Model Predictive Control Exercise - Group 17gr832

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Description of the Problem

The objective is to optimize the profit that can be obtained by running the plant showed in Figure 1.

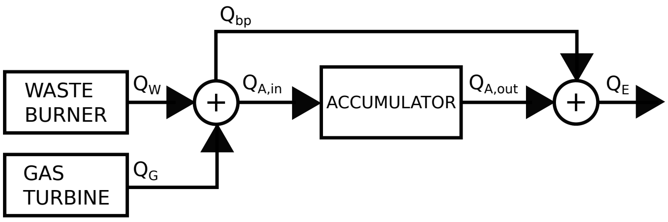

Figure 1: Diagram of the plant, where Q_{G} is the power produced by the gas turbine, Q_{W} is the power produced by the waste burner, $Q_{\mathrm{A,in}}$ is the power feed into the accumulator, Q_{bp} is the power bypassing the accumulator, $Q_{\mathrm{A,out}}$ is the power leaving the accumulator, and Q_{E} is the power leaving the plant. The units of power are given in megawatt [MW]

The power flows in the plant have some constraints shown below:

$$Q_{\mathrm{W}} + Q_{\mathrm{G}} = Q_{\mathrm{bp}} + Q_{\mathrm{A,in}}, \quad Q_{\mathrm{E}} = Q_{\mathrm{bp}} + Q_{\mathrm{A,out}} \tag{1}$$

$$0 \le Q_{\mathrm{W}} \le 40, \quad 0 \le Q_{\mathrm{G}} \le 20 \quad 0, \le Q_{\mathrm{A,in}} \le 50, \quad 0 \le Q_{\mathrm{A,out}} \le 25 \tag{2}$$

And the accumulator have some dynamics and some constraints as well

$$E_{\mathrm{A}}[k+1] = E_{\mathrm{A}}[k] + (Q_{\mathrm{A,in}}[k] - Q_{\mathrm{A,out}}[k])T_{\mathrm{s}} \tag{3}$$

$$0 \le E_{\mathrm{A}} \le 200 \tag{4}$$

The profit can be optimize by scheduling the power production using the knowledge of the plant and future prices of gas, waste burning and electricity (P_{G}, P_{W} and P_{E}). The profit is given by

$$\sum_{i=k}^{k+L-1} (P_{\mathrm{E}}[i]Q_{\mathrm{E}}[i] - (P_{\mathrm{G}}[i]Q_{\mathrm{G}}[i] + P_{\mathrm{W}}[i]Q_{\mathrm{W}}[i]))T_{\mathrm{s}} \tag{5}$$

Problem Formulation in CVX Form

Once the problem has been described, the constraints, the dynamic equation and the cost function need to be rewritten in a suitable way to be able to optimize the problem using CVX.

The dynamic equation, Equation 3, of the accumulator can be rewritten by substituting backwards each $E_{\mathrm{A}}[k]$ until it depends only on the inputs and the initial state.

$$E_{\mathrm{A}}[k+1] = E_{\mathrm{A}}[k] + (Q_{\mathrm{A,in}}[k] - Q_{\mathrm{A,out}}[k])T_{\mathrm{s}} \tag{6}$$

$$\dots$$

$$E_{\mathrm{A}}[k+l] = E_{\mathrm{A}}[k] + \sum_{i=k}^{k+l-1} (Q_{\mathrm{A,in}}[i] - Q_{\mathrm{A,out}}[i])T_{\mathrm{s}} \tag{7}$$

$$\dots$$

$$E_{\mathrm{A}}[k+L] = E_{\mathrm{A}}[k] + \sum_{i=k}^{k+L-1} (Q_{\mathrm{A,in}}[i] - Q_{\mathrm{A,out}}[i])T_{\mathrm{s}} \tag{8}$$

