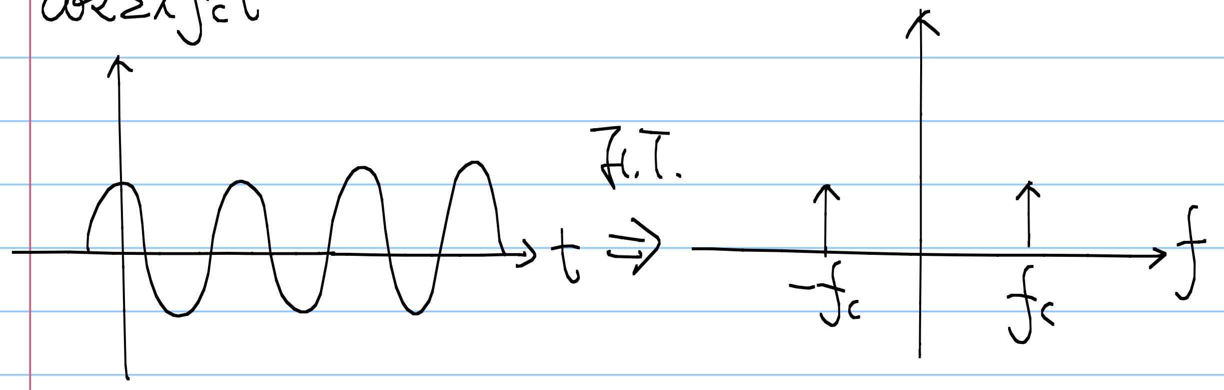
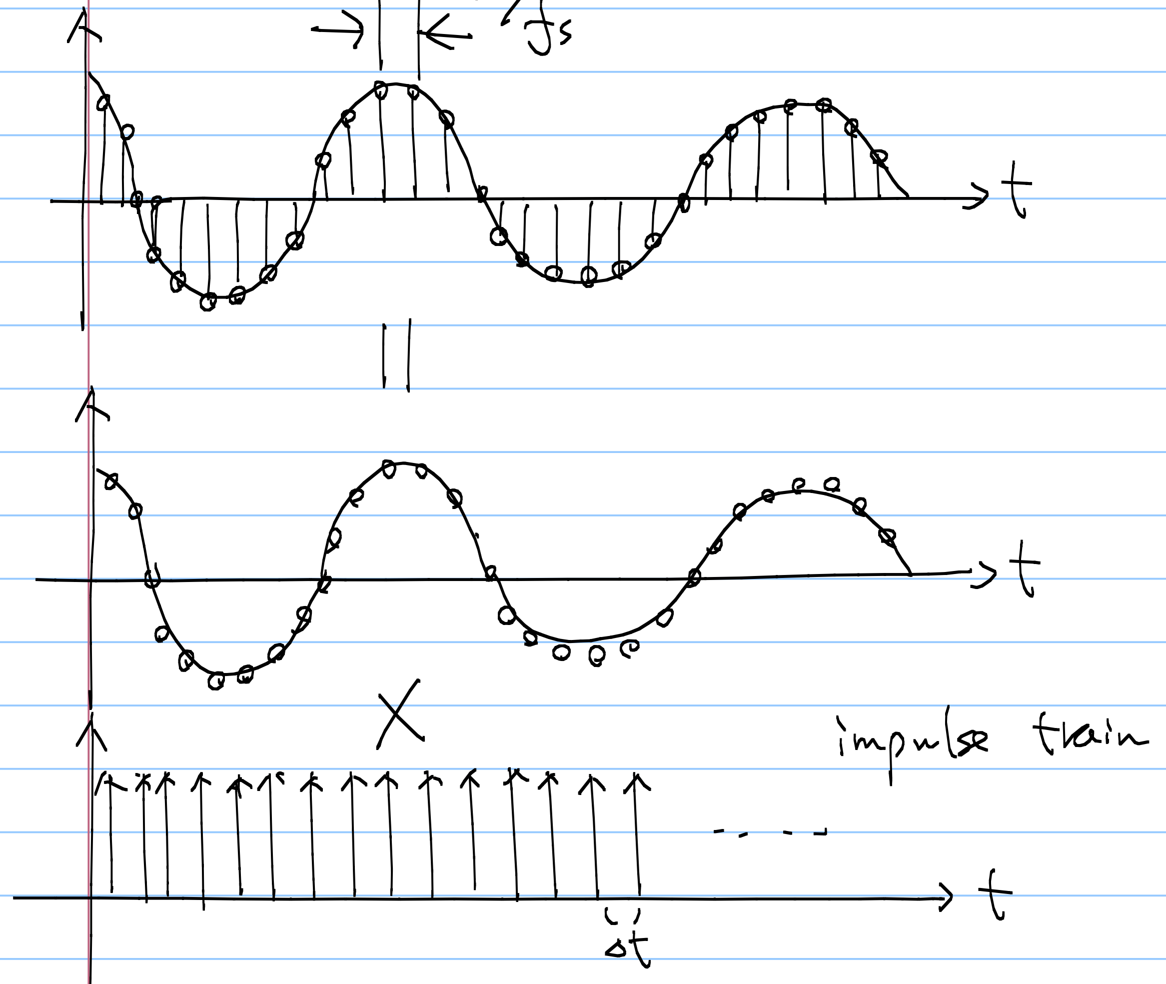
Usage of FFT (fast algorithm of DFT
DFT: discrete Fourier transform)

