Administrivia

Subscribed to zybooks yet?

Zybooks assignments:

do the “participation” and “challenge” exercises online

the “additional exercises” are not part of the assignment, but for you to practice.

Propositions

Definition: A proposition is a declarative statement that is either True or False (but not both).

- 4 + 3 = 7
- 1 + 1 = 3
- Give me an A!
- X + 1 = 2
- Humans are mortal.
- This proposition is true.
- This proposition is false.

- We will use letters to denote propositions:
- H : Humans are mortal
- A : I want to get an A in CS135.

Compound Propositions

Building compound propositions using logical connectives

Negation	$\neg \boldsymbol{G}$	"not $\boldsymbol{G}$"	
Conjunction	$(\boldsymbol{G} \wedge \boldsymbol{H})$	"$\boldsymbol{G}$ and $\boldsymbol{H}$"	True when both $\boldsymbol{G}, \boldsymbol{H}$ are true
Disjunction	$(\boldsymbol{G} \vee \boldsymbol{H})$	"$\boldsymbol{G}$ or $\boldsymbol{H}$"	True when one (or both) of $\boldsymbol{G}, \boldsymbol{H}$ is True
Implication	$(\boldsymbol{G} \Rightarrow \boldsymbol{H})$	"If $\boldsymbol{G}$ then $\boldsymbol{H}$"	

Truth Tables

Tautologies, Contradictions, Logical Equivalence

Logical Connectives: Negation

Compound propositions are built from simpler ones using connectives.

If $\boldsymbol{G}$ is a proposition, then $\neg\boldsymbol{G}$ is a proposition (the negation of $\boldsymbol{G}$).

The truth values of $\boldsymbol{G}$ and $\neg\boldsymbol{G}$ are opposites of each other.

Example: $\boldsymbol{G}$: I am grumpy.

$\neg\boldsymbol{G}$: not(I am grumpy) = I am not grumpy.

Example: $\boldsymbol{H}$: I am hungry. $\neg\boldsymbol{H}$: I am not hungry

What is $\neg(\neg\boldsymbol{H})$?

$\neg(\neg\boldsymbol{H})$: not(I am not hungry) = I am not (not hungry) = I am hungry = $\boldsymbol{H}$.

$\neg(\neg X) = X$ **: Two negatives make a positive**

Logical Connectives: Conjunction

If $\boldsymbol{G}$ and $\boldsymbol{H}$ are propositions, then $(\boldsymbol{G} \wedge \boldsymbol{H})$ is a proposition

$(\boldsymbol{G} \wedge \boldsymbol{H})$ is called the *conjunction* of $\boldsymbol{G}$ and $\boldsymbol{H}$.

$(\boldsymbol{G} \wedge \boldsymbol{H})$ is True when both $\boldsymbol{G}, \boldsymbol{H}$ are True, and False if one (or both) of $\boldsymbol{G}, \boldsymbol{H}$ is False.

Example: $\boldsymbol{G}$**:** I am grumpy

$\boldsymbol{H}$**:** I am hungry

$(\boldsymbol{G} \wedge \boldsymbol{H})$: I am grumpy, and I am hungry.

If I am not grumpy then the proposition $(\boldsymbol{G} \wedge \boldsymbol{H})$ is False

If I am neither grumpy nor hungry then $(\boldsymbol{G} \wedge \boldsymbol{H})$ is False

If I am hungry and I am grumpy then $(\boldsymbol{G} \wedge \boldsymbol{H})$ is True

Logical Connectives: Disjunction

If $\boldsymbol{G}$ and $\boldsymbol{H}$ are propositions, then $(\boldsymbol{G} \vee \boldsymbol{H})$ is a proposition

$(\boldsymbol{G} \vee \boldsymbol{H})$ is called the *disjunction* of $\boldsymbol{G}$ and $\boldsymbol{H}$.

$(\boldsymbol{G} \vee \boldsymbol{H})$ is True when at least one of $\boldsymbol{G}, \boldsymbol{H}$ is True, and False when both $\boldsymbol{G}, \boldsymbol{H}$ are False.

