Seminar 7. Static and dynamic games

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We choose static game with complete information as our benchmark because it's the simplest game. More complex games need stricter conditions to locate the equilibria, i.e. refinement (rule out unrealistic NE).

- Be sure to know which type the game is before answering the questions!

Complete Information

Players have the symmetric information (no private information)
know clearly which "type" their opponents are.

Static (Watson Part II)

Players act simultaneously and independently.

- Rationality & Nash Equilibrium (NE)
- Pure/Mixed strategy
- Repeated game & trigger strategy

Dynamic (Watson Part III)

Empty threat: not every NE are realistic, refinement needed.

- Subgame Perfect Nash Equilibrium (SPNE)
- Imperfect innformation: examine every subgame by hand (Watson pp. 190)
- Perfect information: backward-induction method

Incomplete Information (Watson Part IV)

At least one player does not know which type its opponents are.

- Exogenous move: nature decides player's type.
- Players without private information can only "guess" (assign probability/belief to) the type of its opponent.

Static (Watson pp.327 - 377)

- Bayesian Nash Equilibrium (BNE)
- Bayesian normal form

Dynamic(Watson pp.378 - 406)

More information can be obtained from the opponent's behavior.
$\Rightarrow$ Probability (belief) can be adjusted dynamically (updated).

- Perfect Bayesian (Nash) Equilibrium (PBE)
- Screening: player **without** private information move first (e.g. Insurance scheme to screen risky clients; Contract to rule out low-capacity workers).
- Signaling: player **with** private information move first (High-risk clients pretend to be of low-risk).
- Pooling & Separating equilibrium.

Many students treated a dynamic game as static last year...
Narrative like "player 2 acts before/after player 1" is definitely dynamic. You must refine the NE properly!

- Correctly drawn extensive form for complex dynamic games may gain some points :)
- Do more exercises on screening and signaling games until you can solve it by yourself. It may take 20 points in exam.

1 Problem 1 - Simultaneous and sequential moves with complete information

You and a friend are in a restaurant, and the owner offers both of you an 8-slice pizza under the following condition. Each of you must simultaneously announce how many slices you would like; that is, each player $i \in \{1,2\}$ names his/her desired amount of pizza, $0 \leq s_i \leq 8$. If $s_1 + s_2 \leq 8$, then the players get their demands (and the owner eats any leftover slices). If $s_1 + s_2 > 8$, then the players get nothing. Assume that you each care only about how much pizza you individually consume, preferring more pizza to less.

1.1 What is (are) each player's best response(s) for each of the possible demands for his/her opponent?

Best response set if opponent chooses 0: $\{8\}$
Best response set if opponent chooses 1: $\{7\}$
Best response set if opponent chooses 2: $\{6\}$
Best response set if opponent chooses 3: $\{5\}$
Best response set if opponent chooses 4: $\{4\}$
Best response set if opponent chooses 5: $\{3\}$
Best response set if opponent chooses 6: $\{2\}$
Best response set if opponent chooses 7: $\{1\}$
Best response set if opponent chooses 8: $\{0,1,2,3,4,5,6,7,8\}$

		Player 2								
		0	1	2	3	4	5	6	7	8
	0	(0,0)	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)	(0,8)
	1	(1,0)	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(0,0)
	2	(2,0)	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(0,0)	(0,0)
	3	(3,0)	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(0,0)	(0,0)	(0,0)
Player 1	4	(4,0)	(4,1)	(4,2)	(4,3)	(4,4)	(0,0)	(0,0)	(0,0)	(0,0)
	5	(5,0)	(5,1)	(5,2)	(5,3)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
	6	(6,0)	(6,1)	(6,2)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
	7	(7,0)	(7,1)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
	8	(8,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

1.2 Find all the pure-strategy Nash equilibria

$(0,8),(1,7),(2,6),(3,5),(4,4),(5,3),(6,2),(7,1),(8,0),(8,8)$
Reconsider the situation above, but assume now that player 1 makes her demand before player 2 makes his demand. Player 2 observes player 1's demand before making his choice.

