STA610 Lab06

Yuren Zhou

2024-10-04

- Write down your answers in any blank sheet and submit your work in paper during the lab.
- Your work will not be graded. As long as you submit, you will get a full credit.
- For those who missed the lab today, you can submit it via email to me for half credit.

Conditional Distributions in Linear Mixed Effect model

Consider the linear mixed effect model

$$y_{i,j} = x_{i,j}^\top \beta + z_{i,j}^\top \alpha_j + \epsilon_{i,j}$$

with $\epsilon_{i,j} \overset{iid}{\sim} N(0, \sigma^2)$ and $\alpha_j \overset{iid}{\sim} N(0, \Phi)$ ($\epsilon \perp \alpha$). We consider putting a prior distribution on β as $\beta \sim N(0, I)$. Find the following conditional expectations and variances/covariances:

- $E[y_{i,j}|x, z, \beta, \Phi, \sigma^2]$ and $Cov(y_{i,j}, y_{i',j'}|x, z, \beta, \Phi, \sigma^2)$;
- $E[\beta|z, x, y, \alpha, \sigma^2]$ and $V[\beta|z, x, y, \alpha, \sigma^2]$;
- $E[\beta|z, x, y, \Phi, \sigma^2]$ and $V[\beta|z, x, y, \Phi, \sigma^2]$;
- $E[\alpha_j|z, x, y, \beta, \Phi, \sigma^2]$ and $V[\alpha_j|z, x, y, \beta, \Phi, \sigma^2]$.

Which of the above conditional distributions are normal distributions?

Shrinkage Estimator

Consider the hierarchical model

$$y_{i,j} \overset{iid}{\sim} N(\theta_j, \sigma^2),$$

with j indicating the group index and $\theta_j \overset{iid}{\sim} N(\mu, \tau^2)$.

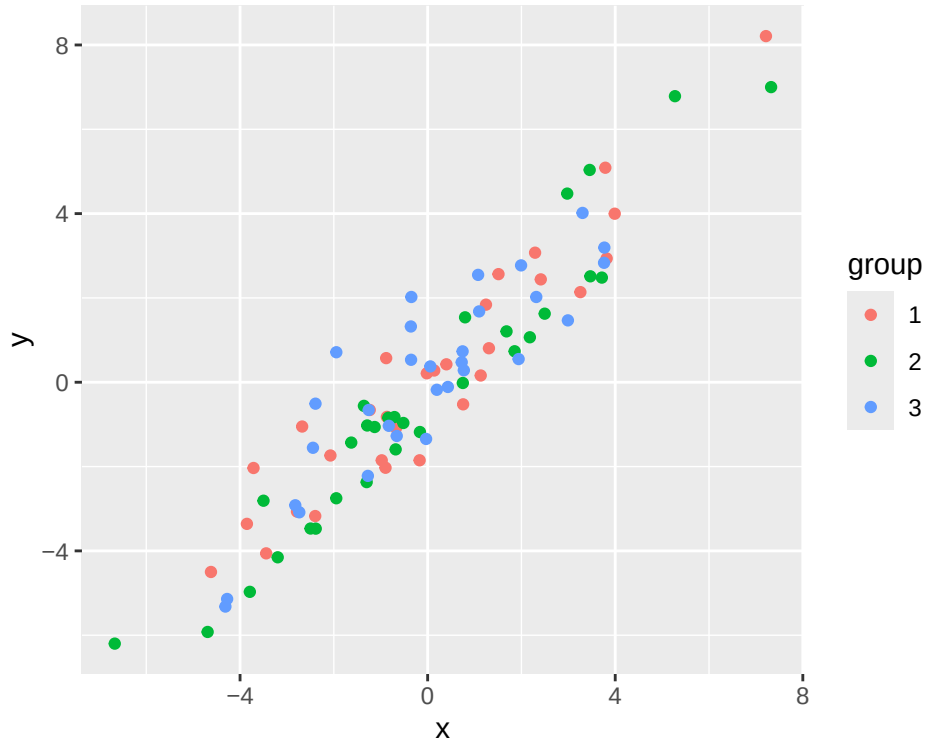
Find the conditional distribution of each $\theta_j|y, \mu, \tau^2, \sigma^2$ and let the estimator $\hat{\theta}_j := E[\theta_j|y, \mu, \tau^2, \sigma^2]$. Compare with the sample mean $\bar{\theta}_j := \frac{1}{n_j}\sum_{i=1}^{n_j} y_{i,j}$ of each group. Which estimator is unbiased? Which has smaller variance? What is the shrinkage weight of $\hat{\theta}_j$?

