Seminar 1 - Preference and Marshallian demand function

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1 Jehle & Reny 1.8. Axioms of consumer choice

Sketch a map of indifference sets that are all **parallel**, **negatively sloped straight lines**, with **preference increasing north-easterly**.We know that preferences such as these satisfy Axioms 1, 2, 3, and 4.

- Prove that they also satisfy Axiom 5'.
- Prove that they do not satisfy Axiom 5.

Review: 5 Axioms of consumer choice (JR pp. 5-12)

The preference (indifference curve) shown in Figure 1 is classical in all economics classes. Why does it look like this way?

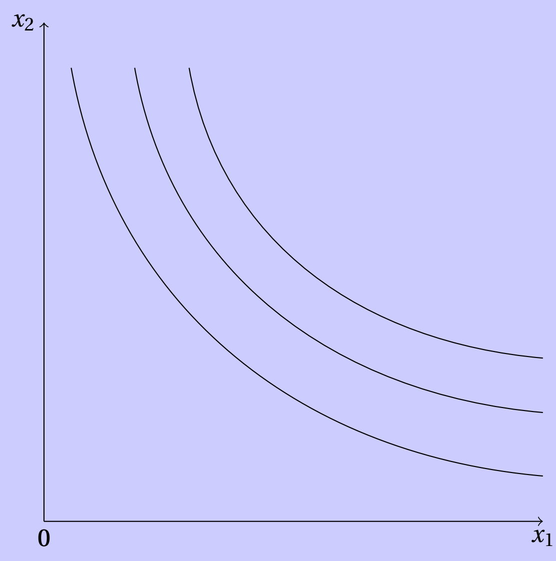

Figure 1: An indifference map

The most basic assumptions about our preference are Axiom 1. and Axiom 2.

- Axiom 1. Completeness (We can always choose) $\forall\ x^1, x^2$ in X, we have: $x^1 \succsim x^2$ or $x^2 \succsim x^3$ or both

- Axiom 2. Transitivity $\forall\ x^1, x^2$, and x^3 in X, if $x^1 \succsim x^2$ and $x^2 \succsim x^3$, then $x^1 \succsim x^3$

With Axiom 1. and Axiom 2. , the preference set can be:

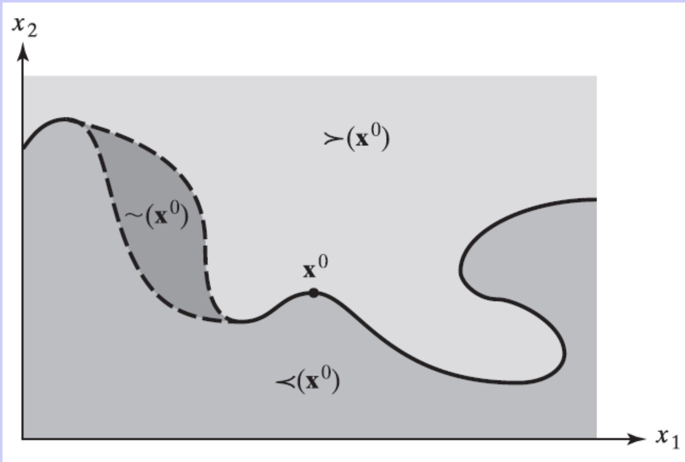

Figure 2: Hypothetical preferences satisfying Axioms 1 and 2.

What happens around the "boundary"?

- Axiom 3. Continuity (define boundary)

  $\succsim(x)$ and $\precsim(x)$ sets are closed in R^n_+ for $x \in R^n_+$.

Once the boundary is properly defined, there is no sudden preference reversal any more. Now the preference set looks like Figure 3

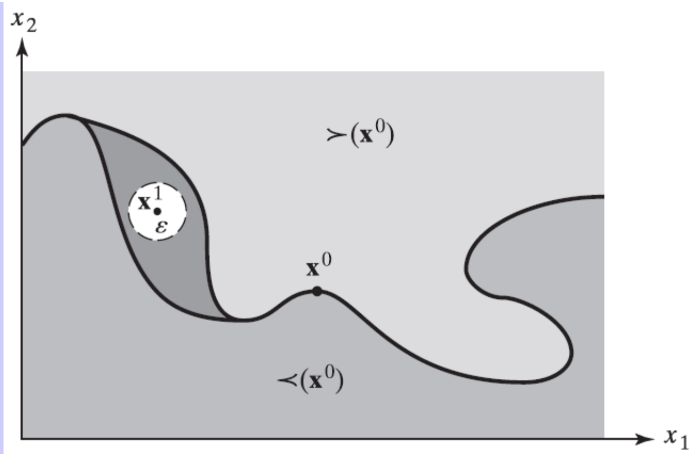

Figure 3: Hypothetical preferences satisfying Axioms 1, 2, and 3.

