Seminar 5. Production Theory

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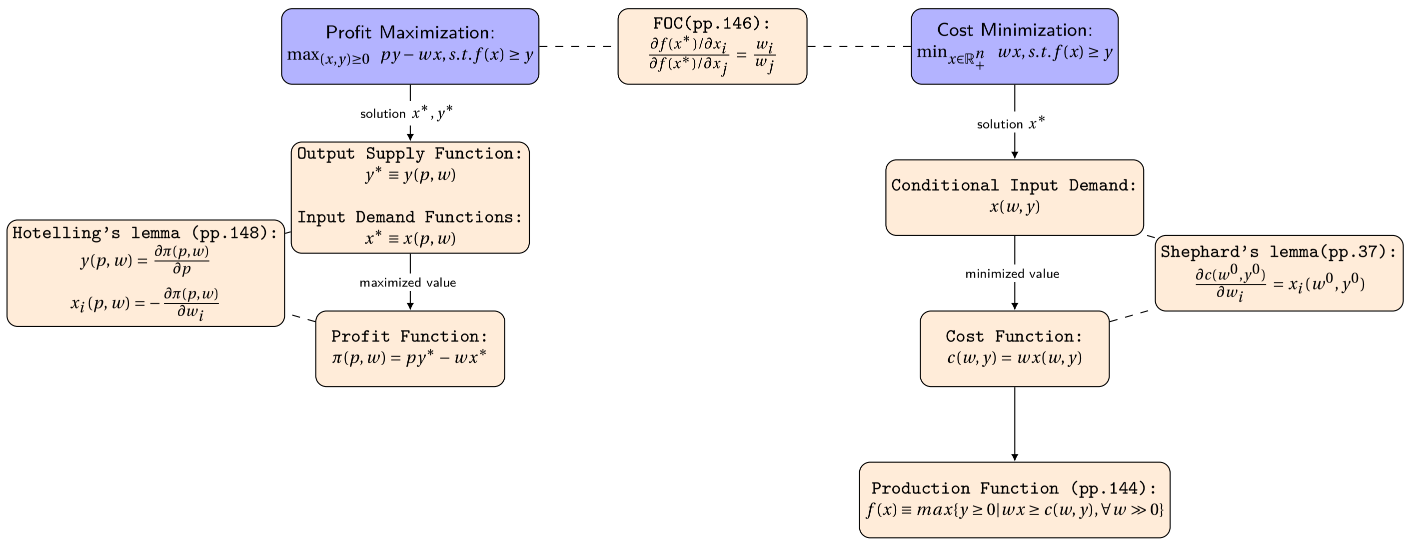
Production Duality



1 Jehle & Reny 3.35

Calculate the **cost function** and the **conditional input demands** for the linear production function, $y = \Sigma_{i=1}^{n} \alpha_i x_i$.

Production Function(Jehle & Reny pp.127)
We use a function $y = f(x)$ to denote y units of a certain commodity is produced using input x, where $x \in \mathbb{R}_+^n, y \in \mathbb{R}_+^1$

ASSUMPTION 3.1 Properties of the Production Function (Jehle & Reny pp.127)
The production function, $f : \mathbb{R}_+^n \to \mathbb{R}_+$, is continuous, strictly increasing, and strictly quasiconcave on $\mathbb{R}_+^n$, and $f(0) = 0$.

DEFINITION 3.5 The Cost Function (Jehle & Reny pp.136)
The cost function, defined for all input prices $w \gg 0$ and all output levels $y \in f(\mathbb{R}_+^n)$ is the minimum-value function,

$$c(w, y) \equiv \min_{x \in \mathbb{R}_+^n} w \cdot x, \quad s.t. \quad f(x) \geq y.$$

The solution $x(w, y)$ is referred to as the firms **conditional input demand**, because it is conditional on the level of output y.

- **Conditional input demand** is similar to Hicksian demands for consumers.

Here the linear production function $y = \Sigma_{i=1}^{n} \alpha_i x_i$ is very similar to the "**perfect substitution**" preference in Seminar 4.

- The product can be produced by any input x_i, the only difference is that for each unit of input, different x_i produces different amount α_i of the output.
- The Marginal Rate of Technical Substitution of input x_j for input x_i is $MRTS_{ij} = \frac{\partial f(x)/\partial x_i}{\partial f(x)/\partial x_j} = \frac{\alpha_i}{\alpha_j}$.

An example: an apple jam factory has 2 types of input, "single apple (x_1)" and "2-apple pack (x_2)".

