Lab 3

CS 135

Problem 1

Show that the following are logically equivalent.

$$\neg \forall x(F(x) \Rightarrow G(x))$$

$$\exists x(F(x) \land \neg G(x))$$

Problem 1 Answer Key

$\neg \forall x(F(x) \Rightarrow G(x))$	Given
$\neg \forall x(\neg F(x) \vee G(x))$	Conditional ID
$\exists x(\neg(\neg F(x) \vee G(x)))$	De Morgan's
$\exists x(\neg \neg F(x) \wedge \neg G(x))$	De Morgan's
$\exists x(F(x) \wedge \neg G(x))$	Double Negation

*note that you can also go from the second logical expression to the first

Problem 2

How many subsets in the following power sets? What are the power sets of the following?

a. $P(\{a, b, c\})$
b. $P(\varnothing)$
c. $P(\{\varnothing\})$

Problem 2 Answer Key

a. $P(\{a, b, c\}) =$

$\{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

b. $P(\varnothing) =$

$\{\varnothing\}$

c. $P(\{\varnothing\}) =$

$\{\varnothing, \{\varnothing\}\}$

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \varnothing = A$	$A \cap U = A$
Domination laws	$A \cap \varnothing = \varnothing$	$A \cup U = U$
Double complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = U$ $\overline{\varnothing} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

Notice anything familiar?

Problem 3

Let A, B, and C be pre-defined sets. Use the set identities to show that

$\overline{(A \cup B)} \cap \overline{(B \cup C)} \cap \overline{(A \cup \overline{C})} = \overline{A} \cap \overline{B} \cap C$

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \varnothing = A$	$A \cap U = A$
Domination laws	$A \cap \varnothing = \varnothing$	$A \cup U = U$
Double complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = U$ $\overline{\varnothing} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

Problem 3 Answer Key

1.	$\overline{(A \cup B)} \cap \overline{(B \cup C)} \cap \overline{(A \cup C)}$	Given
2.	$(\bar{A} \cap \bar{B}) \cap (\bar{B} \cap \bar{C}) \cap (\bar{A} \cap \bar{C})$	De Morgan's x3
3.	$\bar{A} \cap \bar{B} \cap \bar{B} \cap \bar{C} \cap \bar{A} \cap \bar{C}$	Associative x3
4.	$(\bar{A} \cap \bar{A}) \cap (\bar{B} \cap \bar{B}) \cap (\bar{C} \cap \bar{C})$	Commutative/Associative
5.	$\bar{A} \cap \bar{B} \cap \bar{C}$	Idempotent x3

Problem 4

For each of these relations on the set {1, 2, 3, 4}, determine if it's reflexive, symmetric, transitive, or neither.

a. {(2, 4), (4, 2)}
b. {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
c. {(1, 2), (2, 3), (3, 4)}

Problem 4

For each of these relations on the set {1, 2, 3, 4}, determine if it's reflexive, symmetric, transitive, or neither.

a. Symmetric
b. Transitive
c. None