How many Noxious Fumes is too many Noxious Fumes?

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Inspired by this post by /u/irchans, I started to wonder exactly how the number of Noxious Fumes is related to both the overall damage output and the number of turns after having played them.

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The Results and the Math

Let's keep the assumptions the same as in the previous post:

- You play all copies of noxious fumes on the same turn
- The enemy has no existing poison
- You play no other damage cards against that enemy, including AOE or subsequent poison cards

We can in fact find exact relationships between the damage output h (a.k.a. health of the enemy), the number of turns t it takes to deal that amount of damage, and p the amount of poison applied by Noxious Fumes each turn. For those interested in the results alone, here they are:

$$h = \left(pt^2 + pt - t^2 + t\right)/2,$$

$$t = \text{ceil}\left(\frac{\sqrt{8h(p-1) + (p+1)^2} - p - 1}{2(p-1)}\right), \text{ and}$$

$$p \approx \frac{2h + t^2 - t}{t^2 + t}.$$

where ceil means round up to the next highest whole number. (ceil(3)=3, ceil(3.1)=4) and h is the health of the enemy, t is the number of turns needed to kill the monster after having played Noxious Fumes, and p is the poison applied each turn by Noxious Fumes.

For those interested in the process and in double-checking my math, the process of finding these relations starts from recognizing that the damage pattern follows two separate functions of triangular numbers. This can be seen from mapping out the total damage each turn, starting from turn 1.

*With some edits by irchans.

(The pasted spreadsheet graphic looks good to me!) Enemy health lost in total on each turn as a function of poison applied by fumes.

From there, simplifying to the above formula for h is a matter of some refactoring. Getting the other formulas is a result of some algebra as well, either just basic algebra or using the quadratic formula in the case of the formula for t turns. Now that we have our formulas for all three variables, let's take a look at some examples!

Example 1: The Heart

Let's say you want to kill the heart before it buffs for 10 strength on turn 10. The heart has 800 health on A20, so we'll run with that number. While this is obviously unrealistic, let's assume the following:

- you have no damage in your deck other than noxious fumes or Noxious Fumes+,
- you draw all copies of fumes on turn 1, and
- you can block the heart's attacks until turn 11

Since the heart buffs for 10 strength on turn 10, we have until turn 11 for our poison to do damage. Since we have to use turn 1 playing our fumes, that means we have exactly 10 turns for our poison to work. How much poison do we need to apply each turn? Let's run the numbers:

$$p \approx (2h - t + t^2)/(t \cdot (1+t)) = (1600 - 11 + 121)/(11 \cdot 12)) = 1710/132 \approx 12.95$$

Given that we cannot have fractional poison, we must apply 13 poison each turn to kill the heart before it kills us. This would mean 3 copies of fumes+ and 2 copies of unupgraded fumes at minimum. If we plug in $p = 13$ and $t = 11$ into

$$h = \left(pt^2 + pt - t^2 + t\right)/2,$$

we get

$$(13 \cdot 121 + 13 \cdot 11 - 121 + 11)/2 = 803, \text{and}$$
$$13 + 25 + 37 + \cdots + 133 = 803.$$

Example 2: Hexaghost

We keep the same assumptions. This fight is often said to be "solved by fumes," but how many fumes are truly needed to solve hexaghost? Let's assume again that we want to kill hexaghost before it gets off the inferno attack on turn 9. Since we must play all our fumes on turn 1, we have 8 turns to kill this ghost before it burns us alive. Hexaghost has 264 health, and all our previous assumptions are the same.

$$p \approx (2h - t + t^2)/(t + t^2) = (528 - 8 + 64)/(8 + 64) = 584/72 = 8.111111...$$

Again, since we can't have fractional poison, we must apply 9 poison each turn to kill the hexaghost before it kills us. In other words, we need 3 fumes+. In this case,

$$9 + 17 + 25 + \cdots + 65 = 296.$$