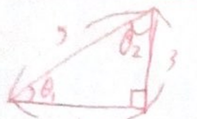
정현파 교류회로의 기초

$\theta = \frac{l}{r}$ ([m/m] = [rad]) , 360[°] = $\frac{2\pi r}{r}$ = 2π [rad]

ω (각속도, 각주파수) : 단위시간당 각이 변하는 비율

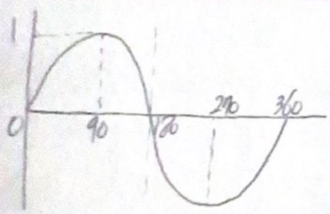
$\omega = \frac{d\theta}{dt}$ [rad/s] = $2\pi f = \frac{2\pi}{T}$

※ [illegible] 삼각함수의 관계 → 삼각함수 공식 ※



$\tan\theta_1 = \frac{3}{4} \rightarrow \theta_1 = \tan^{-1}\left(\frac{3}{4}\right)$

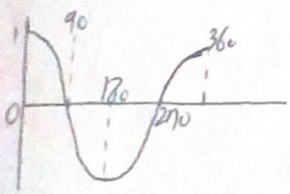
$\sin\theta_1 = \cos\theta_2 = \frac{3}{5}$, $\sin\theta_2 = \cos\theta_1 = \frac{4}{5}$



$\sin\theta$

$\sin\theta = -\sin(-\theta)$

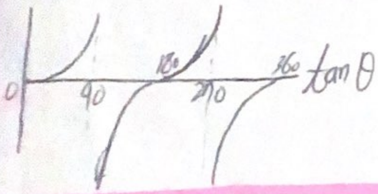
$\sin\theta = \cos(\theta - 90°)$



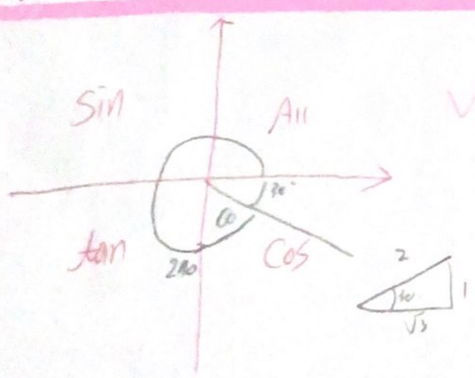
$\cos\theta$

$\cos\theta = \sin(\theta + 90°)$

$\cos\theta = -\cos(-\theta)$



$\tan\theta$



$\cos 330° = \frac{\sqrt{3}}{2}$

T(주기) = $\frac{1}{f(\text{주파수})}$ [s] 또는 $f = \frac{1}{T}$ [Hz]

ω (각속도) 또는 각주파수 = $2\pi f = \frac{2\pi}{T}$ [rad/s]

순시치 : 임의의 시간동안 값이 얼마인가.

$v = V_m \sin(\omega t + \theta) = V_m \sin(2\pi f t + \theta)$ [V]

$i = I_m \sin(\omega t + \varphi) = I_m \sin(2\pi f t + \varphi)$ [A]

ex) $V = 120\sin(377t + \frac{\pi}{6})$ [V]인 전압의 그래프를 그리고, 최대값[V], 최소값[V], 주파수[Hz], 주기[s] 및 $t = 1/120$ [s]에서의 전압[V]를 구하시오.

