Sbírka příkladů z matematiky k přijímacím zkouškám na VŠ - решебник

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1 Algebraické výrazy a jejich úpravy

1.1

$$a^2 \times a^4 \times a^3 = a^{2+4+3} = a^9$$

$$\frac{b^5 \times b^6}{b^2} = b^{5+6-2} = b^9$$

$$(c^{-3})^9 = c^{-3 \times 9} = c^{-27} = \frac{1}{c^{27}}$$

1.2

$$(2x^3)^4 + (x^7)^2 \times x^{-2} = 16x^{12} + x^{14} \times x^{-2} = 16x^{12} + x^{12} = 17x^{12}$$

$$(3x^2)^3 x^4 - (2x^2)^5 = 27x^6 \times x^4 - 32x^{10} = 27x^{10} - 32x^{10} = -5x^{10}$$

$$(x^6)(-2x^{-3})^4 + (4x^{-3})^2 = (x^6)\left(\frac{16}{x^{12}}\right) + \frac{16}{x^6} = \frac{16}{x^{12-6}} + \frac{16}{x^6} = \frac{32}{x^6}$$

1.3

$$\frac{a^0 \times a^{2n-5}}{a^{1-n}} = a^{2n-5-1+n} = a^{3n-6}$$

$$\frac{a^{2n+3} \times a^{3n-2}}{(a^n)^2} = a^{2n+3+3n-2-2n} = a^{3n+1}$$

$$\frac{a^{1-4n} \times (a^{n+2})^2}{a^{3n-1}} = \frac{a^{1-4n} \times a^{2n+4}}{a^{3n-1}} = a^{1-4n+2n+4-3n+1} = a^{6-5n}$$

1.4

$$\frac{18^4 \times 27^{-4} \times 49^2}{14^3} = 3^8 \times 2^4 \times 3^{-12} \times 7^4 \times 7^{-3} \times 2^{-3} =$$

$$= 3^{8-12} \times 2^{4-3} \times 7^{4-3} = \frac{2 \times 7}{3^4} = \frac{14}{81}$$

$$\frac{20^{-3} \times 38^0 \times 100^2}{16^{-2} \times 2^5} = 10^{-3} \times 2^{-3} \times 10^4 \times 2^8 \times 2^{-5} =$$

$$= 10^{-3+4} \times 2^{-3+8-5} = 10 \times 1 = 10$$

$$\frac{88^3 \times 55^{-2} \times 10000^{-1}}{25^{-3} \times 22} = 2^9 \times 11^3 \times 5^{-2} \times 11^{-2} \times 5^{-4} \times 2^{-4} \times 5^6 \times 2^{-1} \times 11^{-1} =$$

$$= 2^{9-4-1} \times 11^{3-2-1} \times 5^{-2-4+6} = 2^4 \times 11^0 \times 5^0 = 16$$

$$\frac{(-4)^3}{3^2} \div \frac{(-2)^4}{3^5} = (-2)^6 \times 3^{-2} \times 3^5 \times (-2)^{-4} =$$

$$= (-2)^{6-4} \times 3^{-2+5} = 4 \times 27 = 108$$

$$\frac{2^{-6} \times 3^{-2} \times 5^3}{(2^{-4} \times 5^3)^2 \div 3^4} = \frac{2^{-6} \times 3^{-2} \times 5^3 \times 3^4}{2^{-8} \times 5^6} = \frac{2^8 \times 5^3 \times 3^4}{2^6 \times 5^6 \times 3^2} =$$

$$= 2^{8-6} \times 5^{3-6} \times 3^{4-2} = 4 \times \frac{1}{125} \times 9 = \frac{36}{125}$$

1.5

$$\left(\frac{a^{-2}b^4c^{-1}}{d^3}\right)^{-3} \div \left(\frac{d^4}{c^5}\right)^{-2} = \frac{d^9}{a^{-6}b^{12}c^{-3}} \times \frac{d^8}{c^{10}} = \frac{a^6d^{17}}{b^{12}c^7}$$

$$\left(\frac{a^2c^3}{b^4d^0}\right)^{-2} \div \left(\frac{b^4}{d^{-3}}\right)^3 = \frac{b^8}{a^4c^6} \times \frac{d^{-9}}{b^{12}} = \frac{b^{-4}d^{-9}}{a^4c^6} = \frac{1}{a^4b^4c^6d^9}$$

$$\left(\frac{b^3}{a^4b^{-1}c^2}\right)^{-4} \div \left(\frac{a^2b^{-3}}{a^{-4}c^2}\right)^{-2} = \left(\frac{a^{16}b^{-4}c^8}{b^{12}}\right) \times \left(\frac{a^4b^{-6}}{a^{-8}c^4}\right) = \frac{a^{16+4+8} \times c^{8-4}}{b^{12+4+6}} = \frac{a^{28}c^4}{b^{22}}$$

