STA610 Lab03

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- Write down your answers in any blank sheet and submit your work in paper during the lab.
- Your work will not be graded. As long as you submit, you will get a full credit.
- For those who missed the lab today, you can submit it via email to me for half credit.

Least Squares Review

How can we find c such that

$$\arg\min_{c\in\mathbb{R}} \sum_{i=1}^{n}(x_i - c)^2$$

One way is to start from

$$\sum_{i=1}^{n}(x_i - c)^2 = \sum_{i=1}^{n}(x_i - \bar{x} + \bar{x} - c)^2$$

Another approach is to use calculus

$$\frac{d}{dc}\sum_{i=1}^{n}(x_i - c)^2 = 0$$

Q1-1: Compute c using both approaches.

Confidence Interval for Group Effects

```
radon<-readRDS(url("https://www2.stat.duke.edu/~pdh10/Teaching/610/Code/radonMN.rds"))
head(radon)
```

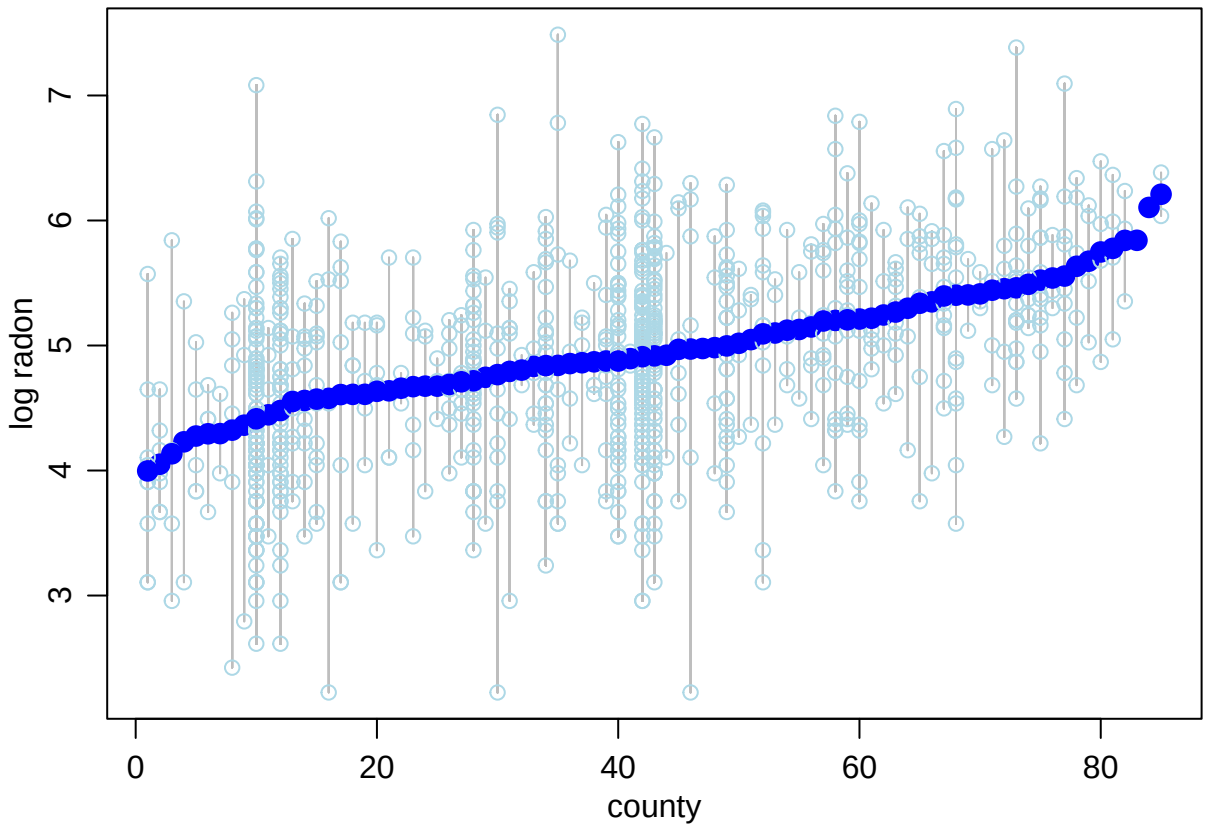
```
##      county     lon    lat     Uppm     radon
## 5081 AITKIN -93.415 46.608 0.502054  82.43790
## 5082 AITKIN -93.415 46.608 0.502054  82.43790
## 5083 AITKIN -93.415 46.608 0.502054 108.09157
## 5084 AITKIN -93.415 46.608 0.502054  39.18363
## 5085  ANOKA -93.246 45.273 0.428565 115.44118
## 5086  ANOKA -93.246 45.273 0.428565  93.41593
```