$\boxed{377 = 2\pi f = 60\,[Hz]}$ = f

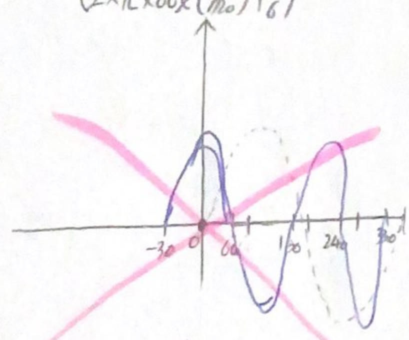
최대값 : $V_m = 120$ [V] , 최소값 : $-V_m = -120$ [V]

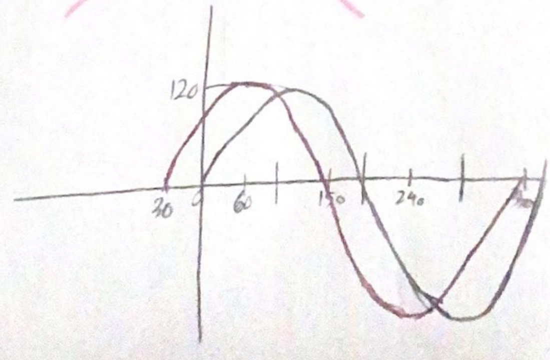
$377t + \frac{\pi}{6} = 0 \rightarrow \omega t = \theta = -\frac{\pi}{6}$가 되어, 이 함수는 $-\frac{\pi}{6}$ [rad]에서 시작하는 sin함수.

$\omega = 2\pi f$에서 $f = \frac{377}{2\pi} = 60$ [Hz] , $T = \frac{1}{f} = 0.01667$ [s]

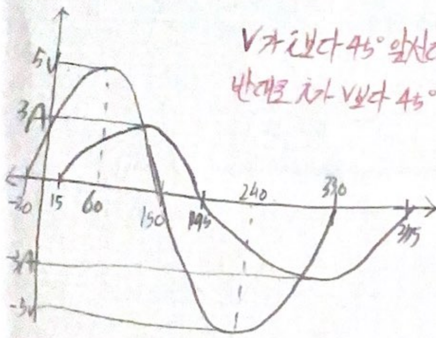
$V\left(\frac{1}{120}\right) = 120\sin\left(377 \times \frac{1}{120} + \frac{\pi}{6}\right) = 120\sin\frac{\pi}{3} = 60\sqrt{3}$ [V]

$\left(2 \times \pi \times 60 \times \left(\frac{1}{120}\right) + \frac{\pi}{6}\right)$





ex) $v = 5\sin(\omega t + 60°)[V]$, $i = 3\sin(\omega t - 15°)[A]$를 시간축이 ωt인 평면에 그려라



V가 i보다 45° 앞선다
반대로 i가 V보다 45° 뒤처진다

정현파 교류의 평균값은 +의 반주기의 평균값이다.

$$V_a = \frac{1}{T}\int_0^T v\,dt\,[V]$$

$$V_a = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta = -\frac{V_m}{\pi}[\cos\theta]_0^{\pi} = \frac{2}{\pi}V_m = 0.637V_m[V]$$

실효값 : 교류전압(전류)의 크기는 그 회로에 직류전압(전류)을 인가할 때 발생하는 열과 같은 크기의 열이 발생하도록 만드는 전압(전류) 값.

교류전압(전류) 값은 실효값 $= \frac{V_m}{\sqrt{2}}$

V_{eff} (실효값) $= \frac{V_m}{\sqrt{2}}$ V_a (평균값) $= \frac{2}{\pi}V_m$

ex) 정현파 교류의 평균값이 450[V]일 때 이 전압의 최대값 및 실효값

$$\underbrace{\text{최대값}}_{V_m}$$

$$V_a = 450[V] = \frac{2}{\pi}V_m \rightarrow V_m = \frac{\pi}{2}\times 450 = 225\pi[V]$$

