Thursday, February 23, 2023 9:30 PM

$$\alpha = E\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right] = E\left[\frac{1}{n}\sum^{n} x_i^2 + \frac{1}{n}\sum^{n}\bar{x}^2 - \frac{2\bar{x}}{n}\sum^{n} x_i\right]$$

$E[x_i^2] = E[x^2]$ bc exp. value of single element has the same value if it is from pop. or sample

$$\frac{\sum^{n} x_i}{n} = \bar{x}$$

$$\Rightarrow \alpha = \frac{1}{n}\sum^{n} E[x^2] + \frac{1}{n}\sum^{n} E[\bar{x}^2] - 2E[\bar{x}^2]$$

$$= \frac{1}{n} n E[x^2] + \frac{1}{n} n E[\bar{x}^2] - 2E[\bar{x}^2]$$

$$\alpha = E[x^2] - E[\bar{x}^2]$$

$$E[\bar{x}^2] = \frac{1}{n}\sum_{i=1}^{n} x_i \frac{1}{n}\sum_{j=1}^{n} x_j = \frac{1}{n^2}(x_1 + x_2 + \cdots + x_n)(x_1 + x_2 + \cdots + x_n)$$

$$= E\left[\frac{1}{n^2}\left\{\sum_{i=j}^{n} x_i^2 + \sum_{i=1}^{n} x_i \sum_{i\neq j}^{n-1} x_j\right\}\right]$$

<-- with i!=j there's no need for the coefficient 2 because the sum will Account for the ij, ji degeneracy. The sum over j is still n-1 to ensure that the i=j element is skipped

$$= \frac{1}{n^2}\left[n E[x^2] + n E[x](n-1)E[x]\right]$$

$$E[\bar{x}^2] = \frac{1}{n} E[x^2] + \frac{n-1}{n} E[x]^2$$

$$\alpha = E[x^2] - E[\bar{x}^2]$$

$$\Rightarrow \alpha = E[x^2] - \frac{1}{n} E[x^2] - \frac{n-1}{n} E[x]^2$$

$$= \left(1 - \frac{1}{n}\right) E[x^2] - \frac{n-1}{n} E[x]^2$$

$$= \frac{n-1}{n} E[x^2] - \frac{n-1}{n} E[x]^2$$

$$\alpha = \frac{n-1}{n}\left(E[x^2] - E[x]^2\right) = \frac{n-1}{n}\sigma^2$$

$$\Rightarrow \alpha = E\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right] = \frac{n-1}{n}\sigma^2$$

$$\frac{E\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]}{n-1} = \frac{\sigma^2}{n}$$