Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

Set Identities

Identity	Name
$A \cap B = B \cap A$ $A \cup B = B \cup A$	**Commutative Laws**
$(A \cap B) \cap C = A \cap (B \cap C)$ $(A \cup B) \cup C = A \cup (B \cup C)$	**Associative Laws**
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	**Distributive Laws**
$A \cap U = A$ $A \cup U = U$	**Identity Law (intersection and Unition with Universal Set)**
$(A')' = A$	**Double Complement Laws**
$A \cap A = A$ $A \cup A = A$	**Idempotent Laws**
$(A \cap B)' = A' \cup B'$ $(A \cup B)' = A' \cap B'$	**De Morgan's Laws**
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	**Absorption Laws**
$A - B = A \cap B'$	**Set Difference Law**

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination laws:	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv F$ $\neg T \equiv F$	$p \vee \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \to q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Operation	Notation	Description
Intersection	$A \cap B$	$\{x : x \in A \text{ and } x \in B\}$
Union	$A \cup B$	$\{x : x \in A \text{ or } x \in B \text{ or both}\}$
Difference	$A - B$	$\{x : x \in A \text{ and } x \notin B\}$
Symmetric difference	$A \oplus B$	$\{x : x \in A - B \text{ or } x \in B - A\}$
Complement	$\bar{A}$	$\{x : x \notin A\}$

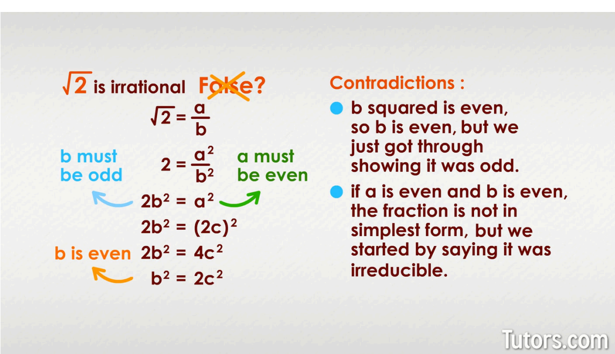


TABLE 7 Logical Equivalences Involving Conditional Statements.

$$p \to q \equiv \neg p \vee q$$
$$p \to q \equiv \neg q \to \neg p$$
$$p \vee q \equiv \neg p \to q$$
$$p \wedge q \equiv \neg(p \to \neg q)$$
$$\neg(p \to q) \equiv p \wedge \neg q$$
$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$
$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$
$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$
$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

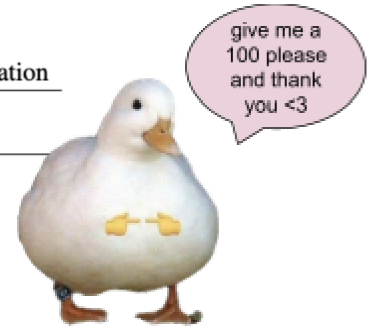
TABLE 8 Logical Equivalences Involving Biconditional Statements.

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$
$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$
$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

$$\sim\forall x \exists y P(x,y) = \exists x \forall y \sim P(x,y)$$

Rules of Inference with Quantifiers

Universal Instantiation	Universal Generalization	Existential Instantiation	Existential Generalization
c is an element $\forall x P(x)$	c is an arbitrary element $P(c)$	 $\exists x P(x)$	c is an element $P(c)$
$P(c)$	$\forall x P(x)$	c is a particular element $\wedge P(c)$	$\exists x P(x)$

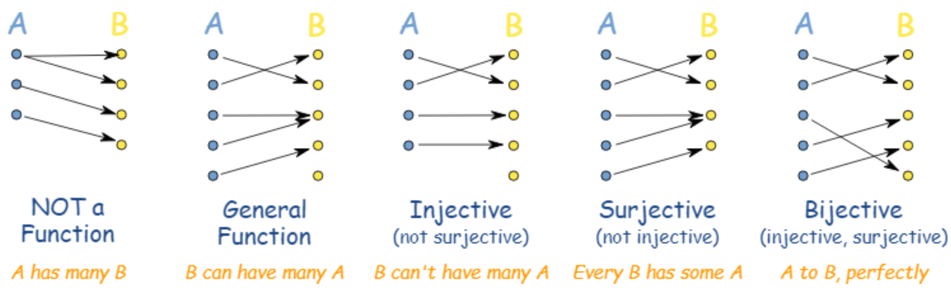


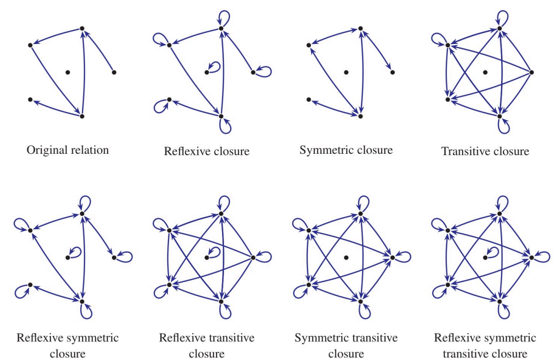
Quantifiers

Domain restrictions go before the colon.

$\forall x P(x) : Q(x) = \forall x : P(x) \to Q(x) \quad \exists x P(x) : Q(x) = \exists x : P(x) \wedge Q(x)$

$\neg \forall x : P(x) = \exists x : \neg P(x) \qquad \neg \exists x : P(x) = \forall x : \neg P(x)$





Reflexive: $\forall x \in A : (x, x) \in R$ — Symmetric: $\forall x, y \in A : (x, y) \in R \leftrightarrow (y, x) \in R$
Transitive: $\forall x, y, z \in A : ((x, y) \in R \wedge (y, z) \in R) \to (x, z) \in R$

- Induction questions: basis, inductive hypothesis, inductive step (k)
  - Strong induction: multiples bases, hypothesis, step (k+1)
- Tree method
  - Negate conclusion, rewrite hypotheses, start at top of hypothesis and make branches - if not all branches are killed, counter-example exists
- Equivalence relation = reflexive, transitive, symmetric connecting classes (a ~ b) ; equivalence class = pairwise disjoint groups that make relations
- relations are subsets of cartesian products
- The composition of injective functions is injective and the composition of surjective functions is surjective, thus the composition of bijective functions is bijective.
  - injective = 1 to 1 ; surjective = b is mapped by at least one a ; bijective = both inj and surj
  - A function f has an inverse if and only if f is a bijection. (inverse is x and y switched)
- Symmetric difference = set of elements that are a member of exactly one of A and B, but not both
- S o R = output of R paired with matching output in S
- Proofs
  - Contrapositive: p → c becomes not c → not p
  - Contradiction (indirect proof): Assume p ∧ ¬q. Follow a series of logical steps to conclude r ∧ ¬r for some proposition r.
  - Proof by cases: universal statement such as ∀x P(x) breaks the domain for the variable x into different classes and gives a different proof for each class

At most one person loves Layla.

$\exists x : Loves(x, Layla) \to \forall y : (y \neq x \to \neg Loves(y, Layla))$

Exactly one person loves Layla.

$\exists x : Loves(x, Layla) \wedge \forall y : (y \neq x \to \neg Loves(y, Layla))$

Exactly two people love Layla.

$\exists\, x, y : (x \neq y) \wedge Loves(x, Layla) \wedge Loves(y, Layla)$
$\wedge \forall z : ((z \neq x \wedge z \neq y) \to \neg Loves(z, Layla))$