Seminar 1 - Preference and Marshallian demand function

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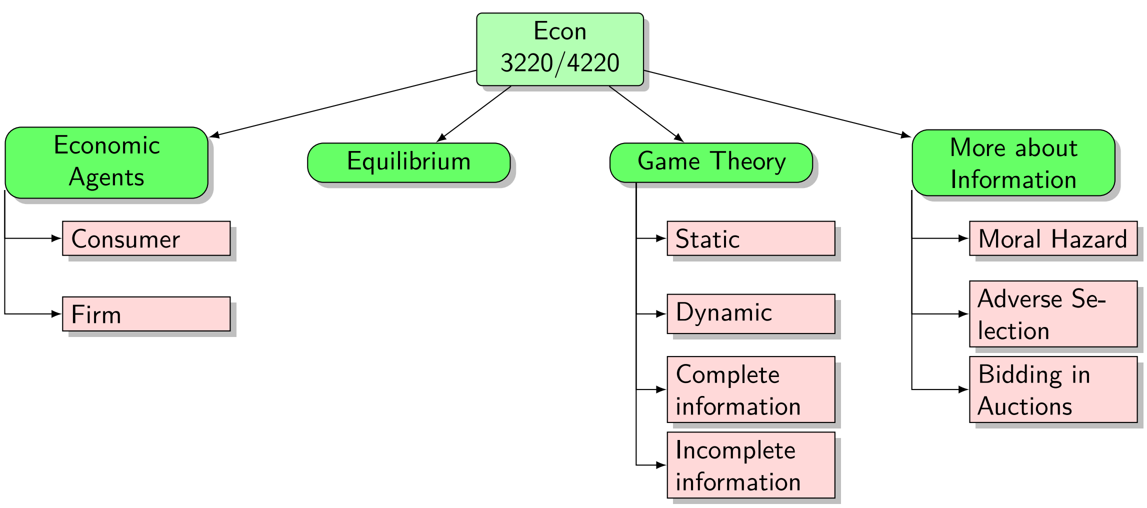
Before we start

- The course/seminar is difficult and time-consuming → help each other
- More details on assumptions we have used in previous economic classes, more complex and interesting questions, more mathematics
- Use your textbook wisely: Hints, Mathematical Appendices, Index
- Open-book exam, can also be difficult. Previous exam: Econ 4220/3220, Econ 4200/3200
- Zoom seminars, solution sketch will be available before every weekend in Canvas.
- Your feedback is important (too fast, unclear, mistakes etc.). Contact me (xiaoguang.ling@econ.uio.no) in time!



1 Jehle & Reny 1.8. Axioms of consumer choice

Sketch a map of indifference sets that are all **parallel**, **negatively sloped straight lines**, with **preference increasing north-easterly**.We know that preferences such as these satisfy Axioms 1, 2, 3, and 4.

- Prove that they also satisfy Axiom 5'.
- Prove that they do not satisfy Axiom 5.

Review: 5 Axioms of consumer choice (JR pp. 5-12)

The preference (indifference curve) shown in Figure 1 is classical in all economics classes. Why does it look like this way?

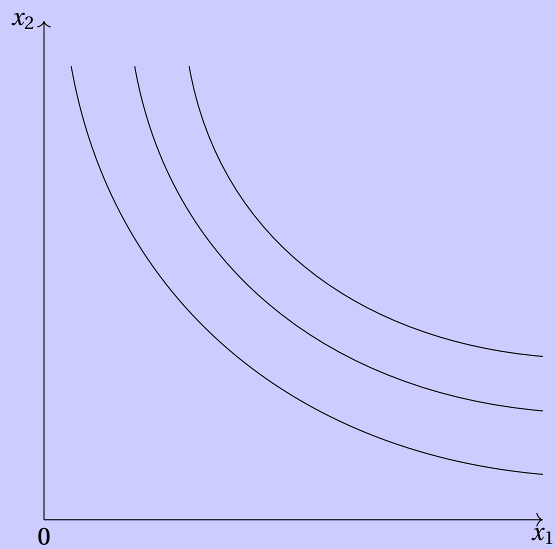


Figure 1: An indifference map

The most basic assumptions about our preference are Axiom 1. and Axiom 2.

- Axiom 1. Completeness (We can always choose) $\forall\ x^1, x^2$ in X, we have: $x^1 \succsim x^2$ or $x^2 \succsim x^3$ or both
- Axiom 2. Transitivity $\forall\ x^1, x^2$, and x^3 in X, if $x^1 \succsim x^2$ and $x^2 \succsim x^3$, then $x^1 \succsim x^3$

With Axiom 1. and Axiom 2. , the preference set can be:

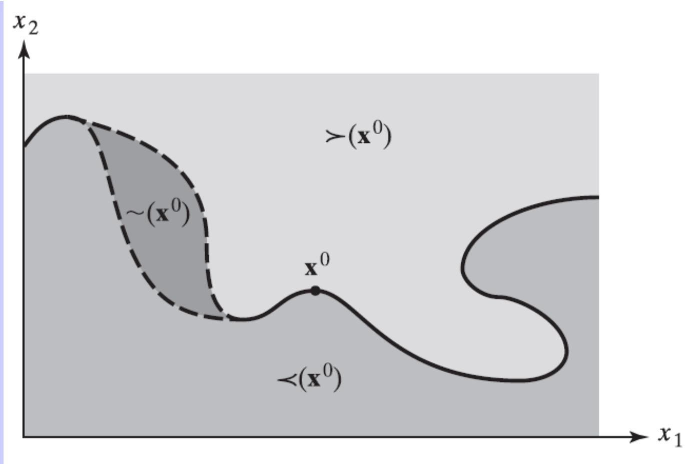


Figure 2: Hypothetical preferences satisfying Axioms 1 and 2.

