Lab 9
CS 135 ~ The finale

Problem 1

Prove that the sum of the degrees of the vertices of any directed graph is even.

Problem 1

Basis: v = 1 means degree of graph is 0 (v being number of vertices)

I.H.: Let G be a finite graph with k edges, and assume that the sum of the degrees of the vertices in G is even.

I.S.:

Case 1: Every step we add an edge and a vertex to G which has even degree based on the IH. The degree of the graph increases by 2! Thus using the I.H. if we have an even degree graph and we add 2 to it, we still have an even degree graph.

Case 2: Add a vertex without an edge connecting it to o.g. G. Add 0 to even # -> even #

By PI we proved that P(k) -> P(k+1)

Problem 2

The set of rooted full binary trees is defined as follows:

I. A full binary tree of height 0 consists of a single vertex (called a leaf)

II. A full binary tree of height k+1 consists of a root vertex with exactly two children, at least one of which is a binary tree of height k and the other a binary tree of height at most k.

Q: Prove by induction, that every full binary tree of height h has at most 2^h leaves.

Problem 2

Basis: h = 0; the tree has one leaf

IH: For some k >= 0 all binary trees of height <= k have at most 2^k leaves

IS: Let T be a b-tree of height k+1. T’s left and right subtrees have height <= k. So by the I.H. have at most 2^k leaves. Combining these subtrees together we have $2^k + 2^k = 2(2^k) = 2^{k+1}$.

By P.I. we proves that P(k) → P(k+1).

Congrats!

You made it through your last 135 lab woohooo :D