# Solving Problems by Uninformed Searching

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Readings: AIMA Sections 3.1∼3.4

# Outline

1. Problem-Solving Agents
2. Problem Formulation
3. Search on Trees and Graphs
4. Uninformed Search
   - Breadth-First
   - Uniform-Cost
   - Depth-First
   - Depth-Limited
   - Iterative Deepening

# Problem-Solving Agents

- A simple problem-solving agent formulates a goal and a problem, searches for a sequence of actions that solves the problem, and then execute the actions one by one.

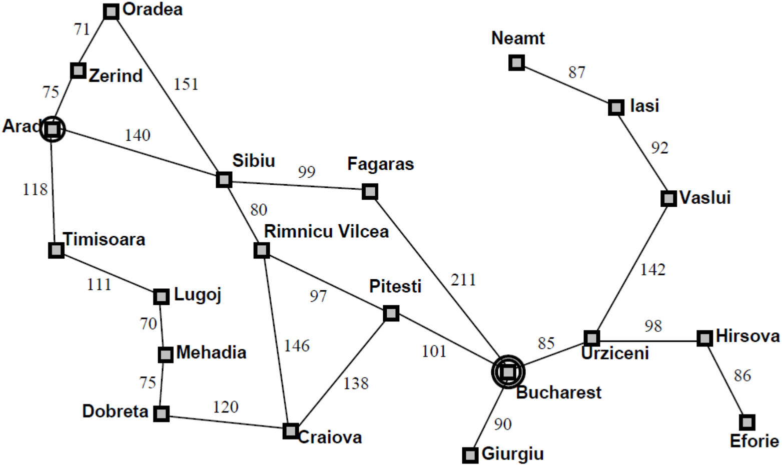
**SIMPLE-PROBLEM-SOLVING-AGENT(*percept*)**

```
state = UPDATE-STATE(state, percept)
if seq == empty
    goal = FORMULATE-GOAL(state)
    problem = FORMULATE-PROBLEM(state, goal)
    seq = SEARCH(problem)
    if seq == failure
        return NIL
action = FIRST(seq)
seq = REST(seq)
return action
```

# Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest.
- Formulate goal
  - Be in Bucharest.
- Formulate problem
  - States: various cities
  - Actions: fly between cities
- Find solution
  - Sequence of cities, *e.g.*, Arad, Sibiu, Fagaras, Bucharest

# Example: Romania



# Problem Formulation

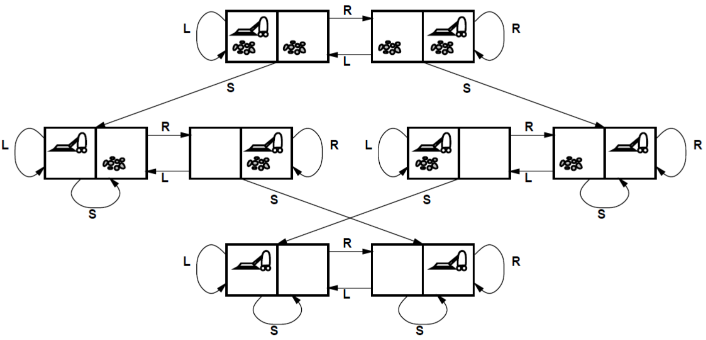
- A problem is defined by five components
  1. Initial state: $In(Arad)$
  2. Actions:
     $\textsc{Action}(In(Arad)) = \{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$
  3. Transition model $\textsc{Result}(s, a)$:
     - $\textsc{Result}(In(Arad), Go(Zerind)) = In(Zerind)$.

     Successor $\mathrm{S}(s)$: states reachable by a single action.
     - $\mathrm{S}(s) = \{s' | \forall a \in \textsc{Action}(s), s' = \textsc{Result}(s, a)\}$
  4. Goal test: $\{In(Bucharest)\}$
  5. Path cost (additive)
     - Sum of distances, number of actions executed, etc.
     - $c(s, a, s')$ is the step cost of taking action $a$ in state $s$ to reach state $s'$, assumed to be $\geq 0$
- A solution is a sequence of actions leading from the initial state to a goal state.

