TEXAS McCombs
The University of Texas at Austin
McCombs School of Business

Project 3 - Testing Asset Pricing Models

Advanced Corporate Finance

Humphrey Hui

Peyton Gibbs

Jack Feen

Motivation

In this project, we test the validity of leading asset pricing models using data from the Ken French Data Library. We test these asset pricing models for multiple reasons. One reason is that we are looking to see if recurring patterns are systematic risk or if there are different explanations. With CAPM, for example, we are looking to see if we can explain the patterns with market risk. For the Fama-French 3-factor model, we are looking at market risk along with the high minus low book-to-market factor, measuring value and growth, along with small minus big, measuring big cap and small cap. When we remove what the factors should earn, we can isolate the alpha and see if there are arbitrage opportunities that can be traded on. A final reason is to examine whether the models have unexplained returns that additional factors could explain.

When we perform our time series regression to determine portfolio exposures to each factor, the portfolio risks we face become clearer. A GRS test is then done to see if there are returns jointly unexplained across our portfolios. Then, we conducted a Fama-MacBeth test to measure the factor risk premia, which represents the percent return the market pays per unit of beta. This can be used to identify which risks are worth owning and to set more effective portfolio allocations. Together, these tests will show whether the strategies we use to evaluate risk are sound and validate benchmarks.

Procedures

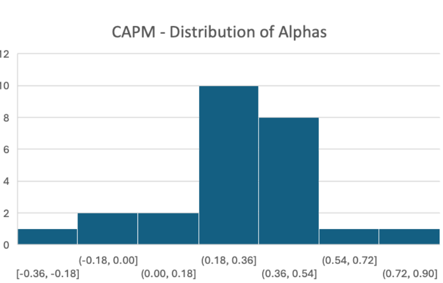
Given that our goal is to understand how various factors (market excess return, small minus big, high minus low) explain portfolio returns, we first test for systemic unexplained portfolio returns. Alpha represents the excess return that can't be explained by exposure to these factors (in other words, the return left over after accounting for systematic risk). We want to test whether alpha is statistically different from zero. Our first test runs time series regressions on individual stock portfolios. For CAPM, this means regressing each portfolio's excess return on the market excess return. For FF3F, we run a multiple line regression on each portfolio's excess return against all three factors. Once we find the alphas for each portfolio, we can find the t-stat and p-value for each to see if they are statistically significant.

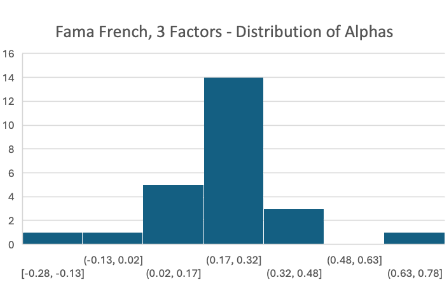
While the time series method tells us whether each individual portfolio is likely to have alpha equal to zero, we encounter the multiple testing problem: when we perform the same test on multiple portfolios, some will appear statistically significant simply by chance. To account for this, we perform an F-GRS test to see if the alphas are jointly zero across all portfolios. We form a vector of the alphas we calculated and compute the GRS statistic. The GRS formula essentially divides a measure of how large the alphas are jointly by a measure of how well the factors explain the returns, adjusted for sample size. Once we calculate this statistic, we compare it to an F-distribution (used instead of the t-distribution because we're testing multiple alphas simultaneously) to determine if the alphas are statistically different from zero as a group.

However, these previous tests do not account for estimation error in the factor risk premiums themselves, i.e., they do not recognize that the factor returns we observe are just one possible outcome and could have been different. This is why we also perform the Fama-Macbeth test, as it runs cross-sectional regressions period by period and averages the results, which produces more reliable standard errors that account for these issues. To perform Fama-Macbeth, we first take the betas estimated previously for each portfolio in the linear regression/multiple linear regression. As these betas are just estimations and not the true values, we need to apply the Shanken correction to account for errors-in-variables bias, i.e., the fact that using estimated betas instead of true betas biases our standard errors downward. Using these betas, we then take a cross-sectional view, looking at multiple portfolios at one point in time (as opposed to one portfolio across multiple time periods in the previous regressions) and run a regression of the portfolios' excess returns on the portfolios' betas to find the lambdas, which represent the price of risk - the extra return you would get for every additional unit of beta. We then average these lambdas and divide by the standard error of the lambdas multiplied by the Shanken correction to obtain the Shanken corrected t-statistics. If this statistic is sufficiently high (e.g. above 2 for a 95% confidence level), then this means the factor is priced in, meaning investors demand compensation for bearing that particular risk.