What happens around the "boundary"?

- Axiom 3. Continuity (define boundary)

  $\succsim(x)$ and $\precsim(x)$ sets are closed in $\mathbb{R}^n_+$ for $x \in \mathbb{R}^n_+$.

Once the boundary is properly defined, there is no sudden preference reversal any more. Now the preference set looks like Figure 3

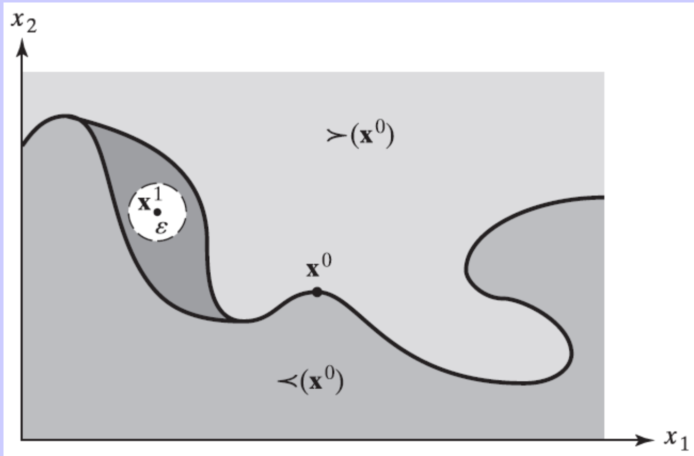


Figure 3: Hypothetical preferences satisfying Axioms 1, 2, and 3.

Further more, we assume "unlimited wants" can be represented by our preference. For example, we can try Axiom 4'.

- Axiom 4'. Local non-satiation (always something better around)

$$\forall\ x^0 \in \mathbb{R}^n_+ \text{ and } \forall\ \epsilon > 0, \exists x \in B_\epsilon(x^0) \cap \mathbb{R}^n_+ \text{ s.t. } x \succ x^0$$

Axiom 4' rulled out the "indifference zone" in Figure 3 and our preference set is deduced into Figure 4.

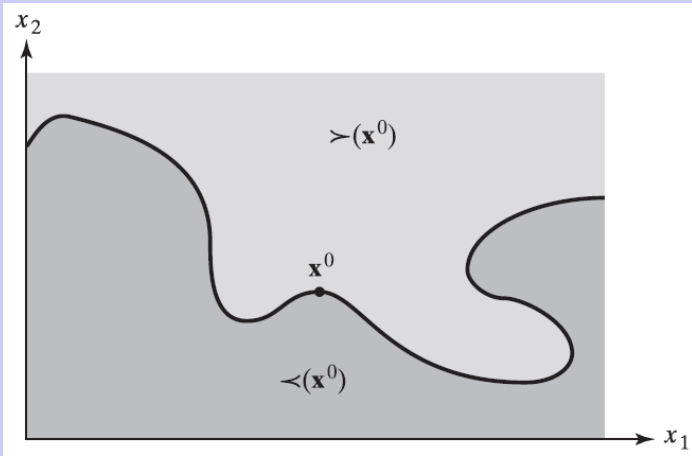


Figure 4: Hypothetical preferences satisfying Axioms 1, 2, 3 and 4'

However, Axiom 4' doesn't mean "the more, the better (at least not worse)" shown in Figure 5.

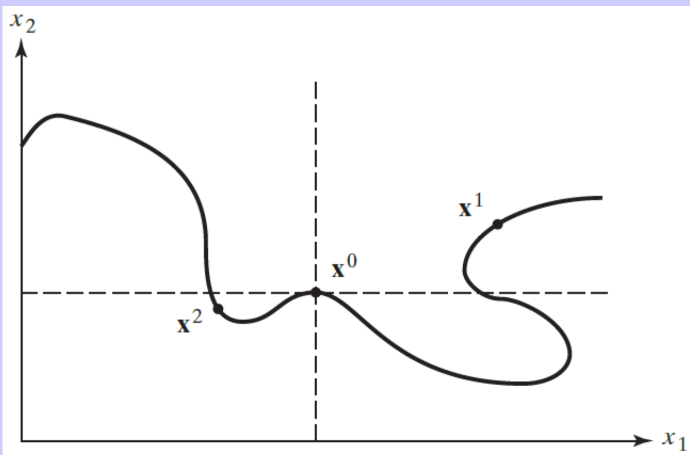


Figure 5: Hypothetical preferences satisfying Axioms 1, 2, 3 and 4' again

To depict this, we assume Axiom 4 instead.

- Axiom 4. Strict monotonicity (the more, the better)
  $\forall\ x^0, x^1 \in \mathbb{R}^n_+$, if $x^0 \geq x^1$, then $x^0 \succsim x^1$, while if $x^0 \gg x^1$, then $x^0 \succ x^1$.