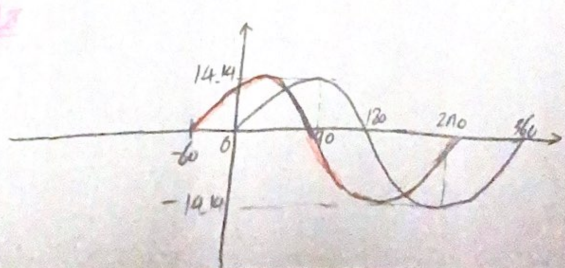
$$V_{eff} = \frac{V_m}{\sqrt{2}} = \frac{255\pi}{\sqrt{2}}$$

ex) $i = 14.14\sin(377t + \frac{\pi}{3})$

1) V_{eff} $\frac{V_m}{\sqrt{2}} = \frac{14.14}{\sqrt{2}} \fallingdotseq 10[A]$

2) 주파수(f), 주기(T)
60[HZ], $\frac{1}{60}$[S]

3) 그래프



ex) $I_{eff} = 220A = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = 220\sqrt{2}$

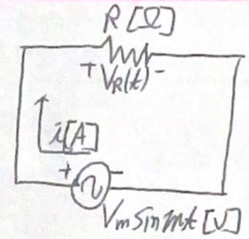
$I_m = 220\sqrt{2}[A]$

$I_a = \frac{2}{\pi}\times 220\sqrt{2}[A]$

ex) $V_a = 220[V] = \frac{2}{\pi}V_m \Rightarrow V_m = \frac{\pi}{2}\times 220[V]$

$V_m = 110\pi[V]$

$V_{eff} = \frac{110\pi}{\sqrt{2}}[V]$



$$v = V_m \sin\omega t[V] \rightarrow i = \frac{v}{R} = \frac{V_m}{R}\sin\omega t = I_m \sin\omega t$$

$$\sqrt{2}I = \frac{\sqrt{2}}{R}V \quad \therefore I = \frac{V}{R}$$

$i(t) = I_m \sin\omega t$

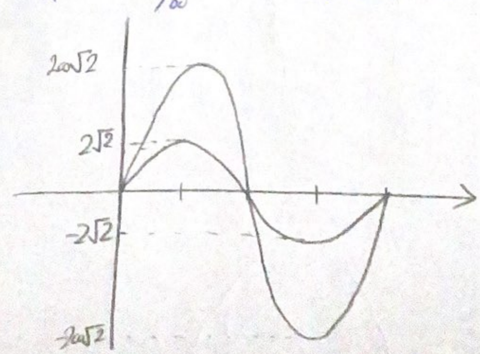
$v(t) = V_m \sin\omega t$

$V_R(t) = V_m \sin\omega t[V]$

ex) $R = 100[\Omega]$, 실효치 200[V], 60[Hz]의 정현파 교류전압을 가했을 때 회로에 흐르는 전류의 순시값 [A]를 구하시오

풀이
$$v(t) = 200\sqrt{2}\sin\omega t = 200\sqrt{2}\sin 377t$$

$$i(t) = \frac{v}{R} = \frac{200\sqrt{2}\sin 377t}{100} = 2\sqrt{2}\sin 377t$$



예제) 132.63[μF] 커패시턴스에 110[V], 60[Hz]의 정현파 전압을 가할 때 전류의 순시값은?

$v(t) = \sqrt{2}V\sin\omega t = 110\sqrt{2}\sin 377t\,[V]$

$X_C = \dfrac{1}{2\pi \times 60 \times 132.63\times10^{-6}} = 20[\Omega] \quad \therefore I = \dfrac{V}{X_C} = 5.5[V]$

$i(t) = 5.5\sqrt{2}\sin(377t + \frac{\pi}{2})[A]$

예제) C[F]의 커패시턴스에 $i = I_m\sin\omega t[A]$의 전류가 흐르는 경우 전압?

$i(t) = I_m\sin\omega t$

$i(t) = C\dfrac{dv}{dt}$에서 $v_C(t) = \dfrac{1}{C}\int i\,dt = \dfrac{I_m}{C}\int\sin\omega t\,dt$

$= -\dfrac{I_m}{\omega C}\cos\omega t = V_m\sin(\omega t - \frac{\pi}{2})[V]$

예제) 6.63[μF] 커패시턴스에 10[A], 2.45[kHz]의 전류가 흐르는 경우 전압의 순시값은?