1.6

$$\sqrt{2} - \sqrt{18} + \sqrt{32} + \sqrt{98} = \sqrt{2} - 3\sqrt{2} + 4\sqrt{2} + 7\sqrt{2} =$$
$$= (1 - 3 + 4 + 7)\sqrt{2} = 9\sqrt{2}$$
$$\sqrt{175} - (\sqrt{8} - \sqrt{7} + \sqrt{28}) = 5\sqrt{7} - 2\sqrt{2} + \sqrt{7} - 2\sqrt{7} =$$
$$= (5 + 1 - 2)\sqrt{7} - 2\sqrt{2} = 4\sqrt{7} - 2\sqrt{2}$$
$$\sqrt{90} + \sqrt{160} - (\sqrt{63} + \sqrt{10}) = 3\sqrt{10} + 4\sqrt{10} - 3\sqrt{7} - \sqrt{10} =$$
$$= (3 + 4 - 1)\sqrt{10} - 3\sqrt{7} = 6\sqrt{10} - 3\sqrt{7}$$
$$2 \times 8^{\frac{1}{2}} - 7 \times 8^{\frac{1}{2}} + 5 \times 72^{\frac{1}{2}} - 50^{\frac{1}{2}} = 4\sqrt{2} - 14\sqrt{2} + 30\sqrt{2} - 5\sqrt{2} =$$
$$= (4 - 14 + 30 - 5)\sqrt{2} = 15\sqrt{2}$$
$$2^{\frac{1}{2}} \times 4^{\frac{1}{3}} \times 8^{\frac{1}{4}} \times 16^{\frac{1}{6}} \times 32^{\frac{1}{12}} = 2^{\frac{1}{2}} \times 2^{\frac{2}{3}} \times 2^{\frac{3}{4}} \times 2^{\frac{4}{6}} \times 2^{\frac{5}{12}} = 2^{\frac{6+8+9+8+5}{12}} = 2^{\frac{36}{12}} = 2^3 = 8$$
$$[2 \times (2 \times 2^{\frac{1}{2}})^{\frac{1}{2}}]^{\frac{1}{2}} = 2^{\frac{1}{2}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{8}} = 2^{\frac{4+2+1}{8}} = \sqrt[8]{2^7}$$

1.7

$$\sqrt[3]{54} + \sqrt[3]{16} - \sqrt[3]{2} + \sqrt[3]{64} = 3\sqrt[3]{2} + 2\sqrt[3]{2} - \sqrt[3]{2} + 4 =$$
$$= (3 + 2 - 1)\sqrt[3]{2} + 4 = 4\sqrt[3]{2} + 4$$
$$\sqrt[3]{24} - (\sqrt[3]{3} - \sqrt[3]{81}) - \sqrt[3]{9} = 2\sqrt[3]{3} - \sqrt[3]{3} + 3\sqrt[3]{3} - \sqrt[3]{9} =$$
$$= (2 - 1 + 3)\sqrt[3]{3} - \sqrt[3]{9} = 4\sqrt[3]{3} - \sqrt[3]{9}$$
$$\sqrt[3]{250} - (\sqrt[3]{40} + \sqrt[3]{16}) + \sqrt[3]{135} = 5\sqrt[3]{2} - 2\sqrt[3]{5} - 2\sqrt[3]{2} + 3\sqrt[3]{5} =$$
$$= (5 - 2)\sqrt[3]{2} + (-2 + 3)\sqrt[3]{5} = 3\sqrt[3]{2} + \sqrt[3]{5}$$

1.8

$$\sqrt{x} \times \sqrt[3]{x} \times \sqrt[7]{x} = x^{\frac{1}{2}+\frac{1}{3}+\frac{1}{7}} = x^{\frac{21+14+6}{42}} = x^{\frac{41}{42}} = \sqrt[42]{x^{41}}$$

$$\sqrt{\frac{\sqrt[3]{x}}{\sqrt[6]{x}}} = x^{\frac{1}{6}-\frac{1}{12}} = x^{\frac{2-1}{12}} = \sqrt[12]{x}$$

$$\frac{\sqrt[4]{x \times \sqrt[3]{x}}}{\sqrt[3]{x^4 \times \sqrt{x}}} = \frac{x^{\frac{1}{4}} \times x^{\frac{1}{12}}}{x^{\frac{4}{3}} \times x^{\frac{1}{3}}} = x^{\frac{3+1-16-4}{12}} = x^{\frac{-16}{12}} = \frac{1}{\sqrt[3]{x^4}}$$

$$\sqrt{a \times \sqrt{a \times \sqrt{a \times \sqrt{a}}}} = a^{\frac{1}{2}} \times a^{\frac{1}{4}} \times a^{\frac{1}{8}} \times a^{\frac{1}{16}} = a^{\frac{8+4+2+1}{16}} = a^{\frac{15}{16}} = a \sqrt[16]{a^{15}}$$

