Lab 2

CS 135

How was the problem set?

Problem 1 - Tree

$$A \land \neg(B \lor C)$$

$$(A \Rightarrow C) \Rightarrow C$$

$$\neg C \Rightarrow D$$

$$\neg D$$

this symbol means "therefore"

$$\therefore \neg(A \land D)$$

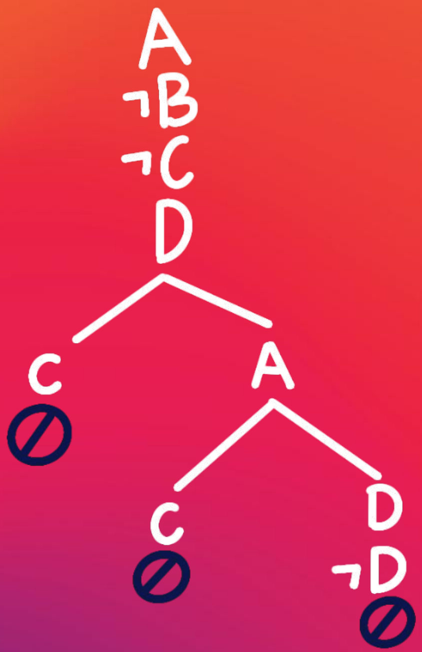
Given this argument use a tree to determine whether it is valid or not

Problem 1 Answer Key

1) $A \land \neg B \land \neg C$
2) $\neg(\neg A \lor C) \lor C$

   $\equiv (A \land \neg C) \lor C$

   $\equiv C \lor A$
3) $C \lor D$
4) $\neg D$
5) $A \land D$



This argument is valid because using the tree method all leaves are cancelled out and no contradictions are found.

Problem 2 - Quantified Proposition Translation

a. **If a person is female and is a parent, then this person is someone's mother.**
   i. Let F(x) represent "x is female"
   ii. Let P(x) represent "x is a parent"
   iii. Let M(x,y) represent "x is the mother of y"
b. **Every student has either asked Professor Bhatt a question or been asked a question by Professor Bhatt.**
   i. Let S(x) represent "x is a student"
   ii. Let A(x,y) represent "x asks y a question"

Problem 2 Answer Key

a. $\forall x((F(x) \wedge P(x)) \Rightarrow \exists y M(x,y))$

$\equiv \forall x \exists y((F(x) \wedge P(x)) \Rightarrow M(x,y))$

b. $\forall x(S(x) \Rightarrow (A(x, \text{Professor Bhatt}) \vee A(\text{Professor Bhatt}, x)))$

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\dfrac{\forall x P(x)}{\therefore\ P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\ \exists x P(x)}$	Existential generalization

Problem 3

Let D(x) represent "x is in this discrete structures class"

Let C(x) represent "x is a computer science major"

Use the laws on inference to prove the following argument.

$$\forall x(D(x) \Rightarrow C(x))$$

$$D(\text{Layla})$$

$$\therefore C(\text{Layla})$$

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

Problem 3 Answer Key

1. $\forall x(D(x) \Rightarrow C(x))$	Premise
2. $D(Layla) \Rightarrow C(Layla)$	Universal instantiation, 1
3. $D(Layla)$	Premise
4. $C(Layla)$	Modus ponens, 2, 3

Racket time!