"Adaptive Reduction..." etc.

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ALgorithm

[A] Noise model

- images = bidimensional functions defined in Ω

☆ Ω = bounded domain of $\mathbb{R}^2$

- 2 dimensional $\rightarrow$ (x, y) where $(x, y \in \mathbb{R})$

image degradation model:

$$f(x, y) = u(x, y) + \eta(x, y)$$

WHERE

- (x, y) = cartesian coordinates
- $f(x, y)$ = observed signal at pixel (x, y)
- $\eta(x, y)$ = stripe noise
- $u(x, y)$ = true signal to be estimated.

This equation solves for the observed signal at a provided pixel (x, y) by adding together the stripe noise associated with the provided pixel (x, y) and the true signal to be estimated associated with the provided pixel (x, y)

- in this case, η represents stripe noise $\Rightarrow$ where geometrical considerations are more valuable than statistical assumptions.

$\boxed{B}$ directional hierarchial decomposition

☆ BV space = space of functions of bounded variation.

- TNV approach: decompose an image in BV space into a sum of images defined in the intermediate (BV, L^2) space.
- method used in this paper: similar to TNV approach but restricted to the L^2 space and exploiting image gradient fields.

variational decomposition:

$$[u_0, v_0] = \underbrace{\inf_{(u,v)/u+v=f} \int_\Omega \|\partial_x v\|^2 \, d\Omega + \lambda_0 \int_\Omega \|\partial_y (u - M.f)\|^2 \, d\Omega}_{\text{known as "energy functional"}}$$

☆ ∂_x = partial derivative in the cross track direction

☆ ∂_y = partial derivative in the across track direction

☆ "." symbol = element-by-element multiplication.

☆ $[u_0, v_0]$: represents a pair of functions

☆ "inf" represents the infimum (greatest lower bound).

- largest value that is less than / equal to every element in set

- optimization is done over all pairs (u,v) such that $u+v=f$, where f is a provided function.

☆ $\int_\Omega \|\partial_x v\|^2 \, d\Omega$ is the functional to be minimized

☆ Ω is the domain of integration, aka the bounded domain of R^2

☆ $\partial_x v$ = partial derivative of v with respect to x

Variational decomposition (continued):

★ objective is to decompse a given function f into two parts: u and v

- you want to minimize the smoothness of v, smoothness measured by $\|\partial_x v\|^2$

★ this whole equation represents finding the decomposition (u_0, v_0) of f into u and v that minimizes the integral of the squared norm of the partial derivative of v over the domain Ω, subject to the constraint $u+v=f$

Matrix M

$$M(x,y) = \begin{cases} 1 & \text{if } \{[\partial_x f(x,y)]^2 + [\partial_y f(x,y)]^2\}^{1/2} \geq \epsilon \\ 0 & \text{if } \{[\partial_x f(x,y)]^2 + [\partial_y f(x,y)]^2\}^{1/2} < \epsilon \end{cases}$$

- alleviates the problem of using the L^2 norm over the L^1 norm.
- M is defined using a threshold on the image gradient norm

★ for SST calcs, $\epsilon = 0.8 \text{ K km}^{-1}$

★ the previous variational decomposition equation splits noisy observation f into...

- u_0 = initial estimate
- v_0 = striping and additional scale-dependent info

★ $\partial_x f(x,y)$ = partial derivative of $f(x,y)$ with respect to x

★ $\partial_y f(x,y)$ = partial derivative of $f(x,y)$ with respect to y

extend variational decomp. with Lagrange:

- Lagrange multiplier = strategy for finding the local maxima and minima of a function subject to equation constraints

$$[u_1, v_1] = \inf_{(u,v)/u+v=v_0} \int_{\Omega} \|\partial_x v\|^2 \, d\Omega$$
$$+ \lambda_0 \cdot 2^{-1} \int_{\Omega} \|\partial_y (u - M.v_0)\|^2 \, d\Omega$$

★ λ_0 = lagrange multiplier

★ noisy observation = $f = u_0 + u_1 + v_0$

★ $\partial_x v$ = partial deriv. of v with respect to x

★ $\partial_y (u - M.v_0)$ = partial deriv. of $(u - M.v_0)$ with respect to y

★ optimization is done over all pairs (u,v) such that $u + v = v_0$, where v_0 represents striping and other scale-dependent info.

★ "." = element-by-element multiplication.

