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Assessment 4: Test of Means and Proportions

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1. A plantita looked at the effect of a certain fertilizer. She tested the fertilizer on one group of plants (A) and did not give fertilizer to another group (B). The growth of the plants, in mm, were as follows:

Group	Growth
A	55 61 33 57 17 46 50 42 71 51 63
B	31 27 12 44 9 25 34 53 33 21 32

On a 1% significance level, test the claim that the fertilizer enhanced growth. Assume that the distributions of growths are approximately normal with equal variances.

Solution. We are to assume that the distribution of growths is approximately normal with equal variances. Since the two groups of plants are independent of each other, and the fertilizer is used only to the first group, A, then we are not going to proceed with paired means. Let μ_A and μ_B be the means of A and B, respectively. Then we have the following hypotheses:

$$H_0 : \mu_A - \mu_B = 0$$
$$H_1 : \mu_A - \mu_B > 0$$

Let s_A and s_B be the sample standard deviation of A and B, respectively. Moreover, $\overline{x}_A = (55 + 61 + 33 + 57 + 17 + 46 + 50 + 42 + 71 + 51 + 63)/11 = 49.63636$ and $\overline{x}_B = (31 + 27 + 12 + 44 + 9 + 25 + 34 + 53 + 33 + 21 + 32)/11 = 29.18182$ It follows that

$$s_A^2 = \frac{\sum_{i=1}^{11}(x_i - 49.63636)^2}{10}$$

$$= 226.25455 s_B^2 \qquad = \frac{\sum_{i=1}^{11}(x_i - 29.18182)^2}{10}$$

$$= 162.76364$$

and $df = 11 + 11 - 2 = 20$ given that $n_A = n_B = 11$. Hence,

$$\begin{aligned} s_p &= \sqrt{\frac{10s_A^2 + 10s_B^2}{df}} \\ &= \sqrt{\frac{s_A^2 + s_B^2}{2}} \\ &= 13.94665175 \end{aligned}$$

Solving for statistic $t-$value, we have

$$\begin{aligned} t &= \frac{\overline{x}_A - \overline{x}_B}{s_p\sqrt{\dfrac{1}{n_A}+\dfrac{1}{n_B}}} \\ &= \frac{49.63636 - 29.18182}{13.94665175\sqrt{2/11}} \\ &= 3.439545867 \end{aligned}$$

Using $\alpha = 0.01$ and $df = 20$, we have the critical value $t_{0.01,\,20} = 2.528$. We reject H_0 if $t > t_{0.01,\,20}$. Since $3.4395 > 2.528$, then we reject H_0. In conclusion, the fertilizer indeed enchanced growth of the plants. □

2. A total of 8605 students are enrolled full-time at a State University this semester, 4134 of whom are women. Of the 6001 students who live on campus, 2915 are women. Can it be argued that the difference in the proportion of men and women living on campus is statistically significant?

   Solution. Out of 8605 students, 4134 are women which follows that 4471 are men. Of these 4134 women, 2915 live on campus. Moreover, 3086 of 4471 male students live on campus. We let p_w and p_m be the proportion of women and male who live on-campus, respectively. We have the following hypotheses:

$$\begin{aligned} H_0 &: p_w - p_m = 0 \\ H_1 &: p_w - p_m \neq 0 \end{aligned}$$

   where $\hat{p}_w = 2915/4134$ and $\hat{p}_m = 3086/4471$. Let n_w and n_m be the number of female and male students in the said university, respectively. Computing for statistic z-value, we have

$$\begin{aligned} z &= \frac{\hat{p}_w - \hat{p}_m}{\sqrt{\overline{p}\,\overline{q}\left(\dfrac{1}{n_w}+\dfrac{1}{n_m}\right)}} \\ &= \frac{2915/4134 - 3086/4471}{\sqrt{\left(\dfrac{2915+3086}{4134+4471}\right)\left(1-\dfrac{2915+3086}{4134+4471}\right)\left(\dfrac{1}{4134}+\dfrac{1}{4471}\right)}} \\ &= 1.5034 \end{aligned}$$

   We reject H_0 if $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$. Using $\alpha = 0.05$, we have $z_{\alpha/2} = 1.96$. Since $1.5034 \not< -1.96$ nor $1.5034 \not> 1.96$, we do not reject H_0. That is, the difference in the proportion of men and women living on campus is not statistically significant. □