CS284: Exercise Booklet 2 - Basic Complexity

Exercise 1

Determine the time growth rate of the following code. You must provide details on how it was established. You may assume that $n > 1$.

```
1  for(int i=0; i<n; i++) {
       for(int j=i; j>0; j--) {
3          System.out.println(i + " " + j);
       }
5  }
```

inner loop: 0 + 1 + ... + n-1

$\frac{n.(n-1)}{2}$

$O(n^2)$ ✓

Exercise 2

Determine the time growth rate of the following code. You must provide details on how it was established. You may assume that $n > 1$.

```
1  for(int i=1; i<n; i++) {
       for(int j=1; j<n; j*=2) {
3          System.out.println(i + " " + j);
       }
5  }
```

n-1 times $O(n \log_2 n)$

$\log_2 n$

indices 1 2 4 8 ...

$2^k \leq n \leq 2^{k+1}$

$k \leq \log_2 n \leq k+1$

$O(n \log n)$ ✓

Exercise 3

Determine the time growth rate of the following code. You must provide details on how it was established. You may assume that $n > 1$.

```
1  for(int i=1; i<n; i++) {
       for(int j=1; j<n; j*=2){
3          break;
       }
5  }
```

n-1 times $O(n)$

1 time

$O(n)$ ✓

Exercise 4

Determine the time growth rate of the following code. You must provide details on how it was established. You may assume that $n > 1$.

```
1  for(int i=n-1; i>=0; i--) {
       for(int j=n-1; j>i; j--) {
3          System.out.println(i + " " + j);
       }
5  }
```

outer index	inner times
i = n-1	0
i = n-2	1
i = n-3	2
⋮	
i = 0	+ n-1

0 + 1 + ... + n-1

$\frac{n(n-1)}{2}$ $\frac{(n-1)(n-1+1)}{2}$ n

$O(n^2)$ ✓

Exercise 5

Determine the time growth rate of the following code. You must provide details on how it was established. You may assume that $n > 1$.

```
1  for(int i=n-1; i>=0; i--) {
       for(int j=9; j>0; j--) {
3          System.out.println("hello");
       }
5  }
```

0 1 2 ... n-1 n times

9 times

$O(9n) = O(n)$

constant

$O(n)$

Exercise 6

Consider the following snippet of code

```
for(int i=0; i<n; i++) {
  ... break;
  }
```

where the missing line is not provided to you. Can you assert that this code will run in $\mathcal{O}(n)$ independently of the missing line of code? If your answer is no, then provide a counterexample.

Exercise 7

Let $f(n) = 100n^4 + 5000n + 3$. Is $f(n) \in \mathcal{O}(n^4)$? If yes, then justify your answer by supplying the appropriate positive constants c and n_0.

Exercise 8

Determine the time growth rate of the following code. You must provide details on how it was established. You may assume that $n > 1$.

```
for (int i=0; i<n; i++) {
  if (i%2==0) {
    for (int j=0; j<n; j++) {
      System.out.println("Hi");
    }
  }
}
```

Exercise 9

Determine the time growth rate of the following code. You must provide details on how it was established. You may assume that $n > 1$.

```
for (int i=0; i<n; i++) {
  if (i%2==0) {
    for (int j=0; j<i; j++) {
      System.out.println("Hi");
    }
  }
}
```

Exercise 10

Consider the code below. Indicate:

