Numerical Analysis
Linear Systems

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1. For $n = 2, 3, \ldots, 10$, generate a Hilbert Matrix H of order n. Note that H is symmetric positive definite matrix. Use Matlab built-in function **hilb(n)** to check the output of the code.

2. In Matlab, **R = chol(A)** factorizes the symmetric positive definite matrix A into an upper triangular R that satisfies $A = R' * R$. Use the built-in function **chol** to decompose H into lower and upper triangular R' and R, respectively.

3. Generate the $n-$vector $b = Hx$, where x is an $n-$vector with all of its components equal to 1. Using the factorization of H, solve the linear system $Hx = b$ using forward and backward substitution to obtain an approximate solution $\hat{x}$.

4. Compute for the $\infty-$norm of the residual $r = b - H\hat{x}$ and the error $\Delta x = \hat{x} - x$, where the $n-$vector x consisting of ones is the true solution of the linear system. Compute also for the condition number of H.

5. Plot n vs the error Δx, residual r, and the condition number in one figure. For the condition number, use the **semilogy** plot function.