⇒ $\cos 2\pi f_c t$

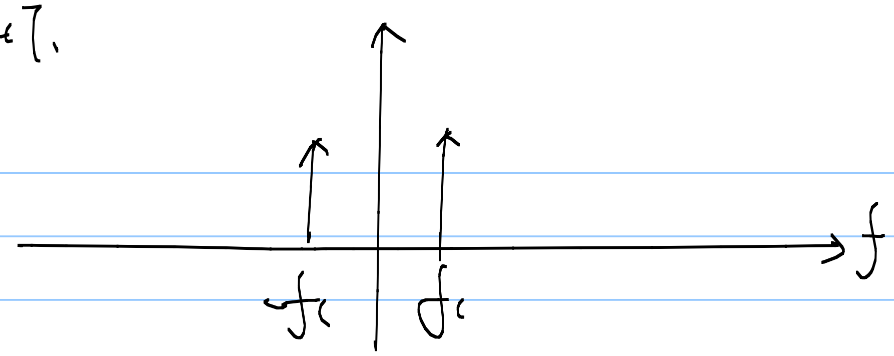


⇒ sampled $\cos 2\pi f_c t$, sampling rate: f_s, $f_s > 2 f_{max}$

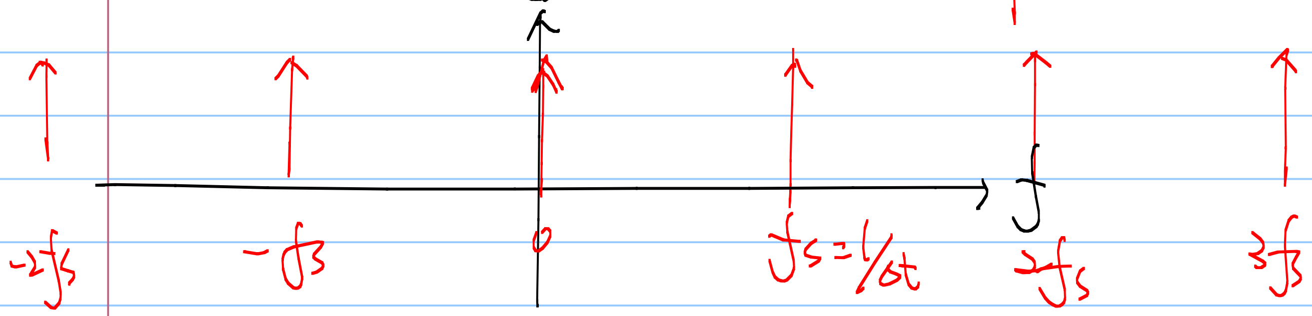
$\Delta t = 1/f_s$



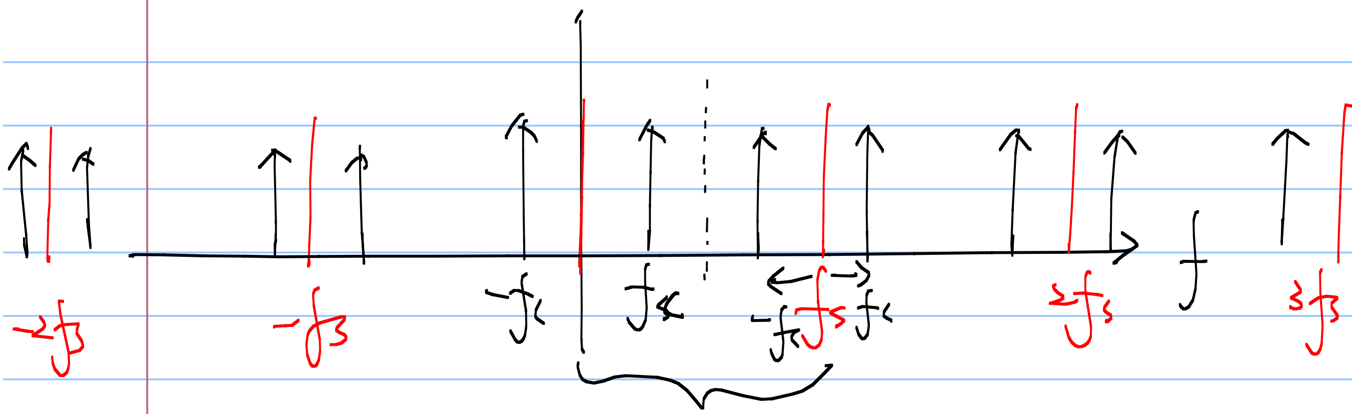
⇒ FFT.



