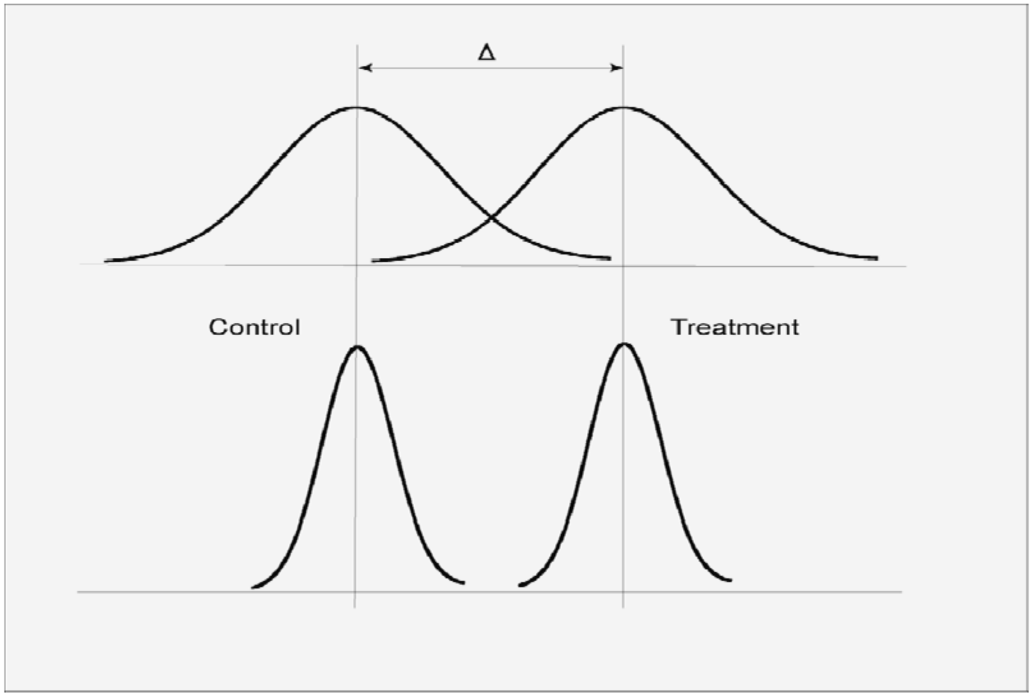
Understanding and Quantifying EFFECT SIZES



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Objective

Effect size comes up in the context of

- sample size calculations for proposals
- reporting results from pilot studies.

By the end of this talk, you will be able to calculate, interpret and report effect sizes in your work.

Outline

- Why are Effect Sizes (ES) important?
- Types of Effect Sizes
- Quantifying Magnitudes of Effect Sizes
- Calculating Effect Sizes
- Use of Softwares for Calculating Effect Sizes
- Specific Guidelines for Reporting Effect Sizes

Definition

- "Effect" - A change or changed state occurring as a direct result of action by somebody or something (Encarta, 2009)
- "Size" - The degree of something in terms of how big or small it is
- 'Effect size' is simply a way of quantifying the size of the difference between two groups.

- It is particularly valuable for quantifying the effectiveness of a particular intervention, relative to some comparison. It allows us to move beyond the simplistic, 'Does it work or not?' to the far more sophisticated, 'How well does it work in a range of contexts?‘

Why is Effect Size Important

- *Knowing the magnitude of an effect allows us to ascertain the practical significance of statistical significance*
  - Can always reach statistical significance if there is a large enough sample size, unless the effect size is 0.
  - Even a large effect may not be statistically significant if the sample size is too small.

Why is Effect Size Important

- *Practical Significance*
  - Even a statistically significant treatment difference may not be practically important if the effect size is too small.
  - However, there could still be practical importance even for small effect sizes, especially in cases where cost and ease make it easy to be implemented on a large scale.

Why is Effect Size Important

- *Sample Size Calculation for Studies*

ES plays a direct role in sample size calculations for any study. It is connected to the power of a test, the level of significance α and sample size (n).