1.3 Explain what a strategy is for player 2 in this game with sequential moves.

Determines a choice for player 2 for each possible choice for player 1. Player 2 has 9^9 strategies.

1.4 Find all the pure-strategy Nash equilibrium outcomes.

(1) $s_1^* \in \{0,1,2,3,4,5,6,7,8\}$ *and*
$s_2^*(s_1) = 8 - s_1$ *if* $s_1 = s_1^*$,
$s_2^*(s_1) > 8 - s_1$ *if* $s_1 > s_1^*$,
$s_2^*(s_1) \in \{0,1,2,3,4,5,6,7,8\}$ *if* $s_1 < s_1^*$.
Example: $(4,(8,7,6,5,4,4,4,4,4))$. *Here player 2 demands the pieces that are left if player 1 does not demand more than 4 pieces, but demands 4 pieces if player 1 demands more than 4 pieces. To demand 4 pieces is a best response for player 1, given that he will not get anything if demands more than 4 pieces. It is a best response for player 2, given that player 1 demands 4 pieces, as his strategy specifies.*
(2) $s_1^* = 8$ *and*
$s_2^*(s_1) > 8 - s_1$ *if* $s_1 \in \{1,2,3,4,5,6,7,8\}$,
$s_2^*(s_1) \in \{0,1,2,3,4,5,6,7,8\}$ *if* $s_1 = 0$.
Eksempel: $(8,(8,8,8,8,8,8,8,8,8))$. *Here player 2 demands all the 8 pieces independently of what player 1 demands. To demand all the 8 pieces is a best response for player 1, given that he will not get anything anyway. It is a best response for player 2, given that player 1 demands all the 8 pieces, as his strategy specifies.*

1.5 Find all the pure-strategy subgame perfect equilibria.

(1) $s_1^* = 7$ *and*
$s_2^*(s_1) = 8 - s_1$ *if* $s_1 \in \{0,1,2,3,4,5,6,7\}$,
$s_2^*(s_1) > 8 - s_1$ *if* $s_1 = 8$.
Example: $(7,(8,7,6,5,4,3,2,1,1))$. *Here player 2 demands the pieces that are left if player 1 demands less that all the 8 pieces, but demands 1 piece if player 1 demands all 8 pieces. This is a best response for player 2, not only if player 1 demands 7 pieces, as his strategy specifies, but also for all other choices that player 1 might do. To demand 7 pieces is a best response for player 1, given that he will not get anything if he demands all the 8 pieces.*
(2) $s_1^* = 8$ *and*
$s_2^*(s_1) = 8 - s_1$ *if* $s_1 \in \{0,1,2,3,4,5,6,7,8\}$.
That is: $(8,(8,7,6,5,4,3,2,1,0))$. *Here player 2 requires the pieces that are left. This is a best response for player 2, not only if player 1 demands all the 8 pieces, as his strategy specifies, but also for all other choices that player 1 might do. To demand all the 8 pieces is a best response for player 1.*

2 Problem 2 - Best response sets

Exercise 6.4
For the game of Figure 6.2 (Watson pp.55), determine the following best-response sets.

FIGURE 6.2 (Watson pp. 55)
An example of best response.

1 \ 2	L	C	R
U	2, 6	0, 4	4, 4
M	3, 3	0, 0	1, 5
D	1, 1	3, 5	2, 3

2.1 $BR_1(\theta_2)$ for $\theta_2 = (1/6, 1/3, 1/2)$

2.2 $BR_2(\theta_1)$ for $\theta_1 = (1/6, 1/3, 1/2)$

2.3 $BR_1(\theta_2)$ for $\theta_2 = (1/4, 1/8, 5/8)$

2.4 $BR_1(\theta_2)$ for $\theta_2 = (1/3, 1/3, 1/3)$

2.5 $BR_2(\theta_1)$ for $\theta_1 = (1/2, 1/2, 0)$

(a) $\{U\}$
(b) $\{R\}$
(c) $\{U\}$
(d) $\{U, D\}$
(e) $\{L, R\}$

3 Problem 3

(Best response functions, Nash equilibria, rationalizable strategies)
Watson Exercise 9.6
Consider a game in which, simultaneously, player 1 selects any real number x and player 2 selects any real number y. The payoffs are given by:

$$u_1(x, y) = 2x - x^2 + 2xy$$

$$u_2(x, y) = 10y - 2xy - y^2$$

3.1 Calculate and graph each players best-response function as a function of the opposing players pure strategy.

(a) $BR_1(y) = 1 + y$, $BR_2(x) = 5 - x$.

3.2 Find and report the Nash equilibria of the game.

(b) $(3,2)$.

3.3 Determine the rationalizable strategy profiles for this game.

For each player the set of rationalizable strategies equals $(-\infty, \infty)$.

4 Problem 4 - True or False?

For each of the statements, if true, try to explain why, and if false, provide a counter-example.

(a) In a finite extensive-form game of perfect information, there always exists a subgame perfect Nash equilibrium.

True. A subgame-perfect Nash equilibrium can be constructed by using backward induction.

(b) In a finite extensive-form game of perfect information, there always exists a unique subgame perfect Nash equilibrium.

False. Let player 1 choose 'out', leading to the payoff vector $(1,1)$ or 'in', whereafter player 2 can choose 'a' leading to the payoff vector $(2,2)$, or 'b' leading to the payoff vector $(0,2)$. Verify that there are two subgame-perfect Nash equilibria depending on what player 2 does if player 1 chooses 'in'.

5 Problem 5- Firm-union bargaining

A firm's output is $L(100 - L)$ when it uses $L \leq 50$ units of labor, and 2500 when it uses $L \geq 50$ units of labor. The price of output is 1. A union that represents workers presents a wage demand (a nonnegative number w), which the firm either accepts or rejects. If the firm accepts the demand, it chooses the number L of workers to employ (which you should take to be a continuous variable, not an integer); if it rejects the demand, no production takes place ($L = 0$). The firm's preferences are represented by its profits, the union's preferences are represented by the value of wL.

5.1 Formulate this situation as an extensive game with perfect information.

Players: $N = \{U, F\}$.
Strategies: U chooses a wage w from the set of non-negative number; F chooses a function that to any non-negative wage w determines a non-negative employment $L(w)$.
Payoffs: The union's payoff is $wL(w)$; the firm's payoff is $L(w)(100 - L(w)) - wL(w)$. No need to have a specific accept/reject decision at $L(w) = 0$ is in effect a rejection by the firm of the demand w, giving both a payoff of 0.

5.2 Find the subgame perfect equilibrium (equilibria?) of the game.

Maximizing the firm's payoff yields $L(w) = \frac{100-w}{2}$ for $w \leq 100$ and $L(w) = 0$ otherwise.
The union's best response to this strategy is setting $w = 50$.

5.3 Is there an outcome of the game that both parties prefer to any subgame perfect equilibrium outcome?

The subgame-perfect equilibrium outcome is $w = 50$ and $L = 25$, yielding a payoff of 1250 for the union and 625 for the firm. Joint surplus is maximized for $L = 50$, yielding a maximized total surplus of 2500. At this employment level, any wage w between 25 and 37.5 would lead to a Pareto-improvement.

5.4 Find a Nash equilibrium for which the outcome differs from any subgame perfect equilbrium outcome.

Consider $L(w) = \frac{100-w}{2}$ for $w \leq 20$ and $L(w) = 0$ otherwise. Then the union's best response is $w = 20$, leading to the employment $L = 40$ and the payoffs 800 for the union and 1600 for the firm. This is a Nash equilibrium, but it is not subgame-perfect.