# Seminar 12. Auctions and previous exam problems

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### 1 Jehle & Reny pp.484, exercise 9.2

Show in two ways that the symmetric equilibrium bidding strategy of a first-price auction with N symmetric bidders each with values distributed according to F, can be written as

$$\hat{b}(v) = v - \int_0^v \left(\frac{F(x)}{F(v)}\right)^{N-1} dx$$

For the first way, use our solution from the text and apply integration by parts. For the second way, use the fact that  $F^{N-1}(r)(v-\hat{b}(r))$  is maximised in r when r=v and then apply the envelope theorem to conclude that  $d(F^{N-1}(v)(v-\hat{b}(v)))/dv = F^{N-1}(v)$ ; now integrate both sides from 0 to v.

See lecture notes for the third lecture on the economics of information (on "Auctions and the revenue equivalence theorem"), pages 17 and 19.

# 2 Jehle & Reny pp.484, exercise 9.1 - Show that the bidding strategy in (9.5) is strictly increasing.

By exercise 9.1, the bid function can be written as:

$$\hat{b}(v_i) = v_i - \frac{\int_0^{v_i} [F(x)]^{N-1} dx}{[F(v_i)]^{N-1}}.$$

Then:

$$\begin{split} \frac{d}{dv_i} \left( \hat{b}(v_i) \right) &= \frac{d}{dv_i} \left( v_i - \frac{\int_0^{v_i} [F(x)]^{N-1} dx}{[F(v_i)]^{N-1}} \right) \\ &= 1 - \frac{[F(v_i)]^{N-1}}{[F(v_i)]^{N-1}} + \frac{\int_0^{v_i} [F(x)]^{N-1} dx}{[F(v_i)]^{2N-2}} \cdot \frac{d}{dv_i} \left( [F(v_i)]^{N-1} \right) \\ &= \frac{\int_0^{v_i} [F(x)]^{N-1} dx}{[F(v_i)]^{2N-2}} \cdot (N-1) [F(v_i)]^{N-2} f(v_i) > 0 \,. \end{split}$$

#### 3 Jehle & Reny pp.485, exercise 9.3

This exercise will guide you through the proof that the bidding function in (9.5) is in fact a symmetric equilibrium of the first-price auction.

(a)

Recall from (9.2) that

$$\frac{du(r,v)}{dr} = (N-1)F^{N-2}(r)f(r)(v-\hat{b}(r)) - F^{N-1}(r)\hat{b}'(r).$$

Using (9.3), show that

$$\frac{du(r,v)}{dr} = (N-1)F^{N-2}(r)f(r)(v-\hat{b}(r)) - (N-1)F^{N-2}(r)f(r)(r-\hat{b}'(r))$$
$$= (N-1)F^{N-2}(r)f(r)(v-r)$$

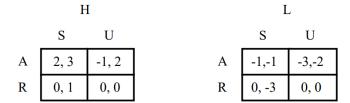
**(b)** 

Use the result in part (a) to conclude that du(r, v)/dr is positive when r < v and negative when r > v, so that u(r, v) is maximised when r = v. See https://www.uio.no/studier/emner/sv/oekonomi/ECO

 $exams/ECON4240-2005V-SENSORVEILEDNING.pdf\ as\ well\ as\ a\ solution\ sketch\ available\ in\ Canvas$ 

# 4 Problems 2 of the exam in ECON4240, Spring 2005

Consider a strategic situation between an employer (E) and a worker (W). E can either accept (A) or reject (R) W. W can either become skilled (S) through education, or remain unskilled (U). W can be of two types; either he is inherently high ability (H) or he is inherently low ability (L). The players' payoffs depending on their actions and W's type is shown below.



#### a)

For each of these games, determine the set of (pure) rationalizable strategies for each player, and the set of pure-strategy Nash equilibria.

#### b)

Assume next that only W knows his own type, while player E thinks that the two types of W are equally likely. Model this situation in an ex ante perspective by specifying the Bayesian normal form.

#### c)

For the Bayesian normal form found in part (b), determine the set of (pure) rationalizable strategies for each player, and the set of pure-strategy and/or mixedstrategy Nash equilibria.

# 5 Problems 3 of the exam in ECON4240, Spring 2005

Problem 3 (20 %) Consider again the strategic situation between an employer (E) and a worker (W) described in Problem 2. Assume (as in parts b and c) of Problem 2) that only W knows his own type, while player E thinks that the two types of W are equally likely.

#### a) (Screening)

Assume now that E acts before W, and that E's choice of A or R can be observed by W before he makes his choice of S or U. Show that there is a unique subgame perfect Nash equilibrium.

#### b) (Signaling)

Assume now that W acts before E, and that W's choice of S or U can be observed by E before she makes her choice of A or R. Show that there is a unique perfect Bayesian equilibrium.