- $\uparrow ES = \uparrow power$
- $\uparrow \alpha = \uparrow power$
- $\uparrow N = \uparrow power$ or $\uparrow reliability = \uparrow power$

- Given any 3 quantities (power, ES, α, n), we can find the 4^{th}.

Why is Effect Size Important

- *Meta-Analysis*
  - pooling information from many studies to verify results of past research and inform future studies.
  - ES is computed in each study and the findings are pooled together to draw overall inferences.

Types of Effect Sizes

- Mean Differences between Groups
  - Effect Size: Cohen's d
- Correlation/Regression
  - Effect Size: Pearson's r and R^2
  - Effect Size: Cohen's f^2
- Contingency tables
  - Effect Size: Odds Ratio or Relative Risk (association between binary variables)

Types of Effect Sizes

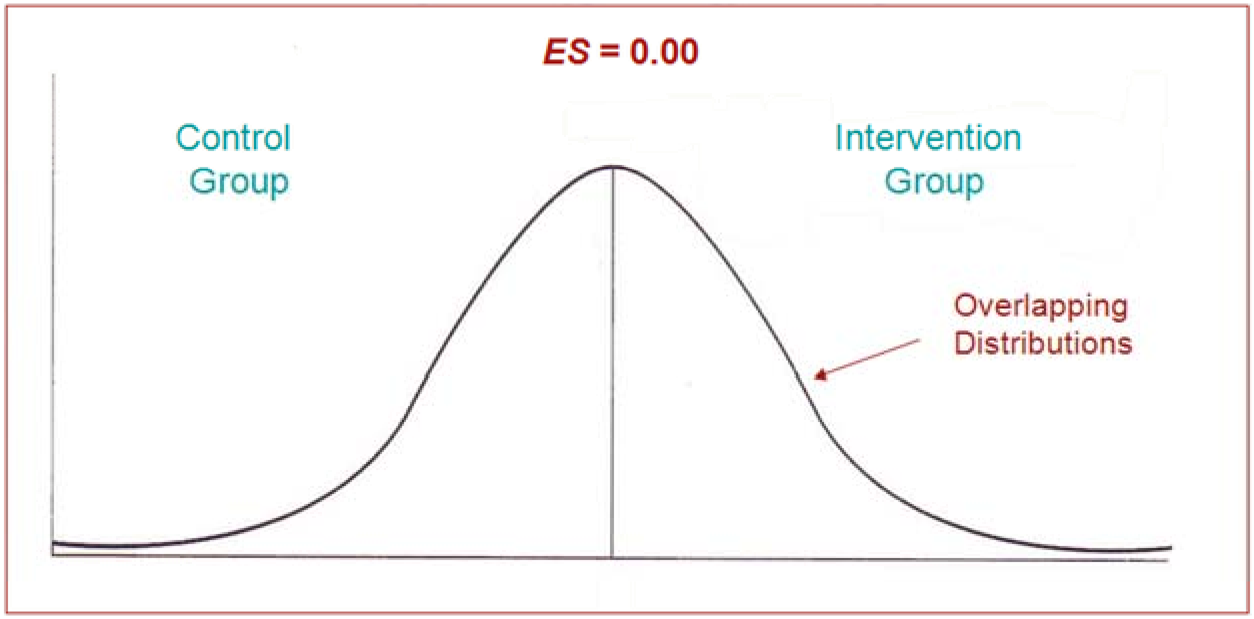
- ANOVA or GLMs
  - Effect Size: Eta-squared
  - Effect Size: Omega squared
  - Effect Size: Intraclass correlation (rater equality)
- Chi-square tests
  - Effect Size: Phi (2 binary variables)
  - Effect Size: Cramer's Phi or V (categorical variables)

Cohen's d

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s_{\text{pooled}}} \quad \text{where} \quad s_{\text{pooled}} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2}}$$