- With a "single apple", the factory can produce a bottle of apple jam;
- with a "2-apple pack", 2 bottles.
- The production function is $y = 1 \cdot x_1 + 2 \cdot x_2$

How will the factory choose? Similarly to consumers' preference substitution preference, the factory will spend all money on the "cheapest per unit" input. Denote the price for x_1 and x_2 as w_1, w_2

- If $\frac{w_1}{1} = \frac{w_2}{2}$, the factory doesn't care which to use;
- If $\frac{w_1}{1} < \frac{w_2}{2}$, single apple is cheaper;
- If $\frac{w_1}{1} > \frac{w_2}{2}$, 2-apple pack is cheaper.

Denote the price for input x_i as w_i. Define $\omega = min\{\frac{w_1}{\alpha_1}, \frac{w_2}{\alpha_2}, \dots, \frac{w_n}{\alpha_n}\}$

If ω is the price of only one input x_j, the firm will only use input x_j to minimize its cost.

- Thus $y = \alpha_j x_j$ can minimize the cost, and $x_j = \frac{y}{\alpha_j}$ is the conditional input demands.
- The cost function $c(w, y) = w_j \frac{y}{\alpha_j} = \omega y$.

If ω is the price of several inputs $x_m, m = 1, 2, \dots, M$, the firm can freely combine x_m to minimize its cost, as long as $\Sigma_{m=1}^{M} \alpha_m x_m = y$.

For cost funtion, let's assume $\frac{w_1}{\alpha_1} = \frac{w_2}{\alpha_2} = \dots = \frac{w_M}{\alpha_M} = \omega$, then ω is the price for 1 single apple, for example. Again, to produce y bottles of jam, you need the same number of single apples. The cost function is thus ωy

2 Jehle & Reny 3.46

- Verify Theorem 3.7 for the profit function obtained in Example 3.5.
- Verify Theorem 3.8 for the associated output supply and input demand functions.

2.1 Verify Theorem 3.7

DEFINITION 3.7 The Profit Function (Jehle & Reny pp.148)
The firms profit function depends only on input and output prices and is defined as the maximum-value function.
THEOREM 3.7 Properties of the Profit Function (Jehle & Reny pp.148)
If f satisfies Assumption 3.1, then for $p \geq 0$ and $w \geq 0$, the profit function $\pi(p, w)$, where well-defined, is continuous and

1. Increasing in p,
2. Decreasing in w,
3. Homogeneous of degree one in (p, w),
4. Convex in (p, w),
5. Differentiable in (p, w)

6. Moreover, under the additional assumption that f is strictly concave (Hotelling's lemma),

$$y(p,w) = \frac{\partial \pi(p,w)}{\partial p}, \ and \ \ x_i(p,w) = -\frac{\partial \pi(p,w)}{\partial w_i}. \quad i = 1,2,\ldots,n.$$

2.2 Verify Theorem 3.8

THEOREM 3.8 Properties of Output Supply and Input Demand Functions (Jehle & Reny pp.149)
Suppose that f is a strictly concave production function satisfying Assumption 3.1 and that its associated profit function, $\pi(p,y)$, is twice continuously differentiable. Then, for all $p > 0$ and $w \gg 0$ where it is well defined:

1. Homogeneity of degree zero:

$$y(tp,tw) = y(p,w), \forall t > 0,$$

$$x_i(tp,tw) = x_i(p,w), \forall t > 0 \ and \ i = 1,\ldots,n.$$

2. Own-price effects:

$$\frac{\partial y(p,w)}{\partial p} \geq 0,$$

$$\frac{\partial x_i(p,w)}{\partial w_i} \leq 0, \ \forall i = 1,\ldots,n.$$

3. The substitution matrix is symmetric and positive semidefinite.

$$\begin{pmatrix} \frac{\partial y(p,w)}{\partial p} & \frac{\partial y(p,w)}{\partial w_1} & \cdots & \frac{\partial y(p,w)}{\partial w_n} \\ \frac{\partial x_1(p,w)}{\partial p} & \frac{\partial x_1(p,w)}{\partial w_1} & \cdots & \frac{\partial x_1(p,w)}{\partial w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_n(p,w)}{\partial p} & \frac{\partial x_n(p,w)}{\partial w_1} & \cdots & \frac{\partial x_n(p,w)}{\partial w_n} \end{pmatrix} \tag{1}$$

3 Jehle & Reny 3.49

1. Derive the **cost function** for the production function in Example 3.5.
2. Solve $\max_y py - c(w,y)$
3. Compare its solution, $y(p,w)$, to the solution in (E.5). Check that $\pi(p,w) = py(p,w) - c(w,y(p,w))$.
4. Supposing that $\beta > 1$, confirm our conclusion that profits are minimised when the first-order conditions are satisfied by showing that marginal cost is decreasing at the solution.

5. Sketch your results.