$i = \sqrt{2}I\sin\omega t = 10\sqrt{2}\sin 15398.2t[A]$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{15398.2\times6.63\times10^{-6}} = 2[\Omega] \quad \therefore V = I X_C = 20[V]$

$v_C(t) = 20\sqrt{2}\sin(15398.2t - \frac{\pi}{2})[V]$

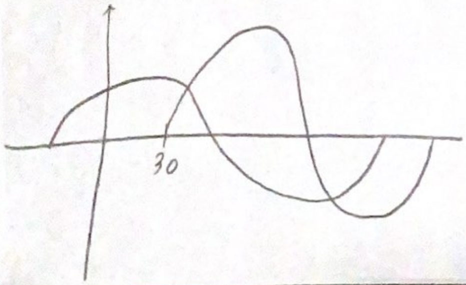
예제) C = 265.26[μF], $i = 10\sqrt{2}\sin(377t + 60^\circ)$

$v(t) =$

$X_C = \dfrac{1}{2\times\pi\times60\times265.26\times10^{-6}} = 10[\Omega]$

$v(t) = 100\sqrt{2}\sin(377t + 60 - 90)$

$= 100\sqrt{2}\sin(377t - 30^\circ)[V]$



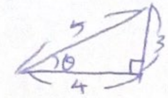
예제) $10\sqrt{2}\sin(\omega t + 30)$

$6\times10\sqrt{2}\sin(\omega t + 30)$

삼각함수식증명

$x\sin\alpha + y\cos\alpha = \sqrt{x^2+y^2}\sin(\alpha+\theta)$

예) $f(\omega t) = 4\sin\omega t + 3\cos\omega t$



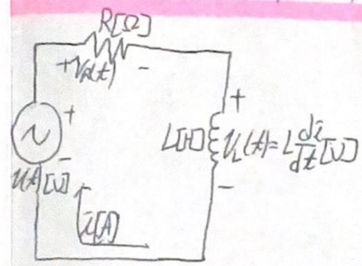
$f(\omega t) = 4\sin\omega t + 3\cos\omega t$

$= \sqrt{4^2+3^2}\left(\underbrace{\dfrac{4}{\sqrt{4^2+3^2}}}_{\cos\theta}\sin\omega t + \underbrace{\dfrac{3}{\sqrt{4^2+3^2}}}_{\sin\theta}\cos\omega t\right)$

$= 5(\sin\omega t\cos\theta + \cos\omega t\sin\theta)$

$= 5\sin(\omega t + \theta) = 5\sin(\omega t + 36.9)$

$\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{3}{4} = 36.9$



$v(t) = v_R(t) + v_L(t) \rightarrow KVL$

$v_R(t) = Ri = RI_m\sin\omega t$, $v_L(t) = L\dfrac{di}{dt} = \omega L I_m\cos\omega t$

$v(t) = v_R(t) + v_L(t) = I_m(R\sin\omega t + \omega L\cos\omega t)$

여기서 공식: $x\sin\alpha + y\cos\alpha = \sqrt{x^2+y^2}\sin(\alpha+\theta)$

$v(t) = I_m\sqrt{R^2+(\omega L)^2}\sin(\omega t + \theta)$, 단 $\theta = \tan^{-1}\dfrac{\omega L}{R}$

$R = 86.6[\Omega]$, $L = 13.26[mH]$, $i = 10\sqrt{2}\sin 377t[A]$
각소자 전압 및 전체전압은?