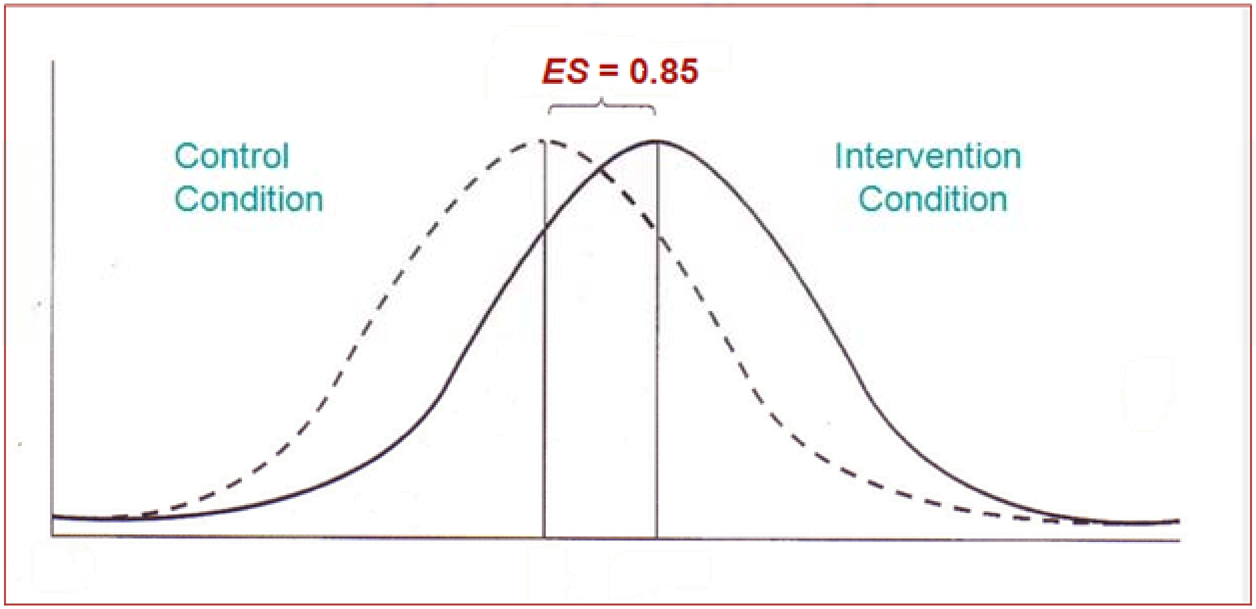
- Standardizes ES of the difference between two means
- Used for two sample independent t-tests
- d ranges from -∞ to +∞
- interpretation: the difference between the mean values is "d" standard deviations, Cohen (1988)

ES Example 1



- <u>*ES* = 0.00</u> means that the *average* treatment participant outperformed <u>50%</u> of the control participants

ES Example 2



- ES = 0.85 means that the *average* treatment participant outperformed 80% of the control participants

General Guidelines

In general, ≤ 0.20 is a small effect size, 0.50 is a moderate effect size and ≥ 0.80 is a large effect size (Cohen, 1992)

	d- standardized mean difference	Percentage of variance explained
• Small	.20	1%
• Moderate	.50	10%
• Large	.80	25%

Cohen's d

- Special Cases
  - For small sample sizes use Hedge's G

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s_{\text{pooled}}} \quad \text{where } s_{\text{pooled}} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

  - For unequal group variances, use Glass's Δ
    - uses sample sd of the control group only so that effect sizes would not differ under equal means and unequal variances

$$\Delta = \frac{\overline{x_1} - \overline{x_2}}{s_2}$$

Pearson's r

$$r_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\Sigma_{i=1}^{n}(y_i-\bar{y})^2}} \quad -1 \le r_{xy} \le 1$$

- used in the context of correlation…measuring association between 2 variables
- Interpretation: For every 1-unit standard deviation change in x, there is a "r-unit" standard deviation change in y

Pearson's R^2

- used in the context of regression…measuring how well a regression line fits to a given data
- R - linear association between 2 continuous variables
- R^2 (Coefficient of Determination) – proportion of shared variability between 2 or more variables
- Interpretation: "R^2 *100%" is percent variance of the outcome y that can be explained by the linear regression model (i.e. indicates how well the linear regression line fits the data)

Odds Ratio

- Used in the context of binary/categorical outcomes
- Odds of being in one group (eg. success) relative to the odds of being in a different group (eg. failure)
- OR ranges from 0 to ∞
- OR>1 indicates an increase in odds relative to the reference group
- OR < 1 indicates a decrease in odds relative to the reference group