# Abstraction

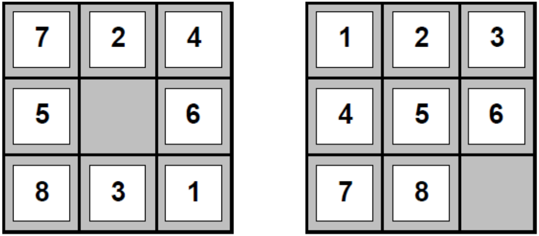
- Real world is absurdly complex
  - State space must be abstracted for problem solving.
- (Abstract) state = subset of real states
- (Abstract) action = complex combination of real actions
  - Go(Zerind) represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"
- (Abstract) solution = set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem!

# Example: Vacuum World State Space Graph




1. Initial state: Any one of the above states. (ignore dirt amounts etc.)
2. Actions: Left, Right, Suck, NoOp
3. Transition model: The above figure.
4. Goal test: no dirt
5. Path cost: 1 per action (0 for NoOp)

# Example: The 8-puzzle

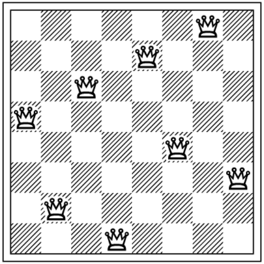



Start State Goal State

1. Initial state: The left figure.
2. Actions: Move blank left, right, up, down.
3. Transition model: Common sense.
4. Goal test: = The right figure.
5. Path cost: One per move.

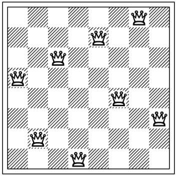
Note: Sliding-block puzzle is $\mathcal{NP}$-hard.

# Example: 8-Queen Puzzle



1. Initial state: No queen on the board.
2. Actions: Add a queen on the board where the square is empty.
3. Transition model: Returns the board with a queen added to the specified square.
4. Goal test: 8 queens are on the board, none attacked.
5. Path cost: Number of trials.

# Example: 8-Queen Puzzle



- States: Any 0$\sim$8 queens on the board.
  - State space: $C_0^{64} + C_1^{64} + C_2^{64} + \cdots + C_8^{64} \simeq 5.1 \times 10^9$
  - Solution space: $64 \cdot 63 \cdots 57 \simeq 1.8 \times 10^{14}$
- States: One queen per column.
  - State space: $8^0 + 8^1 + 8^2 + \cdots + 8^8 \simeq 1.9 \times 10^7$
  - Solution space: $8^8 \simeq 1.6 \times 10^7$
- States: All possible arrangements of $n$ $(0 \leq n \leq 8)$ queens at leftmost $n$ columns with on queen attacked.
  Actions: Add a queen to the next column with no queen attacked, or backtrack.
  - State space: 2057.

# Tree Search Algorithms

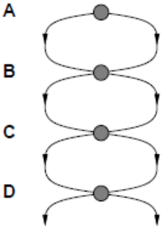
- Offline, simulated exploration of state space by generating successors of already-explored states (a.k.a. expanding states)

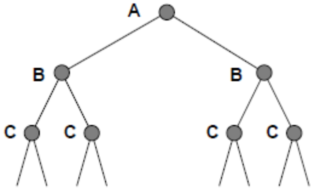
**TREE-SEARCH(*problem*)**

```
initialize the frontier using the initial state of problem
repeat
    if the frontier is empty
        return failure
    choose a leaf node and remove it from the frontier.
    if the node contains a goal state
        return the corresponding solution
    expand the chosen node
    add the resulting nodes to the frontier
```

# Repeated States in Graph Search

- Failure to detect repeated states can turn a linear problem into an exponential one!
- Use a queue to record explored states.
- For fast detection of repeated states, hashing techniques are usually adopted.