Example: $\boldsymbol{G}$**:** I am grumpy

$\boldsymbol{H}$**:** I am hungry

$(\boldsymbol{G} \vee \boldsymbol{H})$: I am grumpy, or I am hungry.

If I am grumpy then the proposition $(\boldsymbol{G} \vee \boldsymbol{H})$ is True

If I am both grumpy and hungry then $(\boldsymbol{G} \vee \boldsymbol{H})$ is True

If I am neither hungry nor grumpy then $(\boldsymbol{G} \vee \boldsymbol{H})$ is False

Inclusive vs. Exclusive OR : Disjunction is Inclusive!

Truth Tables: Basic Connectives

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Logical Connectives: Implication

If $\boldsymbol{G}, \boldsymbol{H}$ are propositions, then $(\boldsymbol{G} \Rightarrow \boldsymbol{H})$ is a proposition

$(\boldsymbol{G} \Rightarrow \boldsymbol{H})$ is read as "If $\boldsymbol{G}$ then $\boldsymbol{H}$" or as "$\boldsymbol{G}$ implies $\boldsymbol{H}$"

Examples

If I am grumpy then I am hungry
If you complete the reading assignment, then you will get full credit
If <antecedent> then <consequent>

When is an implication True and when is it False?

Logical Connectives: Implication

The statement "If I am grumpy then I am hungry"
is FALSE in case I am grumpy but not hungry.

"If you complete the reading assignment then you will get full credit" is FALSE in case you complete the reading assignment but do not get full credit.

If you do not complete the reading assignment, the statement is TRUE – regardless of whether you get full credit or not!

If I am not grumpy the first implication is TRUE – whether I am hungry or not!

$(\boldsymbol{G} \Rightarrow \boldsymbol{H})$ is False when $\boldsymbol{G}$ is True and $\boldsymbol{H}$ is False; otherwise it is True.

The Truth Table for Implications

If 1+1 = 3 then 2 > 10.

When the antecedent is false the implication is …? **TRUE!**

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

When the antecedent is false, the implication is true regardless of the consequent!

If pigs can whistle, then I am king.

If 1 = 2 then (Moriarty escapes and Moriarty is captured).

If the moon shines white, then the moon is made of white cheddar.

Truth Tables Summary

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Logical Tautologies

A proposition that is always True is called a tautology**.**

Examples:

True

2 = 1+1

It is raining or it is not raining

Holmes will catch Moriarty or Holmes will not catch Moriarty

$Q \lor \neg Q$

Logical Contradiction

A proposition that is always False is called a contradiction.

Examples:

False

1 > 1

It is raining and it is not raining

Holmes will catch Moriarty and Holmes will not catch Moriarty

$Q \wedge \neg Q$

Logical Equivalence

Two propositions P and Q that have the same truth table are *logically equivalent*

$$P \equiv Q$$

How do we prove that two propositions are equivalent?

Example: $\neg(\boldsymbol{p} \wedge \boldsymbol{q}) \equiv (\neg \boldsymbol{p} \vee \neg \boldsymbol{q})$ **?**

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

But truth tables are cumbersome – exponential in the number of atomic propositions!

Proving inequivalence

How do we prove that two propositions are NOT equivalent?

Give a counterexample: an assignment of truth values to the atoms that make one proposition true and the other one false.

Example: P: $(x \vee y) \Rightarrow w$ vs. Q: $(x \wedge y) \Rightarrow w$

Counterexample: $x = True,\ \ y = False,\ \ w = False$

P: $(T \vee F) \Rightarrow F$ Q: $(T \wedge F) \Rightarrow F$

FALSE **TRUE**

Equivalence: Conditional Identity

$$(\boldsymbol{p} \Rightarrow \boldsymbol{q}) \equiv (\neg \boldsymbol{p} \vee \boldsymbol{q})$$

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Proving Logical Equivalence using Laws

Table 1.5.1: Laws of propositional logic.

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination laws:	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv F$ $\neg T \equiv F$	$p \vee \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Question of the day!

“If I attend lectures then I will do well in CS135

OR

If I do my homeworks then I will do well in CS135”

If I attend lectures ***and*** do my homeworks then I will do well in CS135

Are these two propositions logically equivalent?