Seminar 2 - Expenditure function

Xiaoguang Ling
xiaoguang.ling@econ.uio.no

September 8, 2020

1 Jehle & Reny 1.38 - Properties of the Expenditure Function

Verify that the expenditure function obtained from the CES direct utility function in Example 1.3 (JR. pp.39) satisfies all the properties given in Theorem 1.7 (JR. pp.37).

Expenditure Function (JR. pp.35)

We define the expenditure function as theminimum-value function:

$$e(p,u) \equiv \min_{x \in \mathbb{R}^n_+} p \cdot x$$

Expenditure Function of CES direct utility function (JR. pp.39)

In Example 1.3 (JR. pp.39), we have a so called CES direct utility function:

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}},\ where\ 0 \neq \rho < 1.$$

To derive the Expenditure Function, we need to solve the expenditure minimisation problem given some utility u. i.e.

$$\min_{x_1, x_2} p_1 x_1 + p_2 x_2 \ \ s.t.\ \ u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}} = u,\ x_1 \leq 0,\ x_2 \leq 0$$

(Read JR. pp.40 to see how to minimize the expenditure with Lagrangian method.)

The solution to the expenditure-minimisation problem is called the consumer's vector of **Hicksian demands**:

$$\begin{cases} x_1^h(p_1, p_2, u) = u(p_1^r + p_2^r)^{\frac{1}{r}-1} p_1^(r-1) \\ x_1^h(p_1, p_2, u) = u(p_1^r + p_2^r)^{\frac{1}{r}-1} p_2^(r-1) \end{cases} \tag{1}$$

Here $r \equiv \frac{\rho}{\rho - 1}$.
Substitute the solution above (Equation 1) into our objective function $p_1 x_1 + p_2 x_2$ to obtain the Expenditure Function:

$$e(p_1,p_2,u) = p_1 x_1^h(p_1,p_2,u) + p_2 x_2^h(p_1,p_2,u) = u(p_1^r + p_2^r)^{\frac{1}{r}}, r \equiv \frac{\rho}{\rho-1}$$

THEOREM 1.7 Properties of the Expenditure Function (JR. pp.37)

If $u(.)$ is continuous and strictly increasing, then $e(p,u)$ defined in (1.14) is

1. Zero when u takes on the lowest level of utility in $\mathscr{U}$,
2. Continuous on its domain $\mathbb{R}^n_{++} \times \mathscr{U}$,
3. For all $p \gg 0$, strictly increasing and unbounded above in u,
4. Increasing in p,
5. Homogeneous of degree 1 in p,
6. Concave in p.

If, in addition, $u(.)$ is strictly quasiconcave, we have

7. Shephard's lemma: $e(p,u)$ is differentiable in p at (p^0,u^0) with $p^0 \gg 0$, and
$$\frac{\partial e(p^0,u^0)}{\partial p_i} = x_i^h(p^0,u^0), \quad i = 1,...,n.$$

Firstly we should know that the CES direct utility function $u(x_1,x_2) = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}}$, $where\, 0 \neq \rho < 1$. is continuous and strictly increasing (can be proved similarly as below), which is the prerequisite of Theorem 1.7.

1.1 $e(p,u)$ is zero when u takes on the lowest level of utility in $\mathscr{U}$

As usual, we assume non-negative consumption, i.e. $x \in R^n_+$. Since $u(.)$ is strictly increasing, $u_{min} = u(0,0) = 0$.
When $u = 0$, we have $e(p,0) = 0 \times (p_1^r + p_2^r)^{\frac{1}{r}} = 0$.
no consumption, no cost

1.2 $e(p,u)$ is continuous on its domain $\mathbb{R}^n_{++} \times \mathscr{U}$,

We can try to take all the (partial) derivatives of function $e(p,u)$ and see if they exist on its domain. If they exist, it's differenciable and thus continuous.

1.3 **For all $p \gg 0$, $e(p,u)$ is strictly increasing and unbounded above in u,**

1.4 **$e(p,u)$ is increasing in p,**

1.5 **$e(p,u)$ is omogeneous of degree 1 in p**

1.6 **$e(p,u)$ is oncave in p.**

1.7 **If $u(.)$ is also strictly quasiconcave, we have Shephard's lemma: $e(p,u)$ is differentiable in p at (p^0, u^0) with $p^0 \gg 0$, and $\frac{\partial e(p^0,u^0)}{\partial p_i} = x_i^h(p^0,u^0), \quad i = 1,...,n$.**

2 Jehle & Reny 1.44 - Inferior and Normal goods

In a two-good case, show that if one good is inferior, the other good must be normal.

3 Jehle & Reny 1.51 - Substitues and Complements

Consider the utility function, $u(x_1, x_2) = (x_1)^{1/2} + (x_2)^{1/2}$.

a. Compute the demand functions, $x_i(p_1, p_2, y),\ i = 1,2$.

b. Compute the substitution term in the Slutsky equation for the effects on x_1 of changes in p_2.

c. Classify x_1 and x_2 as (gross) complements or substitutes.

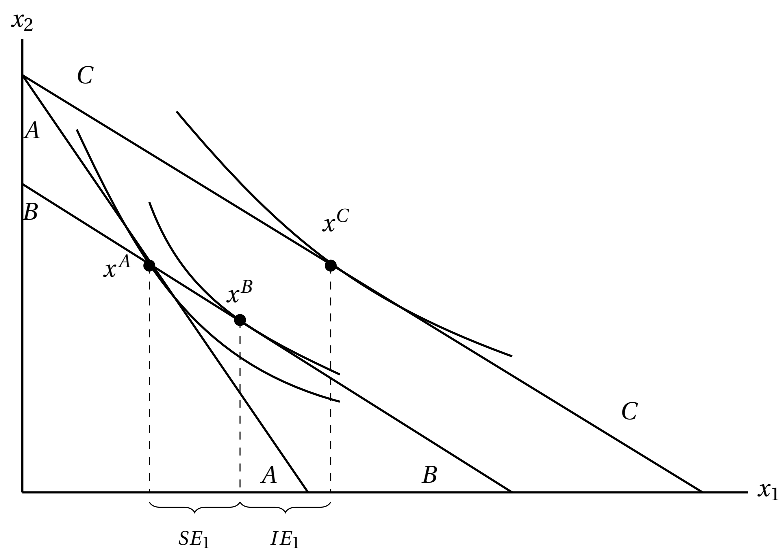

Figure 1: Slutsky decomposition