Administrivia

Hope you completed your first zybooks reading assignment!

Wasn’t that easy?

Last Lecture

Building compound propositions using logical connectives

Negation	$\neg \boldsymbol{G}$	"not $\boldsymbol{G}$"	
Conjunction	$(\boldsymbol{G} \wedge \boldsymbol{H})$	"$\boldsymbol{G}$ and $\boldsymbol{H}$"	True when both $\boldsymbol{G}, \boldsymbol{H}$ are true
Disjunction	$(\boldsymbol{G} \vee \boldsymbol{H})$	"$\boldsymbol{G}$ or $\boldsymbol{H}$"	True when one (or both) of $\boldsymbol{G}, \boldsymbol{H}$ is True
Implication	$(\boldsymbol{G} \Rightarrow \boldsymbol{H})$	"If $\boldsymbol{G}$ then $\boldsymbol{H}$"	

Truth Tables

Tautologies, Contradictions, Logical Equivalence

Proving Logical Equivalence using Laws

Table 1.5.1: Laws of propositional logic.

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination laws:	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv F$ $\neg T \equiv F$	$p \vee \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Pitfalls to Watch For!

“If I attend lectures then I will do well in CS135

OR

If I do my homeworks then I will do well in CS135”

If I attend lectures ***and*** do my homeworks then I will do well in CS135

Are these two propositions logically equivalent?

Proving Logical Equivalence

Method 1: Construct truth tables

Method 2: Use the laws to transform one proposition into the other

Example: Are the following propositions equivalent?

P: If I attend lectures, then I will do well in CS135
OR
If I do my homework, then I will do well in CS135

$$(L \Rightarrow W) \vee (H \Rightarrow W)$$

Q: If I attend lectures AND do my homework then I will do well in CS 135

$$(L \wedge H) \Rightarrow W$$

Let L be the proposition "I attend lectures"

W be the proposition "I will do well in CS 135"

H be the proposition "I do my homework"

Applying the Laws of Propositional Logic

$P \equiv (L \Rightarrow W) \vee (H \Rightarrow W)$	
$\equiv (\neg L \vee W) \vee (\neg H \vee W)$	Conditional Identity Law (applied twice)
$\equiv (\neg L \vee W) \vee (W \vee \neg H)$	Commutative Law
$\equiv \big((\neg L \vee W) \vee W\big) \vee \neg H$	Associative Law
$\equiv \big(\neg L \vee (W \vee W)\big) \vee \neg H$	Associative Law
$\equiv (\neg L \vee W) \vee \neg H$	Idempotent Law
$\equiv \neg L \vee (W \vee \neg H)$	Associative Law
$\equiv \neg L \vee (\neg H \vee W)$	Commutative Law
$\equiv (\neg L \vee \neg H) \vee W$	Associative Law
$\equiv \neg(L \wedge H) \vee W$	De Morgan's Law
$\equiv (L \wedge H) \Rightarrow W$	Conditional Identity

$\equiv Q$ "If I attend lectures AND I do homework then I will do well in CS 135."

Where did intuition fail us?

P:

If I attend lectures, then I will do well in CS135
OR
If I do my homework, then I will do well in CS135

We misinterpreted this instead as: If I want to do well in CS 135, I should attend lectures
OR If I want to do well in CS 135, I should do homework

Which is $(W \Rightarrow L) \vee (W \Rightarrow H)$

This is logically equivalent to $W \Rightarrow (L \vee H)$

Which is our misinterpretation of "If I attend lectures or do homework, I will do well in CS 135"

The Contrapositive

$P \Rightarrow Q$	
$\equiv \neg P \vee Q$	Conditional Identity Law
$\equiv Q \vee \neg P$	Commutative Law
$\equiv \neg\neg Q \vee \neg P$	Double Negation
$\equiv \neg Q \Rightarrow \neg P$	Conditional Identity

Logical Reasoning

A logical argument has the form

$$\begin{array}{ll} H_1 & \\ \underline{H_2} & \text{hypotheses} \\ \therefore C & \text{conclusion} \end{array}$$

Example: Socrates is a man

<u>If Socrates is a man, he is mortal</u>

∴ Socrates is mortal

$$\begin{array}{l} S \\ \underline{S \Rightarrow M} \\ \therefore M \end{array}$$

An argument is valid if the conclusion is true when every hypothesis is true

Logical Reasoning

The form

$$\begin{array}{ll} H_1 & \\ H_2 & \text{hypotheses} \\ \hline \therefore C & \text{conclusion} \end{array}$$

is short-hand for the proposition

$$(H_1 \wedge H_2) \Rightarrow C$$

An argument is valid if and only if its corresponding implication is a tautology.