$v_R(t) = Ri(t) = 86.6\sqrt{2}\sin 377t[V]$

$v_L(t) = L\dfrac{di}{dt} = 13.26\times10^{-3}\times10\sqrt{2}\times377\cos 377t = 50\sqrt{2}\cos 377t[V]$

$v(t) = v_R(t) + v_L(t) = 86.6\sqrt{2}\sin 377t + 50\sqrt{2}\cos 377t$

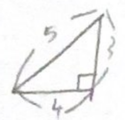
$= \sqrt{2}\sqrt{86.6^2+50^2}\left\{\sin 377t\dfrac{86.6}{\sqrt{86.6^2+50^2}} + \cos 377t\dfrac{50}{\sqrt{86.6^2+50^2}}\right\}$

$= 100\sqrt{2}\{\sin 377t\cos 30^\circ + \cos 377t\sin 30^\circ\}$

단 $\theta = \tan^{-1}\dfrac{\omega L}{R} = \tan^{-1}\dfrac{5}{8.66} = 30^\circ$ $\quad v(t) = 100\sqrt{2}\sin(377t + 30^\circ)[V]$

ex) $R=4[\Omega]$ $L=7.96[mH]$ $v=120\sqrt{2}\sin 377t\,[A]$
저항과 코일에 걸리는 전압 구하기

$X_L = 2\pi f L = 377\times 7.96\times 10^{-3} = 3[\Omega]$



$Z = 5[\Omega]$ $\theta = \tan^{-1}\frac{3}{4} = 36.9$

$i = 24\sqrt{2}\sin(377t-36.9)$

$V_R = R\times i = 96\sqrt{2}\sin(377t-36.9)$

$V_L = 72\sqrt{2}\sin(377t-36.9) = 72\sqrt{2}\sin(377t-36.9+90)$

$= 72\sqrt{2}\sin(377t+53.1)$

ex) $A\sin(\omega t\pm\theta)$ ☆이 문제들은 꼭 풀기☆

1) $10\sqrt{2}\sin\omega t + 10\sqrt{2}\cos\omega t = 20\sin(\omega t+45)$

$\therefore x\sin\alpha + y\cos\alpha = \sqrt{x^2+y^2}\sin(\alpha+\theta)$

2) $8\sin\omega t - 6\cos\omega t = 10\sin(\omega t-36.9)$

$\therefore 8\sin\omega t + (-6\cos\omega t)$

ex) $R=8\Omega$, $L=18.9\,mH$ $v=220\sqrt{2}\sin(377t+60)$ ☆

1) $Z=10\Omega$

2) $i=22\sqrt{2}\sin(377t+23.1)\,[A]$

3) $V_R=176\sqrt{2}\sin(377t+23.1)\,[V]$

4) $V_L=132\sqrt{2}\sin(377t+113.1)\,[V]$

페이저와 복소수 해석

정현파의 위상만큼 가로축에서 회전한 정지한 막대로 표시하는 것을 페이저 표시법이라함 ☆

$v(t) = V_m\sin(\omega t+\theta) = \sqrt{2}V\sin(\omega t+\theta)\,[V]$

$V_m = \sqrt{2}V$: 최대값

$V = \frac{V_m}{\sqrt{2}}$: 실효값

여기서 $V = V_{eff}$ / 예시: 순시치: $v(t) = 10\sqrt{2}(\omega t+45)$ 이면

페이저표기법: V(벡터값) $= 10\angle 45\,[V]$

$10\sqrt{2}\sin(\omega t+45)$

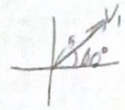
순시치 $v(t) = V_m\sin(\omega t+\theta) = \sqrt{2}V\sin(\omega t+\theta)$

페이저표기법: V(벡터값) $= V_{eff}\angle\theta = \frac{V_m}{\sqrt{2}}\angle\theta\,[V]$

ex) 정현파 전압 또는 전류를 페이저 벡터로 표현하여 그리시오.

