Questions and Answers

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1 Seminar 1

1.1 question 1.26

Q: Should it be "equality" in Kuhn-Tucker condition equation (6): $p_1x_1 + p_2x_2 \leq y$ (slides pp.16)?

A: You can argue it is "equality" for a well defined classical utility function, since the solution is always on the boundary(you can always spend the rest part of your budget to imporve your utility).
But note that Kuhn-Tucker condition describes the most general case for a value maximization problem. If the utility function is weired, for example, in Figure 1, the utility function ($u(x_1, x_2) = 3-(x_1-2)^2-(x_2-2)^2$) looks like a cone, the "peak" of the cone is within the "budget plane($x + y = 6$)". Your utility can therefore be maximized within your budget. "$\leq$" allows this case.

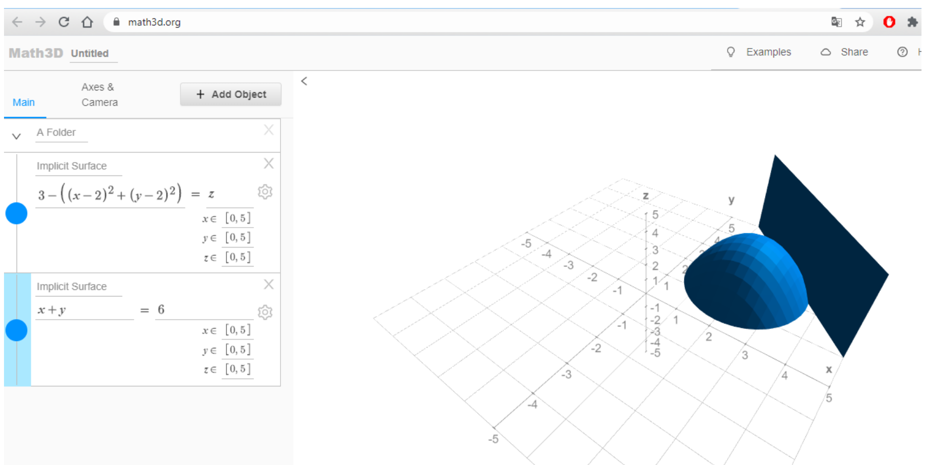

Figure 1: A cone-like utility function and a loose budget

Try to make some graphs yourself on https://www.math3d.org/. Always remember your utility is the extra dimension (z-axis in Figure 1).

2 Seminar 2

3 Previous exams

3.1 Cost minimization

Q: The competitive firm FF has a production function of the form: $F = 2L+5K$, where L denotes labor and K capital. Assume salary $w = 2$ and capital cost is $r = 4$. What is the minimal cost of producing 10 units of output?
(It's from Spring 2019 Exam)

A: You can solve it in the following 3 ways:

1. Kuhn Tucker condition (not recommended)
In this question, you're going to solve the cost minimization problem: $\min 2L + 4K, s.t.\ f(x) >= 10, L \geq 0, K \geq 0$.
You can definitely use Kuhn Tucker condition to solve it, similar to the question in our seminar 1, since the object function is also linear.

2. Substitute the constraint into the object function (be careful with the domain)
Since you're going to minimize the cost, it's not reasonable to produce more than required (i.e. 10, because cost function is increasing in output y), you will let $f(x) = 10$, i.e. $F = 2L + 5K = 10$, or $2L = 10 - 5K$. This relation holds as long as you want to minimize the cost.

Also, don't forget you have $L \geq 0$ and $K \geq 0$. Thus $2L = 10 - 5K$ must also be non-negative, we have $10 - 5K \geq 0, K \leq 2$. The domain of K is $[0, 2]$
You can now substitute $2L = 10 - 5K$ into your object function, $2L + 4K$, and your minimization problem becomes $\min 10 - 5K + 4K$, or $\min 10 - K$. The object function is decreasing in K, you want K to be as great as possible in its domain $[0, 2]$, to minimize the cost.
So you choose $K = 2, L = 0$, minimized cost is 8.

In exam, you can write it concisely:

$\min 2L + 4K\ s.t.\ f(x) >= 10, L \geq 0, K \geq 0$
Since cost function is increasing in output, $f(x) = 10, i.e. F = 2L+5K = 10$. Therefore: $2L = 10 - 5K$. Objection function becomes: 10-5K+4K=10-K, decreasing in K.
Besides, $K \geq 0, L \geq 0$ and $2L = 10 - 5K$ lead to $K \in [0, 2]$.
Therefore $K^* = 2, L^* = 0$,

3. Cheapest per unit of product (recommended)
Since the production function shows perfect substitution between L and K, the firm chooses only the cheapest per unit of product input. Recall that perfect substitution means K and L are the same thing, just with different package, in the view of the firm.

To produce 1 unit output, according to $F = 2L + 5K$, you need either 0.5 unit L or 0.2 unit K. The price for 0.5 unit L is $0.5 \times 2 = 1$; the price for 0.2 unit K is $0.2 \times 4 = 0.8$. For 1 unit output, K is the cheapest input. Therefore the firm chooses $L = 0, F = 5K$. When $F = 10$, $K = 2$, the cost is $2 \times 4 = 8$.