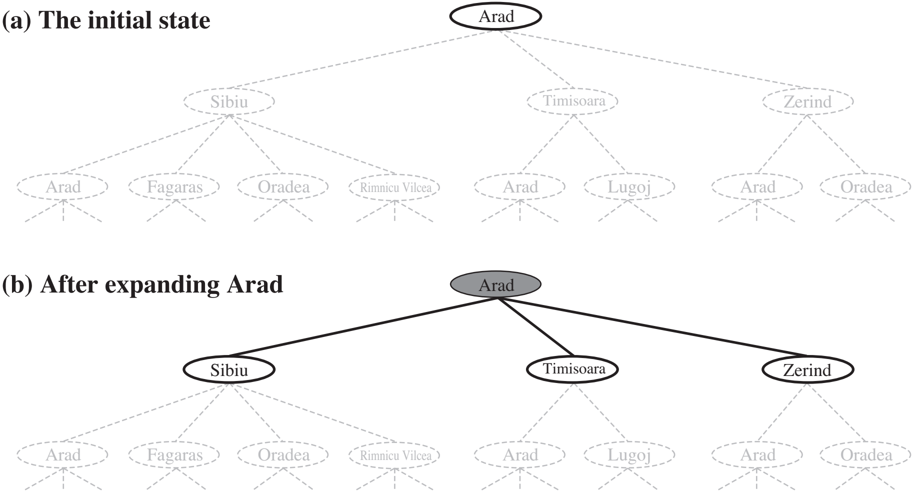



# Graph Search Algorithms

## GRAPH-SEARCH(*problem*)

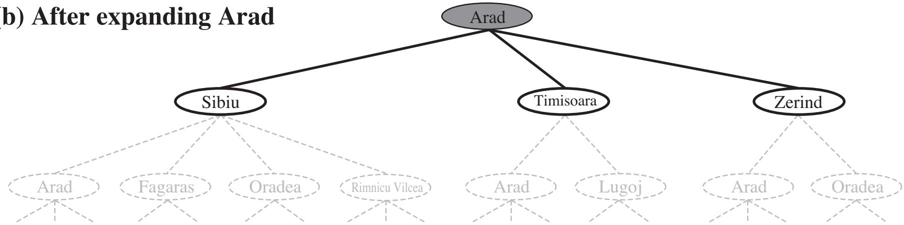
```
initialize the frontier using the initial state of problem
initialize the explored set to be empty
repeat
    if the frontier is empty
        return failure
    choose a leaf node and remove it from the frontier.
    if the node contains a goal state
        return the corresponding solution
    add the node to the explored set
    expand the chosen node
    if not in the frontier or explored set
        add the resulting nodes to the frontier
```

# Partial Search Tree

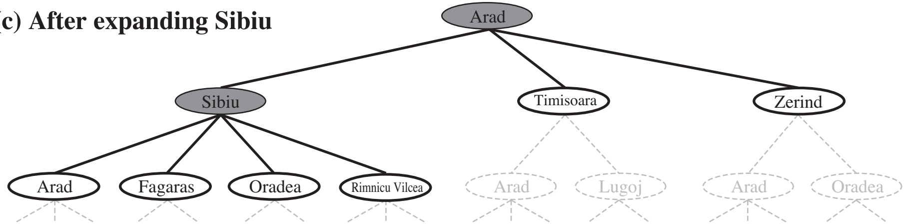


# Partial Search Tree

**(b) After expanding Arad**



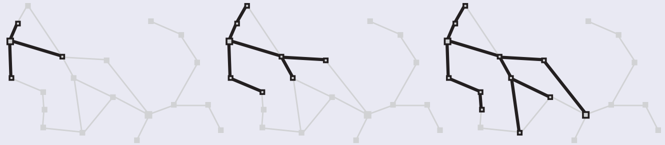

**(c) After expanding Sibiu**



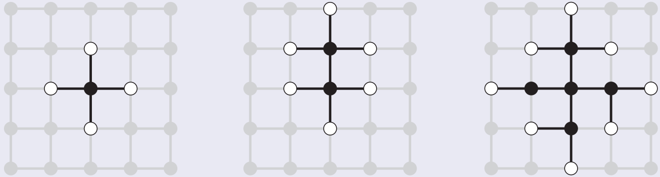
# Graph Search, Search Tree, and Frontier Separation

- The frontier separates the state space into explored and unexplored regions (loop invariant proof).

## Tree generated by graph search on Romania map

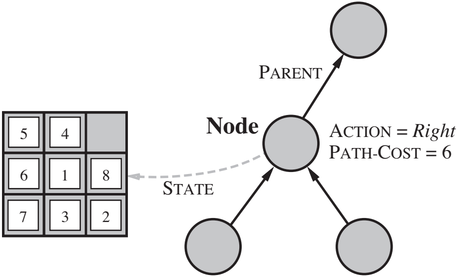


## Separation property of GRAPH-SEARCH



# Implementation: States vs. Nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes parent, children, depth, path cost $g(x)$
- States do not have parents, children, depth, or path cost!