① $v_1 = 5\sqrt{2}\sin(\omega t+60°)\,[V]$

$5\angle 60°$



$\frac{5\sqrt{2}}{\sqrt{2}}\angle(\omega\times 0+60)$

② $i_2 = 3\sqrt{2}\sin(\omega t+30°)\,[A] \rightarrow 3\angle 30°\,[A]$

③ $v_3 = -5\sqrt{2}\sin(\omega t+60°)\,[V] \rightarrow -5\angle 60°\,[V]$

④ $i_4 = 4\sin(\omega t-30°)\,[A] \rightarrow 2\sqrt{2}\angle -30°\,[A]$

⑤ $v_5 = 2.828\sin(\omega t+120°)\,[V] \rightarrow \frac{2.828}{\sqrt{2}}\angle 120 = 2\angle 120°$

⑥ $i_6 = 5\sqrt{2}\sin(\omega t+180°)\,[A] \rightarrow 5\angle 180 \rightarrow -5\angle 0° = -5\,[V]$

ex) 페이저를 사인함수로 변환

① $V_1 = 5\angle 45°\,[V] \rightarrow 5\sqrt{2}\sin(\omega t+45°)\,[V]$

② $I_2 = -3\angle 45°\,[A] \rightarrow 3\sqrt{2}(\omega t+225°)\,[A]$

③ $V_3 = 20\angle -30°\,[V] \rightarrow 20\sqrt{2}\sin(\omega t-30°)\,[V]$

④ $I_4 = 6\angle 240°\,[A] \rightarrow 3\sqrt{2}\sin(\omega t+240°)\,[A]$

☆꼭 풀기☆

복소수 → $x^2+1=0$ 또는 $x^2=-1$ ($x^2=-1$ 또는 i(허수)$=\sqrt{-1}$)

$x^2=-1=i^2$, $x^2-i^2=0$, $(x-i)(x+i)=0$ $\therefore x=i, -i$

x^2-4의 해

$\Rightarrow x^2=-4=-1\times 4=i^2\times 4=(2i)^2$

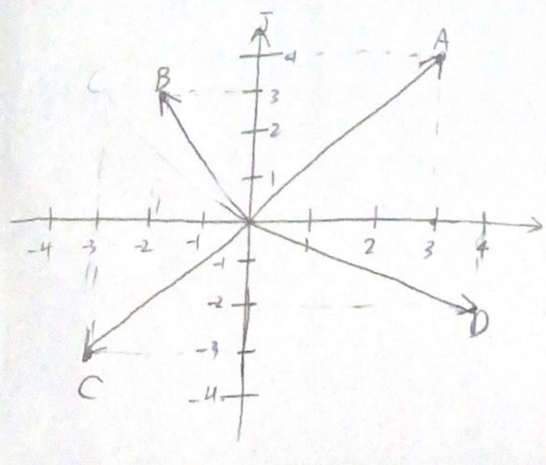
$x^2-(2i)^2=0$, $(x-2i)(x+2i)=0$

$\therefore x=2i, -2i$

A(복소수) $= a+jb$

실수 허수

ex) $A=3+j4$ $B=-2+j3$ $C=-3-j3$ $D=4-j2$

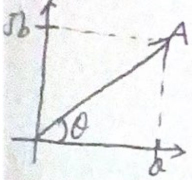


공액복소수 $\rightarrow A = m + jn$
(켤레복소수) $\bar{A} = m - jn$

$X = a + jb,\ Y = c + jd \quad \therefore \frac{X}{Y} \neq \frac{Y}{X}$

$$\frac{X}{Y} = \frac{a+jb}{c+jd} = \frac{(a+jb)(c-jd)}{(c+jd)(c-jd)} = \frac{ac+bd}{c^2+d^2} + j\,\frac{bc-ad}{c^2+d^2}$$

복소수의 극좌표형식



$|A| = A = \sqrt{a^2+b^2}$

$\tan\theta = \frac{b}{a},\ \therefore \theta = \tan^{-1}\frac{b}{a}$

$A = \sqrt{a^2+b^2}\angle\tan^{-1}\frac{b}{a} = A\angle\theta$ (극좌표형식)

ex) ① $A = 4+j3 \longrightarrow \sqrt{4^2+3^2}\angle\tan^{-1}\frac{3}{4}$ 6주 (10분)