Relative Risk

$$RR = \frac{a/(a+b)}{c/(c+d)} \approx \frac{ad}{bc} = OR$$

Risk	Disease status	
	Present	Absent
Smk	a	b
Non-smk	c	d

- RR measures the risk of an event relative to an independent variable
- For small probabilities, the relative risk is approximately equal to the odds ratio
- Interpretation: If RR > 1 then the risk of disease X among smokers is "RR" times the risk of disease X among non-smokers (vice versa if RR < 1)

Eta-Squared (η^2) and partial Eta-Squared ($\eta_p{}^2$)

$$\eta^2 = \frac{SS_{treatment}}{SS_{total}} \quad \eta_p^2 = \frac{SS_{treatment}}{SS_{treatment} + SS_{error}} \quad 0 \leq \eta^2 \leq 1$$

- Used with ANOVA family and GLMs
- Measures the degree of association in the sample
- Standardizes Effect Sizes of the shared variance between a continuous outcome and categorical predictors
- Partial eta-squared is the proportion of the total variability attributable to a given factor.

Eta-Squared (η^2) and partial Eta-Squared ($\eta_p^{\ 2}$)

- Interpretation: "η^2 *100%" is percent of the variance in y explained by the variance in x (similar to the R^2 interpretation for linear regression (Dattalo, 2008)).
- η^2 is biased and on average overestimates the variance explained in the population, but decreases as the sample size gets larger.
- Caution: these effect sizes depend on the number and magnitude of the other effects

Cohen's f²

$$f^2 = \frac{R^2}{1 - R^2} = \frac{\eta^2}{1 - \eta^2}$$

- Used in multiple linear regression, $R^2 = \eta^2$
- Standardized effect size is the proportion of explained variance over unexplained variance
- Estimate is biased and overestimates the effect size for ANOVA (unbiased estimate is Omega-Squared)

Omega-Squared

$$\hat{\omega}^2 = \frac{SS_{treatment} - df_{treatment} * MS_{error}}{SS_{total} + MS_{error}}$$

- Estimates the proportion of variance in the *population* that is explained by the treatment
- $\hat{\omega}^2$ is always smaller than η^2 or η_p^2 since Omega measures the population variance and Eta measures the sample variance

Intraclass Correlation

- ICC is used to measure inter-rater reliability for two or more raters. It may also be used to assess test-retest reliability. ICC may be conceptualized as the ratio of between-groups variance to total variance.

$$ICC = \frac{MS_{Treatment} - MS_{Error}}{MS_{Treatment} + (n-1)MS_{Error}}$$

- Can be used in ANOVA
- Similar interpretation to Omega-Squared

Phi

$$\phi = \sqrt{\frac{\chi^2}{n}}$$

- Used for crosstabs or for chi-square tests (equality of proportions or tests of independence between 2 binary variables)… φ= 0 indicates independence
- Phi are related to correlation and Cohen's d (for 2 binary variables)
- Interpreted like Pearson's r and R^2

Cramer's Phi or V

- Cramer's Phi (Cramer's V) can be used with categorical variables with more than 2 categories ($m \geq 2$) (R x C tables)

$$\varphi_c = \sqrt{\frac{\chi^2}{N(k-1)}} \quad ; k = \min(R, C)$$

- measures the inter-correlation of the variables, but is biased since it increases with the number of cells. Increase in R and C will indicate a strong association, which is just an artifact of the type of variable used.