A set of preferences satisfying Axioms 1, 2, 3, and 4 is given in Figure 6

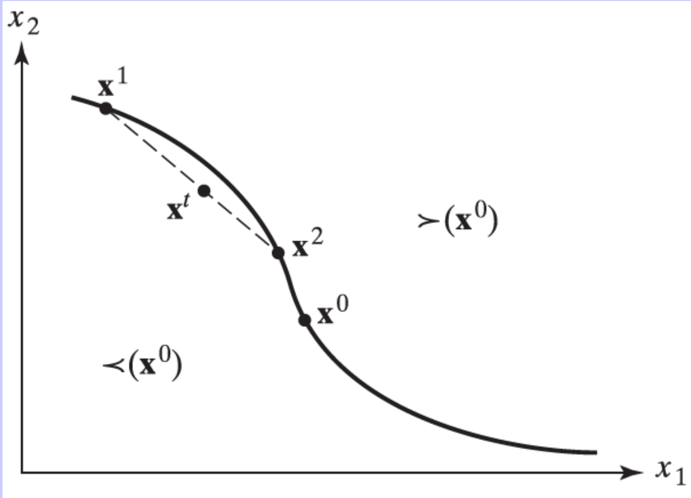


Figure 6: Hypothetical preferences satisfying Axioms 1, 2, 3 and 4

In addition, we assume people prefer "balanced" than "extreme" bundles in consumption. Either Axiom 5' or Axiom 5 can guarantee this, but Axiom 5 will make our analysis easier in the future.

- Axiom 5'. Convexity

  If $x^1 \succsim x^0$, then $tx^1 + (1-t)x^0 \succsim x^0$ for all $t \in [0,1]$

- Axiom 5. Strict convexity

  If $x^1 \neq x^0$ and $x^1 \succsim x^0$, then $tx^1 + (1-t)x^0 \succ x^0$ for all $t \in (0,1)$

Both Axiom 5' and Axiom 5 can rule out the concave-to-the-origin segments in Figure 6. Finally, we our indifference curve looks the same as in Figure 1 and Figure 7

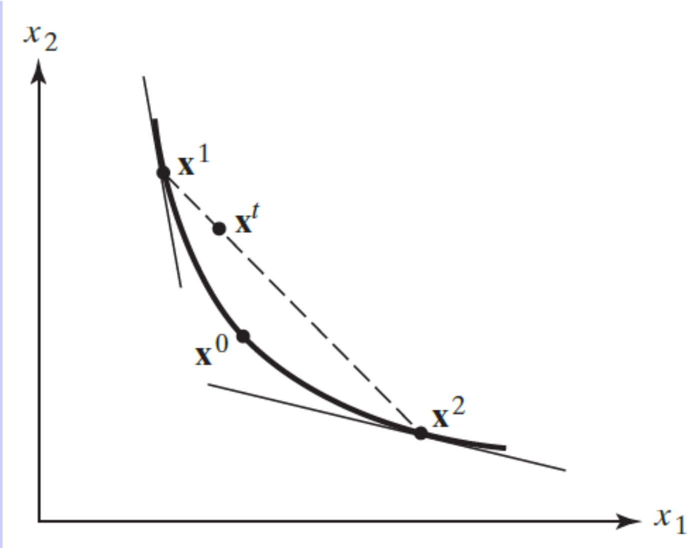


Figure 7: Hypothetical preferences satisfying Axioms 1, 2, 3, 4 and 5'/5

As required by question 1.8, a map of the indifference sets is showed in Figure 8

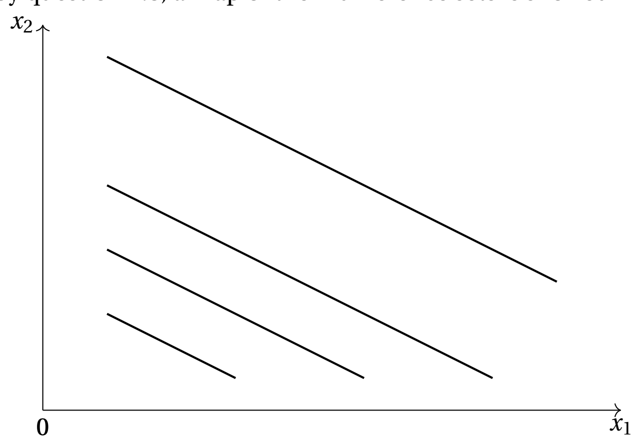


Figure 8: A map of the indifference sets for Q.1.8

1.1 Prove that they also satisfy Axiom 5'

Read JR. pp. 501 for the definition Convex combination.

For any given bundle x^0 in Figure 9, we can always find another bundle x^1 either on the same indifference curve with x^0 lying on or to the northeast of x^0 s.t. $x^1 \succsim x^0$. No matter which case, the convex combination of x^0 and x^1 is always at least as good as x^0

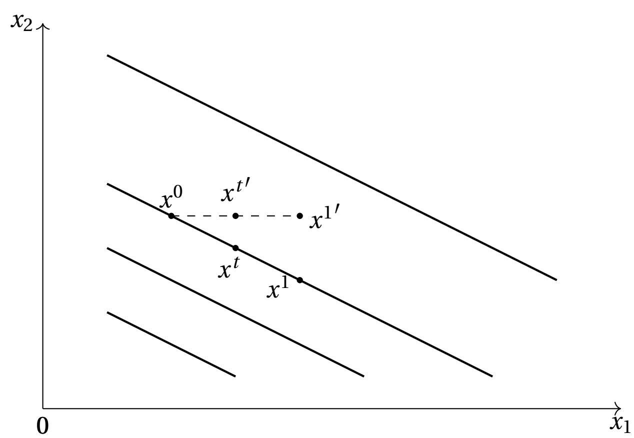


Figure 9: Axiom 5' Convexity

1.2 Prove that they do not satisfy Axiom 5

To prove the preferences do not satisfy Axiom 5, we only need to give one example of the violation.
In Figure 10, $x^1 \neq x^0$ and $x^1 \succsim x^0$, but $x^t = tx^1 + (1-t)x^0 \not\succ x^0$ for any $t \in (0,1)$