1.9

$$\sqrt{\frac{b\sqrt[3]{b}}{\sqrt{b}}} \div \left(\frac{a^{-\frac{1}{3}}b^{\frac{1}{2}}}{b^2}\right)^3 = \frac{b^{\frac{1}{2}}b^{\frac{1}{6}}}{b^{\frac{1}{4}}} \times \frac{b^6 a}{b^{\frac{3}{2}}} = b^{\frac{6+2-3}{12}} \times ab^{\frac{72-18}{12}} = ab^{\frac{5+54}{12}} = ab^{\frac{59}{12}} = ab^4 \sqrt[12]{b^{11}}$$

$$\sqrt[3]{\frac{a\sqrt[3]{a}}{\sqrt{b}}} \times \left(\frac{a^{-\frac{1}{2}}b^{\frac{1}{3}}}{a^2}\right)^6 = \frac{a^{\frac{1}{3}}a^{\frac{1}{9}}}{b^{\frac{1}{6}}} \times \frac{a^{-3}b^2}{a^{12}} = a^{\frac{3+1-27-108}{9}}b^{\frac{-1+12}{6}} = a^{-\frac{131}{9}}b^{\frac{11}{6}} = \frac{b\sqrt[6]{b^5}}{a^{14}\sqrt[9]{a^5}}$$

$$\sqrt[3]{\frac{\sqrt{a}}{\sqrt[3]{b}}} \div \left(\frac{a^{\frac{1}{3}}b^{-1}}{a^{-\frac{1}{2}}}\right)^{\frac{1}{2}} = \frac{a^{\frac{1}{6}}}{b^{\frac{1}{9}}} \times \frac{b^{\frac{1}{2}}}{a^{\frac{1}{6}}a^{\frac{1}{4}}} = a^{\frac{2-2-3}{12}}b^{\frac{-2+9}{18}} = \frac{b^{\frac{7}{18}}}{a^{\frac{1}{4}}} = \frac{\sqrt[18]{b^7}}{\sqrt[4]{a}}$$

1.10

$$\left(2^{\sqrt{3}} \times 2^2\right)^{2-\sqrt{3}} = 2^{2\sqrt{3}-3+4-2\sqrt{3}} = 2$$

$$\left(\frac{3^{\sqrt{7}}}{3^{\sqrt{5}}}\right)^{\sqrt{5}+\sqrt{7}} = 3^{\sqrt{35}+7-5-\sqrt{35}} = 3^2 = 9$$

$$\left[(1+\sqrt{2})^{1-\sqrt{2}}\right]^{1+\sqrt{2}} = (1+\sqrt{2})^{-1} = \frac{1}{1+\sqrt{2}}$$

$$\left(\sqrt{a^{1+\sqrt{3}}}\right)^{1-\sqrt{3}} \times a^{\frac{3}{2}} = \left(a^{\frac{1+\sqrt{3}}{2}}\right)^{1-\sqrt{3}} \times a^{\frac{3}{2}} = a^{\frac{-2+3}{2}} = \sqrt{a}$$

$$\frac{\left(\sqrt{2}+1\right)^3 + \left(\sqrt{2}-1\right)^3}{2\sqrt{2}} = \frac{2\sqrt{2}+6+3\sqrt{2}+1+2\sqrt{2}-6+3\sqrt{2}-1}{2\sqrt{2}} =$$

$$= \frac{10\sqrt{2}}{2\sqrt{2}} = 5$$

$$[4^{-\frac{1}{4}} + (2^{1.5})^{-\frac{4}{3}}] \times [4^{-0.25} - (2\sqrt{2})^{-\frac{4}{3}}] = \left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) \times \left(\frac{1}{\sqrt{2}} - \frac{1}{4}\right) =$$

$$= \frac{(2\sqrt{2}+1) \times (2\sqrt{2}-1)}{16} = \frac{7}{16}$$

$$a^{\sqrt{5}+1} \times \left(\frac{1}{a}\right)^{\sqrt{5}-1} = a^{\sqrt{5}+1-\sqrt{5}+1} = a^2$$