Magnitude of Effect Summary Table

Effect Size	Small	Medium	Large
r	0.10	0.30	0.50
r^2	0.01	0.09	0.25
η^2	0.01	0.06	0.14
R^2	0.01	0.06	0.14
Cohen's d	0.20	0.50	0.80
φ / Cramer's V	0.10	0.30	0.50
Cohen's f^2	0.02	0.15	0.35
OR	1.44	2.47	4.25

Effect Size Conversions

Effect Size	Converted to Cohen's d
Correlation	$d = 2r \Big/ \sqrt{1 - r^2}$
Chi-Square df =1 df > 1	$d = \sqrt{4\chi^2/(N - \chi^2)}$ $d = \sqrt{\dfrac{4\chi^2}{N}}$
Odds Ratio (Chinn, 2000)	$d = \dfrac{\ln(OR)}{1.81}$

Software

- Calculate effect size
  - http://www.uccs.edu/~faculty/lbecker/
  - http://faculty.vassar.edu/lowry/newcs.html
  - Statistical softwares such as SAS, R, STATA, SPSS can calculate most of the standard Effect Sizes

Calculating Cohen’s d – online calculator http://www.uccs.edu/~faculty/lbecker/

Effect Size Calculators

Calculate Cohen's *d* and the effect-size correlation, $r_{Y\lambda}$, using --

- **means and standard deviations**
- **independent groups *t* test values and *df***

For a discussion of these effect size measures see Effect Size Lecture Notes

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Calculate *d* and *r* using means and standard deviations

Calculate the value of Cohen's *d* and the effect-size correlation, $r_{Y\lambda}$, using the means and standard deviations of two groups (treatment and control).

Cohen's $d = M_1 - M_2 / s_{pooled}$
where s_{pooled} = �[(s 1� ' s 2�) / 2]

$r_{Y\lambda}$ = d / �(d� + 4)

Note: d and $r_{Y\lambda}$ are positive if the mean difference is in the predicted direction.

Group 1		Group 2	
M_1	6.25	M_2	7.36
SD_1	3.27	SD_2	4.65

Compute Reset

Cohen's *d*	effect-size *r*

top

Calculating Cohen's d – online calculator http://www.uccs.edu/~faculty/lbecker/

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	Group 1		Group 2
M_1	6.25	M_2	7.36
SD_1	3.27	SD_2	4.65
	Compute		Reset
	Cohen's *d*		effect-size *r*
	-0.27614:		-0.13677:

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Calculating Cramer's Phi – online calculator http://www.uccs.edu/~faculty/lbecker/

Chi-Square, Cramer's V, and Lambda

For a Rows by Columns Contingency Table

For a contingency table containing up to 5 rows and 5 columns, this unit will:

- ~ perform a chi-square analysis [the logic and computational details of chi-square tests are described in Chapter 8 of Concepts and Applications];
- ~ calculate Cramer's V, which is a measure of the strength of association among the levels of the row and column variables [for a 2x2 table, Cramer's V is equal to the absolute value of the phi coefficient];
- ~ and calculate the two asymmetrical versions of lambda, the Goodman- Kruskal index of predictive association, along with some other measures relevant to categorical prediction. [Click here for a brief explanation of lambda.]

To begin, select the number of rows and the number of columns by clicking the appropriate buttons below; then enter your data into the appropriate cells of the data-entry matrix. After all data have been entered, click the «Calculate» button.

Select the number of rows:	2	3	4	5	---
Select the number of columns:	2	3	4	5	---

Calculating Cramer's Phi – online calculator http://www.uccs.edu/~faculty/lbecker/

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Calculating Cramer's Phi – online calculator http://www.uccs.edu/~faculty/lbecker/

Data Entry

	B_1	B_2	B_3	B_4	B_5	Totals
A_1	12	7	-----	-----	-----	19
A_2	8	4	-----	-----	-----	12
A_3	-----	-----	-----	-----	-----	-----
A_4	-----	-----	-----	-----	-----	-----
A_5	-----	-----	-----	-----	-----	-----
Totals	20	11	-----	-----	-----	31

