STA610 Lab02

2024-09-06

Review of Expectation, Variance, and Covariance

Recall for random variables X, Y and constant c, we have

$$E[X+Y] = E[X] + E[Y], \quad E[cX] = cE[X],$$

and

$$Var[X+Y] = Var[X] + 2Cov(X,Y) + Var[Y], \quad Var[cX] = cVar[X].$$

Now let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ be random variables. Let $c_0, c_1, \ldots, c_n$ and $a_1, \ldots, a_m$ be constants. Try simplifying the following:

1. $E[c_0 + \sum_{i=1}^n c_i X_i]$;
2. $Var[c_0 + \sum_{i=1}^n c_i X_i]$;
3. $Cov(c_0 + \sum_{i=1}^n c_i X_i, \sum_{j=1}^m a_j Y_j)$.

What if each pair of X_i and X_j are independent?

What if each pair of X_i and Y_j are independent?

Further thoughts - what if X_i, Y_j are random vectors and c_i, a_j are constant vectors?

Some Comments from Office Hours on HW1 Q1a

If random variables $Y_1 \sim N(\mu_1, \sigma_1^2), \ldots, Y_n \sim N(\mu_n, \sigma_n^2)$ are independent, why does $\sum_{i=1}^n Y_i$ also follow a normal distribution?

Note: **Not** because of central limit theorem.

Using lme4 Package

Install and load the package

```
# Install lme4 package if you haven't already
if (!require(lme4)){
  install.packages("lme4")
  # Load the lme4 package
  library(lme4)
}
```

Load the dataset *wheat*

```
library(tidyverse)
URL <- "https://campus.murraystate.edu/academic/faculty/cmecklin/STA565/wheat.txt"
wheat <- read.table(URL,header=TRUE)
```

```
str(wheat)
```

```
## 'data.frame':    30 obs. of  3 variables:
##  $ variety : chr  "A" "A" "A" "A" ...
##  $ location: int  1 2 3 4 5 6 1 2 3 4 ...
##  $ yield   : num  35.3 31 32.7 36.8 37.2 33.1 33.7 32.2 31.4 32.7 ...
```

Fit a one-way ANOVA using lme4

```
model <- lmer(yield ~ 1 + (1 | location), data = wheat, REML = FALSE)
summary(model)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: yield ~ 1 + (1 | location)
##    Data: wheat
##
##      AIC      BIC   logLik deviance df.resid
##    138.7    142.9    -66.3    132.7       27
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.88282 -0.73603 -0.04646  0.77515  1.77514
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  location (Intercept) 0.7281   0.8533
##  Residual             4.3167   2.0777
## Number of obs: 30, groups:  location, 6
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)   34.183      0.515   66.38
```

Get confidence intervals:

```
confint(model)
```

```
## Computing profile confidence intervals ...
```

```
##                 2.5 %    97.5 %
## .sig01       0.000000  2.355073
## .sigma       1.604296  2.839204
## (Intercept) 32.988602 35.378064
```

