the network or provide them with an optimal information structure.

Paths Paths are sequels of roads that join the different states of the network together. Throughout this document, the cost associated to every road that composes a path is expressed in multiple ways. The implementation of the simulation considers paths as being described by a constant cost x, a variable cost y,a share of agents $s\,(p)$ traveling on that path and an uncertain state of the world ω such that the total cost of a path is expressed by the following formula:

$$x + ys\,(p) \quad s.t. \quad x, y \in \mathbb{N}_{\geq 0}, \quad s\,(p) \in [0, 1]$$

Routing priority We supposed networks could be crowded simultaneously by agents of different kinds. For this purpose, the selection of agent-types is not mutually exclusive. Consequently, the results that are yielded can vary depending on the type of agent that routes first.

Routing Policy The First-Best selection method is used to distribute controlled agents amongst the different existing paths. This method uses linear-programming in order to find the minimum total cost of the current network by estimating possible future costs of paths when traveled on by new agents. Results yielded by the linear-programming algorithm enables the central planner to know in what proportion controlled agents should be distributed amongst the different paths given their current state.

The Full Information method is one the methods that enables selfish users to select the path they will travel on. In this setting, agents will dispose of full information concerning the state of the network. By retrieving the current cost of each possible path and estimating the updated cost when traveled on, the agent disposes of all information he requires to select the path that translate in lowest travel cost. As mentioned previously, this choice is made non-cooperatively.

The Optimal Information method is the second method that can be used by selfish users to select the path they will travel on. This method can be interpreted as the social planner giving advice to selfish agents on whether it would be more cost-efficient to take a specific path to reach their destination. It is crucial that the information disclosed to agents is constructed correctly as some information structures could invert benefits (Arnott et al., 1991).

Networks The results of the paper this work is based on were obtained by running simulations on the networks represented on Figure 1 and Figure 3. For this reason, the implemented simulator disposes of the three identical network structures. As mentioned previously, the implementation of the paths composing the network makes to possible to alter the fixed and variable costs associated to each path.

Simulation The simulation of a network setting is composed by elements described previously in this section. Hence, the parameters of each simulation are defined by:

- number of selfish agents
- number of controlled agents
- routing priority
- routing policy
- one of three networks

Additionally, the fixed and variable costs of each path can be edited to liking.

Discussion

This experiment about analyzing congestion trough information design was conducted on fairly basic network structures. We believe it would be interesting to expand the scope of this research to networks with more complex features.

Realistic networks Networks are often used to model roads related traffic information. This being said, the considered networks could be modeled to include parameters that are not directly related to a fixed cost $(e.g. kilometer)$ and a variable cost $(traffic)$. For instance, we could include the effects of time on networks. Consequently, the fixed cost related to a path would increase over time. By extending this logic, we could also implement periodic maintenance of some roads, resulting in periodic drops of the fixed costs associated to these.

Modeling networks in such a way they will reflect reality also requires additional cost parameters. We understand the current experiments fix the cost as being the required time to travel on a path, as it sometimes depends on the share of agents taking that path (traffic). However, in real life situations agents also base their decisions on the exact distance of their itinerary to choose the most time-efficient path. In real life situations, this choice might be based on information provided by GPS-based applications (Acemoglu et al., 2016).

Realistic agents The current experiments on networks model agents as being individuals that require to go from a point A to a point B. However, real life situations prove that agents have more complex behaviors. Modeling real-life situations should require the networks to have agents that can start from any state and go to any other state.

The simulation could also be improved by adding additional constraints on the behavior of the agents of the network. Hence, we could imagine specify different itineraries corresponding to an agent and the number of times he would have to accomplish each one of these. By doing this, we could model the impact of patters in transportation. Agents could also display preferences regarding preferred sources for information acquisition. If such a system was to be implemented the simulations could relate to existing work on wisdom of the crowd(Kremer et al., 2013).

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