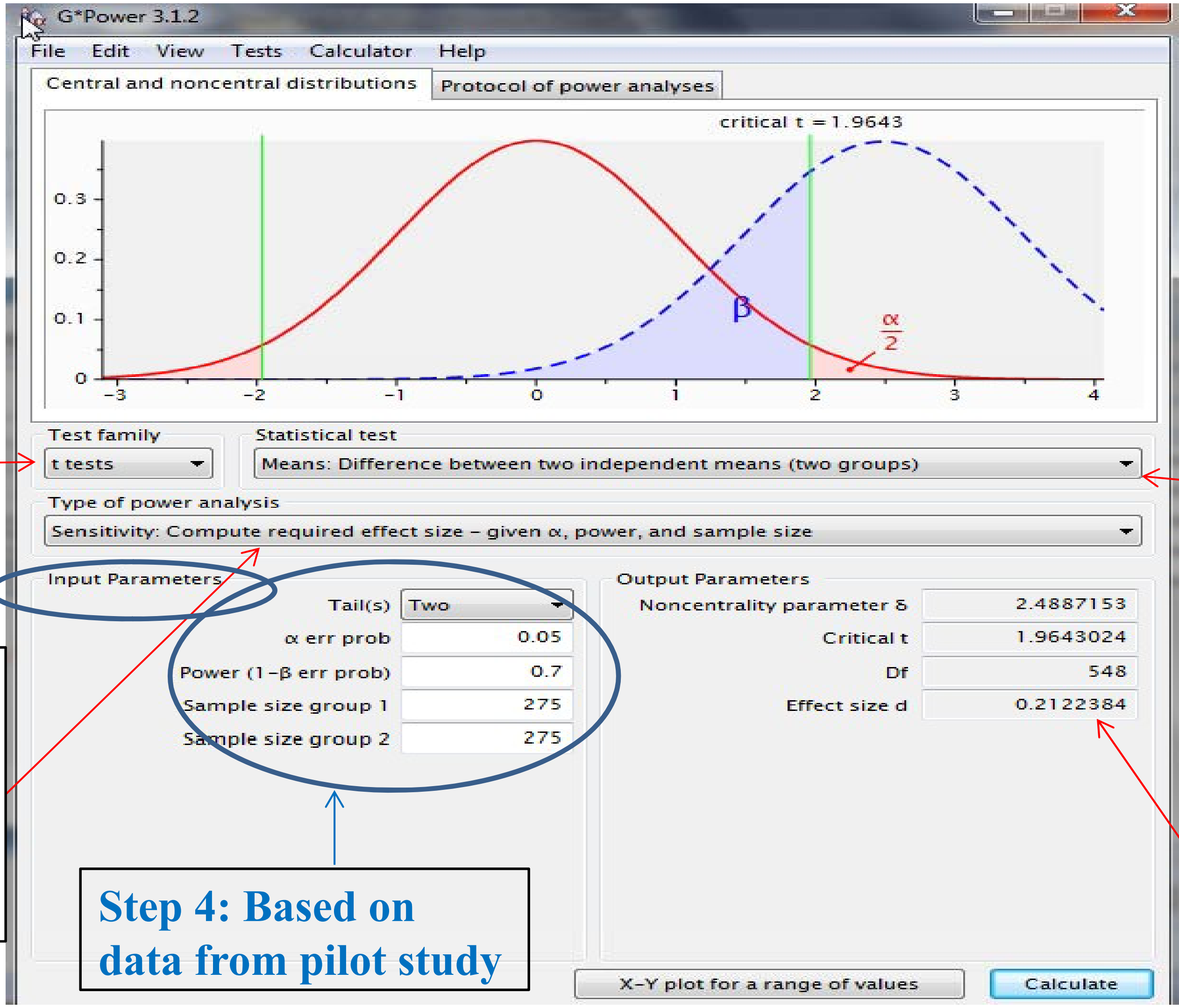
Reset Calculate

Chi-Square	df	P
0.03	1	0.8625

Cramer's V = 0.0359

Note that one of your expected cell frequencies is smaller than 5. For a rows by columns chi-square test, at least 80% of the cells must have an expected frequency of 5 or greater, and no cell may have an expected frequency smaller than 1.0. For a 2x2 table, the chi-square test is valid only if all expected cell frequencies are equal to or greater than 5. If this requirement is not met for a 2x2 table, use instead the Fisher Exact Probability Test. The Fisher Exact Test is also available for 2x3, 2x4, and 3x3.