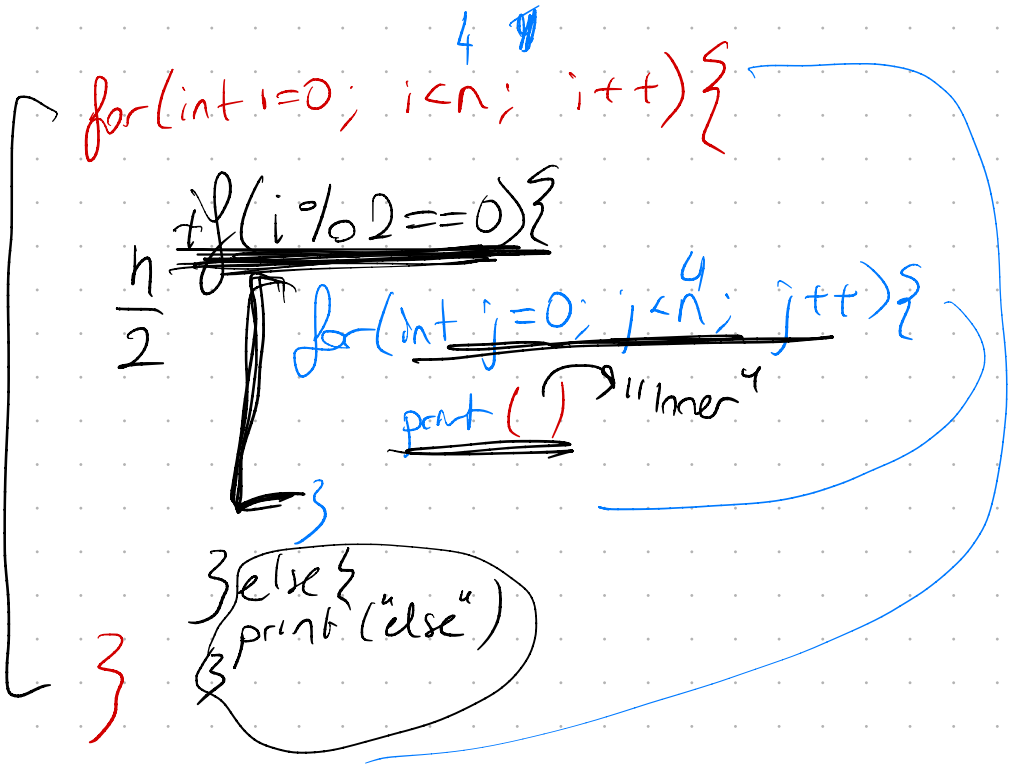
1. How many times it prints a message.
2. Its complexity.

You may assume that $n > 1$.

```
for (int i=0; i<n; i++) {
    for (int j=0; j<n; j++) {
        if (i%2==0) {
            System.out.println("Hi");
        }
    }
}
```

Exercise 11

Provide an example of code that has time growth rate of $\mathcal{O}(n \log n)$.



$$T(n) = \frac{n}{2} * n + \frac{n}{2} \qquad \Theta(n^2)$$

$$\frac{n^2}{2} + \frac{n}{2}$$

$\frac{n}{2}$ $n + \frac{n}{2}$

$2 * 4 + 2$

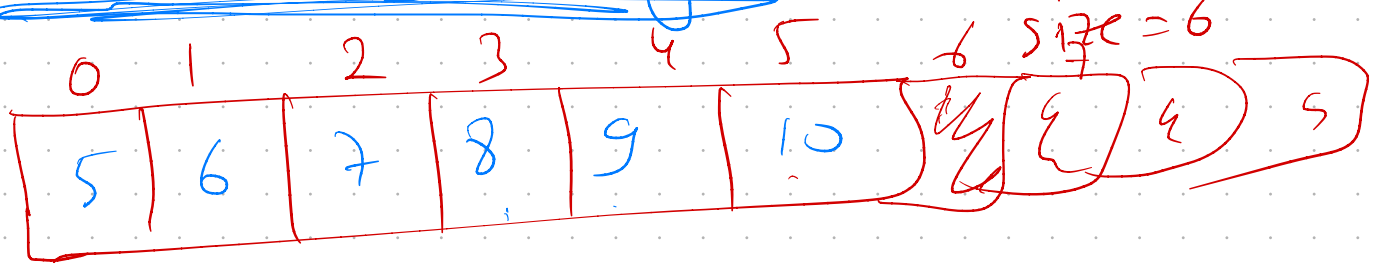
i=0 → inner 4 times

i=1 → 1 if check

i=2 → inner 4 times

i=3 → 1 if check

```
for(int i=size; i>index; i--){
    theData[i] = theData[i-1]
}
theData[index] = entry;
```



add(3, 100)