Further more, we assume "unlimited wants" can be represented by our preference. For example, we can try Axiom 4'.

- Axiom 4'. Local non-satiation (always something better around)
  $\forall\ x^0 \in R^n_+$ and $\forall\ \epsilon > 0, \exists x \in B_\epsilon(x^0) \cap R^n_+$ s.t. $x \succ x^0$

Axiom 4' rulled out the "indifference zone" in Figure 3 and our preference set is deduced into Figure 4.

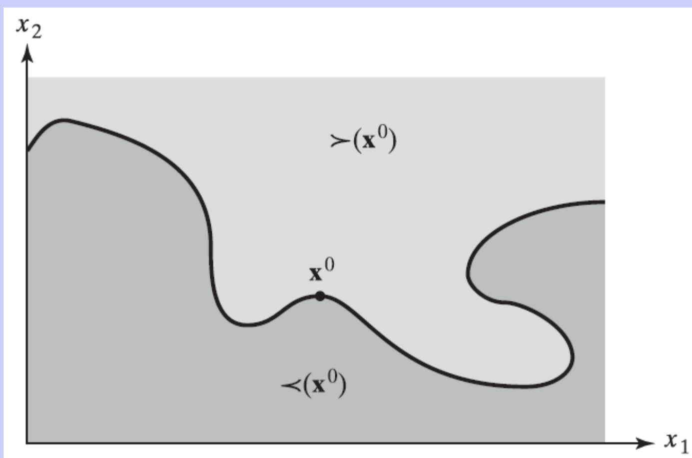

Figure 4: Hypothetical preferences satisfying Axioms 1, 2, 3 and 4'

However, Axiom 4' doesn't mean "the more, the better (at least not worse)" shown in Figure 5.

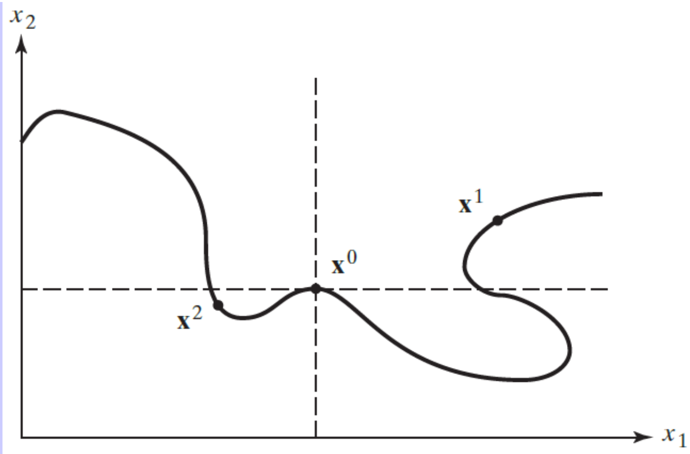

Figure 5: Hypothetical preferences satisfying Axioms 1, 2, 3 and 4' again

To depict this, we assume Axiom 4 instead.

- Axiom 4. Strict monotonicity (the more, the better)
  $\forall\ x^0, x^1 \in R^n_+$, if $x^0 \geq x^1$, then $x^0 \succsim x^1$, while if $x^0 \gg x^1$, then $x^0 \succ x^1$.