★ Ω = domain of integration + bounded domain of R^2

directional hierarchical decomposition derived from extended variational decomp.:

$$[u_K, v_K] = \inf_{(u,v)/u+v=v_{K-1}} \int_\Omega \|\partial_x v\|^2 d\Omega$$
$$+ \lambda_0 . 2^{-K} \int_\Omega \|\partial_y (u - M.v_{K-1})\|^2 d\Omega$$

using this, f can also be expressed now as...

$$\left.\begin{aligned} f &= u_0 + v_0 \\ &= u_0 + u_1 + v_1 \\ &= \\ &\vdots \end{aligned}\right\} = \sum_{i=0}^{K} u_i + v_K .$$

- DHD acts as a directional filter in the spacial domain that progressively retrieves cross-track variations in the term v_K while isolating the stripe noise in its high-frequency domain.

- we know that as the number of iterations approaches ∞, the Lagrange multiplier $\lambda_0 . 2^{-K}$ converges to 0

$\downarrow$

$[u_K, v_K]$ converge into one sol. of the problem:

$$[u_K, v_K] = \inf_{(u,v)/u+v=v_{K-1}} \int_\Omega \|\partial_x (u - v_{K-1})\|^2 d\Omega$$

knowing that

$u_K = v_K + A$

$\forall (x,y) \in \mathbb{R}^2, \partial_x A(x,y) = 0$

☆ $\forall$ = for all

☆ A = matrix that represents one constant per line.

for all (x,y) that is an element of $\mathbb{R}^2$ (dim. 2), the partial derivative of $A(x,y)$ with respect to x is 0.

$\boxed{C}$ Nonlocal filtering

weighting function that measures the radiometric similarity between pixels (x,y) and (x',y'):

$w(x,y,x',y')$

$$w(x,y,x',y') = \exp\left\{\frac{-[v_N(x,y) - v_N(x',y')]^2}{2\sigma^2}\right\}$$

★ σ controls the decay of weighting coefficients with respect to the radiometric distance between pixels + depends on the NEΔD.

★NEΔT = noise equivalent delta temperature – represents the radiometric resolution and sensitivity of a radiometer.

★ $\sigma^2 = NE\Delta T/2$

val. of noisy pixel! ★ v_N = remaining information, also contains a low-freq. component that has to be retrieved for the estimation of the true scene.

Yaroslavsky neighborhood filter (YNF).

$$YNF[v_N(x,y)] = \frac{1}{C(x,y)} \int_{N(x,y)} v_N(x,y)\, w(x,y,x',y')\, dx'\, dy'$$

★ $C(x,y)$ = normalization term represented by...

$$C(x,y) = \int_{N(x,y)} w(x,y,x',y')\, dx'\, dy'$$

★ $N(x,y)$ = spatial neighborhood of pixel (x,y)

defining $N(x,y)$ in YNF

$$N(x,y) = \{(x', y') \in \Omega' : |y - y'| \leq \Delta y \text{ and } |x - x'| = 0\}$$

* Ω' = subdomain of Ω defined with respect to the binary matrix M as...

$$\Omega' = \{(x,y) \in \Omega : M(x,y) = 0\}.$$

* assuming an even # of detectors, $\Delta y = D/2$
  * D = # of detectors. (16 for VIIRS "M" bands")
* to account for possible mirror side differences, $\Delta y = D$ (no mirror banding in VIIRS banding).

nonlocal filtering Yaroslavsky neighborhood filter (YNF).

$$YNF[v_N(x,y)] = \frac{1}{C(x,y)} \int_{y-\Delta x}^{y+\Delta x} v_N(x, y') \exp\left(\frac{-B^2}{2\sigma^2}\right) dy'.$$

* exp = exponential function ().
* $B = v_N(x,y) - v_N(x,y')$

- for N iterations of PHD, an estimate of true signal is...

$$\hat{u} = \sum_{k=0}^{N} u_k + YNF[v_N(x,y)].$$

meaning that the estimate of true signal is equal to the summation of all u values from k = 0 to N added to the YNF of provided value of noisy pixel (x,y).

D optimization

redeclaring energy functional:

$$E_0(f, \lambda_0) = \int_\Omega \| \partial_x (u-f) \|^2 \, d\Omega$$

$$+ \lambda_0 \int_\Omega \| \partial_y (u - M.f) \, d\Omega$$

this satisfies the below Euler-Lagrange eq.

$$\frac{\partial E_0}{\partial f} - \frac{\partial}{\partial x}\left[\frac{\partial E_0}{\partial(\partial_x f)}\right] - \lambda_0 \frac{\partial}{\partial y}\left[\frac{\partial E_0}{\partial(\partial_y f)}\right] = 0$$

- ☆ E_0 = energy functional
- ☆ f = noisy observation.
- ☆ x and y represents a pixel (x,y)
- ☆ λ_0 = lagrange multiplier previously defined.