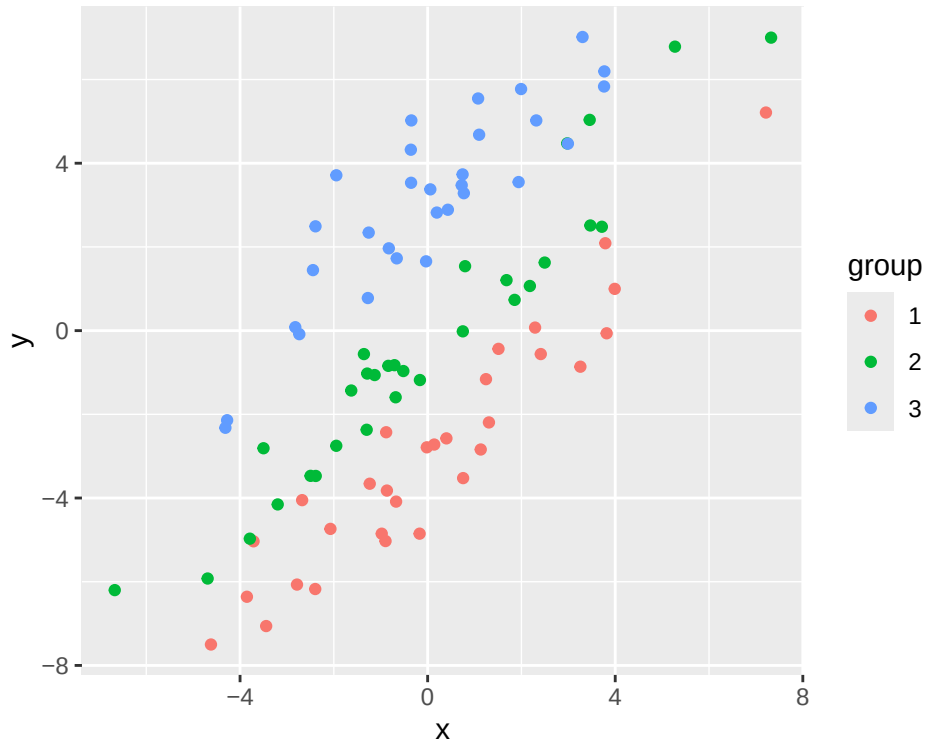
Recognize Heterogeneity Effects from Scatter Plots

