Seminar 10. Incomplete Information in Dynamic Games

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1 Problem 1 - Screening and signaling

Consider again the strategic situation described in Problem 1 of the set for the nineth seminar,

Game 1

	L	R
U	$0,0$	$4,2$
D	$2,6$	$0,8$

Game 2

	L	R
U'	$0,2$	$0,0$
D'	$2,0$	$2,2$

where only player 1 knows which game is being played, while player 2 thinks that the two games are **equally likely**.

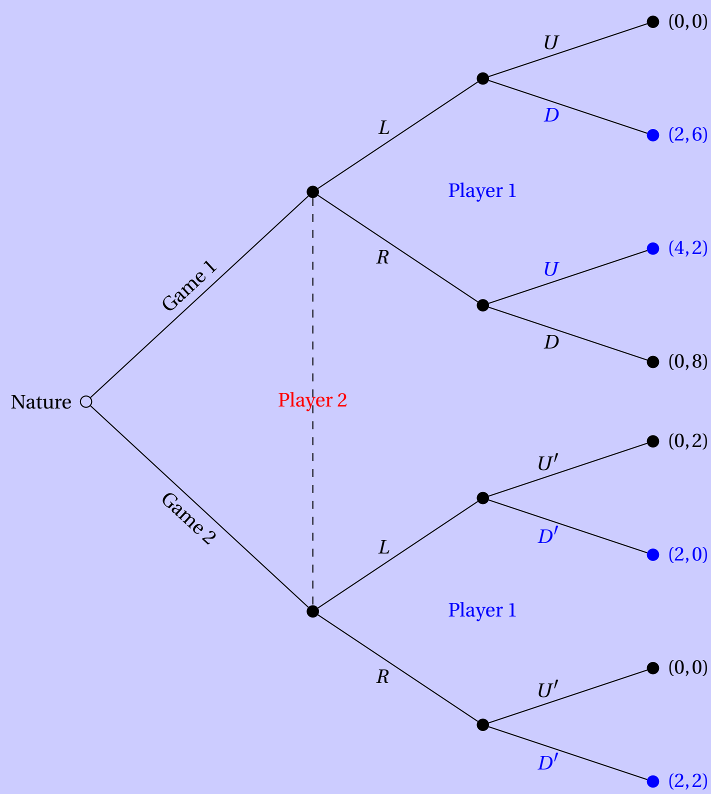
1.1 (a) Screening

Assume now that **player 2 acts before player 1**, and that 2's choice can be observed by 1 before he makes his choice. Show that there is a unique subgame perfect Nash equilibrium.

Screening: Player **without** private information moves first ⇒ there is nothing to infer (since player 1 knows everything, and player 2 has no chance to infer anything)

- Player 1 has private information: contingent strategy
- Player 2 acts before player 1: can only choose between L and R

Extensive form:



Player 1 acts contingently; Player 2 acts according to the expected payoff based on some belief $(\frac{1}{2}, \frac{1}{2})$;

- Note: use backward induction method to calculate player 2's expected payoff (payoff in blue)
- We can also find when Game 2 is played, for player 1, D' dominates U'

The contingent strategy for player 1 is easy to express, since there is no incomplete information.

The strategy of player 1:

- If Game 1 is played:
  - when player 2 chooses L, player 1 chooses D
  - when player 2 chooses R, player 1 chooses U
- If Game 2 is played:
  - when player 2 chooses L, player 1 chooses D'

– when player 2 chooses R, player 1 chooses D'

For player 2:

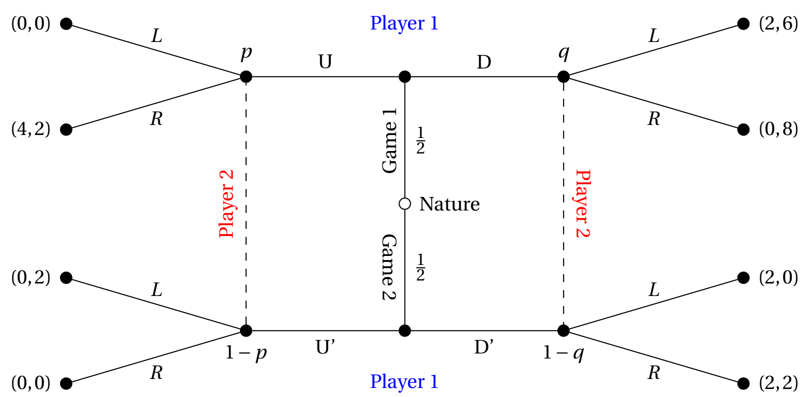
$$E(U_L^2) = \frac{1}{2} \times 6 + \frac{1}{2} \times 0 = 3$$
$$E(U_R^2) = \frac{1}{2} \times 2 + \frac{1}{2} \times 2 = 2$$
$$\Rightarrow E(U_L^2) > E(U_R^2)$$

