STA 610 Homework 6

Yuren Zhou

November 1, 2024

Question 1.

(a) (2 points)

$$\mathbb{E}[(y_{i,j,k} - \mu)^2] = \mathbb{E}[(a_k + b_{j,k} + \epsilon_{i,j,k})^2] = \tau_a^2 + \tau_b^2 + \sigma^2.$$
$$\mathbb{E}[(y_{i,j,k} - \mu - a_k)^2] = \mathbb{E}[(b_{j,k} + \epsilon_{i,j,k})^2] = \tau_b^2 + \sigma^2.$$
$$\mathbb{E}[(y_{i,j,k} - \mu - a_k - b_{j,k})^2] = \mathbb{E}[\epsilon_{i,j,k}^2] = \sigma^2.$$

(b) (3 points)

(i) For $k = k', j = j'$,

$$\mathbb{E}[(y_{i,j,k} - \mu)(y_{i',j',k'} - \mu)] = \mathbb{E}[(a_k + b_{j,k} + \epsilon_{i,j,k})(a_k + b_{j,k} + \epsilon_{i',j,k})] = \tau_a^2 + \tau_b^2.$$

(ii) For $k = k', j \neq j'$,

$$\mathbb{E}[(y_{i,j,k} - \mu)(y_{i',j',k'} - \mu)] = \mathbb{E}[(a_k + b_{j,k} + \epsilon_{i,j,k})(a_k + b_{j',k} + \epsilon_{i',j',k})] = \tau_a^2.$$

(iii) For $k \neq k', j \neq j'$,

$$\mathbb{E}[(y_{i,j,k} - \mu)(y_{i',j',k'} - \mu)] = \mathbb{E}[(a_k + b_{j,k} + \epsilon_{i,j,k})(a_{k'} + b_{j',k'} + \epsilon_{i',j',k'})] = 0.$$