Seminar 12. Auctions and previous exam problems

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1 Jehle & Reny pp.484, exercise 9.2

Show in two ways that the symmetric equilibrium bidding strategy of a first-price auction with N symmetric bidders each with values distributed according to F, can be written as

$$\hat{b}(v) = v - \int_0^v \left(\frac{F(x)}{F(v)}\right)^{N-1} dx$$

For the first way, use our solution from the text and apply integration by parts. For the second way, use the fact that $F^{N-1}(r)(v - \hat{b}(r))$ is maximised in r when $r = v$ and then apply the envelope theorem to conclude that $d(F^{N-1}(v)(v - \hat{b}(v))/dv = F^{N-1}(v)$; now integrate both sides from 0 to v.

See lecture notes for the third lecture on the economics of information (on "Auctions and the revenue equivalence theorem"), pages 17 and 19.

2 Jehle & Reny pp.484, exercise 9.1 - Show that the bidding strategy in (9.5) is strictly increasing.

By exercise 9.1, the bid function can be written as:

$$\hat{b}(v_i) = v_i - \frac{\int_0^{v_i} [F(x)]^{N-1} dx}{[F(v_i)]^{N-1}}.$$

Then:

$$
\begin{aligned}
\frac{d}{dv_i}\left(\hat{b}(v_i)\right) &= \frac{d}{dv_i}\left(v_i - \frac{\int_0^{v_i}[F(x)]^{N-1}dx}{[F(v_i)]^{N-1}}\right) \\
&= 1 - \frac{[F(v_i)]^{N-1}}{[F(v_i)]^{N-1}} + \frac{\int_0^{v_i}[F(x)]^{N-1}dx}{[F(v_i)]^{2N-2}} \cdot \frac{d}{dv_i}\left([F(v_i)]^{N-1}\right) \\
&= \frac{\int_0^{v_i}[F(x)]^{N-1}dx}{[F(v_i)]^{2N-2}} \cdot (N-1)[F(v_i)]^{N-2} f(v_i) > 0\,.
\end{aligned}
$$

3 Jehle & Reny pp.485, exercise 9.3

This exercise will guide you through the proof that the bidding function in (9.5) is in fact a symmetric equilibrium of the first-price auction.

(a)

Recall from (9.2) that

$$
\frac{du(r,v)}{dr} = (N-1)F^{N-2}(r)f(r)(v-\hat{b}(r)) - F^{N-1}(r)\hat{b}'(r).
$$

Using (9.3), show that

$$
\begin{aligned}
\frac{du(r,v)}{dr} &= (N-1)F^{N-2}(r)f(r)(v-\hat{b}(r)) - (N-1)F^{N-2}(r)f(r)(r-\hat{b}'(r)) \\
&= (N-1)F^{N-2}(r)f(r)(v-r)
\end{aligned}
$$

(b)

Use the result in part (a) to conclude that $du(r,v)/dr$ is positive when $r < v$ and negative when $r > v$, so that $u(r,v)$ is maximised when $r = v$. See https://www.uio.no/studier/emner/sv/oekonomi/ECO

exams/ECON4240-2005V-SENSORVEILEDNING.pdf as well as a solution sketch available in Canvas

4 Problems 2 of the exam in ECON4240, Spring 2005

Consider a strategic situation between an employer (E) and a worker (W). E can either accept (A) or reject (R) W. W can either become skilled (S) through education, or remain unskilled (U). W can be of two types; either he is inherently high ability (H) or he is inherently low ability (L). The players' payoffs depending on their actions and W's type is shown below.

H

	S	U
A	2, 3	-1, 2
R	0, 1	0, 0

L

	S	U
A	-1,-1	-3,-2
R	0, -3	0, 0

a)

For each of these games, determine the set of (pure) rationalizable strategies for each player, and the set of pure-strategy Nash equilibria.

b)

Assume next that only W knows his own type, while player E thinks that the two types of W are equally likely. Model this situation in an ex ante perspective by specifying the Bayesian normal form.

c)

For the Bayesian normal form found in part (b), determine the set of (pure) rationalizable strategies for each player, and the set of pure-strategy and/or mixedstrategy Nash equilibria.

5 Problems 3 of the exam in ECON4240, Spring 2005

Problem 3 (20 %) Consider again the strategic situation between an employer (E) and a worker (W) described in Problem 2. Assume (as in parts b and c) of Problem 2) that only W knows his own type, while player E thinks that the two types of W are equally likely.

a) (Screening)

Assume now that E acts before W, and that E's choice of A or R can be observed by W before he makes his choice of S or U. Show that there is a unique subgame perfect Nash equilibrium.

b) (Signaling)

Assume now that W acts before E, and that W's choice of S or U can be observed by E before she makes her choice of A or R. Show that there is a unique perfect Bayesian equilibrium.