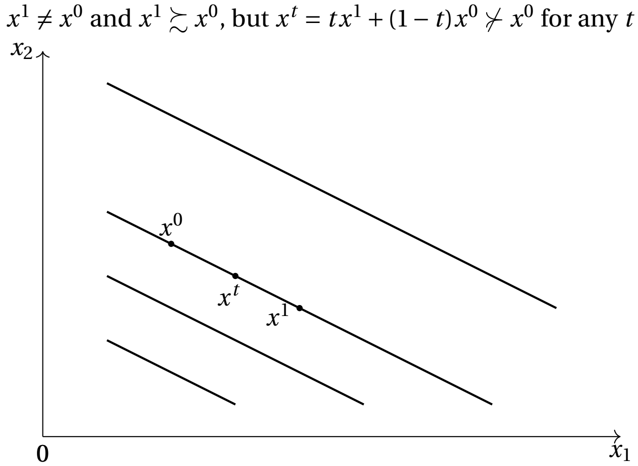


Figure 10: Violation of Axiom 5 Strict Convexity

2 Jehle & Reny 1.9 - Leontief preferences

Sketch a map of indifference sets that are **all parallel right angles that kink on the line** $x_1 = x_2$. If **preference increases north-easterly**, these preferences will satisfy Axioms 1, 2, 3, and 4'.

- Prove that they also satisfy Axiom 5'.
- Do they satisfy Axiom 4?
- Do they satisfy Axiom 5?

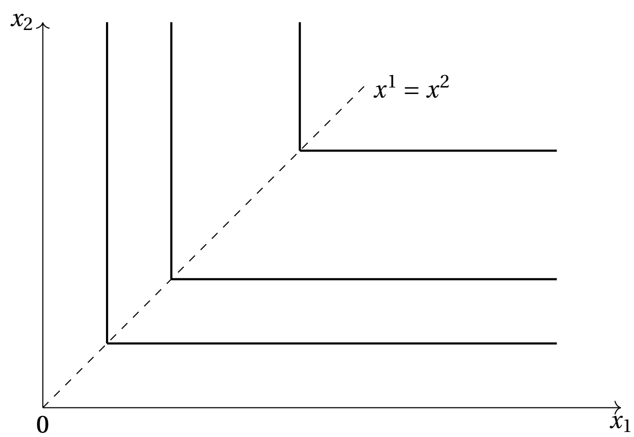


Figure 11: A map of the indifference sets for Q.1.9

2.1 Prove that they also satisfy Axiom 5'

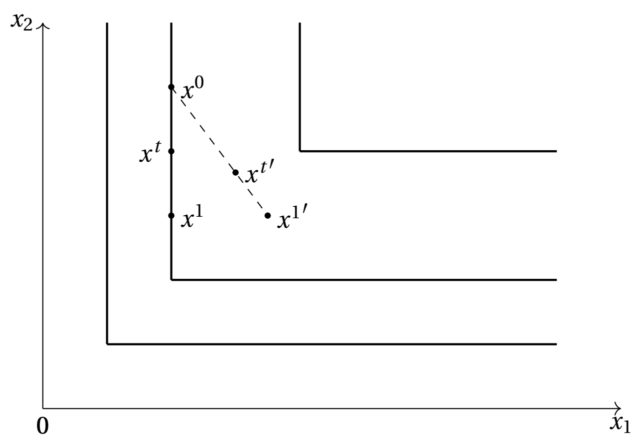


Figure 12: Axiom 5' Convexity

2.2 Do they satisfy Axiom 4?

Yes. Any bundle $x^{0'}$ that contains at least as much of every good as x^1 does (i.e. $x^{0'} \geq x^1$)can only lies in the shaded area including the border. Obviously, $x^{0'} \succsim x^1$. In addition, for any x^0 contains strictly more of every good than x^1 does (i.e. $x^{0'} \gg x^1$), we have $x^0 \succ x^1$

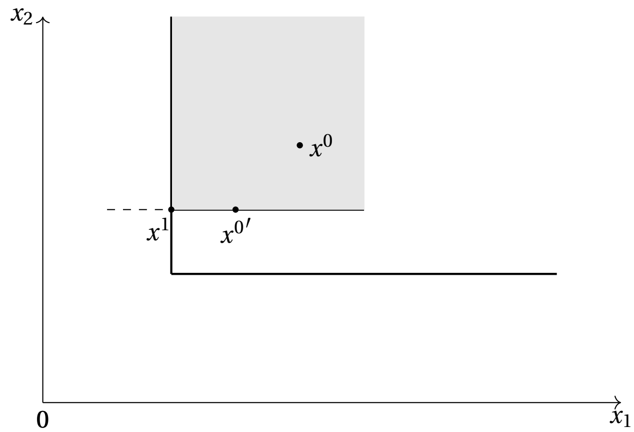


Figure 13: Axiom 4 Strict Monotonicity

2.3 Do they satisfy Axiom 5?

No. In Figure 14, $x^1 \neq x^0$ and $x^1 \succsim x^0$, but $x^t = tx^1 + (1-t)x^0 \not\succ x^0$ for any $t \in (0,1)$