This expression can also be rewritten in matrix form as follows

$$\begin{bmatrix} E_{\mathrm{A}}[k+1] \\ E_{\mathrm{A}}[k+2] \\ \dots \\ E_{\mathrm{A}}[k+L] \end{bmatrix} = \begin{bmatrix} E_{\mathrm{A}}[k] \\ E_{\mathrm{A}}[k] \\ \dots \\ E_{\mathrm{A}}[k] \end{bmatrix} + \begin{bmatrix} Q_{\mathrm{A,in}}[k] - Q_{\mathrm{A,out}}[k] \\ Q_{\mathrm{A,in}}[k+1] - Q_{\mathrm{A,out}}[k+1] \\ \dots \\ Q_{\mathrm{A,in}}[k+L-1] - Q_{\mathrm{A,out}}[k+L-1] \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{9}$$

Since there exists a constraint on the amount of charge that can be stored in the accumulator at each time, Equation 9 can be used to write that constraint in an appropriate form.

$$0 \leq \begin{bmatrix} E_{\mathrm{A}}[k] \\ E_{\mathrm{A}}[k] \\ \dots \\ E_{\mathrm{A}}[k] \end{bmatrix} + \begin{bmatrix} Q_{\mathrm{A,in}}[k] - Q_{\mathrm{A,out}}[k] \\ Q_{\mathrm{A,in}}[k+1] - Q_{\mathrm{A,out}}[k+1] \\ \dots \\ Q_{\mathrm{A,in}}[k+L-1] - Q_{\mathrm{A,out}}[k+L-1] \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \leq 200 \tag{10}$$

The equality constraints in Equation 1 can be used to rewrite the cost function as a function of Q_{G}, Q_{W}, $Q_{\mathrm{A,in}}$ and $Q_{\mathrm{A,out}}$. This results in Equation 14

$$\sum_{i=k}^{k+L-1} (P_{\mathrm{E}}[i]Q_{\mathrm{E}}[i] - (P_{\mathrm{G}}[i]Q_{\mathrm{G}}[i] + P_{\mathrm{W}}[i]Q_{\mathrm{W}}[i]))T_{\mathrm{s}} \tag{11}$$

$$\sum_{i=k}^{k+L-1} (P_{\mathrm{E}}[i](Q_{\mathrm{bp}}[i] + Q_{\mathrm{A,out}}[i]) - (P_{\mathrm{G}}[i]Q_{\mathrm{G}}[i] + P_{\mathrm{W}}[i]Q_{\mathrm{W}}[i]))T_{\mathrm{s}} \tag{12}$$

$$\sum_{i=k}^{k+L-1} (P_{\mathrm{E}}[i](Q_{\mathrm{W}}[i] + Q_{\mathrm{G}}[i] - Q_{\mathrm{A,in}}[i] + Q_{\mathrm{A,out}}) - (P_{\mathrm{G}}[i]Q_{\mathrm{G}}[i] + P_{\mathrm{W}}[i]Q_{\mathrm{W}}[i]))T_{\mathrm{s}} \tag{13}$$

$$\sum_{i=k}^{k+L-1} ((P_{\mathrm{E}}[i] - P_{\mathrm{W}}[i])Q_{\mathrm{G}}[i] + (P_{\mathrm{E}}[i] - P_{\mathrm{G}}[i])Q_{\mathrm{G}}[i] + P_{\mathrm{E}}[i]Q_{\mathrm{A,out}}[i] - P_{\mathrm{E}}[i]Q_{\mathrm{A,in}}[i])T_{\mathrm{s}} \tag{14}$$

This expression can also be written in matrix form to be suitable to use with CVX.

$$
\begin{bmatrix} P_{\mathrm{E}}[k]-P_{\mathrm{W}}[k] \\ P_{\mathrm{E}}[k+1]-P_{\mathrm{W}}[k+1] \\ \dots \\ P_{\mathrm{E}}[k+L-1]-P_{\mathrm{W}}[k+L-1] \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_{\mathrm{W}}[k] \\ Q_{\mathrm{W}}[k+1] \\ \dots \\ Q_{\mathrm{W}}[k+L-1] \end{bmatrix} + \begin{bmatrix} P_{\mathrm{E}}[k]-P_{\mathrm{G}}[k] \\ P_{\mathrm{E}}[k+1]-P_{\mathrm{G}}[k+1] \\ \dots \\ P_{\mathrm{E}}[k+L-1]-P_{\mathrm{G}}[k+L-1] \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_{\mathrm{G}}[k] \\ Q_{\mathrm{G}}[k+1] \\ \dots \\ Q_{\mathrm{G}}[k+L-1] \end{bmatrix} +
$$

