STA 610 Lab 11

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- Write down your answers in any blank sheet and submit your work in paper during the lab.
- Your work will not be graded. As long as you submit, you will get a full credit.
- For those who missed the lab today, you can submit it via email to me for half credit.

Consider the hierarchical normal model with $j \in [m], i \in [n_j]$:

$$y_{i,j} = \theta_j + \epsilon_{i,j},$$

$$\theta_j \overset{iid}{\sim} N(\mu, \tau^2),$$

$$\epsilon_{i,j} \overset{iid}{\sim} N(0, \sigma^2).$$

1. Specify reasonable conjugate priors for μ, τ^2, σ^2.
2. Derive the Gibbs sampler for posterior inference of this model.
3. Implement the Gibbs sampler in R.
4. Visualize the trace plots and examine the mixing of the Gibbs sampler.
5. Plot histograms for the posterior distribution of each parameter.
6. (Optional) Derive a Metropolis-Hastings algorithm for posterior inference with appropriate proposal distributions.
7. (Optional) Implement the Metropolis-Hastings algorithm in R.
8. (Optional) Compare the effectiveness of Gibbs sampler vs. the Metropolis-Hastings algorithm.
9. (Optional) What if the model is multi-dimensional, i.e. $y_{i,j}, \theta_j, \epsilon_{i,j}$ are d-dimensional vectors?