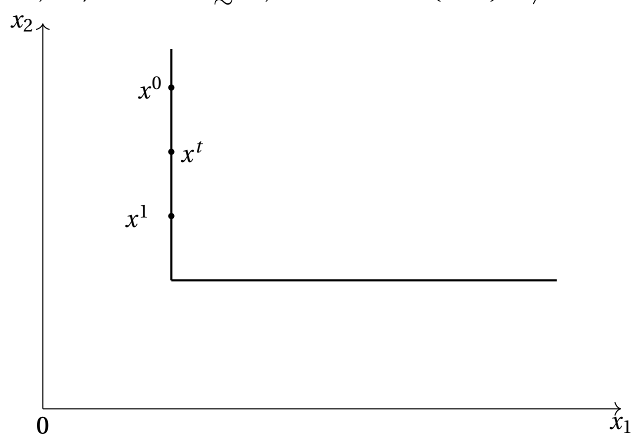


Figure 14: Axiom 5 Strict Convexity

3 Jehle & Reny 1.13 - Lexicographic preferences

A consumer has lexicographic preferences over $\mathbb{R}^2_+$ if the relation satisfies x_1, x_2 whenever $x_1^1 > x_1^2$, or $x_1^1 = x_1^2$ and $x_1^1 \geq x_1^2$.

- Sketch an indifference map for these preferences.
- Can these preferences be represented by a continuous utility function? Why or why not?

$\forall x^1, x^2 \in \mathbb{R}^2_+$, Lexicographic preferences can be defined as:

$$x^1 \succsim x^2 \Leftrightarrow \begin{cases} x^1_1 > x^2_1 \\ \text{or} \\ x^1_1 = x^2_1 \ and \ x^1_2 \geq x^2_2 \end{cases} \tag{1}$$

- x_1 is critical
- like a dictionary

3.1 Sketch an indifference map for these preferences.

There is no indifference map for Lexicographic preferences. To draw the indifference map, we must have different bundles lying on indifference curves.
Assume there are two different budles x^1, x^2 s.t. $x^1 \sim x^2$, i.e. $x^1 \succsim x^2$ and $x^2 \succsim x^1$. According to the definition in formula 1, this requires

$$Both \begin{cases} x^1_1 > x^2_1 \\ \text{or} \\ x^1_1 = x^2_1 \ and \ x^1_2 \geq x^2_2 \end{cases} \quad and \begin{cases} x^2_1 > x^1_1 \\ \text{or} \\ x^2_1 = x^1_1 \ and \ x^2_2 \geq x^1_2 \end{cases} \tag{2}$$

Obviously, the only possible condition is $x^1_1 = x^2_1 \ and \ x^1_2 = x^2_2$, which contradicts with our assertion x^1, x^2 are different.

3.2 Can these preferences be represented by a continuous utility function? Why or why not?

No. Because Lexicographic preferences are not continuous.
According to our Axiom 3, continuity means $\succsim$ and $\precsim$ sets are closed. In Figure 15, we can see the $\succsim$ set for any bundle x^0 is not closed.

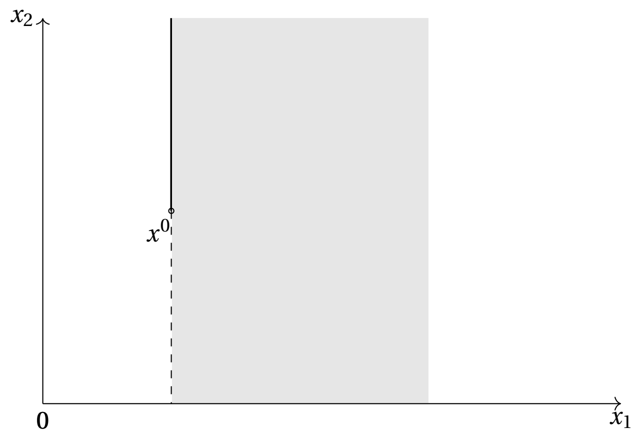


Figure 15: $\succsim x^0$ set for Lexicographic preferences

4 Jehle & Reny 1.15 - compact and convex

Prove that the budget set, B, is a **compact, convex set whenever** $p \gg 0$.

- A budget set B can be difined as $B \equiv x \in \mathbb{R}^n_+ | p_1x_1 + p_2x_2 + \ldots + p_nx_n \leq y$
-
$$S\ is\ Compact\ (JR.\ pp.514): \begin{cases} \text{Closed: not open} \\ \text{Bounded: } \exists\epsilon > 0 \text{ s.t. } S \subset B_\epsilon(x) \end{cases} \tag{3}$$
- Open: S is open if $\forall x \in S, \exists \epsilon > 0$ s.t. $B_\epsilon(x) \subset S$
- Convex: S is convex if for any $x^1, x^2 \in S$, we have $tx^1 + (1-t)x^2 \in S, \forall t \in [0,1]$

4.1 Budget set B is compact when $p \gg 0$

(1) B is closed(not open).