$$\left[\left(\frac{1}{3}\right)^{\sqrt{2}}\right]^{-\sqrt{8}} = 3^4 = 81$$

$$\left[\left(\sqrt[3]{3}\right)^{\sqrt{3}}\right]^{-2\sqrt{3}} = \left(\frac{1}{3^{\frac{1}{3}}}\right)^6 = \frac{1}{9}$$

$$52^2 - 49^2 = 2704 - 2401 = 303$$

$$205^2 - 204^2 = 42025 - 41616 = 409$$

$$\frac{10^{10} - 10^6}{10^4 + 10^2} = \frac{9.999 \times 10^9}{10100} = 99 \times 10^4$$

1.11

$$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\frac{1}{\sqrt{8}} = \frac{1}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{8}}{8} = \frac{\sqrt{2}}{4}$$

$$\frac{1}{\sqrt{12}} = \frac{1}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{12}}{12} = \frac{\sqrt{3}}{6}$$

$$\frac{2}{6 - \sqrt{10}} = \frac{2}{6 - \sqrt{10}} \times \frac{6 + \sqrt{10}}{6 + \sqrt{10}} = \frac{12 + 2\sqrt{10}}{26} = \frac{6 + \sqrt{10}}{13}$$

$$\frac{6}{\sqrt{5} - \sqrt{3}} = \frac{6}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{6(\sqrt{5} + \sqrt{3})}{2} = 3(\sqrt{5} + \sqrt{3})$$

$$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2} - \sqrt{3}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2} - \sqrt{3}} \times \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} =$$

$$= \frac{\sqrt{2}}{2} + \sqrt{2} + \sqrt{3} = \frac{3\sqrt{2} + 2\sqrt{3}}{2}$$

$$\frac{1}{\sqrt{3}} - \frac{1}{3 + \sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}} =$$

$$= \frac{\sqrt{3}}{3} - \frac{3 - \sqrt{3}}{6} = \frac{3(\sqrt{3} - 1)}{6} = \frac{\sqrt{3} - 1}{2}$$

$$\frac{1}{\sqrt{8}} + \frac{1}{8 - \sqrt{8}} = \frac{1}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} + \frac{1}{8 - \sqrt{8}} \times \frac{8 + \sqrt{8}}{8 + \sqrt{8}} = \frac{2\sqrt{2}}{8} + \frac{8 + 2\sqrt{2}}{56} =$$

$$= \frac{\sqrt{2}}{4} + \frac{4 + \sqrt{2}}{28} = \frac{8\sqrt{2} + 4}{28} = \frac{2\sqrt{2} + 1}{7}$$

2 Rovnice a nerovnice

3 Kvadratické rovnice a nerovnice

4 Iracionální rovnice

4.1

$$\sqrt{7 - 3x} = 2;$$
$$7 - 3x = 4;$$
$$-3x = -3;$$
$$x = 1;$$

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$$\sqrt{-7x - 5} = 3;$$
$$-7x - 5 = 9;$$
$$-7x = 14;$$
$$x = -2;$$

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$$\sqrt{10 - 2x} = 1 - x; \ (x \leq 1)$$
$$10 - 2x = 1 - 2x + x^2;$$
$$-x^2 - 2x + 2x = 1 - 10;$$
$$x^2 = 9;$$
$$x = -3; \ (x \leq 1)$$

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$$\sqrt{4x - 29} = x + 2; \ (x \geq 2)$$
$$4x - 29 = x^2 + 4x + 4;$$
$$-x^2 + 4x - 4x = 4 + 29;$$
$$x^2 = -33; \ \varnothing$$

$$\sqrt{x + 10} = x + 4; (x \geq -4)$$
$$x + 10 = x^2 + 8x + 16;$$
$$-x^2 + x - 8x + 10 - 16 = 0;$$
$$x^2 + 7x + 6 = 0;$$
$$D = 49 - 24 = 25$$

$$x_{1,2} = \frac{-7 \pm 5}{2} = \begin{cases} x_1 = -6; \varnothing \ (x \geq -4) \\ x_2 = -1; \end{cases}$$

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$$\sqrt{x + 8} = x + 2; (x \geq -2)$$
$$x + 8 = x^2 + 4x + 4;$$
$$-x^2 + x - 4x - 4 + 8 = 0;$$
$$x^2 + 3x - 4 = 0;$$
$$D = 9 + 16 = 25;$$

$$x_{1,2} = \frac{-3 \pm 5}{2} = \begin{cases} x_1 = -4; \varnothing \ (x \geq -2) \\ x_2 = 1; \end{cases}$$