A (not-so-great) Alternative: Calculating Cohen's d using GPower



Step 1: Choose "t-test" from drop down menu

Step 2: Choose "means" from drop down menu

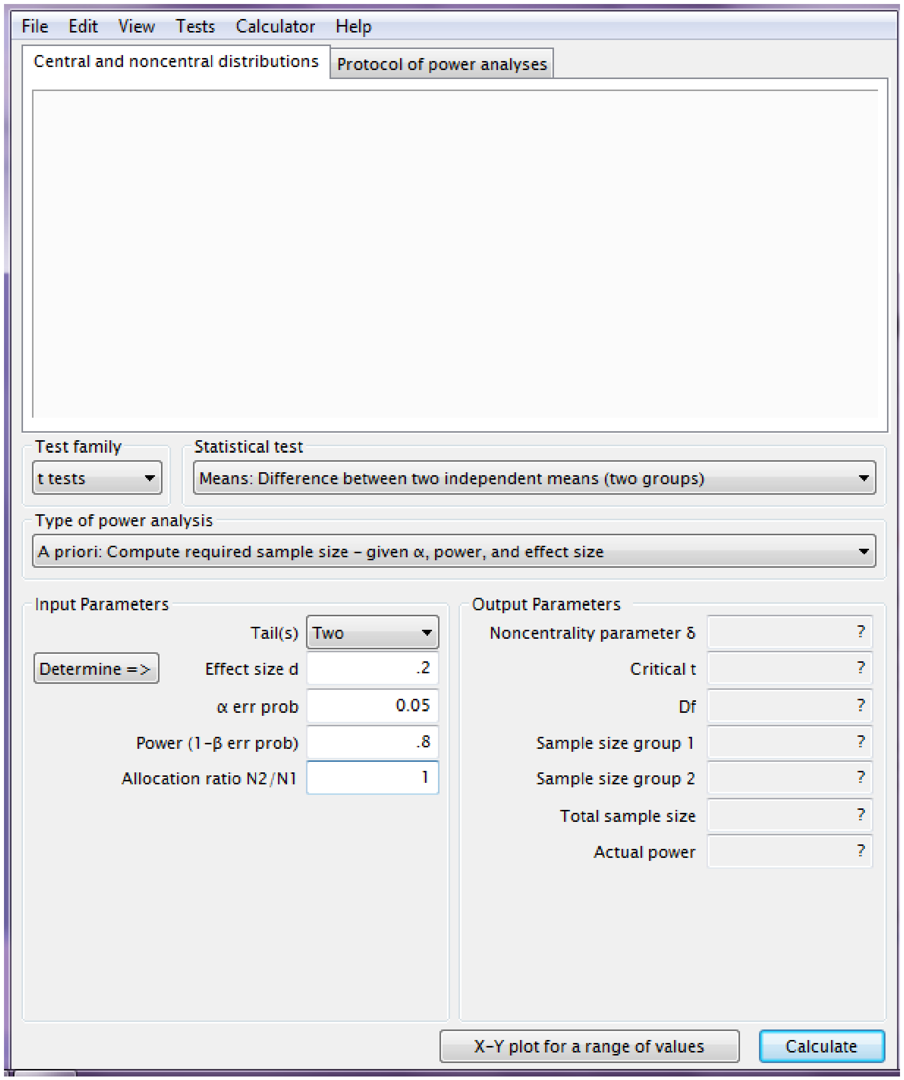
Step 3: Choose "Sensitivity" from the drop down menu