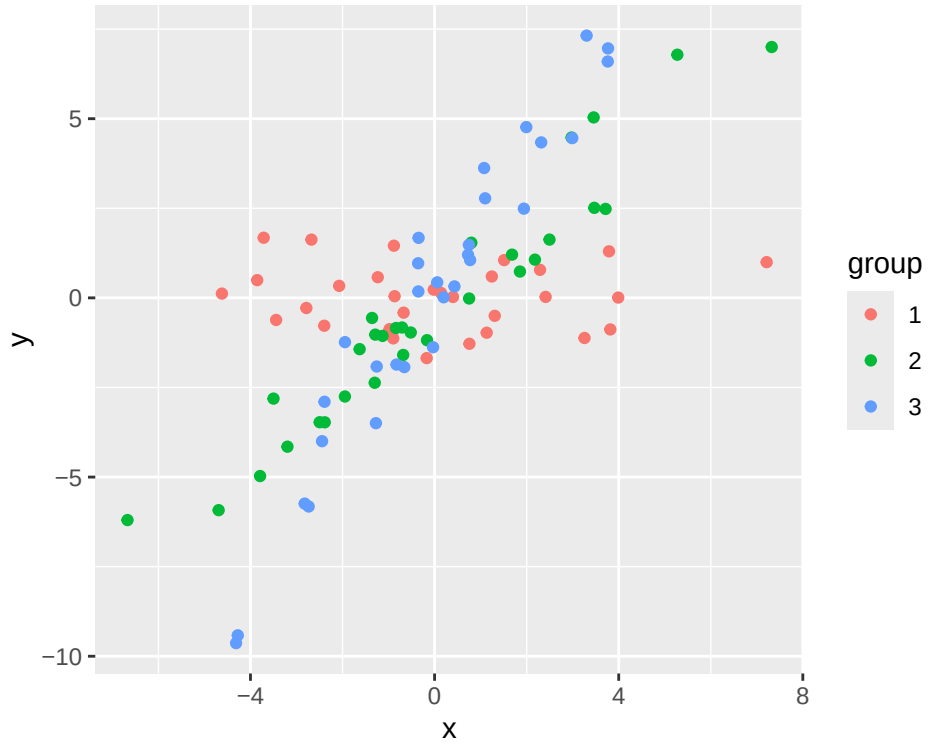
Consider data $\{(x_{i,j}, y_{i,j})\}$ with $j \in \{1, 2, 3\}$ indicating the group index. State the correct linear fixed effects model for each scatter plot of $y \sim x$.

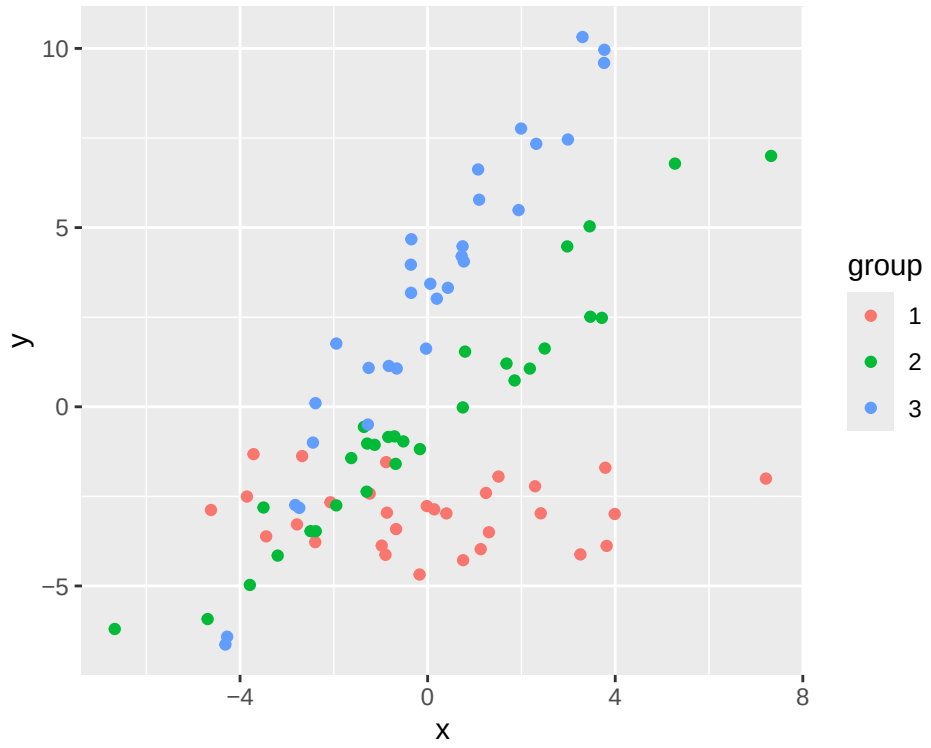
```
library(ggplot2)
library(lme4)
```

```
## Loading required package: Matrix
```




4
0
−4
−8
y
−4
0
4
8
x
group
1
2
3


5
0
−5
−10
y
−4
0
4
8
x
group
1
2
3


10
5
0
−5
y
−4
0
4
8
x
group
1
2
3