How to deal with this S4 class:

```r
# Check the class of the fitted model
class(model)
```

```
## [1] "lmerMod"
## attr(,"package")
## [1] "lme4"
```

```r
# Explore the structure of the object
str(model)
```

```
## Formal class 'lmerMod' [package "lme4"] with 13 slots
##   ..@ resp   :Reference class 'lmerResp' [package "lme4"] with 9 fields
##   .. ..$ Ptr    :<externalptr>
##   .. ..$ mu     : num [1:30] 33.8 33.5 33.6 34.6 34.9 ...
##   .. ..$ offset : num [1:30] 0 0 0 0 0 0 0 0 0 0 ...
##   .. ..$ sqrtXwt: num [1:30] 1 1 1 1 1 1 1 1 1 1 ...
##   .. ..$ sqrtrwt: num [1:30] 1 1 1 1 1 1 1 1 1 1 ...
##   .. ..$ weights: num [1:30] 1 1 1 1 1 1 1 1 1 1 ...
##   .. ..$ wtres  : num [1:30] 1.52 -2.51 -0.86 2.19 2.31 ...
##   .. ..$ y      : num [1:30] 35.3 31 32.7 36.8 37.2 33.1 33.7 32.2 31.4 32.7 ...
##   .. ..$ REML   : int 0
##   .. ..and 28 methods, of which 14 are  possibly relevant:
##   .. ..  allInfo, copy#envRefClass, initialize, initialize#lmResp,
##   .. ..  initializePtr, initializePtr#lmResp, objective, ptr, ptr#lmResp,
##   .. ..  setOffset, setResp, setWeights, updateMu, wrss
##   ..@ Gp     : int [1:2] 0 6
##   ..@ call   : language lmer(formula = yield ~ 1 + (1 | location), data = wheat, REML = FALSE)
##   ..@ frame  :'data.frame':  30 obs. of  2 variables:
##   .. ..$ yield   : num [1:30] 35.3 31 32.7 36.8 37.2 33.1 33.7 32.2 31.4 32.7 ...
##   .. ..$ location: int [1:30] 1 2 3 4 5 6 1 2 3 4 ...
##   .. ..- attr(*, "terms")=Classes 'terms', 'formula'  language yield ~ 1 + (1 + location)
##   .. .. .. ..- attr(*, "variables")= language list(yield, location)
##   .. .. .. ..- attr(*, "factors")= int [1:2, 1] 0 1
##   .. .. .. .. ..- attr(*, "dimnames")=List of 2
##   .. .. .. .. .. ..$ : chr [1:2] "yield" "location"
##   .. .. .. .. .. ..$ : chr "location"
##   .. .. .. ..- attr(*, "term.labels")= chr "location"
##   .. .. .. ..- attr(*, "order")= int 1
##   .. .. .. ..- attr(*, "intercept")= int 1
##   .. .. .. ..- attr(*, "response")= int 1
##   .. .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
##   .. .. .. ..- attr(*, "predvars")= language list(yield, location)
##   .. .. .. ..- attr(*, "dataClasses")= Named chr [1:2] "numeric" "numeric"
##   .. .. .. .. ..- attr(*, "names")= chr [1:2] "yield" "location"
##   .. .. .. ..- attr(*, "predvars.fixed")= language list(yield)
##   .. .. .. ..- attr(*, "varnames.fixed")= chr "yield"
##   .. .. .. ..- attr(*, "predvars.random")= language list(yield, location)
##   .. ..- attr(*, "formula")=Class 'formula'  language yield ~ 1 + (1 | location)
##   .. .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
##   ..@ flist  :List of 1
##   .. ..$ location: Factor w/ 6 levels "1","2","3","4",..: 1 2 3 4 5 6 1 2 3 4 ...
##   .. ..- attr(*, "assign")= int 1
##   ..@ cnms   :List of 1
```

```
##   .. ..$ location: chr "(Intercept)"
##   ..@ lower  : num 0
##   ..@ theta  : num 0.411
##   ..@ beta   : num 34.2
##   ..@ u      : num [1:6] -0.984 -1.63 -1.519 1.043 1.712 ...
##   ..@ devcomp:List of 2
##   .. ..$ cmp : Named num [1:10] 3.67 2.79 117.65 11.85 129.5 ...
##   .. .. ..- attr(*, "names")= chr [1:10] "ldL2" "ldRX2" "wrss" "ussq" ...
##   .. ..$ dims: Named int [1:12] 30 30 1 29 6 1 1 1 0 0 ...
##   .. .. ..- attr(*, "names")= chr [1:12] "N" "n" "p" "nmp" ...
##   ..@ pp     :Reference class 'merPredD' [package "lme4"] with 18 fields
##   .. ..$ Lambdat:Formal class 'dgCMatrix' [package "Matrix"] with 6 slots
##   .. .. .. ..@ i       : int [1:6] 0 1 2 3 4 5
##   .. .. .. ..@ p       : int [1:7] 0 1 2 3 4 5 6
##   .. .. .. ..@ Dim     : int [1:2] 6 6
##   .. .. .. ..@ Dimnames:List of 2
##   .. .. .. .. ..$ : NULL
##   .. .. .. .. ..$ : NULL
##   .. .. .. ..@ x       : num [1:6] 0.411 0.411 0.411 0.411 0.411 ...
##   .. .. .. ..@ factors : list()
##   .. ..