



Lab 3

CS 135

Problem 1

Show that the following are logically equivalent.

$$\neg \forall x(F(x) \Rightarrow G(x))$$

$$\exists x(F(x) \wedge \neg G(x))$$

Problem 1 Answer Key

$$\neg \forall x(F(x) \Rightarrow G(x))$$

Given

$$\neg \forall x(\neg F(x) \vee G(x))$$

Conditional ID

$$\exists x(\neg(\neg F(x) \vee G(x)))$$

De Morgan's

$$\exists x(\neg \neg F(x) \wedge \neg G(x))$$

De Morgan's

$$\exists x(F(x) \wedge \neg G(x))$$

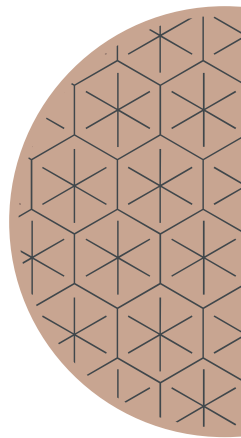
Double Negation

*note that you can also go from the second logical expression to the first

Problem 2

How many subsets in the following power sets? What are the power sets of the following?

- a. $P(\{a, b, c\})$
- b. $P(\emptyset)$
- c. $P(\{\emptyset\})$



Problem 2 Answer Key

a. $P(\{a, b, c\}) =$

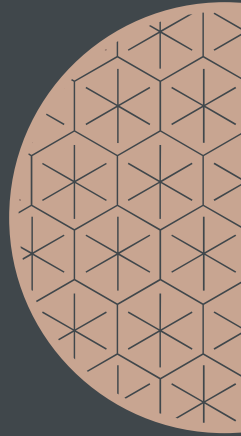
$$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

b. $P(\emptyset) =$

$$\{\emptyset\}$$

c. $P(\{\emptyset\}) =$

$$\{\emptyset, \{\emptyset\}\}$$



Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \emptyset = A$	$A \cap U = A$
Domination laws	$A \cap \emptyset = \emptyset$	$A \cup U = U$
Double complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \emptyset$ $\overline{\overline{U}} = U$	$A \cup \overline{A} = U$ $\overline{\emptyset} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

Notice
anything
familiar?

Problem 3

Let A , B , and C be pre-defined sets. Use the set identities to show that

$$\overline{(A \cup B) \cap (B \cup C) \cap (A \cup C)} = \overline{\overline{A \cap B \cap C}}$$

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \emptyset = A$	$A \cap U = A$
Domination laws	$A \cap \emptyset = \emptyset$	$A \cup U = U$
Double complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \emptyset$ $\overline{\overline{U}} = U$	$A \cup \overline{A} = U$ $\overline{\emptyset} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

Problem 3 Answer Key

1.	$\overline{(A \cup B)} \cap \overline{(B \cup C)} \cap \overline{(A \cup C)}$
2.	$(\bar{A} \cap \bar{B}) \cap (\bar{B} \cap \bar{C}) \cap (\bar{A} \cap \bar{C})$
3.	$\bar{A} \cap \bar{B} \cap \bar{B} \cap \bar{C} \cap \bar{A} \cap \bar{C}$
4.	$(\bar{A} \cap \bar{A}) \cap (\bar{B} \cap \bar{B}) \cap (\bar{C} \cap \bar{C})$
5.	$\bar{A} \cap \bar{B} \cap \bar{C}$

Given

De Morgan's x3

Associative x3

Commutative/Associative

Idempotent x3

Problem 4

For each of these relations on the set $\{1, 2, 3, 4\}$, determine if it's reflexive, symmetric, transitive, or neither.

- a. $\{(2, 4), (4, 2)\}$
- b. $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- c. $\{(1, 2), (2, 3), (3, 4)\}$

Problem 4

For each of these relations on the set $\{1, 2, 3, 4\}$, determine if it's reflexive, symmetric, transitive, or neither.

- a. Symmetric
- b. Transitive
- c. None