STA610 Lab10

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- Write down your answers in any blank sheet and submit your work in paper during the lab.
- Your work will not be graded. As long as you submit, you will get a full credit.
- For those who missed the lab today, you can submit it via email to me for half credit.

Linear Algebra Stuffs

Consider matrices $A \in \mathbb{R}^{n\times n}$, $U \in \mathbb{R}^{n\times k}$, $C \in \mathbb{R}^{k\times k}$, $V \in \mathbb{R}^{k\times n}$ $(k \leq n)$ with A and C invertible. Sherman–Morrison–Woodbury formula (https://en.wikipedia.org/wiki/Woodbury_matrix_identity) gives

$$(A + UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}.$$

1. Verify the formula by computing the following:

$$(A + UCV)(A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}).$$

2. Use the formula to simplify the following $n \times n$ matrix:

$$(I - a11^\top)^{-1}.$$

3. For a matrix A with $||A||_2 < 1$ (i.e. all eigenvalues of A are between -1 and 1), we have power series

$$(I - A)^{-1} = I + A + A^2 + \cdots = \sum_{\ell=0}^{\infty} A^\ell.$$

   Now use this formula to simplify $(I - a11^\top)^{-1}$ when $-\frac{1}{n} < a < \frac{1}{n}$. Do you get the same result as in part 2?

Consider an invertible matrix A and vectors u, v. The matrix determinant lemma gives

$$\det(A + uv^\top) = (1 + v^\top A^{-1}u)\det(A).$$

1. Prove this lemma. Hint: check out https://en.wikipedia.org/wiki/Matrix_determinant_lemma.
2. Compute the determinant

$$\det(I - a11^\top).$$

nlme package

Load package.

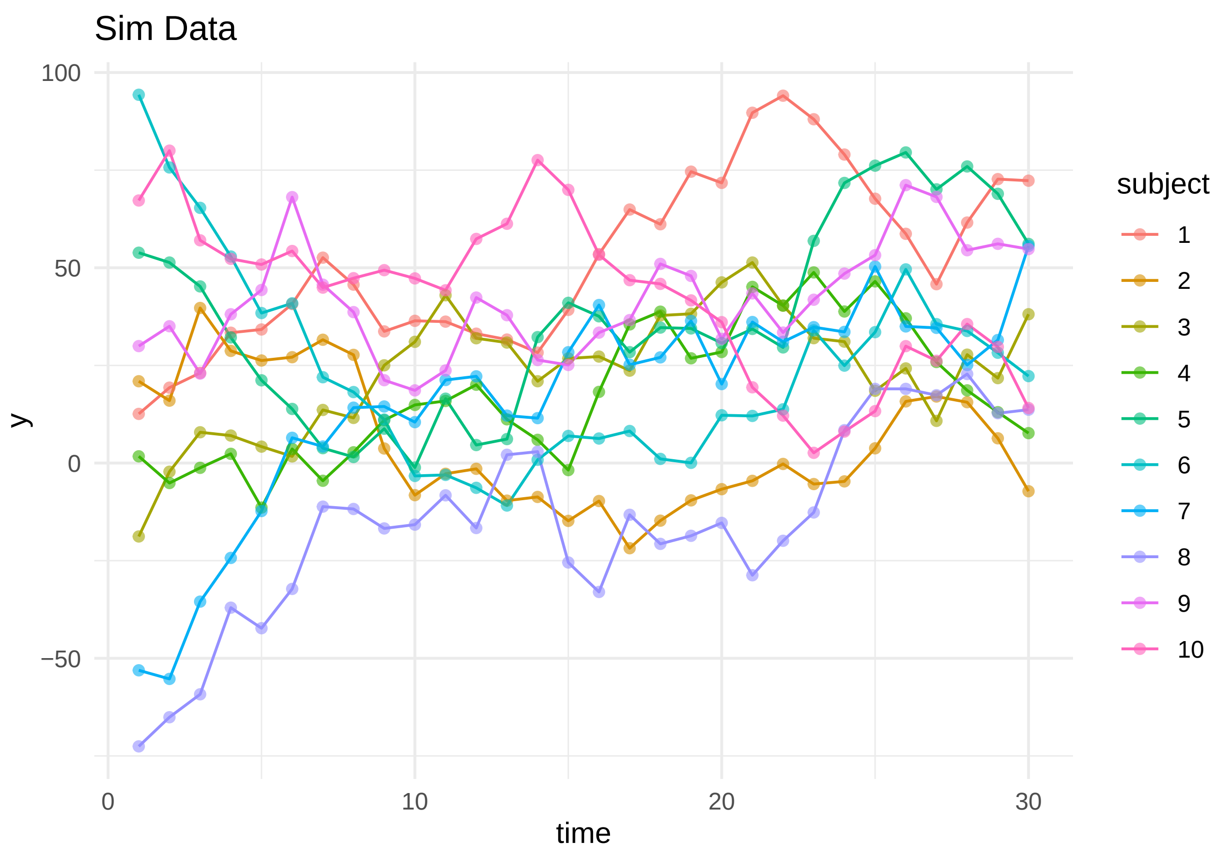
```r
library(nlme)
```

Data simulation.

```r
set.seed(0)

N <- 10
TT <- 30
subject <- as.factor(rep(1:N, each = TT))
time <- rep(1:TT, N)

intercept <- rnorm(N, 10, 10)
slope <- rnorm(N, 1, 1)

whitenoise <- rnorm(N * TT, 0, 10)
phi <- 0.9
ar_noise <- rep(0, N * TT)
for (n in 1:N){
  for (t in 1:TT){
    i <- (n - 1) * TT + t
    if (t == 1) ar_noise[i] <- whitenoise[i] / (1 - phi ^ 2)
    else ar_noise[i] <- whitenoise[i] + phi * ar_noise[i - 1]
  }
}

y <- intercept[subject] + slope[subject] * time + ar_noise

data <- data.frame(subject = subject, time = time, y = y)
```

Plot data.

```r
library(ggplot2)
ggplot(data, aes(x = time, y = y, color = subject, group = subject)) +
  geom_line() +
  geom_point(alpha = 0.6) +
  labs(title = "Sim Data", x = "time", y = "y") +
  theme_minimal()
```



1. Fit the model with a fixed effect of time, random intercepts and slopes for each subject, and independent noise.

2. Fit the model with a fixed effect of time, random intercepts and slopes for each subject, and autocorrelated noise.

3. Report the BIC for both models. Which model would you choose?

```
model1 <- lme(y ~ time, random = ~ 1 + time | subject,
              data = data, method = "ML")

BIC(model1)
```

```
model2 <- lme(y ~ time, random = ~ 1 + time | subject,
              correlation = corAR1(), data = data, method = "ML")

BIC(model2)
```