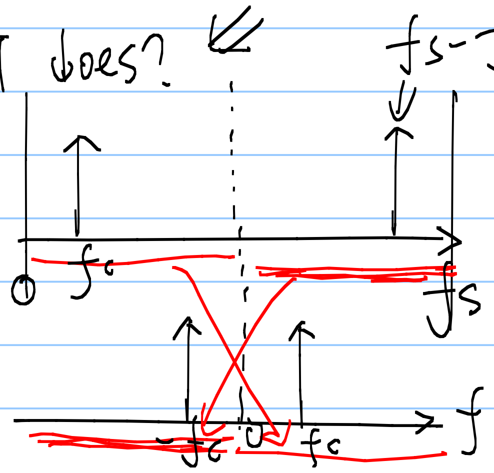
⊗ impulse train



⇓



⇒ What FFT does?



DFT : fft() (sampled DTFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}k\cdot n}$$

IDFT : ifft()

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j\frac{2\pi}{N}kn}$$

(1) vs.

DTFS

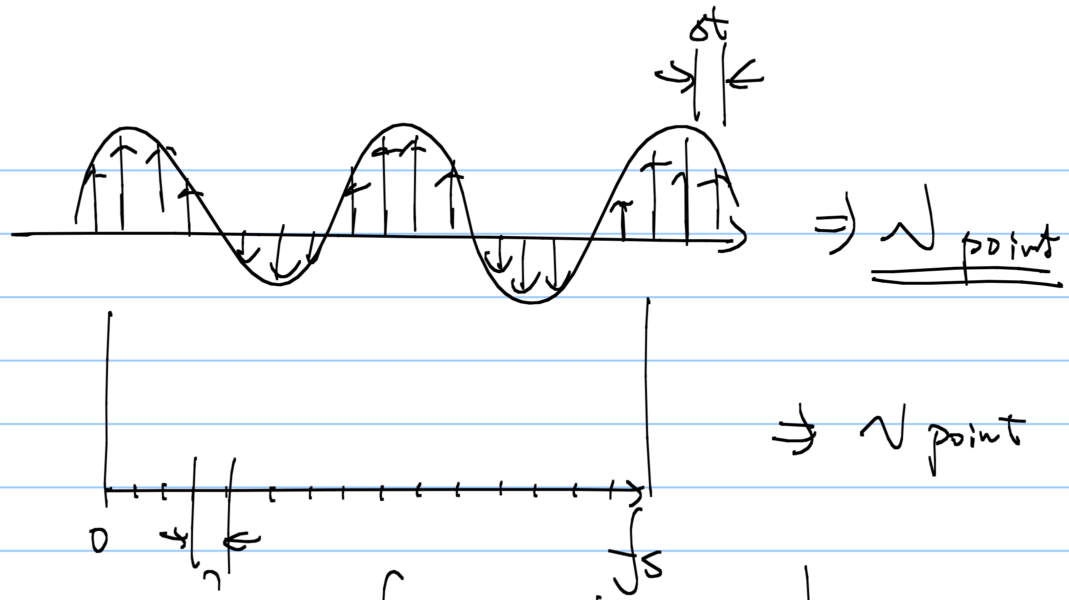
$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}k\cdot n}$$

$$x[n] = \sum_{k=0}^{N-1} a_k\, e^{+j\frac{2\pi}{N}k\cdot n}$$

(2) vs. the implemented CTFT in computer HW3.

$$X(F_k) = \sum_{n=0}^{N-1} x(nT)\cdot e^{-j2\pi F_k \cdot nT}\cdot T$$

$$F_k = \frac{F_s}{N}k \Rightarrow = \sum_{n=0}^{N-1} x(nT)\, e^{-j\frac{2\pi}{N}kn\underbrace{F_s\cdot T}_{1}}\cdot T$$



$$\hookrightarrow df = \frac{f_s}{N} = \frac{1}{N \cdot \delta t} = \frac{1}{\text{total time}}$$

Example of Matlab codes: df: freq resolution (spectral resolution, i.e., sampling interval in freq domain)

df ↓, N ↑

↳ by zero padding (to artificially create periodic signals)

e.g., fft(sampled_cos, N')

if N' > N, zero padding is done by fft().

