Midterm Review

CS 135

Problem 1

Use induction to prove that the statement $P(n) = 1^2 + 2^2 + \ldots + n^2 = (n(n+1)(2n+1))/6$ for all positive integers.

Problem 1 Answer Key

Basis: $n=1$

$1^2=(1)(2)(3)/6$

$1=1$

Inductive Hypothesis: Let k be an arbitrary integer $k \geq 1$. $P(k)=1^2+2^2+\ldots+k^2 = k(k+1)(2k+1)/6$

Inductive Step: We assume $P(k)$ is true for an arbitrary integer k and must now prove that $P(k+1)$ is true. In other words: we must prove $P(k+1) = P(k+1)=1^2+2^2+\ldots+(k+1)^2 = (k+1)(k+1+1)(2(k+1)+1)/6$ from the inductive hypothesis $P(k)=1^2+2^2+\ldots+k^2 = k(k+1)(2k+1)/6$

$P(k+1) = P(k+1)=1^2+2^2+\ldots+(k+1)^2 = (k+1)(k+1+1)(2(k+1)+1)/6$

$P(k+1)=1^2+2^2+\ldots+k^2+(k+1)^2$

$[(k(k+1)(2k+1))/6]+(k+1)^2$ By I.H.

$(k(k+1)(2k+1)+6(k+1)^2)/6$

$((k+1)(k(2k+1)+6(k+1)))/6$

$((k+1)(2k^2+7k+6))/6 =(k+1)(2k^2+7k+6) \rightarrow (k+1)(k+2)\ (2k+3)$

$(k+1)(k+2)(2k+3)/6$

Thus by the principal of induction the statement is true for every positive integer n.

Problem 2

Use mathematical induction to prove that $P(n) = 2^n < n!$ for every integer $n \geq 4$.

*Note: This is not true for integers less than 4

Problem 2 Answer Key

Basis: $n=4$

$2^4<4!$

$16<24$

Inductive Hypothesis: Let k be an arbitrary integer $k \geq 4$. $P(k) = 2^k < k!$

Inductive Step: We assume $P(k)$ is true for an arbitrary integer k and must now prove that $P(k+1)$ is true. In other words: we must prove $P(k+1) = 2^{k+1} < (k+1)!$ from the inductive hypothesis $P(k) = 2^k < k!$

Since $2^{k+1}=2 \cdot 2^k$ by the def. of exponent, we can use the <u>inductive hypothesis</u> to say that $2^{k+1}<2 \cdot k!$. Then since $2 < k+1$ we can now say $2^{k+1}<(k+1) \cdot k!$ which equals $2^{k+1}<(k+1)!$ by the definition of factorial function.

Induction Proof Writing Overview

1. Theorem $\forall n >= \_$: here will be the equation
2. Basis: $n=\_$ (then prove it holds)
3. Inductive Hypothesis: $\exists k >= \_$: here you write equation from theorem in terms of K
4. Inductive Step: Take the equation from IH but replace all the k's with k+1. Then we want to simplify this down using Inductive Hypothesis and a bunch of math to show the left side is equal to the right
5. Conclusion: make sure you state something along the lines of "Therefore by the principle of induction I've shown that for all numbers >= _ …

Problem 3

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a. No one is perfect
b. Not everyone is perfect
c. All your friends are perfect
d. At least one of your friends is perfect
e. Everyone is your friend and is perfect
f. Not everybody is your friend or someone is not perfect

1.4 #25

Problem 3 Answer Key

$P(x)$ "x is perfect"
$F(x)$ "x is your friend"
Domain is all people

$\forall x \neg \exists \rightarrow \wedge$

a) $\forall x \neg P(x)$
b) $\neg \forall x P(x)$
c) $\forall x (F(x) \rightarrow P(x))$
d) $\exists x (F(x) \wedge P(x))$
e) $\forall x (F(x) \wedge P(x))$ or $(\forall x\ F(x)) \wedge (\forall x\ P(x))$
f) $(\neg \forall x F(x)) \vee (\exists x \neg P(x))$

Remember with quantifiers,

- Make sure to state domain
- $\forall x P(x)$: universal means for all x (everyone), $P(x)$ is true for very value x in the domain
- $\exists x P(x)$: existential means there exists (someone), $P(x)$ is true for at least one x in the domain
- De Morgan's Law, push negation through. $\sim \forall x \exists y P(x,y) = \exists x \forall y \sim P(x,y)$

Problem 4

Use tree method to show whether valid or invalid. If invalid, give a counterexample.

$A \rightarrow B$

$\neg C \rightarrow A$

$\neg(B \rightarrow C)$

Problem 4 Answer Key

Remember with tree method:

1. Negate conclusion
2. Rewrite each hypothesis to get rid of the conditional (->) so they only have NOTs ANDs ORs
3. Look for counterexample
   - Branch when OR in proposition
   - Stack when AND
   - All leaves (paths) blocked off then no counterexample, so valid
   - Leaf not blocked off (path) then counter example, so invalid

apply conditional

$A \to B \equiv \neg A \lor B$

$\neg C \to A \equiv \neg\neg C \lor A \equiv C \lor A$

$\neg(B \to C)$

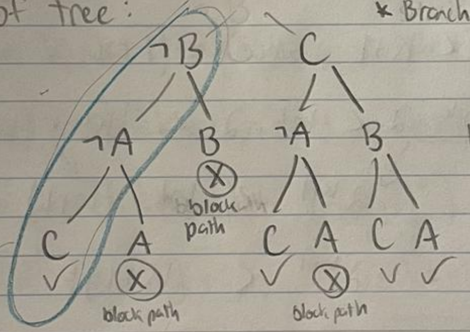
negate conclusion: $\neg(\neg(B \to C))$

$\neg(\neg(\neg B \lor C))$ conditional

$\neg(B \land \neg C)$ de morgans

$\neg B \lor C$ de morgans

root of tree: * Branch OR, Stack AND



~~1st hypothesis added to tree~~

2nd hypothesis added to tree

there is a valid path so valid counter example so argument is INVALID

Counter example: C - true

A - false

B - false

Things to know:

- Logic (predicates, propositions, laws, logical arguments, ...)
- Set properties (power sets, union, intersection, difference, ...)
- Function properties (injective, surjective, bijective, composite, ...)
- Countability
- Relations (equivalence relations, closures, pairwise disjoint, ...)
- Tree method (how to prove logical argument)
- Induction
- Quantified propositions (instantiation, generalization, rules of inference)
- etc...

Resources:

- Lecture Slides
- Problem Sets
- Lab Slides
- Zybooks
- Rosen book

CS-135 is a pretty difficult class, it's your own fault for starti thin

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rea help with and asking others for help

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effort! (edited)

Memes make the pain go away...
GOODLUCK EVERYONE!