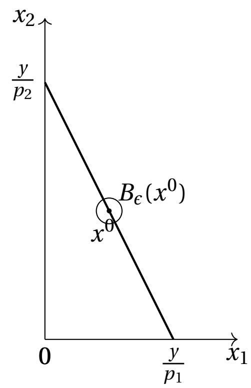


Figure 16: An example of closed budget set with 2 dimentions

We can find some x^0, s.t. $p'x^0 = p_1x_1^0 + p_2x_2^0 + \ldots + p_nx_n^0 = y$.
Where $p' = (p_1, p_2, \ldots, p_n)$ and $x^{0\prime} = (x_1^0, x_2^0, \ldots, x_n^0)$
Obviously, $x^0 \in S$ (actually it's on the boundary).

Define $B_\epsilon(x^0)$ as the ball with x^0 as center and $\epsilon > 0$ as radius.
For any $\epsilon > 0$, we can always find some $e \in (0, \epsilon)$ s.t. bundle $x^1 = (x_1^0 + e, x_2^0, \ldots, x_n^0)$ lies in ball $B_\epsilon(x^0)$, while since $p'x^1 > y$, x^1 is out of the budget set B.

In conclusion: we can never find an $\epsilon > 0$ s.t. $B_\epsilon(x^0) \subset B$. Therefore B is closed.

(2) B is bounded
Define $N \equiv max\{\frac{y}{p_1}, \frac{y}{p_2}, \ldots, \frac{y}{p_n}\} > 0$.
$\forall x \in B$, we have $C = (N, N, \ldots, N) > x$. We can thus argue that B can be contained by some ball $B_{(\epsilon)}(x^0)$, i.e. bounded.

For example:

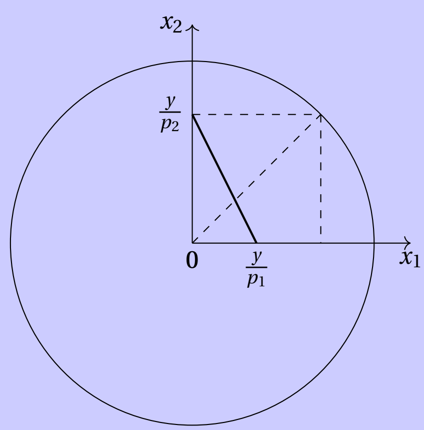


Figure 17: An example of closed budget set with 2 dimentions

$B \subset C \subset B_{(nN^n)^{\frac{1}{n}}}(origin)$.

4.2 Budget set B is convex when $p \gg 0$

Again, define price vector $p' = (p_1, p_2, \ldots, p_n)$ and bundle $x' = (x_1, x_2, \ldots, x_n)$
For any $x^1, x^2 \in B$, we have

$$\begin{aligned} p'x^1 &\leq y \\ p'x^2 &\leq y \end{aligned}$$

Define $x^t = tx^1 + (1-t)x^2$, $t \in [0,1]$
We have

$$\begin{aligned} p'x^t &= p'tx^1 + p'(1-t)x^2 \\ &= tp'x^1 + (1-t)p'x^2 \\ &\leq ty + (1-t)y \end{aligned}$$

$p'x^t \leq y \Rightarrow x^t \in B$. Therefore B is compact.

5 Jehle & Reny 1.26 - Marshallian demand function

A consumer of **two goods** faces **positive prices** and has a **positive income**. His utility function is

$$u(x_1, x_2) = x_1$$

- Derive the Marshallian demand functions.

Marshallian demand functions $x^* = x(p, y)$ is the solutions to the utility maximisation problem (JR. pp.21).

Here we have a 2-commodities consumption problem:

$$\begin{cases} Commodities: x_1, x_2 \geq 0 \\ Price: p_1, p_2 > 0 \\ Income: y > 0 \\ Utility: u(x_1, x_2) = x_1 \end{cases} \tag{4}$$

A consumer wants to

$$\max_{x_1, x_2} u(x_1, x_2) = x_1 \;\; s.t. \; p_1x_1 + p_2x_2 \leq y, \;\; and \; x_1, x_2 \geq 0$$

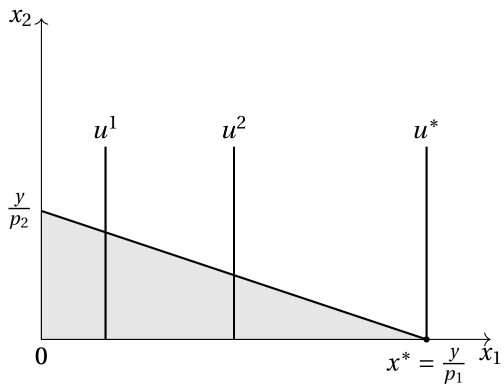


Figure 18: Corner solution

$x_1^*(p_1, y) = \frac{y}{p_1}, x_2^* = 0$, note it's a corner solution.

