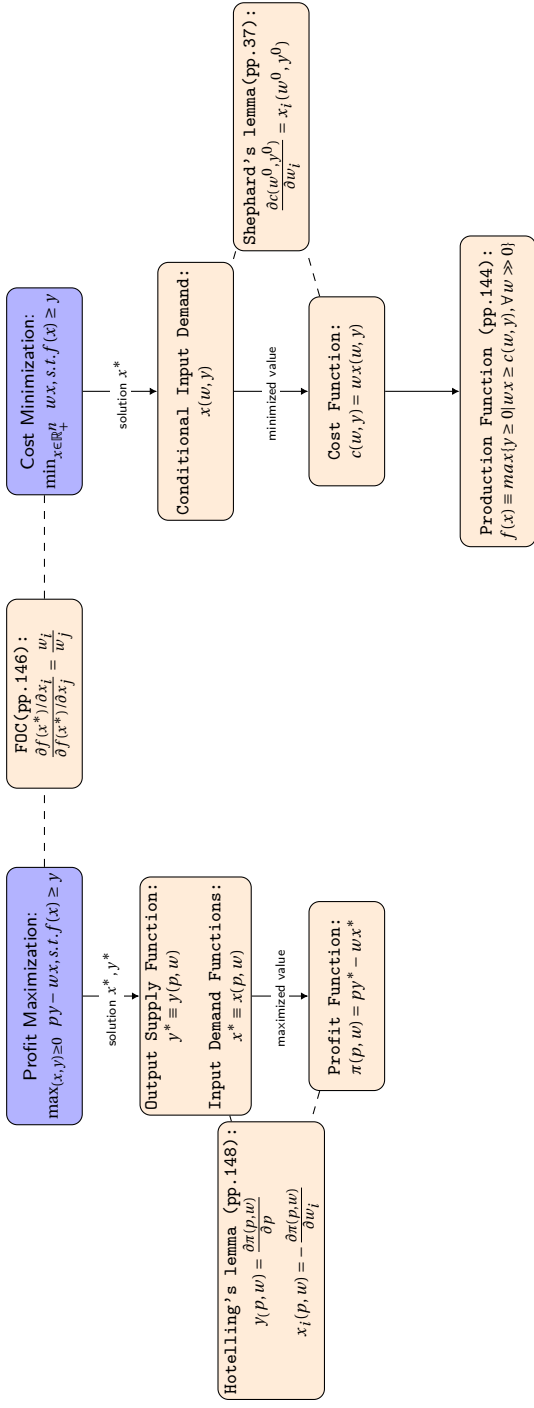


Seminar 5. Production Theory

Xiaoguang Ling
xiaoguang.ling@econ.uio.no

September 26, 2020

Production Duality



1 Jehle & Reny 3.35

Calculate the **cost function** and the **conditional input demands** for the linear production function, $y = \sum_{i=1}^n \alpha_i x_i$.

Production Function(Jehle & Reny pp.127)

We use a function $y = f(x)$ to denote y units of a certain commodity is produced using input x , where $x \in \mathbb{R}_+^n, y \in \mathbb{R}_+^1$

ASSUMPTION 3.1 Properties of the Production Function (Jehle & Reny pp.127)

The production function, $f : \mathbb{R}_+^n \rightarrow \mathbb{R}_+$, is continuous, strictly increasing, and strictly quasiconcave on \mathbb{R}_+^n , and $f(0) = 0$.

DEFINITION 3.5 The Cost Function (Jehle & Reny pp.136)

The cost function, defined for all input prices $w \gg 0$ and all output levels $y \in f(\mathbb{R}_+^n)$ is the minimum-value function,

$$c(w, y) \equiv \min_{x \in \mathbb{R}_+^n} w \cdot x, \text{ s.t. } f(x) \geq y.$$

The solution $x(w, y)$ is referred to as the firms **conditional input demand**, because it is conditional on the level of output y .

- **Conditional input demand** is similar to Hicksian demands for consumers
- The difference is that cost minimization may not lead to profit maximization.

2 Jehle & Reny 3.46

- Verify Theorem 3.7 for the profit function obtained in Example 3.5.
- Verify Theorem 3.8 for the associated output supply and input demand functions.

DEFINITION 3.7 The Profit Function (Jehle & Reny pp.148)

The firms profit function depends only on input and output prices and is defined as the maximum-value function,

THEOREM 3.7 Properties of the Profit Function (Jehle & Reny pp.148)

THEOREM 3.8 Properties of Output Supply and Input Demand Functions (Jehle & Reny pp.149)

3 Jehle & Reny 3.49

1. Derive the **cost function** for the production function in Example 3.5.

2. Solve $\max_y py - c(w, y)$
3. Compare its solution, $y(p, w)$, to the solution in (E.5). Check that $\pi(p, w) = py(p, w) - c(w, y(p, w))$.
4. Supposing that $\beta > 1$, confirm our conclusion that profits are minimised when the first-order conditions are satisfied by showing that marginal cost is decreasing at the solution.
5. Sketch your results.