The following is the code to produce the first plot of the lecture slide s5GroupEstNP. We will modify this code to plot various confidence intervals of the group means.

```r
gdotplot<-function(y,g,xlab="group",ylab="response",mcol="blue",
                   ocol="lightblue",sortgroups=TRUE,...)
{
  m<-length(unique(g))
  rg<-rank( tapply(y,g,mean),ties.method="first")
  if(sortgroups==FALSE){ rg<-1:m ; names(rg)<-unique(g)}
  plot(c(1,m),range(y),type="n",xlab=xlab,ylab=ylab)

  for(j in unique(g))
  {
    yj<-y[g==j]
    rj<-rg[ match(as.character(j),names(rg)) ]
    nj<-length(yj)
    segments( rep(rj,nj) ,max(yj),rep(rj,nj),min(yj),col="gray")
    points( rep(rj,nj), yj,col=ocol,...)
    points(rj,mean(yj),pch=16,cex=1.5,col=mcol)
  }
}

par(mar=c(3,3,1,1), mgp=c(1.75,.75,0))
gdotplot(log(radon$radon),
         radon$county,
         xlab="county", ylab="log radon")
```



Q2-1: Identify the code lines that 1) compute the mean of each group, 2) order the

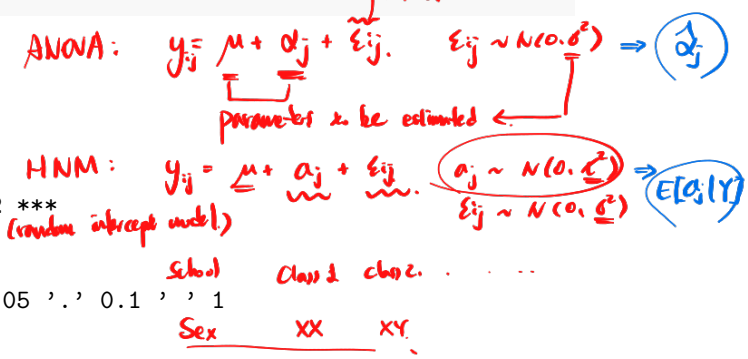
counties, 3) plot data points, 4) plot vertical lines.

Q2-2: Modify the plot so that counties are sorted according to the number of observations within county and draw a horizontal line marking the grand mean.

We first use ANOVA to evaluate statistical evidence of heterogeneity in across-county means.

```
anova(lm(log(radon) ~ county, data = radon))
```

```
## Analysis of Variance Table
##
## Response: log(radon)
##            Df Sum Sq Mean Sq F value    Pr(>F)
## county     84 126.93 1.51103  2.6406 2.981e-12 ***
## Residuals 834 477.23 0.57222
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



Now we fit the hierarchical normal model to the data. Descriptions about the package with many useful examples can be found in https://www.jstatsoft.org/article/view/v067i01.

```
library(lme4)
```

Q2-3: Write down the model in the following code and identify the parameters and the random variables with corresponding distribution.

```
mod <- lmer(log(radon) ~ 1 + (1 | county), data = radon, REML = FALSE)
summary(mod)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: log(radon) ~ 1 + (1 | county)
##    Data: radon
##
##      AIC      BIC   logLik deviance df.resid
##   2164.1   2178.5  -1079.0   2158.1      916
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6165 -0.6141  0.0292  0.6526  3.4932
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  county   (Intercept) 0.08804  0.2967
##  Residual             0.57154  0.7560
## Number of obs: 919, groups:  county, 85
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  4.94656    0.04664   106.1
```

We can extract the estimates of μ, τ and σ as below. Here, the estimate of σ is a pooled estimate.

```r
fixef(mod)
```

```
## (Intercept)
##    4.946557
```

```r
data.frame(VarCorr(mod))
```

```
##        grp        var1 var2       vcov     sdcor
## 1   county (Intercept) <NA> 0.08804027 0.2967158
## 2 Residual        <NA> <NA> 0.57153536 0.7559996
```

There are two different types of confidence interval we learned in class. One is the t-interval for group mean with pooled variance estimate:

$$\bar{y}_j \pm \frac{t_{1-\alpha/2}}{\sqrt{n_j/\hat{\sigma}^2}}$$

and the other is the Empirical Bayes interval

$$\left(\frac{\hat{\tau}^2}{\hat{\tau}^2 + \hat{\sigma}^2/n_j}\hat{\mu} + \frac{\hat{\sigma}^2/n_j}{\hat{\tau}^2 + \hat{\sigma}^2/n_j}\bar{y}_j\right) \pm \frac{t_{1-\alpha/2}}{\sqrt{n_j/\hat{\sigma}^2 + 1/\tau^2}}$$

Q2-4: Describe the differences between two intervals.

```r
# pooled estimates
muhat = fixef(mod)
sighat = data.frame(VarCorr(mod))$sdcor[1]
tauhat = data.frame(VarCorr(mod))$sdcor[2]

# group-wise statistics
ybars = aggregate(log(radon) ~ county, data = radon, mean)[,2]
njs = aggregate(log(radon) ~ county, data = radon, length)[,2]
```

Q2-5: Plot two credible intervals for each group using the code above.