# Infrastructure for Search Algorithms

$\textsc{Child-Node}(\mathit{problem}, \mathit{parent}, \mathit{action})$

**return** a node $n$ with

$$
\begin{aligned}
&n.\mathit{state} = \mathit{problem}.\textsc{Result}(\mathit{parent}.\mathit{state}, \mathit{action})\\
&n.\mathit{parent} = \mathit{parent}\\
&n.\mathit{action} = \mathit{action}\\
&n.\mathit{path\_cost} = \mathit{parent}.\mathit{path\_cost}\\
&\qquad + \mathit{problem}.\textsc{Step-Cost}(\mathit{parent}.\mathit{state}, \mathit{action})
\end{aligned}
$$

- The appropriate data structure to maintain the frontier is a queue.
- Can be
  - FIFO.
  - LIFO (a.k.a. stack)
  - Priority queue

# Tree Search Algorithms

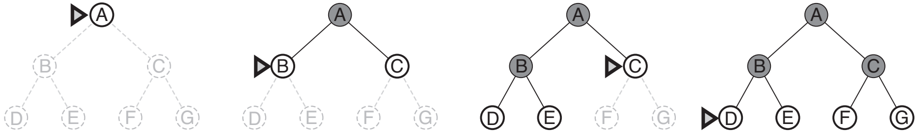
- A strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
    - Completeness - does it always find a solution if one exists?
    - Optimality - does it always find a least-cost solution?
    - Time complexity - number of nodes generated/expanded
    - Space complexity - maximum number of nodes in memory
- Time and space complexity are measured in terms of
    - $b$ - maximum branching factor of the search tree
    - $d$ - depth of the least-cost solution
    - $m$ - maximum depth of the state space (may be $\infty$)

# Uninformed Search Strategies (Blind Search)

- Uninformed strategies use only the information available in the problem definition.
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

# Breadth-First Search (BFS)

- Expand the shallowest unexpanded node.
- FIFO queue.



# Properties of BFS

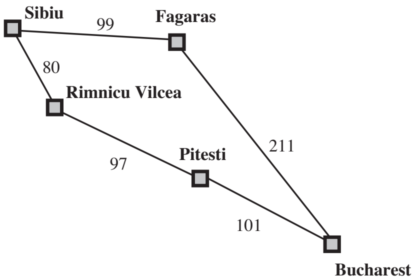
- Completeness: Yes (if $b$ is finite)
- Optimality: No in general; yes when the path cost is a non-decreasing function of the depth of the node.
- Time complexity: $1 + b + b^2 + \cdots + b^d = O(b^d)$.
  Or $O(b^{d+1})$ if goal test is applied after expansion.
- Space complexity: $O(b^d)$ (keeps every node in memory)

Space is the big problem; can easily generate nodes at 100MB/sec so 24hrs = 8640GB.

# Uniform-Cost Search

- Expand the unexpanded node with the lowest path cost.
- Priority queue ordered by $g(n)$.
- Equivalent to BFS if step costs all equal.

- For TREE-SEARCH, priority queue gives the cheapest path first.
- For GRAPH-SEARCH, if the node is already in the frontier, need to find the minimum cost, and call DECREASEKEY as needed.

