

Noxious Fumes : Guaranteed Death in $\text{ceil} \left(\left(-3 + \sqrt{8h + 9} \right) / 2 \right)$ turns

Hein Hundal

November 14, 2023

I love Noxious Fumes. I love that once I play Noxious Fumes, the enemy will die in square root time no matter what he does and no matter what I do. In fact, every monster with h or less hit points will be dead in no more than

$$\text{deathTime} = \text{ceil} \left(\left(-3 + \sqrt{8h + 9} \right) / 2 \right)$$

turns.¹

Definition 1. *The notation*

$$\text{ceil}(x)$$

is defined to be the smallest integer which is greater than or equal to x . For example, $\text{ceil}(13) = 13$, $\text{ceil}(3.2) = 4$, and $\text{ceil}(\pi) = 4$. “ceil” is an abbreviation for “ceiling” and $\text{ceil}(x)$ is x rounded upward.

¹Alright, technically this formula does not work if the monster can heal or if the monster can remove the noxious fumes. The list of monsters that can heal or remove debuffs includes: Centurion/Mystic, Spheric Guardian, Reptomancer, The Collector, the Awakened One, and the Time Eater.

Let's try out the formula. If the monster has 100 health points, then it will be dead in

$$\begin{aligned}
\text{deathTime} &= \text{ceil} \left((-3 + \sqrt{8 \cdot 100 + 9}) / 2 \right) \\
&= \text{ceil} \left((-3 + \sqrt{809}) / 2 \right) \\
&= \text{ceil} ((-3 + 28.44) / 2) \\
&= \text{ceil} (25.44 / 2) \\
&= \text{ceil} (12.72) \\
&= 13
\end{aligned}$$

turns.

- If you play the Noxious Fumes on turn 4, then on turn 4+1=5, the monster will have 2 poison, and 2 total poison damage.
- On turn 4+2=6, the monster will have 3 poison, and 2+3 total poison damage.
- On turn 4+3=7, the monster will have 4 poison, and 2+3+4 total poison damage.
- ...
- On turn 4+12=16, the monster will have 13 poison and $2 + 3 + 4 + \dots + 13 = 90$ total poison damage.
- On turn 4+13=17, the monster will have 14 poison and $2 + 3 + 4 + \dots + 14 = 104$ total poison damage and hence will be dead.

Theorem 1. *If a monster has h health points when a Noxious Fumes card is played and that monster has no way to heal or remove the Noxious Fumes, then it will be dead*

$$\text{deathTime} = \text{ceil} \left((-3 + \sqrt{8h + 9}) / 2 \right)$$

turns later.²

For the few of you who like proofs, here it is.

Proof. The sum $2 + 3 + 4 + \dots + n$ is the sum of $n - 1$ numbers whose average value is $\frac{n+2}{2}$, so that sum is

$$2 + 3 + 4 + \dots + n = (n - 1) \frac{n + 2}{2}.$$

For example,

$$2 + 3 + 4 + 5 = (5 - 1) \cdot (5 + 2) / 2 = (4 \cdot 7) / 2 = 28 / 2 = 14.$$

t turns after the Noxious Fumes card is played, the monster will have $t + 1$ poison and a total poison damage of

$$\text{totalPoisonDamage} = 2 + 3 + 4 + \dots + (t + 1) = t(t + 3) / 2.$$

If the monster had h health points when the Noxious Fumes card was played, then t turns later the monster will be dead if

$$h \leq \text{totalPoisonDamage} = t(t + 3) / 2.$$

²Possibly earlier if there is any other source of damage.

Now let's try to find the value of t where the left hand side equals the right hand side. The following are equivalent

$$\begin{aligned} h &= t(t+3)/2 \\ 0 &= \frac{t^2}{2} + \frac{3t}{2} - h. \end{aligned}$$

According to the quadratic formula, the equation above will be satisfied by

$$\begin{aligned} t &= \frac{-3/2 \pm \sqrt{(3/2)^2 - 4(1/2)(-h)}}{2 \cdot (1/2)} \\ &= -3/2 \pm \sqrt{9/4 + 2h} \\ &= \frac{-3 \pm \sqrt{9 + 8h}}{2}. \end{aligned}$$

The solution $t = \frac{-3 - \sqrt{9 + 8h}}{2}$ can be disregarded because it is negative. So,

$$h = t(t+3)/2$$

if $t = \frac{-3 + \sqrt{9 + 8h}}{2}$. The fact that poison damage is always increasing implies that

$$h \leq t(t+3)/2 \quad \text{if } t \geq \frac{-3 + \sqrt{9 + 8h}}{2}.$$

If t is an integer, then

$$h \leq t(t+3)/2 \quad \text{if } t \geq \text{ceil}\left(\frac{-3 + \sqrt{9 + 8h}}{2}\right)$$

which implies the monster is dead if

$$t \geq \text{ceil}\left(\frac{-3 + \sqrt{9 + 8h}}{2}\right).$$

□

That's it. Watch out for the sequel,

“Noxious Fumes+ : Guaranteed Death in <Spoiler Removed> turns or less”.