The strategy of player 2 is to choose L.

Therefore there is only one SPNE:
{(For player 1) In Game 1: D after L, U after R. In Game 2, D' after L, D' after R ; (For player 2) L}

1.2 (b) Signaling

Assume now that player 1 acts before player 2, and that 1's choice can be observed by 2 before she makes her choice. Show that there is a unique separating perfect Bayesian equilibrium.



2 Problem 2 - Sequential moves and incomplete information; Perfect Bayesian equilibrium

Consider the situation of Problem 1 of the eighth seminar, but assume now in addition that the pizza comes in 5 different sizes, each with x slices, where $x \in \{4, 6, 8, 10, 12\}$. Player 1 observes x before making her demand, while players 2 only observes player 1's demand, but not x, before having to make his own demand. Before observing player 1's demand, player 2 thinks that the 5 different pizza sizes are equally likely, but he may infer something from her demand.

2.1 (a) Explain what a strategy is for player 1 in this game of incomplete information.

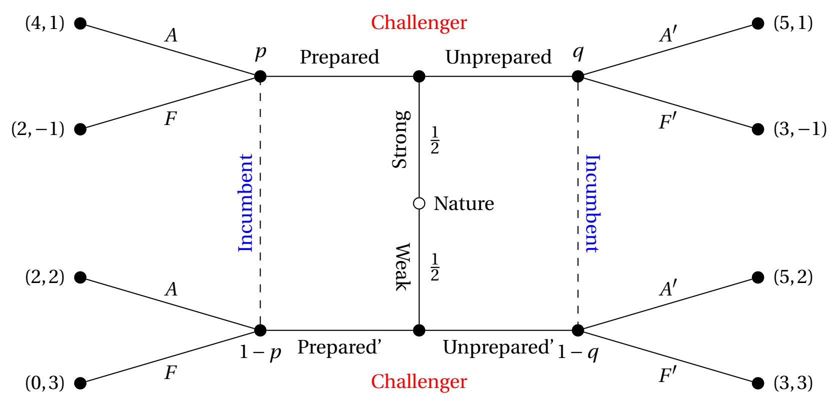
2.2 (b) perfect Bayesian equilibrium

Show that the following strategy for player 1 can be part of a perfect Bayesian equilibrium: $s_1(4) = 2$, $s_1(6) = 3$, $s_1(8) = 4$, $s_1(10) = 5$, $s_1(12) = 11$. Specify both player 2's strategy and player 2's beliefs.

2.3 (c) Are there other perfect Bayesian equilibria in this game?

3 Problem 3 - Challenging an incumbent

Consider a market where there is an incumbent firm and a challenger. The challenger is *strong* with probability $\frac{1}{2}$ and *weak* with probability $\frac{1}{2}$; it knows its type, but the incumbent does not. The challenger may either *prepare* itself for battle or remain *unprepared*. The incumbent observes the challenger's preparedness, but not its type, and chooses whether to *fight* (F) or *acquiesce* (A). The extensive form and the payoffs are given by the following figure. The challenger's payoff is listed first, the incumbent's second.



3.1 (a) What are the (pure) strategies for the challenger?

3.2 (b) Why is there no perfect Bayesian equilibrium where the weak challenger chooses *Prepared′* ?

3.3 (c) Separating

Show that there is a perfect Bayesian equilibrium where the strong challenger chooses *Prepared* and the weak challenger chooses *Unprepared′*.

3.4 (d) Pooling

Show that there is a perfect Bayesian equilibrium where the strong challenger chooses *Unprepared* and the weak challenger chooses *Unprepared*$'$. What do we call such an equilibrium?