If you want to use Lagrangian method, be sure don't forget $x_1, x_2 \geq 0$. This is important for corner solutions.
More general, we can use Kuhn-Tucker conditions to solve the problem.
$\max_{x_1,x_2} \ u(x_1, x_2) = \ \ s.t. \ p_1x_1 + p_2x_2 \leq y, \ \ and \ x_1, x_2 \geq 0$ are: Lagrangian function:

$$L = x_1 + \lambda_1(y - p_1x_1 - p_2x_2) + \lambda_2(x_1 - 0) + \lambda_3(x_2 - 0)$$

The Kuhn-Tucker conditions are:

$$\begin{cases} \frac{\partial L}{\partial x_1} = 1 - p_1\lambda_1 + \lambda_2 = 0 \\ \frac{\partial L}{\partial x_1} = -p_2\lambda_1 + \lambda_3 = 0 \end{cases} \tag{5}$$

$$\begin{cases} p_1x_1 + p_2x_2 \leq y \\ \lambda_1 \geq 0 \\ \lambda_1(y - p_1x_1 - p_2x_2) = 0 \end{cases} \tag{6}$$

$$\begin{cases} x_1 \geq y \\ \lambda_2 \geq 0 \\ \lambda_2 x_1 = 0 \end{cases} \tag{7}$$

$$\begin{cases} x_2 \geq y \\ \lambda_3 \geq 0 \\ \lambda_3 x_2 = 0 \end{cases} \tag{8}$$

- If $x_1, x_2 > 0$, by condition 7 and 8, $\lambda_2 = \lambda_3 = 0$. Contradicts with condition 5

- If $x_1 = 0, x_2 > 0$, by condition 8, $\lambda_3 = 0$. With condition 5, we have $\lambda_1 = 0, \lambda_2 = -1$, condition 7 violated.

- If $x_1 > 0, x_2 = 0$, by condition 7, $\lambda_2 = 0$. With condition 5, we have $\lambda_1 = \frac{1}{p_1}, \lambda_3 = \frac{p_2}{p_1}$. With condition 6, $x_1^* = \frac{y}{p_1}$

Tips for exam:

- Drawing a sketch of the budget set and the indifference curves can be very helpful when the utility function is "bizarre". See also the next question.

6 Jehle & Reny 1.27 - Masshallian demand function

A consumer of **two goods** faces **positive** prices and has a **positive income**. His utility function is

$$u(x_1, x_2) = max[ax_1, ax_2] + min[x_1, x_2], \quad where \ \ 0 < a < 1.$$

- Derive the Marshallian demand functions.

The utility $u(x_1, x_2) = max[ax_1, ax_2] + min[x_1, x_2]$ depends on the relation between ax_1, ax_2 and x_1, x_2

Since $0 < a < 1$, we know $ax_1 \geq ax_2 \iff x_1 \geq x_2$. Thus:

$$\begin{cases} If \ \ x_2 >= x_1, & u(x_1, x_2) = ax_2 + x_1 \\ If \ \ x_1 >= x_2, & u(x_1, x_2) = ax_1 + x_2 \end{cases} \tag{9}$$

Any indifference curve in a rectangular coordinate system below is actually the graph of a function $x_2 = f_(x_1)$

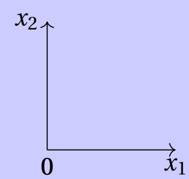


Figure 19: A rectangular coordinate system

More specifically, since bundles on the same indifference curve can provide the same level of utility, an indifference curve is the graph of a function $x_2 = f_(x_1)$ given some utility $\bar{u}$

Given $u = \bar{u}$, We can rewrite equation 9 as:

$$\begin{cases} If \ \ x_2 >= x_1, & x_2 = -\frac{1}{a}x_1 + \frac{1}{a}\bar{u} \\ If \ \ x_1 >= x_2, & x_2 = -ax_1 + \bar{u} \end{cases} \tag{10}$$

Now we can draw the sketch of the indifference curves:

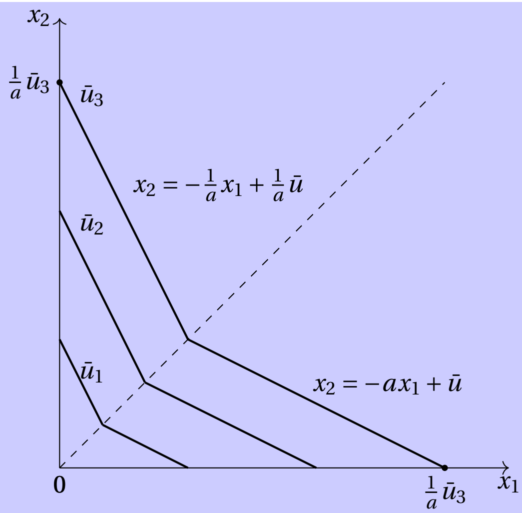


Figure 20: Indifference curves are seperated by 45 ° line

If we have a budget set similar to the one in Figure 21, we can move our indifference curve until we have a tangent point. The affordable bundle maximazing the utility is thus found.

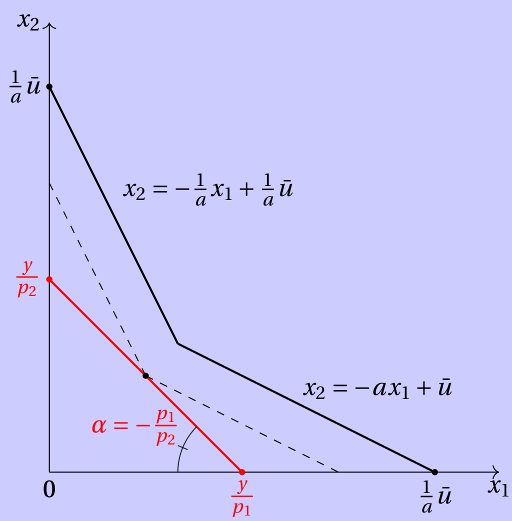


Figure 21: A budget set example

However, there can be other tricky conditions, depending on the shape of the budget set. For example, in Figure 22