Rules of Inference

p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
p $\therefore p \lor q$	Addition
$p \land q$ $\therefore p$	Simplification

p q $\therefore p \land q$	Conjunction
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	Resolution

Proving Validity

If Alfred studies, he will get good grades
If Alfred does not study, then he will enjoy college
<u>If Alfred does not get good grades, he will not enjoy college</u>
∴ Alfred will get good grades

$$S \Rightarrow G$$
$$\neg S \Rightarrow E$$
$$\underline{\neg G \Rightarrow \neg E}$$
$$\therefore G$$

We will use the laws of propositional logic and the rules of inference to prove that this inference is valid

Proof of Validity

A. $S \Rightarrow G$
B. $\neg S \Rightarrow E$
C. $\underline{\neg G \Rightarrow \neg E}$

$\therefore G$

1.	$\neg G \Rightarrow \neg E$	hypothesis, C
2.	$E \Rightarrow G$	contrapositive, 1
3.	$\neg S \Rightarrow E$	hypothesis, B
4.	$\neg S \Rightarrow G$	hypothetical syllogism, 3,2
5.	$S \Rightarrow G$	hypothesis, A
6.	$G \vee \neg S$	conditional identity, 5
7.	$G \vee S$	conditional identity, 4
8.	G	resolution, 6,7

Another Example

If Holmes does not act, Moriarty will escape
<u>We shall rely on Watson only if Holmes does not act</u>
∴ If Holmes does not act, then if we don't rely on Watson, Moriarty will escape

$\neg H \Rightarrow M$
$\underline{W \Rightarrow \neg H}$
$\therefore \neg H \Rightarrow (\neg W \Rightarrow M)$

Proof of Validity

A. $\neg H \Rightarrow M$
B. $\underline{\quad W \Rightarrow \neg H}$
C. $\therefore \neg H \Rightarrow (\neg W \Rightarrow M)$

1.	$\neg H \Rightarrow M$	hypothesis, A
2.	$H \vee M$	conditional identity, 1
3.	$H \vee M \vee W$	addition, 2
4.	$\neg H \Rightarrow (W \vee M)$	conditional identity, commutative law, 3
5.	$\neg H \Rightarrow (\neg W \Rightarrow M)$	conditional identity, 4

Hypothesis B was never used in the proof!

Is there a simpler systematic method to prove validity?
What if the argument is invalid? How do we find a counterexample?

A different method

Is the following argument valid?

$$\frac{(H \lor W) \Rightarrow M}{\therefore (M \lor H)}$$

Looking for a counterexample

$$\frac{(H \lor W) \Rightarrow M}{\therefore (M \lor H)}$$

✓	$(H \lor W) \Rightarrow M$	*hypothesis*
✓	$\neg(M \lor H)$	*negate the conclusion*
	$\neg M$	
	$\neg H$	

✓ $\neg(H \lor W)$	M 🚫
$\neg H$	
$\neg W$	

All compound propositions have been checked off.
Counterexample: $M, H, W = F$ since all are negated along the open path.

Yet another example

Is the following argument valid?

Babies are illogical.
Nobody who can manage a crocodile is despised.
Illogical persons are despised.
$\therefore$ Babies cannot manage crocodiles.

$$
\begin{array}{l}
B \Rightarrow I \\
M \Rightarrow \neg D \\
\underline{I \Rightarrow D \qquad} \\
\therefore B \Rightarrow \neg M
\end{array}
$$

Looking for a counterexample

$B \Rightarrow I$

$M \Rightarrow \neg D$

$\underline{I \Rightarrow D \quad\quad}$

$\therefore B \Rightarrow \neg M$

✓ $B \Rightarrow I$

✓ $M \Rightarrow \neg D$

✓ $I \Rightarrow D$

✓ $\neg(B \Rightarrow \neg M)$ *negate the conclusion*

✓ $\neg(\neg B \vee \neg M)$

B

M

$\neg B$ $\quad$ I

$\neg I$ $\quad$ D

$\neg M$ $\quad$ $\neg D$

All compound propositions are checked off.

All paths are blocked off.

No counterexample exists.

The argument is valid!

The Tree Method

1. Write down each premise
2. Negate the conclusion
3. Look for a counterexample
    a) If all leaves are blocked off: no counterexample exists
    b) Else, if some leaf is not blocked off but all compound propositions are checked off: counterexamples exist
    c) If there an unchecked compound proposition, expand it **at every leaf below** and check off the proposition
    d) Close off every leaf whose path to the top contains a contradiction