A set of preferences satisfying Axioms 1, 2, 3, and 4 is given in Figure 6

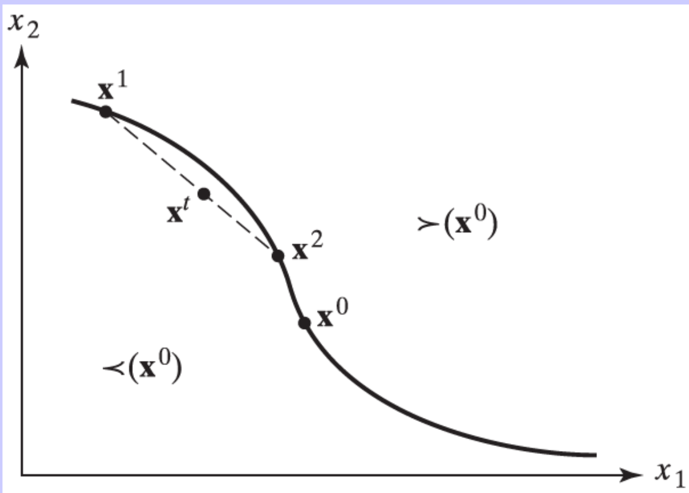

Figure 6: Hypothetical preferences satisfying Axioms 1, 2, 3 and 4

In addition, we assume people prefer "balanced" than "extreme" bundles in consumption. Either Axiom 5' or Axiom 5 can guarantee this, but Axiom 5 will make our analysis easier in the future.

- Axiom 5'. Convexity

  If $x^1 \succsim x^0$, then $tx^1 + (1-t)x^0 \succsim x^0$ for all $t \in [0,1]$

- Axiom 5. Strict convexity

  If $x^1 \neq x^0$ and $x^1 \succsim x^0$, then $tx^1 + (1-t)x^0 \succ x^0$ for all $t \in (0,1)$

Both Axiom 5' and Axiom 5 can rule out the concave-to-the-origin segments in Figure 6. Finally, we our indifference curve looks the same as in Figure 1 and Figure 7

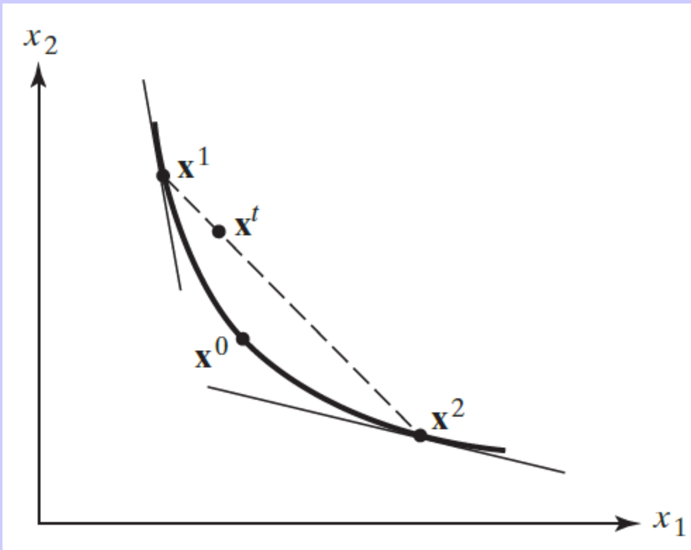

Figure 7: Hypothetical preferences satisfying Axioms 1, 2, 3, 4 and 5'/5

As required by question 1.8, a map of the indifference sets is showed in Figure 8

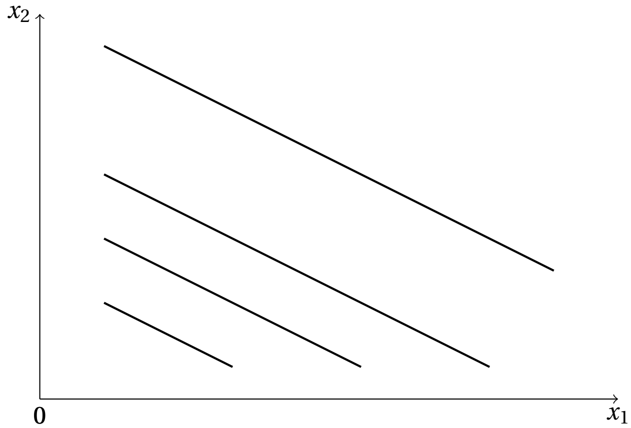

Figure 8: A map of the indifference sets for Q.1.8

1.1 Prove that they also satisfy Axiom 5'

- Read JR. pp. 501 for the definition of Convex combination.