② $F = -2+j2 \longrightarrow \sqrt{(-2)^2+2^2}\angle\tan^{-1}\frac{2}{(-2)}$

③ $Q = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \longrightarrow \sqrt{(\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\angle\tan^{-1}\frac{(-1/2)}{(\sqrt{3}/2)}$

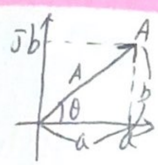
④ $N = \sqrt{3} - j \longrightarrow$

복소평면도 그리기

ex) $X = 4-j3,\ Y = -2+j3$ 일 때 ☆ 시험나올수도

① $X - 2Y$ ② $4X - 3Y$ ③ X^3

복소수의 삼각함수 형식과 지수함수 형식



$A = a + jb = A\angle\theta$

$\sin\theta = \frac{b}{A},\ b = A\sin\theta$

$\cos\theta = \frac{a}{A} = a = A\cos\theta$

$A = A\angle\theta = a + jb = A\cos\theta + jA\sin\theta = A(\cos\theta + j\sin\theta)$

ex) $12\angle 34.9° + 10\angle 53.1° - 13\angle 202.62°$ 극좌표 형식으로

$= 21 + j17 = \sqrt{21^2+17^2}\angle\tan^{-1}\frac{7}{26} = 26.93\angle 15.1°$

$\cos\theta + j\sin\theta$ 형태로 바꿔서 계산하고 ??

✓ 소수점 3번째 까지 구해서 반올림해서
답은 2번째까지 적기

$e^{j\theta} = \cos\theta + j\sin\theta$

$A = A\angle\theta = A(\cos\theta + j\sin\theta) = Ae^{j\theta}$

ex) 직각좌표 형식으로 $\rightarrow a + jb$

① $4e^{j\frac{\pi}{3}} \longrightarrow 2 - j3.46$

② $(2\angle 30°)^3 \longrightarrow j8$

③ $(3\angle 60°)^4 \longrightarrow$ j70.16

④ $(2\angle 120°)^5 \longrightarrow 16 + j27.71$

순시치 : $v(t) = V_m\sin(\omega t + \theta) = \sqrt{2}V\sin(\omega t + \theta)$

극좌표형식 (페이저 표기법) : $V = V_{eff}\angle\theta = \frac{V_m}{\sqrt{2}}\angle\theta$

직각좌표 형식 : $V = a + jb$

지수함수 형식 : $V = Ve^{j\theta}$

극좌표 → 직각좌표

$V = V\angle\theta = V(\cos\theta + j\sin\theta) = V\cos\theta + jA\sin\theta = a + jb$

직각좌표 → 극좌표

$V = a + jb = \sqrt{a^2+b^2}\angle\tan^{-1}\frac{b}{a} = V\angle\theta$ (극좌표)

$V = a + jb = V\angle\theta = Ve^{j\theta}$

ex) 복소수를 직각좌표 형식으로 변환

① $A = 5e^{j\frac{\pi}{3}} \longrightarrow 2.5 + j4.33$

② $B = -5e^{j\frac{\pi}{3}} \longrightarrow -2.5 + j4.33$

③ $C = 5e^{j\frac{3\pi}{4}} \longrightarrow -3.54 + j3.54$

④ $D = -5e^{j\frac{2\pi}{3}} \longrightarrow 2.5 - j4.33$

ex) 직각좌표형식으로

1) $A = (2\angle 30°)^3 = (2e^{j30})^3 = 8e^{j90} = 8(\cos 90 + j\sin 90) = j8$

2) $B = (4+j3) \times 5e^{j30} = (4+j3)\times(4.35+j2.5) = (17.4+j10+j13.05-7.5$

$= 9.9 + j23.05$

$5(\cos 30 + j\sin 30)$

$0.87 \quad j0.5$