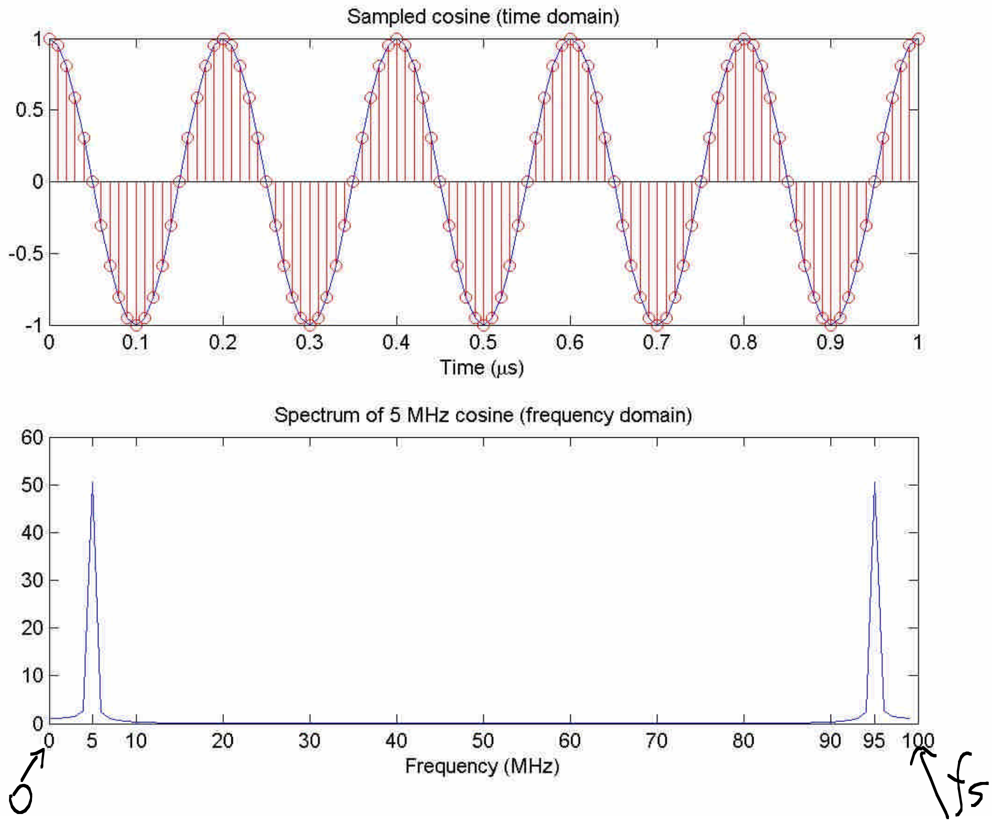
```
%
%   Matlab script - FFT Example
%                                       Edited by Meng-Lin Li, 10/05/2007
%

% Generate sampled consine
fc = 5; % in MHz
fs = 100; % in MHz
Ncycle = 5; % number of cycles of sampled cosine
dt = 1/fs;  % time resolution
t_axis = (0:dt:Ncycle/fc);  % time axis
sampled_cos = cos(2*pi*fc*t_axis);  % sampled cosine ,time domain
Npoint = length(sampled_cos);   % number of points in sampled cosine

% Fourier transform
df = fs/Npoint; % frequency resolution
f_axis = (0:1:(Npoint-1))*df;   % frequency axis
SAMPLED_COS = fft(sampled_cos); % spectrum of sampled cosine, freqeuncy domain, complex
mag_SAMPLED_COS = abs(SAMPLED_COS);   % magnitude
pha_SAMPLED_COS = angle(SAMPLED_COS); % phase

figure
subplot(2,1,1)
plot(t_axis, sampled_cos);
hold
stem(t_axis, sampled_cos,'r');
xlabel('Time (\mus)');
title('Sampled cosine (time domain)');

subplot(2,1,2)
plot(f_axis, mag_SAMPLED_COS);
xlabel('Frequency (MHz)');
title('Spectrum of 5 MHz cosine (frequency domain)')
set(gca,'Xtick',[0 5 10 20 30 40 50 60 70 80 90 95 100]);
print -djpeg fft_example.jpg
```



⇒ how about fftshift(mag_Sampled_COS)?