For any given bundle x^0 in Figure 9, we can always find another bundle x^1 either on the same indifference curve with x^0 lying on or to the northeast of x^0 s.t. $x^1 \succsim x^0$. No matter which case, the convex combination of x^0 and x^1 is always at least as good as x^0

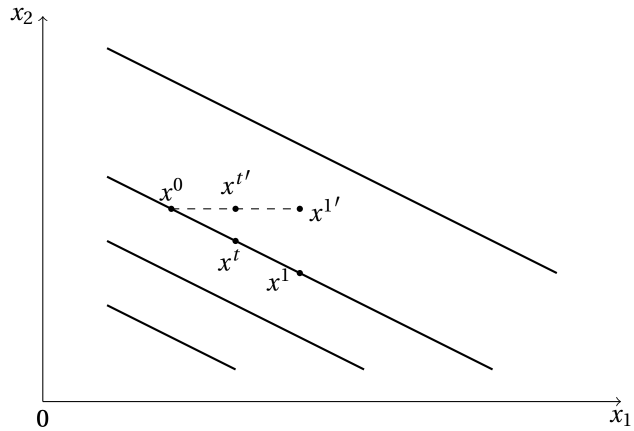

Figure 9: Axiom 5' Convexity

1.2 Prove that they do not satisfy Axiom 5

To prove the preferences do not satisfy Axiom 5, we only need to give one example of the violation.
In Figure 10, $x^1 \neq x^0$ and $x^1 \succsim x^0$, but $x^t = tx^1 + (1-t)x^0 \not\succ x^0$ for any $t \in (0,1)$

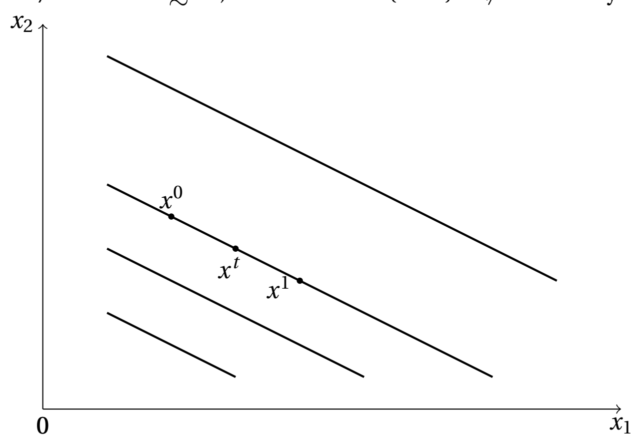

Figure 10: Violation of Axiom 5 Strict Convexity

2 Jehle & Reny 1.9 - Leontief preferences

Sketch a map of indifference sets that are **all parallel right angles that 'kink' on the line** $x_1 = x_2$. If **preference increases north-easterly**, these preferences will satisfy Axioms 1, 2, 3, and 4'.

- Prove that they also satisfy Axiom 5'.
- Do they satisfy Axiom 4?
- Do they satisfy Axiom 5?