$ LamtUt :Formal class 'dgCMatrix' [package "Matrix"] with 6 slots
##   .. .. .. ..@ i       : int [1:30] 0 1 2 3 4 5 0 1 2 3 ...
##   .. .. .. ..@ p       : int [1:31] 0 1 2 3 4 5 6 7 8 9 ...
##   .. .. .. ..@ Dim     : int [1:2] 6 30
##   .. .. .. ..@ Dimnames:List of 2
##   .. .. .. .. ..$ : NULL
##   .. .. .. .. ..$ : chr [1:30] "1" "2" "3" "4" ...
##   .. .. .. ..@ x       : num [1:30] 0.411 0.411 0.411 0.411 0.411 ...
##   .. .. .. ..@ factors : list()
##   .. ..$ Lind   : int [1:6] 1 1 1 1 1 1
##   .. ..$ Ptr    :<externalptr>
##   .. ..$ RZX    : num [1:6, 1] 1.51 1.51 1.51 1.51 1.51 ...
##   .. ..$ Ut     :Formal class 'dgCMatrix' [package "Matrix"] with 6 slots
##   .. .. .. ..@ i       : int [1:30] 0 1 2 3 4 5 0 1 2 3 ...
##   .. .. .. ..@ p       : int [1:31] 0 1 2 3 4 5 6 7 8 9 ...
##   .. .. .. ..@ Dim     : int [1:2] 6 30
##   .. .. .. ..@ Dimnames:List of 2
##   .. .. .. .. ..$ : chr [1:6] "1" "2" "3" "4" ...
##   .. .. .. .. ..$ : chr [1:30] "1" "2" "3" "4" ...
##   .. .. .. ..@ x       : num [1:30] 1 1 1 1 1 1 1 1 1 1 ...
##   .. .. .. ..@ factors : list()
##   .. ..$ Utr    : num [1:6] 68.4 67.2 67.4 72.1 73.3 ...
##   .. ..$ V      : num [1:30, 1] 1 1 1 1 1 1 1 1 1 1 ...
##   .. ..$ VtV    : num [1, 1] 30
##   .. ..$ Vtr    : num 1026
##   .. ..$ X      : num [1:30, 1] 1 1 1 1 1 1 1 1 1 1 ...
##   .. .. ..- attr(*, "dimnames")=List of 2
##   .. .. .. ..$ : chr [1:30] "1" "2" "3" "4" ...
##   .. .. .. ..$ : chr "(Intercept)"
##   .. .. ..- attr(*, "assign")= int 0
##   .. .. ..- attr(*, "msgScaleX")= chr(0)
##   .. ..$ Xwts   : num [1:30] 1 1 1 1 1 1 1 1 1 1 ...
##   .. ..$ Zt     :Formal class 'dgCMatrix' [package "Matrix"] with 6 slots
##   .. .. .. ..@ i       : int [1:30] 0 1 2 3 4 5 0 1 2 3 ...
```

```
##   .. .. .. ..@ p       : int [1:31] 0 1 2 3 4 5 6 7 8 9 ...
##   .. .. .. ..@ Dim     : int [1:2] 6 30
##   .. .. .. ..@ Dimnames:List of 2
##   .. .. .. .. ..$ : chr [1:6] "1" "2" "3" "4" ...
##   .. .. .. .. ..$ : chr [1:30] "1" "2" "3" "4" ...
##   .. .. .. ..@ x       : num [1:30] 1 1 1 1 1 1 1 1 1 1 ...
##   .. .. .. ..@ factors : list()
##   .. ..$ beta0  : num 0
##   .. ..$ delb   : num 34.2
##   .. ..$ delu   : num [1:6] -0.984 -1.63 -1.519 1.043 1.712 ...
##   .. ..$ theta  : num 0.411
##   .. ..$ u0     : num [1:6] 0 0 0 0 0 0
##   .. ..and 45 methods, of which 31 are  possibly relevant:
##   .. ..  b, beta, CcNumer, copy#envRefClass, initialize, initializePtr,
##   .. ..  installPars, L, ldL2, ldRX2, linPred, P, ptr, RX, RXdiag, RXi,
##   .. ..  setBeta0, setDelb, setDelu, setTheta, setZt, solve, solveU, sqrL, u,
##   .. ..  unsc, updateDecomp, updateL, updateLamtUt, updateRes, updateXwts
##   ..@ optinfo:List of 8
##   .. ..$ optimizer: chr "nloptwrap"
##   .. ..$ control  :List of 1
##   .. .. ..$ print_level: num 0
##   .. ..$ derivs   :List of 2
##   .. .. ..$ gradient: num -2.3e-08
##   .. .. ..$ Hessian : num [1, 1] 23.8
##   .. ..$ conv     :List of 2
##   .. .. ..$ opt : num 0
##   .. .. ..$ lme4: list()
##   .. ..$ feval    : int 13
##   .. ..$ message  : chr "NLOPT_XTOL_REACHED: Optimization stopped because xtol_rel or xtol_abs (above
##   .. ..$ warnings : list()
##   .. ..$ val      : num 0.411
```

```
# Access the slots of the S4 object
slotNames(model)
```

```
##  [1] "resp"    "Gp"      "call"    "frame"   "flist"   "cnms"    "lower"
##  [8] "theta"   "beta"    "u"       "devcomp" "pp"      "optinfo"
```

```
# Example: Access fixed effects estimates using the @ operator
model@beta
```

```
## [1] 34.18333
```

```
# Example: Access random effects variance-covariance estimates
model@theta
```

```
## [1] 0.4106842
```

Now try again with *variety* being the group. What is the fixed effect? What is the random effect variance? And what are their 95% confidence intervals?