



LAB 5

CS 135

PROBLEM 1

Prove that if n is an **odd** positive integer, then $n^2 \equiv 1 \pmod{8}$.

PROBLEM 1 ANSWER KEY

$$n = 2k+1$$

definition of an odd #

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1$$

Since 4 divides $4k(k+1)$

And 2 divides $k(k+1)$

two consecutive #s, one is bound to be even

Then $2 \cdot 4 \mid 4k(k+1)$

And $4k(k+1) + 1 \equiv 1 \pmod{8}$ adding 1 to above

PROBLEM 2

Use Euclid's Algorithm to solve the following:

$$\text{gcd}(101, 4620)$$

PROBLEM 2 ANSWER KEY

gcd(4620, 101)

gcd(101, 75)

$$4620 = 101 \cdot 45 + 75$$

gcd(75, 26)

$$101 = 75 \cdot 1 + 26$$

gcd(26, 23)

$$75 = 26 \cdot 2 + 23$$

gcd(23, 3)

$$26 = 23 \cdot 1 + 3$$

gcd(3, 2)

$$23 = 3 \cdot 7 + 2$$

gcd(2, 1)

$$3 = 2 \cdot 1 + 1$$

gcd(1, 0)

PROBLEM 3

Use the Pulverizer to express $\gcd(1529, 14039)$ as a linear combination of 1529 and 14039.

PROBLEM 3 ANSWER KEY

$$\gcd(14039, 1529)$$

$$\gcd(1529, 278)$$

$$\gcd(278, 139)$$

$$\gcd(139, 0)$$

$$139$$

$$14039 = 9 \cdot 1529 + 278$$

$$1529 = 5 \cdot 278 + 139$$

$$278 = 2 \cdot 139$$

Putting it all together:

$$139 = 1529 - 5 \cdot 278$$

$$= 1529 - 5(14039 - 9 \cdot 1529)$$

$$= -5 \cdot 14039 + 46 \cdot 1529$$

simplify