Interpretation

Using data from 1990 to 2024, the 3 Factor Fama French performed better than the CAPM model. The CAPM demonstrated an R-squared value of 0.70, indicating that the model explained 70% of a portfolio's returns. The Fama-French, however, returned an R^2 of 0.94. Additionally, the average alpha of the Fama-French portfolio was lower, at 0.21, compared to CAPM's alpha of 0.29. Neither of these models fully explained the portfolio returns; an average alpha of 0 was 5.11 and 5.39 standard errors away from CAPM and FF3F, respectively.





Examining portfolio mean returns, it appears that higher book-to-market ratios are associated with higher returns. On average, the high book-to-market portfolios had 42% higher mean returns than the low book-to-market portfolios. Additionally, lower market capitalization portfolios appeared to magnify the differences in expected returns between high and low book-to-market ratios. The Small HiBM portfolio had returns 131% higher than the Small LowBM portfolio, whereas the Big HiBM portfolio performed only 8% better than the Big LowBM portfolio. Although we only examined a multiple linear regression, this suggests that including interactions between variables could enhance model performance.

Portfolio	Mean Return (%)	Std Dev (%)	Sharpe Ratio
SMALL LoBM	0.76187298	8.679299387	0.06381482
ME1 BM2	1.183836364	7.274937403	0.134136043
ME1 BM3	1.328001768	6.229361426	0.179793183
ME1 BM4	1.421981818	5.646716336	0.214988091
SMALL HiBM	1.757510354	6.150857298	0.251916965
ME2 BM1	1.045318939	7.626443628	0.109790871
ME2 BM2	1.26835	6.198085974	0.171076193
ME2 BM3	1.348410859	5.676440426	0.200901573
ME2 BM4	1.273561869	5.546917587	0.192098909
ME2 BM5	1.387983081	6.937846934	0.170078418
ME3 BM1	1.064661364	7.032370784	0.121816147
ME3 BM2	1.362433586	5.614191056	0.20562687
ME3 BM3	1.246736364	5.312434974	0.195528288
ME3 BM4	1.310865909	5.610058709	0.196586331
ME3 BM5	1.475649495	6.618769654	0.191522671
ME4 BM1	1.205624495	6.037384096	0.165240347
ME4 BM2	1.249131566	5.184450028	0.200817157
ME4 BM3	1.247761869	5.304364602	0.196019108
ME4 BM4	1.309143434	5.421049801	0.203122721
ME4 BM5	1.278582323	6.523457748	0.164111935
BIG LoBM	1.135980808	5.096717897	0.182073204
ME5 BM2	1.198554545	4.593646058	0.215634701
ME5 BM3	1.228945202	4.646040324	0.219744143
ME5 BM4	1.10317803	5.190817909	0.172453165
BIG HiBM	1.234131818	6.119465784	0.16768241

The stability between portfolios in the high market capitalization group also appeared to reflect lower standard deviations of returns, with high market capitalization portfolios maintaining an average standard deviation 24% lower than that of low market capitalization portfolios.

These trends were also reflected in the portfolio alphas; portfolios with higher mean returns tended to have higher alphas. The same trend of larger variances of alphas in low market capitalization portfolios occurred for both the CAPM and Fama-French portfolios, where higher book-to-market ratios were associated with higher alphas.

Interestingly, the small market capitalization portfolios noticeably suffered in model performance for both CAPM and FF3F. The average R^2 for CAPM was 24% lower than the total portfolio average, and 10% for FF3F, corroborating the observation that the low market capitalization models were more unstable.

Interestingly, the only factor that achieved statistical significance was the Market Risk factor. The Small minus Big and High minus Low factors ended up receiving T-statistics of 0.74 (standard error = 0.19) and 1.99 (standard error = 0.17), respectively (0.72 and 1.94 with the Shanken corrections, with standard errors of 0.19 and 0.18).

Even though these factors did not reach statistical significance, that does not mean that they are invalid. The 2-standard error rule is somewhat arbitrary, and the drastic increase in R^2 between CAPM and FF3F shows use usefulness of these factors.