- this equation simplifies to...

$$\partial_x^2 (u-f) + \lambda_0 \partial_y^2 (u - M.f) = 0$$

☆ u is derived from (u_0, v_0) in original energy functional

- the 2nd partial derivative of (u-f) with respect to x plus the 2nd partial derivative of (u-M.f) with respect to y multiplied by the lagrange multiplier = 0.

Spatial frequency + fourier domain:

☆ (ξ_x, ξ_y) = spatial frequency vars in Fourier domain

- for a Fourier domain image, each point represents a particular frequency contained in the spatial domain image.

☆ i = imaginary unity

☆ $\mathcal{F}$ = Fourier transform of a derivative function.

☆ f = noisy observation,

$\mathcal{F}(\partial_x f) = i \cdot \xi_x \mathcal{F}(f)$ and $\mathcal{F}(\partial_y f) = i \cdot \xi_y \mathcal{F}(f)$

$\mathcal{F}(\partial_x^2 f) = -\xi_x^2 \mathcal{F}(f)$ and $\mathcal{F}(\partial_y^2 f) = -\xi_y^2 \mathcal{F}(f)$.

Fourier transform applied to simplified Euler-Lagrange equation derived from energy functional:

☆ $\tilde{a}$ = Fourier transform of a

☆ a^M = element-by-element multiplication of a with matrix M

$$\tilde{u}_0 = (\xi_x^2 \tilde{f} + \lambda_0 \xi_y^2 \tilde{f}^M)(\xi_x^2 + \lambda_0 \xi_y^2)^{-1}$$

$$\tilde{v}_0 = [\lambda_0 \xi_y^2 (\tilde{f} - \tilde{f}^M)](\xi_x^2 + \lambda_0 \xi_y^2)^{-1}$$

- inverse Fourier transform → estimate of $[u_0, v_0]$ in spatial domain minimizing energy functional $E_N(v_{N-1}, \lambda_N)$ at Nth iteration

$$E_N(v_{N-1}, \lambda_N) = \int_\Omega \| \partial_x (u - v_{N-1}) \|^2 d\Omega + \lambda_0 \cdot 2^{-N} \int_\Omega \| \partial_y (u - M \cdot v_{N-1}) \|^2 d\Omega$$

★ λ_N = Lagrange multiplier associated with N

★ v_N = val. of noisy pixel

★ Ω = bounded domain of $\mathbb{R}^2$.

Simplify Euler-Lagrange equation:

$$\partial_x^2 (u - v_{N-1}) + \lambda_0 \cdot 2^{-N} \partial_y^2 (u - M \cdot v_{N-1}) = 0$$

• we understand all the variables ☺

$$-\xi_x^2 (\widetilde{v_{N-1}} - \tilde{u}) - \lambda_0 \cdot 2^{-N} \xi_y^2 (\widetilde{v_{N-1}^M} - \tilde{u}) = 0$$

shifting to Fourier space

define Nth iterations in Fourier space:

$$\tilde{u}_N = (\xi_x^2 \widetilde{v_{K-1}} + \lambda_0 \cdot 2^{-K} \xi_y^2 \widetilde{v_{K-1}^M})(\xi_x^2 + \lambda_0 \cdot 2^{-K} \xi_y^2)^{-1}$$

$$\tilde{v}_N = [\lambda_0 \cdot 2^{-K} \xi_y^2 (\widetilde{v_{K-1}} - \widetilde{v_{K-1}^M})](\xi_x^2 + \lambda_0 \cdot 2^{-K} \xi_y^2)^{-1}$$

[c] Image quality

NIF and NDF

★ NIF = metric that quantifies relative change in gradients across the scan

★ NDF = quantifies the relative change with destriping in the gradients along the scan direction

$$NIF(f,u) = \left(\int_{\Omega'} (|\partial_y f| - |\partial_y u|) dy \right) \left(\int_{\Omega'} |\partial_y f| dy \right)^{-1}$$

$$NDF(f,u) = 1 - \left[\int_{\Omega'} |\partial_x (f-u)| dy \right] \left(\int_{\Omega'} |\partial_x f| dx \right)^{-1}$$

★ Ω' = subdomain of Ω defined with respect to the binary matrix M as...

$$\Omega' = \{(x,y) \in \Omega : M(x,y) = 0\}$$