CS 135 Spring 2023: Mid-Term Exam B.

Problem 1. Indicate whether each of the following statements is True or False.

1. For all sets A, B, C: If $A \cap C = B \cap C$ then $A = B$.
2. For all sets A, B: $\wp(A \cup B) = \wp(A) \cup \wp(B)$, where $\wp(X)$ denotes the power set of X.
3. The composition of two surjective functions is always surjective.
4. If $f: A \to B$ is surjective then there must exist a function $g: A \to B$ that is injective.
5. For every pair of sets A, B, if $A \subset B$, then there is no bijective function from A to B.
6. The transitive closure of every symmetric relation is an equivalence relation.
7. For every equivalence relation R, $R \circ R \supset R$.
8. Every function is a relation.
9. If an argument is valid, then the conclusion is always true.
10. If an argument is invalid, then one of the hypotheses is false.

Problem 2. Either prove that the following argument is valid or provide a counterexample. Provide a key to translate phrases into symbols.

Today it is cloudy, but it is not raining.
Bob will bring an umbrella only if it is raining.
If Bob does not bring an umbrella, then Alice will lend him one.
<u>Bob will thank Alice whenever she lends him her umbrella.</u>
Therefore, Bob thanked Alice.

Problem 3. Translate the following sentences from English to predicate logic. The domain CA that you are working over all CAs who have taught CS135. You may use the predicates $S(x)$, meaning that "x has been a student of CS135," $A(x)$, meaning that "x got an A in CS135," and $E(x, y)$, meaning that "x and y are the same person."

a. At least one CA has taken CS135 and got an A in CS135.
b. If a CA did not get an A in CS135 then the CA did not take CS135.
c. If there is a CA who did not take CS 135 then there is another one who did.
d. There were at least two CAs who had not taken CS135.

Problem 4. Consider the following function:

$$f(n) = \begin{cases} 1, & n = 0 \\ 2, & n = 1 \\ f(n-1) + 2f(n-2), & n \geq 2 \end{cases}$$

a) What are the values of $f(2), f(3), f(4)$?
b) From part (a) infer a simple formula for $f(n)$.
c) Prove, using the principle of induction, that the formula you inferred in part (b) is correct for all $n \in \mathbb{N}$. Be sure to include all 3 steps of the proof.

Problem 5. Lem E. Hackett has done it again! After an all-nighter he finally "proved" a truly remarkable but truly false result. Here is his argument. Point out the flaw in his reasoning.

Claim: $\forall n \in \mathbb{N}: n^2 \leq n.$

Base Case: When $n = 0$, the statement $0^2 \leq 0$ is true.

Inductive Hypothesis: For some $k \in \mathbb{N}$: $k^2 \leq k.$

Inductive Step: Working backwards, we have that:

$$\begin{aligned} &(k+1)^2 \leq k+1 \\ \Rightarrow\ &k^2 + 2k + 1 \leq k+1 \\ \Rightarrow\ &k^2 + 2k \leq k \end{aligned}$$

Now, because $2k \geq 0$, it follows that:

$$k^2 \leq k^2 + 2k \;\; \Rightarrow \;\; k^2 \leq k,$$

which, by the inductive hypothesis, the last inequality is true. Therefore, the inductive step is established.