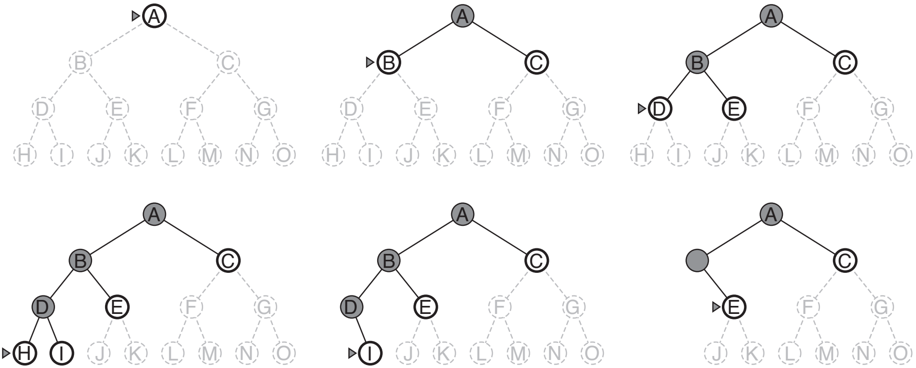


# Properties of Uniform-Cost Search

- Completeness: Yes, if step cost $\geq \epsilon > 0$.
- Optimality: Yes - nodes expanded in increasing order of $g(n)$.
- Time complexity: # of nodes with $g \leq$ cost of optimal solution. Maximum depth is given by $1 + \lfloor C^*/\epsilon \rfloor$, where $C^*$ is the cost of the optimal solution.
  $O(b^{1+\lfloor C^*/\epsilon \rfloor})$.
- Space complexity: # of nodes with $g \leq$ cost of optimal solution, $O(b^{1+\lfloor C^*/\epsilon \rfloor})$.

# Depth-First Search (DFS)

- Expand the deepest unexpanded node.
- LIFO queue, *i.e.*, put successors at front.



# Depth-First Search (DFS)

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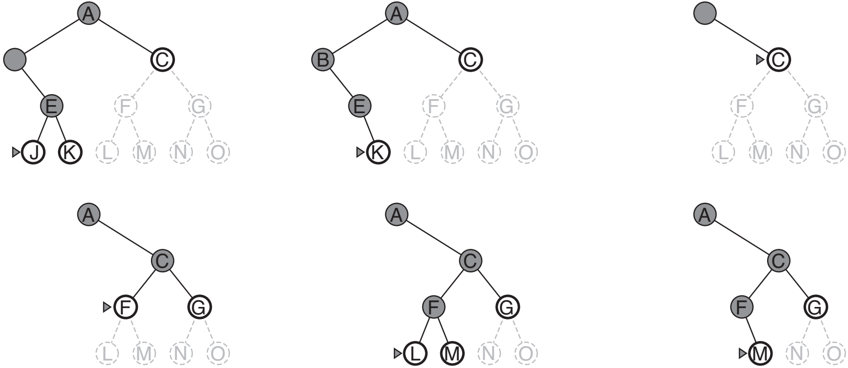


# Properties of DFS

- Completeness: No, fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path $\rightarrow$ complete in finite spaces.
- Optimality: No.
- Time complexity: $O(b^m)$, terrible if $m$ is much greater than $d$.
  - But if solutions are dense, may be much faster than breadth-first
- Space complexity: $O(bm)$ , linear space!
  - Backtracking technique only generate one successor instead of all successors $\rightarrow O(m)$.

# Depth-Limited Search (DLS)

- DFS never terminates if $m \to \infty$.
- DLS $=$ DFS with depth limit $\ell$,
- Nodes at depth $\ell$ have no successors
- Recursive implementation:

**DEPTH-LIMITED-SEARCH(*problem*, *limit*)**

**return** RECURSIVE-DLS(MAKE-NODE(*problem.initial_state*), *problem*, *limit*)

# Depth-Limited Search (DLS)

**RECURSIVE-DLS(*node*, *problem*, *limit*)**

```
if problem.GOAL-TEST(node.state)
    return SOLUTION(node)
elseif limit == 0
    return cutoff
else
    cutoff_occurred = FALSE
    for each action in problem.ACTIONS(node.state)
        child = CHILD-NODE(problem, node, action)
        result = RECURSIVE-DLS(child, problem, limit − 1)
        if result == cutoff
            cutoff_occurred = TRUE
        elseif result ≠ failure
            return result
    if cutoff_occurred
        return cutoff
    else
        return failure
```

# Properties of DLS

- Completeness: Not complete if $\ell < d$; complete otherwise.
- Optimality: Not optimal in general (even if $\ell > d$).
- Time complexity: $O(b^\ell)$
- Space complexity: $O(b\ell)$, linear space.
- Two termination conditions:
    - *failure*: no solution.
    - *cutoff*: no solution within the depth limit.

