# COMP760 - Self Assessment Quiz

August 2022

## Preamble and Important Information

This is a ungraded self-assessment quiz, **without answers**, which should aid in your own understanding of whether this course is the best fit for you. Many concepts in this courses require a high level of comfort with fundamental machine learning ideas such as deep learning, including coding in a familiar framework. Furthermore, the nature of differential geometry requires a certain mathematical appetite and maturity to digest and power through challenging but intellectually stimulating mathematical ideas that will be presented in the paper readings. Any curious mind is invited to register and stay in this course, but this is fair warning to anyone else who thinks this is an easy course. It is not; you will be challenged and encouraged to execute a high level of research achievement that hopefully invites fun class collaborations and eventually a novel insight that the larger community can benefit from. Of course, we your instructors will be here to assist and guide you every step of the way but much like in research the onus is on you to take control of your own mastery and learning.

## Q.1 Basic Math

**Part 1.** Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function on standard Euclidan space equipped with the usual inner product (dot-product) $\langle \cdot, \cdot \rangle$. Define the derivative of this function at a point $x$, what is its Hessian?

**Part 2.** You are given a coin which you would like to decide whether it is biased or not. You think it may be rigged to only land on heads. You have no other prior information about this coin so you assume that it is equally likely to be a fair coin or a rigged coin. You conduct an experiment and observe in 10 trials 10 heads appear. What is the probability that the coin is rigged?

**Part 3.** Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a function that is bijective—i.e. it is surjective and injective and thus invertible—and $z \sim q(z)$ be a sample from some distribution defined on $\mathbb{R}^n$. If $z' = f(z)$, what is the density $p(z')$ expressed in terms of $q(z)$?

**Part 4.** You are given two identical circular coins which are placed side-by-side. Fixing the location of one coin, you rotate the second coin around the perimeter of the first coin for exactly 1 revolution. How many times would the coin that is moving spin around its own axis after 1 revolution?

## Q.2 Machine Learning

**Part 1.** If we parametrize a $d$-dimensional categorical distribution using a $d$-dimensional vector $\theta$ as $p(c|\theta) = \theta_c$. What is the allowable range for the vector $\theta$? Hint: what happens when we set all $\theta_c = 0$?

**Part 2.** Suppose we are given a parametrized Gaussian distribution $\mathcal{N}(\mu, \sigma^2)$, where the mean and variance are parametrized $\theta = \{\mu, \sigma\}$. How might we sample from this distribution in such a way that is amenable to backpropagation?

**Part 3.** Qualitatively explain the difference between the forward KL-divergence and the reverse KL-divergence in terms of "mode"-seeking versus "mode"-averaging behavior.

**Part 4.** We would like to estimate the log density of the data distribution $\log p(x)$ in a VAE setting. Derive the VAE objective—i.e. the Evidence Lower Bound(ELBO)—given a variational distribution $q$.