$$
+ \begin{bmatrix} P_{\mathrm{E}}[k] \\ P_{\mathrm{E}}[k+1] \\ \dots \\ P_{\mathrm{E}}[k+L-1] \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_{\mathrm{A_out}}[k] \\ Q_{\mathrm{A_out}}[k+1] \\ \dots \\ Q_{\mathrm{A_out}}[k+L-1] \end{bmatrix} - \begin{bmatrix} P_{\mathrm{E}}[k] \\ P_{\mathrm{E}}[k+1] \\ \dots \\ P_{\mathrm{E}}[k+L-1] \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_{\mathrm{A_in}}[k] \\ Q_{\mathrm{A_in}}[k+1] \\ \dots \\ Q_{\mathrm{A_in}}[k+L-1] \end{bmatrix} \tag{15}
$$

Implementation

The implementation of the optimization problem is shown in Listing 1. It includes the inequality constraints in Equation 2 and 10 as well as teh cost defined in Equation 15.

```
cvx_begin quiet % The beginning of the optimization problem

    % Define the variables
    variable Q_W(L,1);
    variable Q_G(L,1);
    variable Q_A_in(L,1);
    variable Q_A_out(L,1);

    % Specify the optimization of cost
    minimize(-(P_E(k:k+L-1)'*Q_A_out+(P_E(k:k+L-1)'-P_G(k:k+L-1)')*Q_G+...
    (P_E(k:k+L-1)'-P_W(k:k+L-1)')*Q_W-P_E(k:k+L-1)'*Q_A_in)*Ts);

    % Constraints
    subject to
    Q_W>=Q_W_min;
    Q_W<=Q_W_max;
    Q_G>=Q_G_min;
    Q_G<=Q_G_max;
    Q_A_in>=Q_A_in_min;
    Q_A_in<=Q_A_in_max;
    Q_A_out>=Q_A_out_min;
    Q_A_out<=Q_A_out_max;
    (Q_A_in-Q_A_out)'*triu(ones(L,L))>=E_A_min-E_A_sys(k);
    (Q_A_in-Q_A_out)'*triu(ones(L,L))<=E_A_max-E_A_sys(k);

cvx_end     % The end of the optimization problem
```

Listing 1: Matlab code for the implementation of the problem with CVX

In each iteration of the loop it tries to minimize the cost along the horizon L knowing the corresponding prices and the inequality constraints, by finding the optimal Q_{W}, Q_{W}, $Q_{\mathrm{A,in}}$ and $Q_{\mathrm{A,out}}$.

Results

The results obtained are presented in Figure 2, 3, 4 and 5.

In Figure 2 it can be seen the state of charge of the accumulator. It shall be noticed that the accumulator charges when the price of electricity is low and discharges when it is high. For this to be seen it is neccesary to look also at Figure 3, where the power production and price is depicted. Good examples appear at 90 hours and between 170 and 180 hours. The accumulator also charges when the power production of gas is cheaper, this is shown in Figure 5. The clearest example of this occurs between 110 and 120 hours.

From the power production seen in Figure 3, it can be said that the power produced is low when electricity is cheap and high otherwise. This is clear in the graph as when the price peaks up, so does the production and vice versa

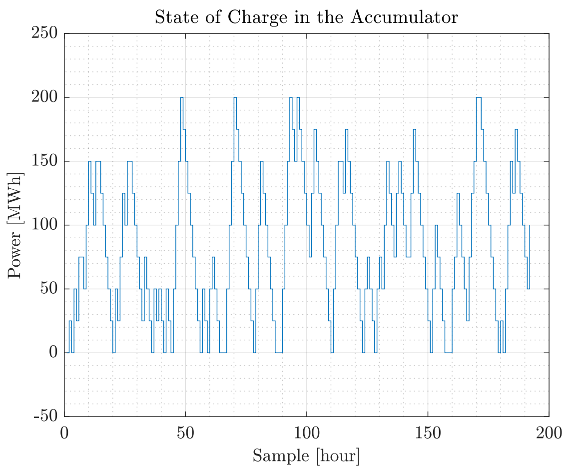

Figure 2: State of charge in the accumulator.

