Ordinary and Partial Derivatives

Explained with Bugs

1 Ordinary Differentiation $\frac{d}{dx}$

Let's assume that we have a bug, 🐌, that depends on another two bugs, 🐛 and 🐜. We can define this relationship like so;

$$\text{🐌} = 3\text{🐛}^2 + 2\text{🐜}$$

In an ordinary derivative, we assume all bugs are bugs. We want to take the derivative of 🐌 with respect to 🐛, so let's also say 🐜 has a relationship with 🐛 defined as;

$$\text{🐜} = 2\text{🐛}^2 + 3$$

Now, to differentiate. To do so, we must take the derivative of every bug with respect to the bug we're differentiating with respect to, which in our case is 🐛;

$$\frac{d\text{🐌}}{d\text{🐛}} = \frac{d}{d\text{🐛}}3\text{🐛}^2 + \frac{d}{d\text{🐛}}2\text{🐜}$$

Here we can use the power rule; if we have 🐛^n, the derivative with respect to 🐛 is $n\text{🐛}^{n-1}$. For example, with $n = 2$, $\frac{d}{d\text{🐛}}\text{🐛}^2 = 2\text{🐛}$. In our case, we get $2 \times 3\text{🐛}^{2-1}$, or just 6🐛.

$$\frac{d\text{🐌}}{d\text{🐛}} = 6\text{🐛} + \frac{d}{d\text{🐛}}2\text{🐜}$$

We still need to find the derivative of 2🐜 with respect to 🐛. When taking the ordinary, or total, derivative, every bug that depends on 🐛 must be differentiated using the chain rule. We know that 🐜 depends on 🐛, so we must include $\frac{d\text{🐜}}{d\text{🐛}}$ in our expression. Their relationship here is $\text{🐜} = 2\text{🐛}^2 + 3$. Let's just find the derivative of that with respect to 🐛 first!

$$\frac{d\text{🐜}}{d\text{🐛}} = \frac{d}{d\text{🐛}}2\text{🐛}^2 + \frac{d}{d\text{🐛}}3$$

For the first term, we can use the power rule again, so it becomes 4🐛. 3, however, is a constant and not a bug! It has no relationship with 🐛, so the derivative is just 0. Our result is as follows;

$$\frac{d🐜}{d🐛} = 4🐛$$

Now, back to the main point. We had to differentiate 2🐜 with respect to 🐛—that's just our previous answer multiplied by 2, or 8🐛.

$$\frac{d🐌}{d🐛} = 6🐛 + 8🐛$$

$$= 14🐛$$

Great! That's our answer for the **ordinary derivative**.

2 Partial Differentiation $\frac{\partial}{\partial x}$

Now, what about our **partial derivative**? This one is surprisingly more simple. The only difference is that we assume all bugs that aren't the one we're differentiating with respect to don't have a relationship with the respective bug, and are constants. Let's go ahead and define 🐌 the same way as before; $3🐛^2 + 2🐜$. Now, we'll differentiate with respect to 🐛 again—we already know what the first term will become!

$$\frac{\partial🐌}{\partial🐛} = 6🐛 + \frac{\partial}{\partial🐛}2🐜$$

Remember how in a partial derivative we pretend every other bug is a constant? We'll treat 🐜 as a constant, so the term 2🐜 also becomes a constant. But now we encounter the same with we did last time with 3. 2🐜 is a constant, not a bug, so our derivative becomes 0 again! Now, we've simplified everything, and our final answer is...

$$\frac{\partial🐌}{\partial🐛} = 6🐛$$

[0]For fun, we can also write the ordinary derivative of 🐌 with respect to 🐛 in multivariable form, as $\frac{d🐌}{d🐛} = \frac{\partial🐌}{\partial🐛} + \frac{\partial🐌}{\partial🐜}\frac{d🐜}{d🐛}$