Numerical Analysis
Linear Systems

I. Consider the linear system

$$Ax = \begin{bmatrix} 0.913 & 0.659 \\ 0.457 & 0.330 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.254 \\ 0.127 \end{bmatrix} = b$$

and the two approximate solutions $\hat{x}_1 = [0.6391 \ -0.5]^T$ and $\hat{x}_2 = [0.999 \ -1.001]^T$

(a) Calculate the $1-$norms of the respective residuals of $\hat{x}_1$ and $\hat{x}_2$.

(b) Calculate the condition number of A.

(c) Which is better solution to the system if it is known that the exact solution is $[1 \ -1]^T$

II. Any matrix A can be expressed as a product of an orthogonal matrix Q and an upper triangular matrix R. This QR factorization can be used to solve linear systems:

$$Ax = b \longleftrightarrow QRx = b$$

(a) Determine the orthogonal matrix Q and the upper triangular matrix R such that

$$QR = \begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} =: A$$

(b) Given $Ax = b$ with $b = [0 \ 1 \ 3]^T$, determine y using Gaussian elimination from the linear system $Qy = b$. If there is an error, use the Matlab built-in function.

(c) Calculate x from $Rx = y$ using back substitution.