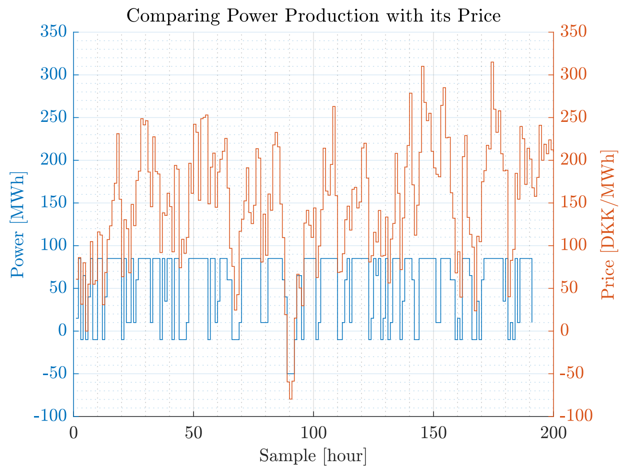

Figure 3: Comparison of the power production and the price of electricity.

The information presented in Figure 4 and 5 shows how the electric power is produced. It can be observed how waste is used in almost all the simulation as its price is negative. The only exception occurs when the price of electricity is also negative. This leads to a stop in the electricity production and from gas and waste and a charging period of the accumulator using electric power from outside the plant. The use of gas is more interesting as it is used as long as its price is lower than that of electricity, else the production of electric power with gas is zero. Examples of this take place between 65 and 70 hours, between 90 and 95 hours and at 160 hours.

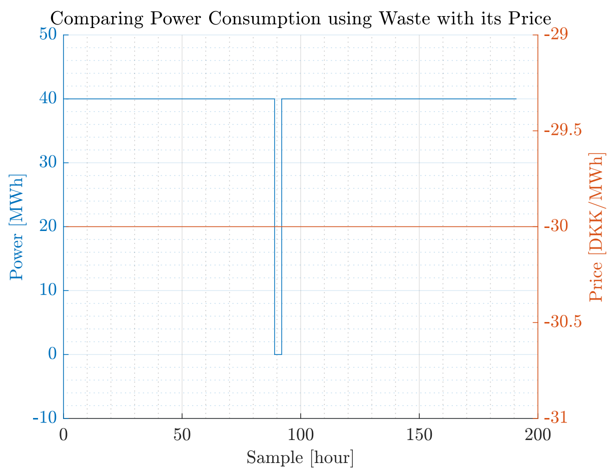

Figure 4: Comparison of the power produced using waste and the price of waste burning.

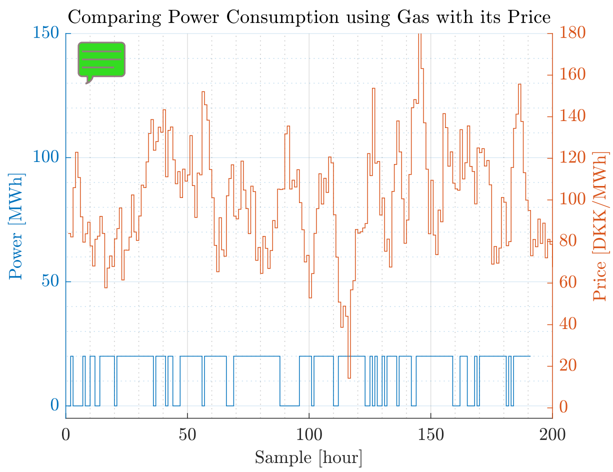

Figure 5: Comparison of the power produced using gas and the price of gas.

All the information presented suggest that the plant operates taking into account the different prices and adjusting the power production in order to maximize the profit obtained when running.