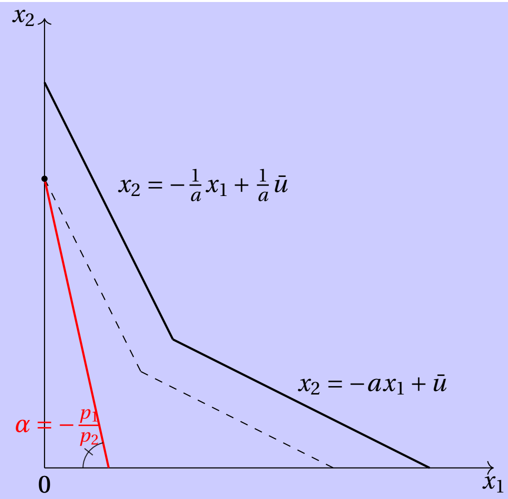


Figure 22: A budget set example 2

- The position of the optimal consumption bundle depends on the slope $\alpha = -\frac{p_1}{p_2}$
- We need to discussion the relation between $\frac{p_1}{p_2}$ and the slop of the indifference curve $[a, \frac{1}{a}]$, for $a \in (0,1)$

1. When $0 < \frac{p_1}{p_2} < a$, the budget set is showed in Figure 23. The Marshallian demand is $\{x_1 = \frac{y}{p_1}, x_2 = 0\}$

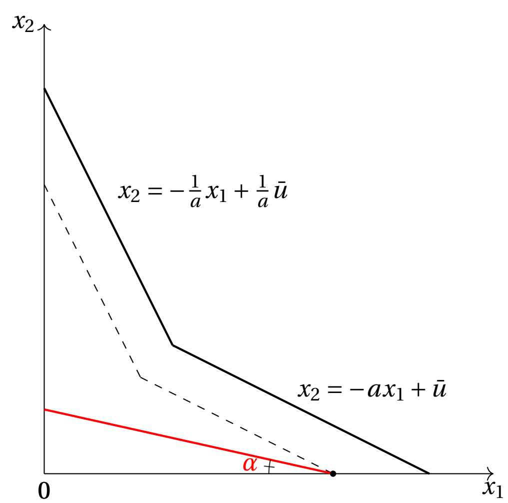


Figure 23: A budget set when $\frac{p_1}{p_2} < a$

2. When $\frac{p_1}{p_2} = a$, the budget set is showed in Figure 24. The Marshallian demand is in a set $x(p_1, p_2, y) = \{(x_1, x_2) | x_2 \leq x_1 and x_2 = -ax_1 + \frac{y}{p_2}\}$

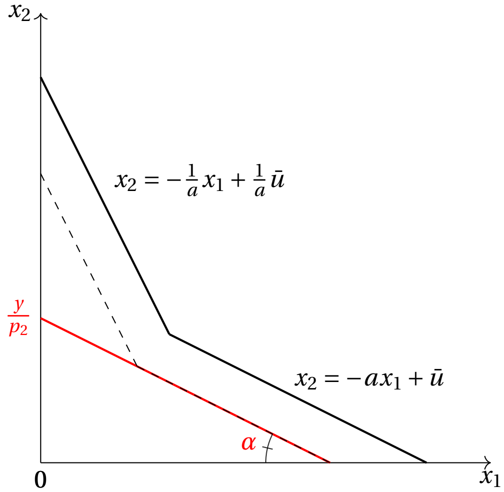


Figure 24: A budget set when $\frac{p_1}{p_2} = a$

3. When $\frac{p_1}{p_2} \in (a, \frac{1}{a})$, the budget set was already showed in Figure 21. In this case, $x_1 = x_2$ and $p_1x_1 + p_2x_2 = y$, the Marshallian demand is therefore $\{x_1 = \frac{y}{p_1+p_2}, x_2 = \frac{y}{p_1+p_2}\}$

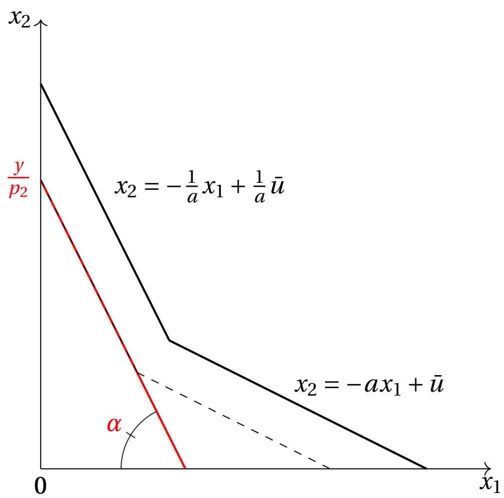


Figure 25: A budget set when $\frac{p_1}{p_2} = \frac{1}{a}$

4. When $\frac{p_1}{p_2} = \frac{1}{a}$, the budget set is showed in Figure 25. The Marshallian demand is in a set $x(p_1, p_2, y) = \{(x_1, x_2) | x_2 \geq x_1 and x_2 = -\frac{1}{a}x_1 + \frac{y}{p_2}\}$

5. When $1 > \frac{p_1}{p_2} > \frac{1}{a}$, the budget set was already showed in Figure 22. The Marshallian demand is $\{x_1 = 0, x_2 = \frac{y}{p_2}\}$