

# Multivariable Differentiation

## Explained with Bugs

Say we have a bug, 🐞, that depends on three other bugs; 🐛, 🐝 and 🐝 (by now, we've established that the snail is the main function bug). We will define this relationship as follows;

$$\text{螬} = 3\text{螬}^3 - 2\text{螬}^2 + 4\text{螬}$$

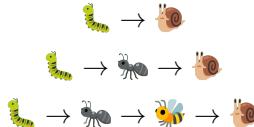
Let's also define 🐝 and 🐝 as functions of 🐛.

$$\text{螬} = \text{螬}^2$$

$$\text{螬} = -\text{螬}$$

If you saw the last episode on *Ordinary and Partial Derivatives*, you might've noticed that 🐝 here has a different relationship with 🐛—the ant and caterpillar colonies had some discussions since last time. This is some important lore in the Explained with Bugs universe, and is definitely not so I can do less math.

Suppose 🐛 is nudged a little, maybe it ate a leaf and got bigger... or smaller. Perhaps this place has negative matter. Nevertheless, since 🐛 changed, 🐞, which depends on 🐛, must also change by  $\frac{\partial \text{螬}}{\partial \text{螬}}$ . We use a partial derivative here because we only want what 🐛 did to 🐞, not what every bug did to it. For the entire sequence of bugs, we will be following a path from 🐛 to 🐞, as 🐛 is the bug we are differentiating with respect to. A good way to view this is a dependency tree;



Continuing the **chain** (thus the name of this rule I secretly just explained, the **chain rule**), 🐝 also changes due to 🐛 by  $\frac{d\text{螬}}{d\text{螬}}$ . We use an ordinary derivative here because the only bug 🐝 depends on is 🐛, so it doesn't matter if we take a partial or ordinary derivative. Following the trend, 🐝 also changes due to 🐛, but because it relies on 🐝, which itself changes due to 🐛. Here, 🐝 changes by  $\frac{d\text{螬}}{d\text{螬}} \frac{d\text{螬}}{d\text{螬}}$ . We use ordinary derivatives here for the same reason as before.

You may think, 🐞 depends on all of these bugs, so it should also be changed due to 🐝 and 🐝. Indeed, we can do what we did with the 🐛 term, and change

by  $\frac{\partial \text{bug}}{\partial \text{ants}}$  and  $\frac{\partial \text{bug}}{\partial \text{bees}}$ . These are partial derivatives for aforementioned reason in the last paragraph. We must multiply each  $\frac{\partial \text{bug}}{\partial x}$  by how the bug placed in  $x$ 's place changes with respect to  $\text{ants}$ , as those bugs change indirectly when  $\text{ants}$  changes.

Now that we've got every term resolved, we can construct our derivative;

$$\frac{d \text{bug}}{d \text{ants}} = \frac{\partial \text{bug}}{\partial \text{ants}} + \frac{\partial \text{bug}}{\partial \text{ants}} \frac{d \text{ants}}{d \text{ants}} + \frac{\partial \text{bug}}{\partial \text{bees}} \frac{d \text{bees}}{d \text{ants}}$$

Also, notice how  $\frac{d \text{ants}}{d \text{ants}}$  neatly simplifies to  $1$ , like how a pair of fractions with a matching numerator and denominator cancel out. There are many cases in which these fraction-like behaviours will occur, the beauty of this notation.

Let us get to solving these terms now.

$$\frac{\partial \text{bug}}{\partial \text{ants}} = 9 \text{ants}^2$$

$$\frac{\partial \text{bug}}{\partial \text{bees}} = -4 \text{bees}$$

$$\frac{\partial \text{ants}}{\partial \text{ants}} = 4$$

Here, I simply used the power rule, mentioned in the previous episode.  $3\text{ants}^3$ ,  $-2\text{ants}^2$  and  $4\text{bees}$  become their respective terms above. We can use this same rule to find the rest of our terms.

$$\frac{d \text{ants}}{d \text{ants}} = 2 \text{ants}$$

$$\frac{d \text{bees}}{d \text{ants}} = -2 \text{ants}$$

$\text{bees}$  is defined as  $-\text{ants}$ , so we can just take the negative derivative of  $\text{ants}$  with respect to  $\text{ants}$  for the derivative of  $\text{bees}$  with respect to  $\text{ants}$ . You'll notice this would be the same result as solving  $\frac{d \text{ants}}{d \text{ants}} \frac{d \text{bees}}{d \text{ants}}$ , as it evaluates to  $-1 \times 2 \text{ants}$ . Substituting these for the terms in our main derivative, we get;

$$\frac{d \text{bug}}{d \text{ants}} = 9 \text{ants}^2 - 8 \text{ants} \text{bees} - 8 \text{ants}$$

Here, we can substitute  $\text{ants}$  for its definition to simplify the equation;

$$\frac{d \text{bug}}{d \text{ants}} = 9 \text{ants}^2 - 8 \text{ants}(\text{ants}^2) - 8 \text{ants}$$

$$= 9\text{L}^2 - 8\text{L}^3 - 8\text{L}$$

Great, we have solved the derivative! If you're feeling extra special, or you're just very cool like me, you'll transform this cubic equation into its standard form.

$$\frac{d\text{L}^0}{d\text{L}} = -8\text{L}^3 + 9\text{L}^2 - 8\text{L}$$

You may also factor the result as follows, but beware that this will officially make you a *nerd*;

$$\frac{d\text{L}^0}{d\text{L}} = -\text{L}(8\text{L}^2 - 9\text{L} + 8)$$