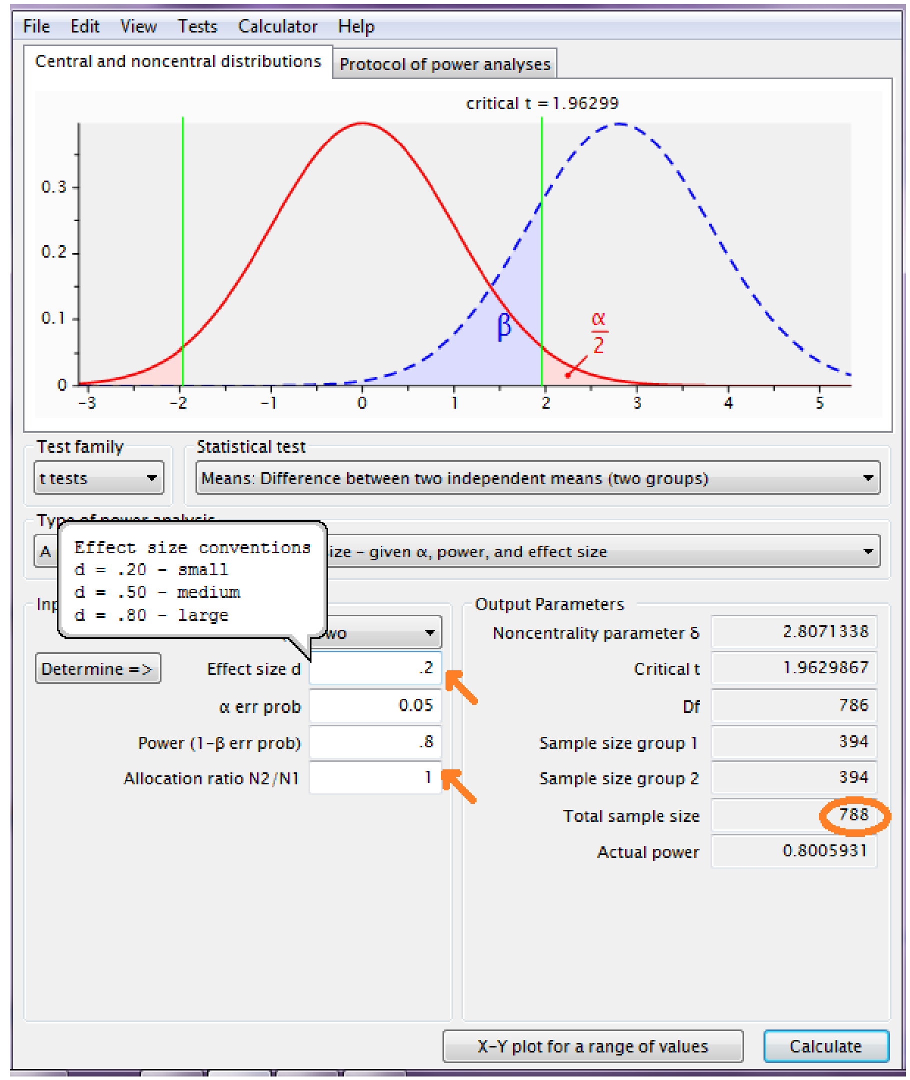
Step 4: Based on data from pilot study

Solution: Calculated effect size

Calculating SS given Cohen's d - GPower

A better use of Gpower is to calculate sample sizes.





Reporting Guidelines and Trends

- Reporting effect sizes has three important benefits (APA, 1999):
  - Meta-analysis
  - Informing subsequent research
  - Interpretation and evaluation of results within the context of related literature

Reporting Guidelines and Trends

- What to report (APA, 2010):
  - Type of effect size
  - Value of the effect size (in original units, such as lbs., or mean differences on a scale, and/or the effect size statistic)
  - Interpretation of the effect size
  - Practical significance of the effect size

References

- Chinn, S.(2000). A simple method for converting an odds ratio to effect size for use in meta-analysis. Statistics in Medicine, 19, 3127-3131.
- Cohen, J.(1992). A power primer. Psychological Bulletin, 112(1), 155-159.
- Cohen, J. (1988) “Statistical power analysis for the behavioral sciences”. New Jersey: Lawrenced Erlbaum Associates, Inc. Publishers. pp 283-286.

References

- Dunst, Carl J. et al. (2004) Guidelines for Calculating Effect Sizes for Practice-Based Research Syntheses. Centerscope 3(1) http://courseweb.unt.edu/gknezek/06spring/5610/centerscopevol3no1.pdf
- A presentation on effect size: http://www.family.umaryland.edu/ryc_research_and_evaluation/publication_product_files/selected_presentations/presentation_files/pdfs/effect%20size%20and%20intervention%20research.pdf
- Lee B.R., Bright C.L., Svobo da, D., Fakunmoju, S. and Barth R.P. Outcomes of group care for youth: A review of comparative literature. Research on Social Work Practice