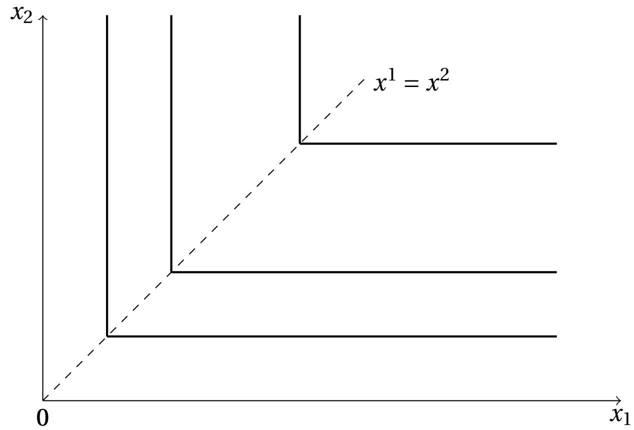

Figure 11: A map of the indifference sets for Q.1.9

2.1 Prove that they also satisfy Axiom 5'

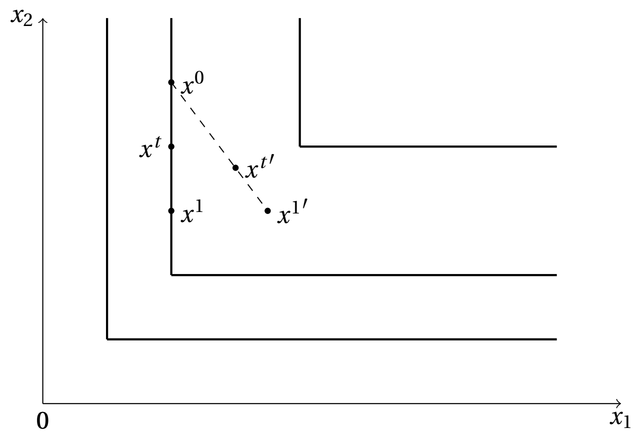

Figure 12: Axiom 5' Convexity

2.2 Do they satisfy Axiom 4?

Yes. Any bundle $x^{0'}$ that contains at least as much of every good as x^1 does (i.e. $x^{0'} \geq x^1$)can only lies in the shaded area including the border. Obviously, $x^{0'} \succsim x^1$. In addition, for any x^0 contains strictly more of every good than x^1 does (i.e. $x^{0'} \gg x^1$), we have $x^0 \succ x^1$

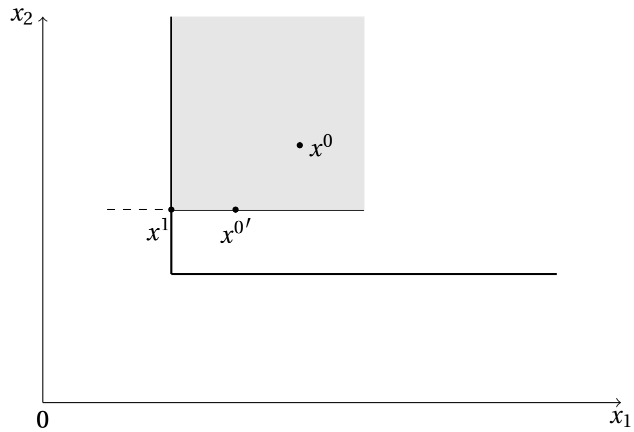

Figure 13: Axiom 4 Strict Monotonicity

2.3 Do they satisfy Axiom 5?

No. In Figure 14, $x^1 \neq x^0$ and $x^1 \succsim x^0$, but $x^t = tx^1 + (1-t)x^0 \not\succ x^0$ for any $t \in (0,1)$

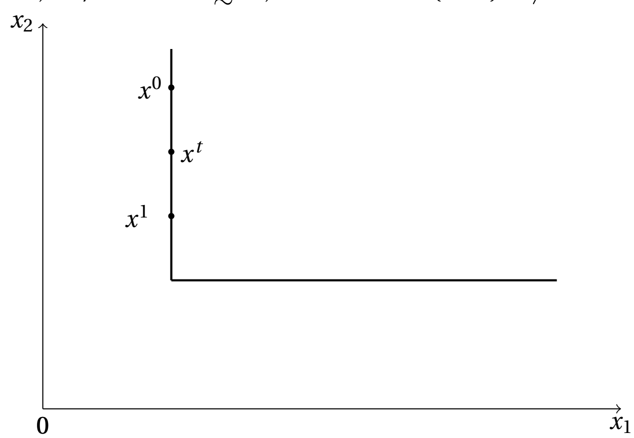

Figure 14: Axiom 5 Strict Convexity

3 Jehle & Reny 1.13 - Lexicographic preferences

A consumer has lexicographic preferences over $xR2$ if the relation satisfies x_1, x_2 whenever $x_1^1 > x_1^2$, or $x_1^1 = x_1^2$ and $x_1^1 \geq x_1^2$.

- Sketch an indifference map for these preferences.
- Can these preferences be represented by a continuous utility function? Why or why not?

4 Jehle & Reny 1.15 - compact and convex

Prove that the budget set, B, is a **compact, convex set whenever** $p \gg 0$.

5 Jehle & Reny 1.26 - Masshallian demand function

A consumer of **two goods** faces **positive prices** and has a **positive income**. His utility function is

$$u(x_1, x_2) = x_1$$

Derive the Marshallian demand functions.

6 Jehle & Reny 1.27 - Masshallian demand function

A consumer of **two goods** faces **positive** prices and has a **positive income**. His utility function is

$$u(x_1, x_2) = max[ax_1, ax_2] + min[x_1, x_2], \ \ where \ 0 < a < 1.$$

Derive the Marshallian demand functions.