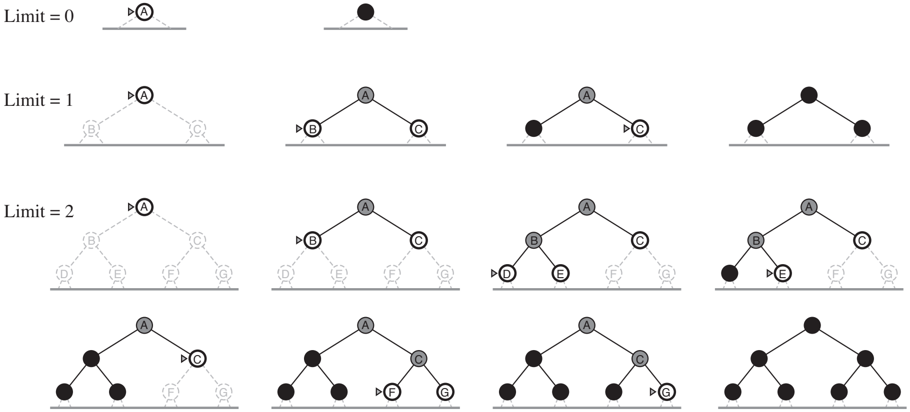
# Iterative-Deepening Search (IDS)

- Call DLS iteratively with increasing depth limit.
- Seems to be wasteful, but actually not.
- Combine the benefits of BFS and DFS.

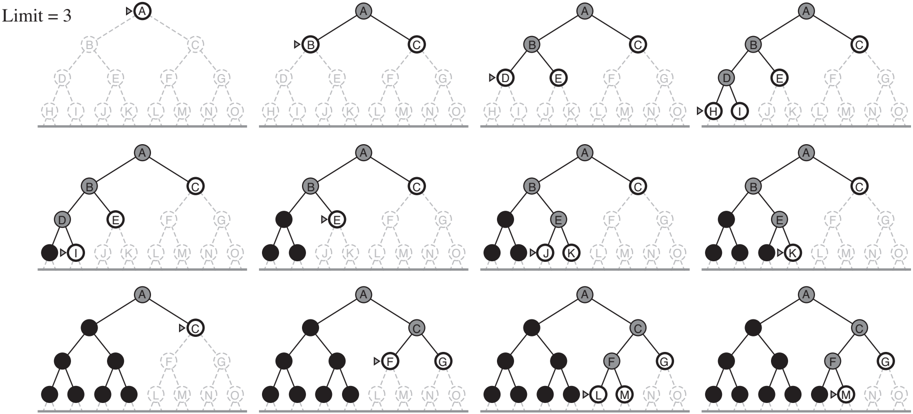
**ITERATIVE-DEEPENING-SEARCH(*problem*)**

```
for depth = 0 to ∞
    result = DEPTH-LIMITED-SEARCH(problem, depth)
    if result ≠ cutoff
        return result
```

# Iterative Deepening Search



# Iterative Deepening Search



# Properties of Iterative Deepening Search

- Completeness: Yes
- Optimality: No in general; yes when the path cost is a non-decreasing function of the depth of the node.
  - Can be modified to explore uniform-cost trees (optimal).
- Time complexity: $db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space complexity: $O(bd)$
- Numerical comparison for $b = 10$, $d = 5$, solution at far right leaf.
  - N(IDS) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450
  - N(BFS) = 10 + 100 + 1,000 + 10,000 + 100,000 = 111,100
- Repeated search in IDS is not severe.
- In general, IDS is preferred when search space is large and depth is unknown.
- We'll talk about more advantages of IDS in adversarial search.

# Summary of Algorithms

| Criterion | BFS | Uniform-Cost | DFS | DLS | IDS |
|---|---|---|---|---|---|
| Completeness | Yes[a] | Yes[b] | No | No[c] | Yes[a] |
| Optimality | No[d] | Yes | No | No | No[e] |
| Time Complexity | $O(b^d)$ | $O(b^{1+\lfloor C^*/\epsilon \rfloor})$ | $O(b^m)$ | $O(b^\ell)$ | $O(b^d)$ |
| Space Complexity | $O(b^d)$ | $O(b^{1+\lfloor C^*/\epsilon \rfloor})$ | $O(bm)$ | $O(b\ell)$ | $O(bd)$ |

[a] if $b$ is finite
[b] if $b$ is finite and step cost $\geq \epsilon$
[c] unless $\ell \geq d$
[d] unless the path cost is a non-decreasing function of the depth of the node

# Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored.
  - Initial state.
  - Actions.
  - Transition model.
  - Goal test.
  - Path cost.
- Graph search can be exponentially more efficient than tree search, but usually impractical due to memory requirement.
- Variety of uninformed search strategies judged on the basis of
  - Completeness
  - Optimality
  - Time and space complexity.
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms.