Personal Savings Analysis

Andrew Brown, Melissa Hooke, Frances Hung, Mai Nguyen, Brenner Ryan

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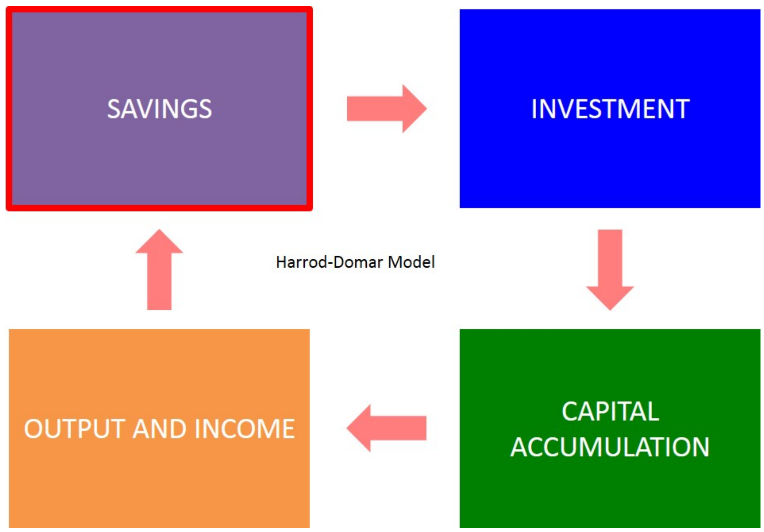
Abstract: *This paper explores personal savings data in the United States from 1955 - 1980; a span of 26 years. First, we examined the trend of our data by using various time series analysis tools such as model specification and model fitting. Through these in-depth analyses, we found three candidate models:* $AR(1)$, $AR(1) \times AR(1)_4$, *and* $ARIMA(0,1,0) \times ARIMA(1,0,1)_6$. *Further examination and diagnostic analysis of the three models, we concluded that an* $AR(1) \times AR(1)_4$ *was the most appropriate model for the data set. With this finding, we were able to create a forecast of the U.S. personal savings rate. These analyses allowed us to form a greater understanding of personal savings data, gain insights into the U.S.'s economic performance and perform predictions on future savings rates.*

Introduction

The personal savings rate of consumers is one of many indicators of how a country's economy is performing. Moreover, savings drive long-run economic growth as they provides funds for investment in capital or projects, which then drives future economic growth. Typically, household savings are invested either directly (i.e. when purchasing equity) or indirectly, (i.e. putting it into a bank, which uses those funds for lending) (Carroll and Mowry). These investments lead to economic growth in free-market economies. Additionally, the Harod-Domar Model of economic growth suggests that economic growth rates is driven by the level of savings. The model reasons that increased savings will lead to increased investment which result in higher capital stock, and thus, higher economic growth.



To understand the factors that influence personal savings rate, it is helpful to know how the savings rate is calculated. Personal savings can be understood as one minus the ratio of personal outlays (spending) to disposable income (personal income minus personal taxes). This calculation is expressed in the formula below.

$$\text{Personal Savings Rate (\%)} = 100 \times (1 - \frac{\text{Personal Outlays}}{\text{Personal Income - Personal Taxes}})$$

Moreover, this shows that an increase in the personal savings rate can be associated with one of the following factors: increase in personal income, decrease in personal outlays, or decrease in personal taxes.(Carroll and Mowry)

The personal savings rate typically decreases when individuals spend more than they initially save, be it due to inflation, consumer habits, or reliance on other financial assets. Because the personal savings rate can only go so low, a low rate can be a sign of a looming recession. In contrast, after financial crises, consumers are often more cautious and tend to raise the personal savings rate. For example, in 2008, personal savings rate rose from 1.4 percent to 2.6 percent and in 2009, it reached 4.3 percent, highest since 1998. (Carroll and Mowry) This increase can be explained by the 2008 recession, after which consumers became more cautious with their spending.

Given that saving rates is an indicator of a country's economic performance, it is important to explore personal saving rates data in order to gain insights on the performance of economy. In this project, we are most interested in exploring the U.S. personal savings rate. By analyzing past data, we are able to form a mathematical model that will allow us to predict, monitor, and control the data. This understanding will enable us to take observe the projection of the personal savings rate and take precautionary measures to ensure economic growth.

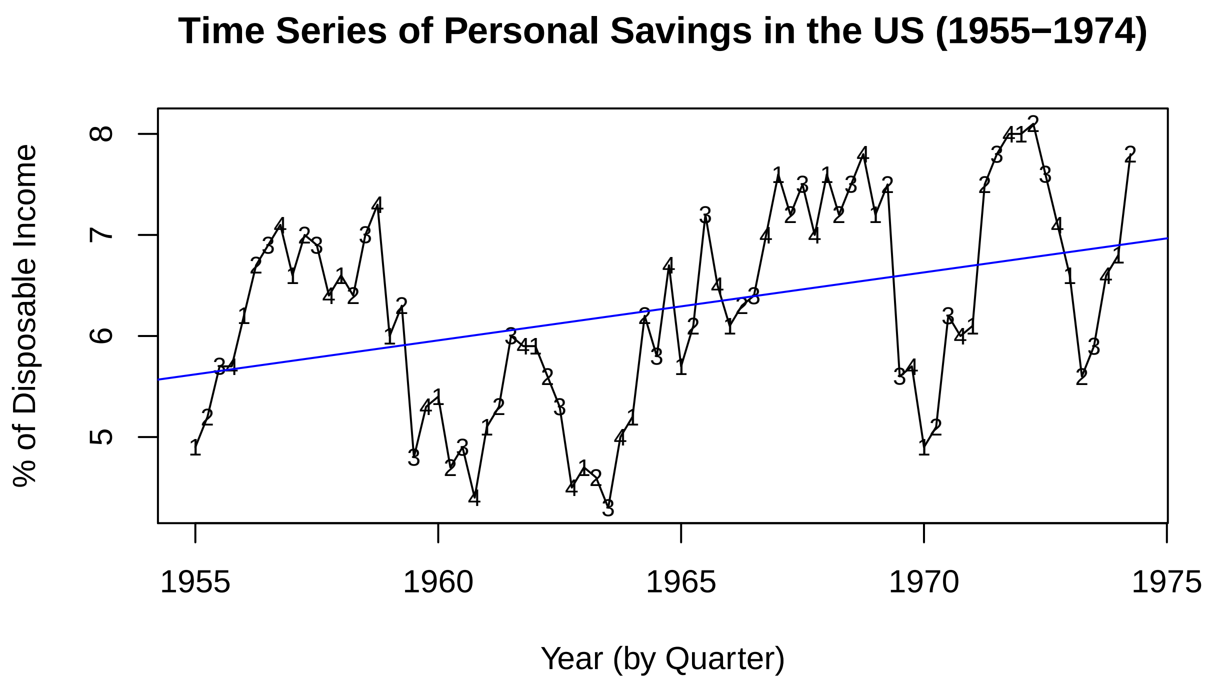
The time series spans over 26 years with each year divided into financial quarters, amounting to 104 total observations. From our original time series plot, it is difficult to see any obvious seasonal trends. However, there seems to be a positive correlation between terms close to one another. The general increasing trend and volatility in the 70s may be linked to the economic crash in the early 70s and oil energy crisis in 1979. The 1970's were marked by high inflation and growing expenses due to rising interest rates.

Time Series Exploration

The original time series, as pulled from DataMarket (https://datamarket.com/), spans 26 years of personal savings as percent of disposible income in the United States. Each year of the time series is divided into financial quarters, amounting to 104 total observations, which we plot in the time series below:

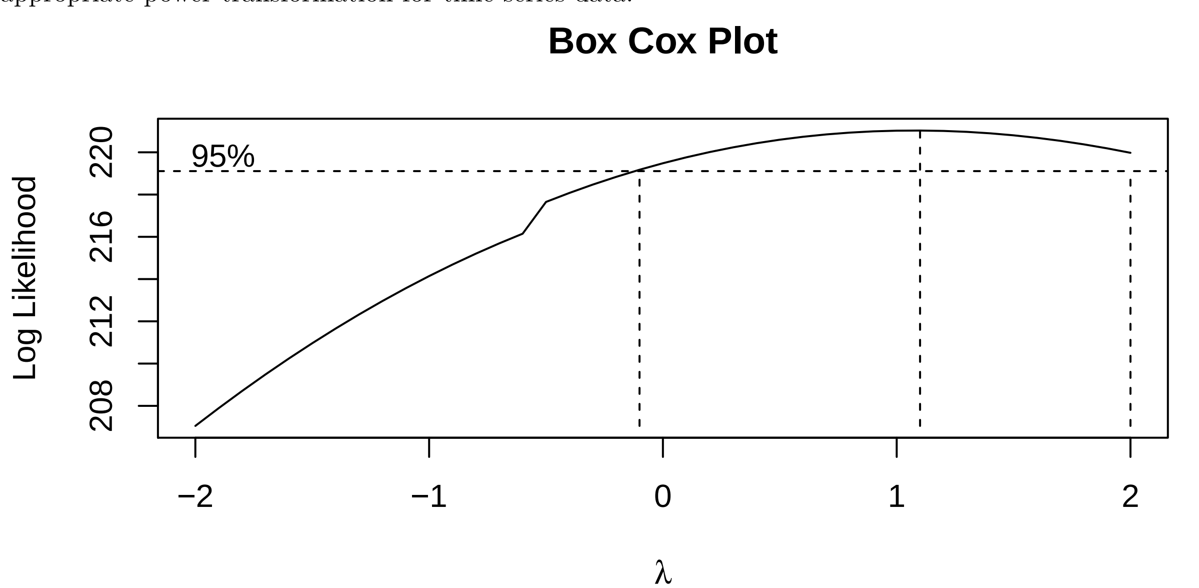
From our original time series plot, we see that there is a general upward trend with some sudden volatility in the 1970s that may be linked to the economic crash in the early 70s and oil energy crisis in 1979. Since the 1970s were marked by high inflation and growing expenses due to rising interest rates, we made the decision to remove the last 5 years of the time sereies since they would likely follow a different time series trend than the rest of the data.

In addition, we set aside the last 6 observations in order to use them as test points to compare with our forecasts at the end of our analysis. The resulting series of 78 observations is plotted below:

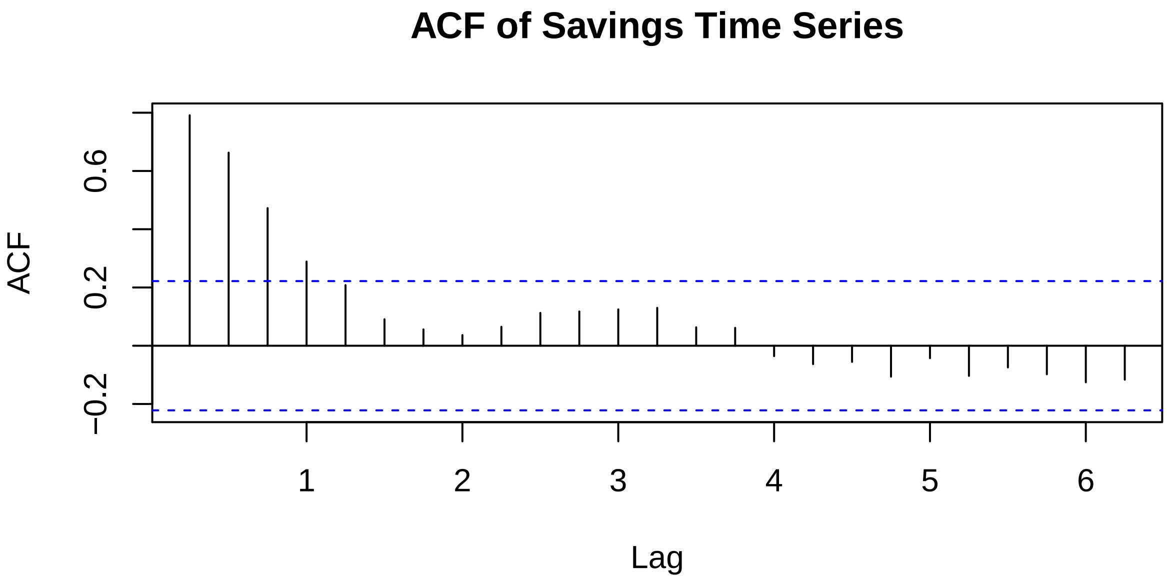


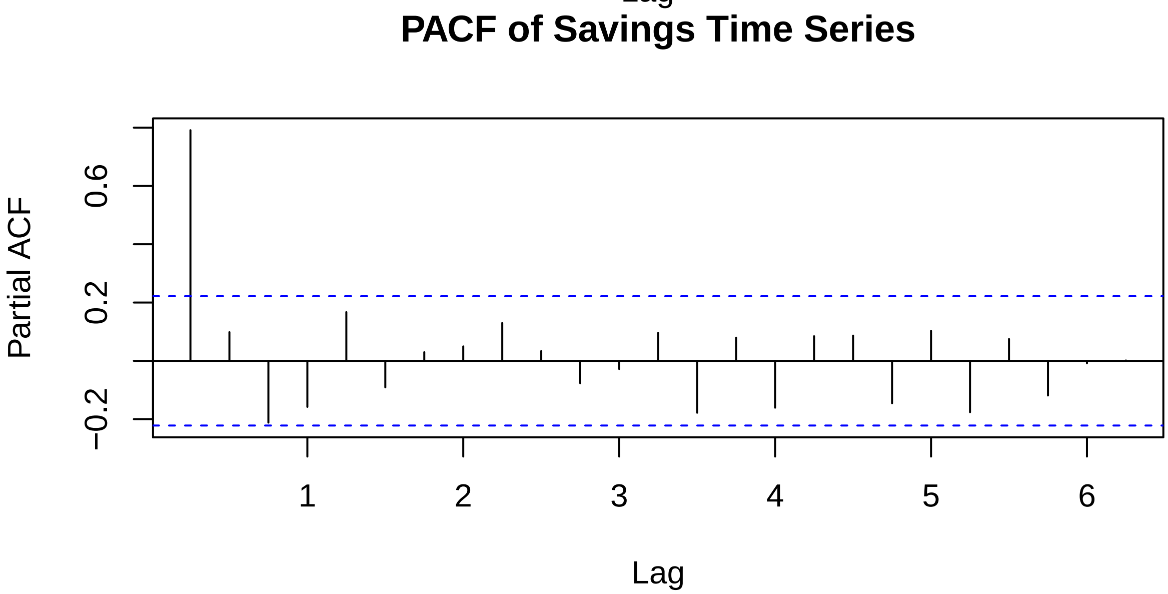
In the time series, we see a general upward trend in the data, which indicates that the time series may not be stationary and we may want to consider taking the first difference of the data. Also, while do not see any *obvious* seasonal trends, given that the data is divided into financial quarters we may want to consider the possibility of taking a seasonal difference to make our time series stationary. First, however, let's explore without taking the difference.

The first step in analyzing our time series is to consider the possible need for a transformation to stabilize the variance of the series over time. In order to do this, we use the function BoxCox.ar to determine the appropriate power transformation for time-series data.



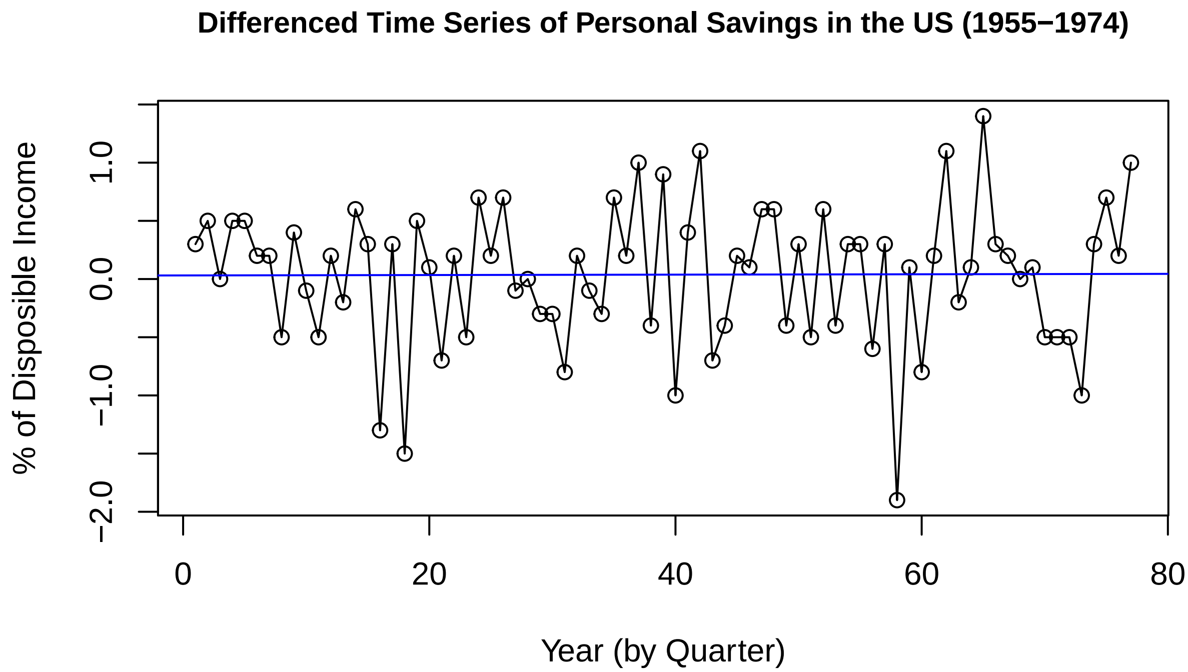
The Boxcox output indicates that a transformation is not necessary in order to stabilize the variance since λ is about equal to 1. Therefore, we proceed by examining the acf and pacf of the series.



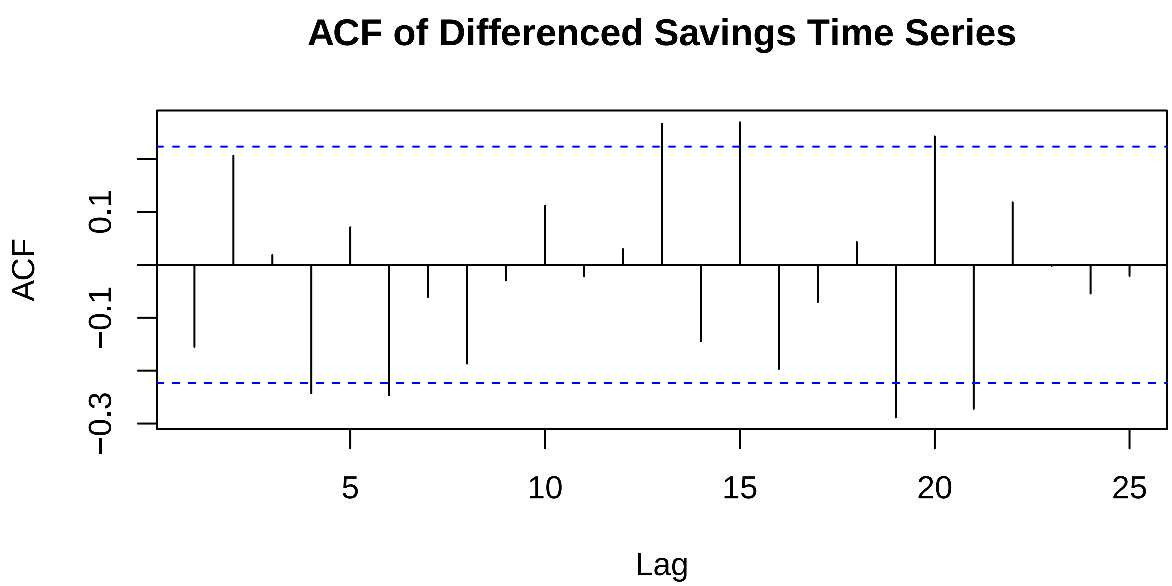


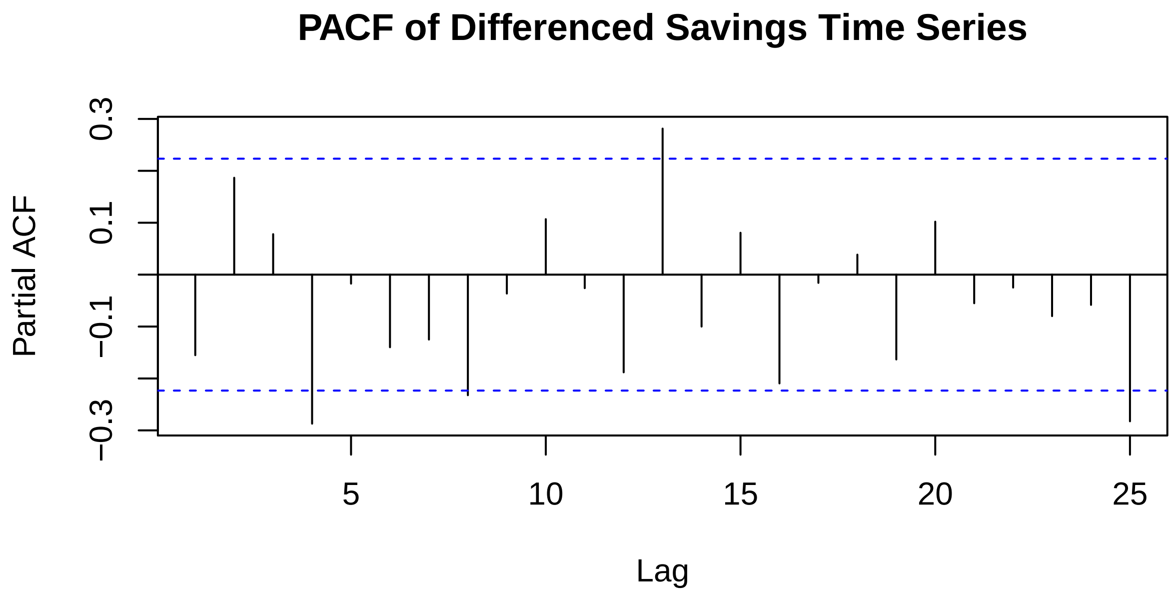
The ACF starts out large and then slowly decreases until it ends up within the white noise bounds. This kind of behavior is usually seen in ARMA and AR models. Looking at the PACF could give us more information about the nature of our data. The PACF seems to indicate that an AR(1) process may be a good candidate model because the only non-zero sample partial autocorrelation is at lag $k = 1$. For lags $k \geq 1$, the partial autocorrelations appear to reduce to white-noise.

Next, we consider the differenced time series, which is plotted below:



The plot of the first difference of our time series indicates that the upward trend in the data has been removed and the mean of the differenced series is about equal to zero.





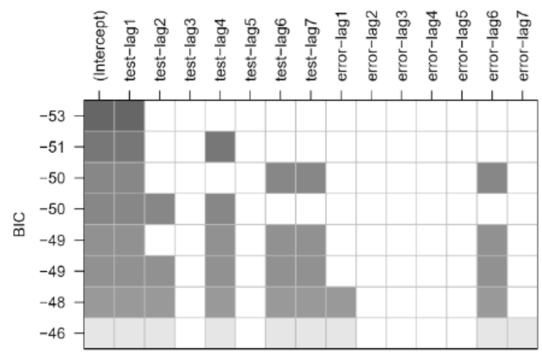
The ACF and PACF of the differenced series appear to suggest a seasonal trend in the differenced series, however, the period of this trend is unclear.

Model Specification

Based on the preliminary explorations of our original time series and the differenced series, the candidate models we have in mind are an AR(1) process or some type of seasonal model based on either the original or differenced series. We suspect that a seasonal model for the original series would have a seasonality of 4, but we are not sure if this seasonality will be retained in the differenced series.

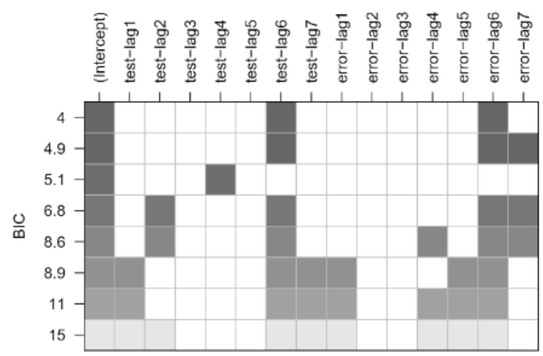
In order to validate these candidate models and determine the period of possible seasonal trends, we turn to the EACF and the best subsets methods.

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x o o o o o o o  o  o  o
## 1 o x o o o o o o o o o  o  o  o
## 2 x x o o o o o o o o o  o  o  o
## 3 x x o o o o o o o o o  o  o  o
## 4 x o o o o o o o o o o  o  o  o
## 5 x x o o o o o x o o o  o  o  o
## 6 x o o o o o o o o o o  o  o  o
## 7 x o o o o o o o o o o  o  o  o
```



While the EACF for the original (non-differenced) time series is inconclusive, this is not a concern. EACF is often inconclusive and not the best way of determining a useful model. We now look at the best subsets method, which indicates that an AR(1) process or a multiplicative $AR(1) \times AR(1)_4$ with a seasonal period of 4 are good candidate models.

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o o o x o x o o o o o  o  x  o
## 1 x o o x o o o o o o o  o  o  o
## 2 x x o x o o o o o o o  o  o  o
## 3 x x o o o o o o o o o  o  o  o
## 4 o x o o o o o x o o o  o  o  o
## 5 o x o o o o o o o o o  o  o  o
## 6 x x o o o o o o o o o  o  o  o
## 7 x o x o o o o o o o o  o  o  o
```



The EACF for the differenced series also seemed to be inconclusive, and the best subsets method indicated a trend with a seasonal period of 6. This is curious, as seasonality should theoretically be retained across a first difference. There are a couple factors we think could be contributing to this change in seasonal period. First is that a seasonal model with a period of 4 does not perfectly capture our original data. If our original data is far off enough from a true seasonal 4 model, then the first difference could have an entirely different seasonality. The second factor is that our model includes an autoregressive component, which could also affect the first differencing process.

Model Fitting

Given the results in the previous section, we have decided to fit and compare 3 different models: an AR(1), a multiplicative $AR(1) \times AR(1)_4$, and an $ARIMA(0,1,0) \times ARIMA(1,0,1)_6$ model. The parameters for each model are given in the table below:

Model	Intercept	se	ar1	se	sar1	se	sma1	se	sigma^2	log likelihood	aic
AR(1)	6.28	0.35	.83	0.07	x	x	x	x	0.336	-68.71	141.43
Seasonal 4	6.27	0.35	0.86	0.06	-0.28	0.12	x	x	0.3198	-66.87	139.75
Seasonal 6	x	x	x	x	-0.18	0.38	-0.08	0.38	0.3402	-67.96	139.92

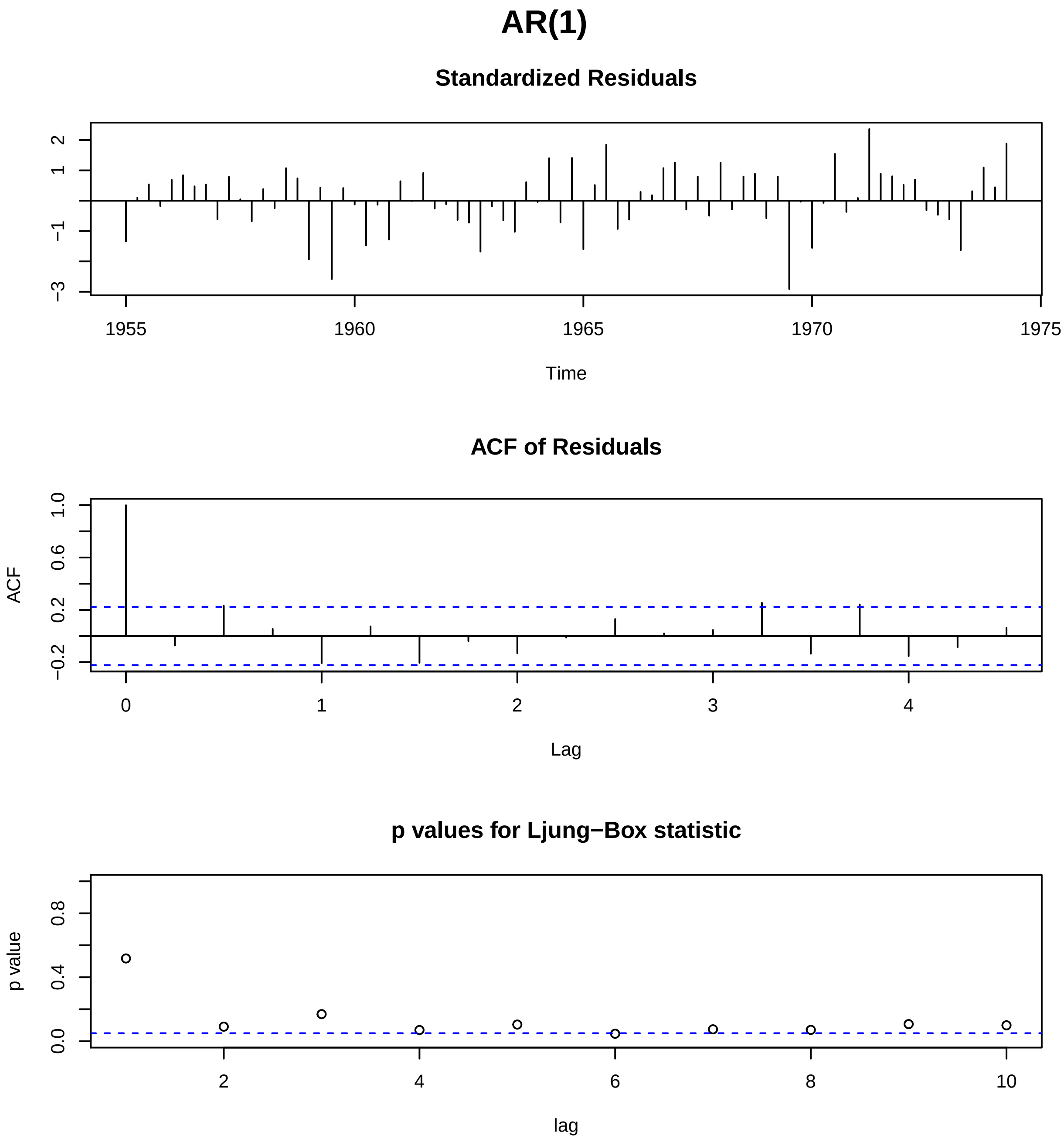
Thus the equations of our 3 models are:

1. AR(1): $Y_t - 6.28 = .825(Y_{t-1} - 6.28) + e_t$
2. $AR(1) \times AR(1)_4$: $(Y_t - 6.27)(1 - .861(B - 6.27))(1 + .228(B - 6.27)^4) = e_t$
3. $ARIMA(0,1,0) \times ARIMA(1,0,1)_6$: Y_t

Diagnostics

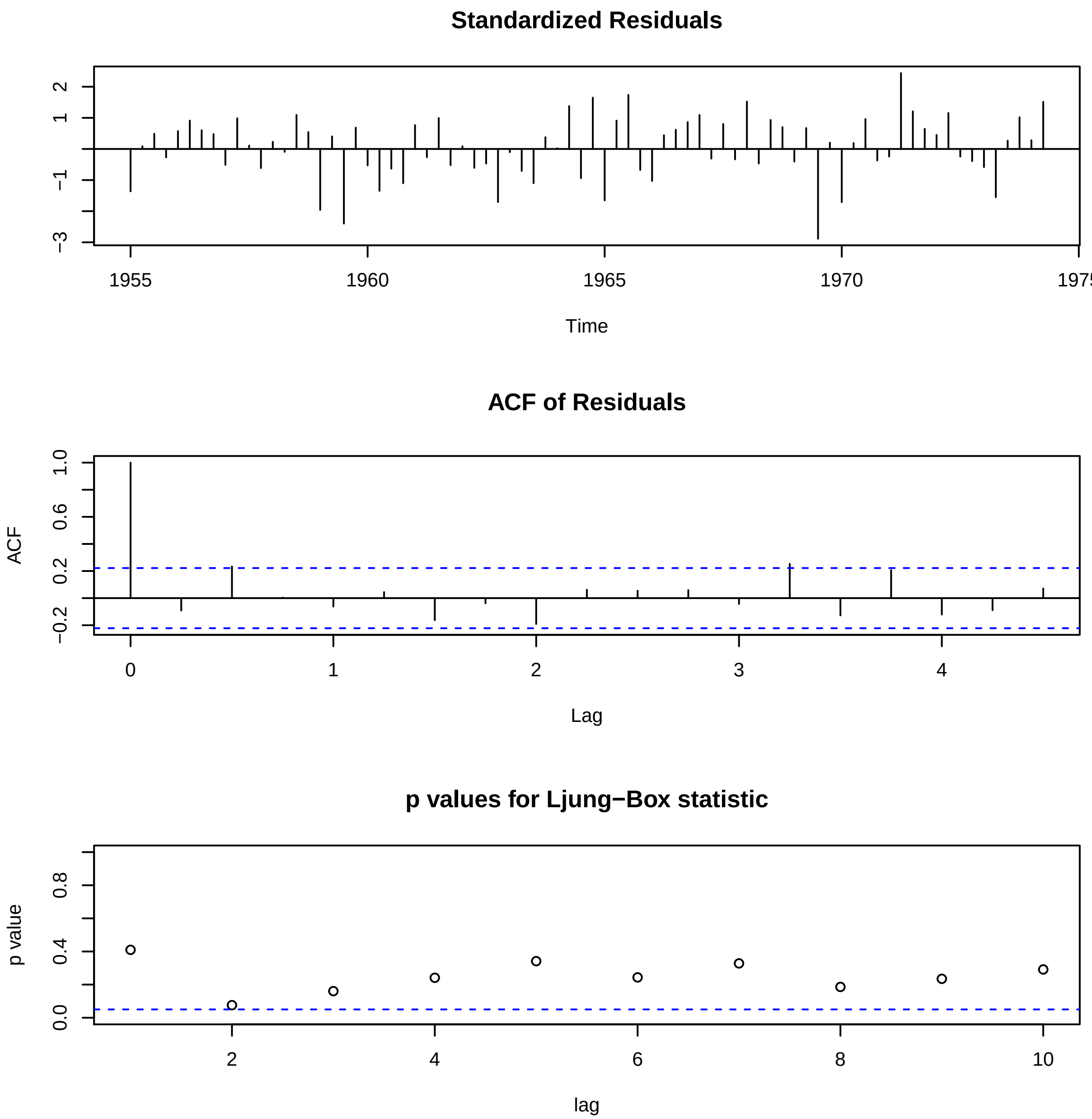
Given the output from fitting the three models, we see that different error criterion point us to different model selections. While, the AR(1) model has the lowest BIC, but the seasonal model with a period of 4 has a lower standard error and AIC. Meanwhile the differenced seasonal model with a period of 6 has a similar AIC to the seasonal 4 model, but has the highest standard error. Thus, we turn to residual analysis to see if any of our models show abnormalities.

In our residual analysis we are looking for 3 things: residual nonnormality, residual dependence, and residual structure. The presence of any of these 3 things may indicate that our model has not sufficiently identified the structure of the data and is not an adequate model.

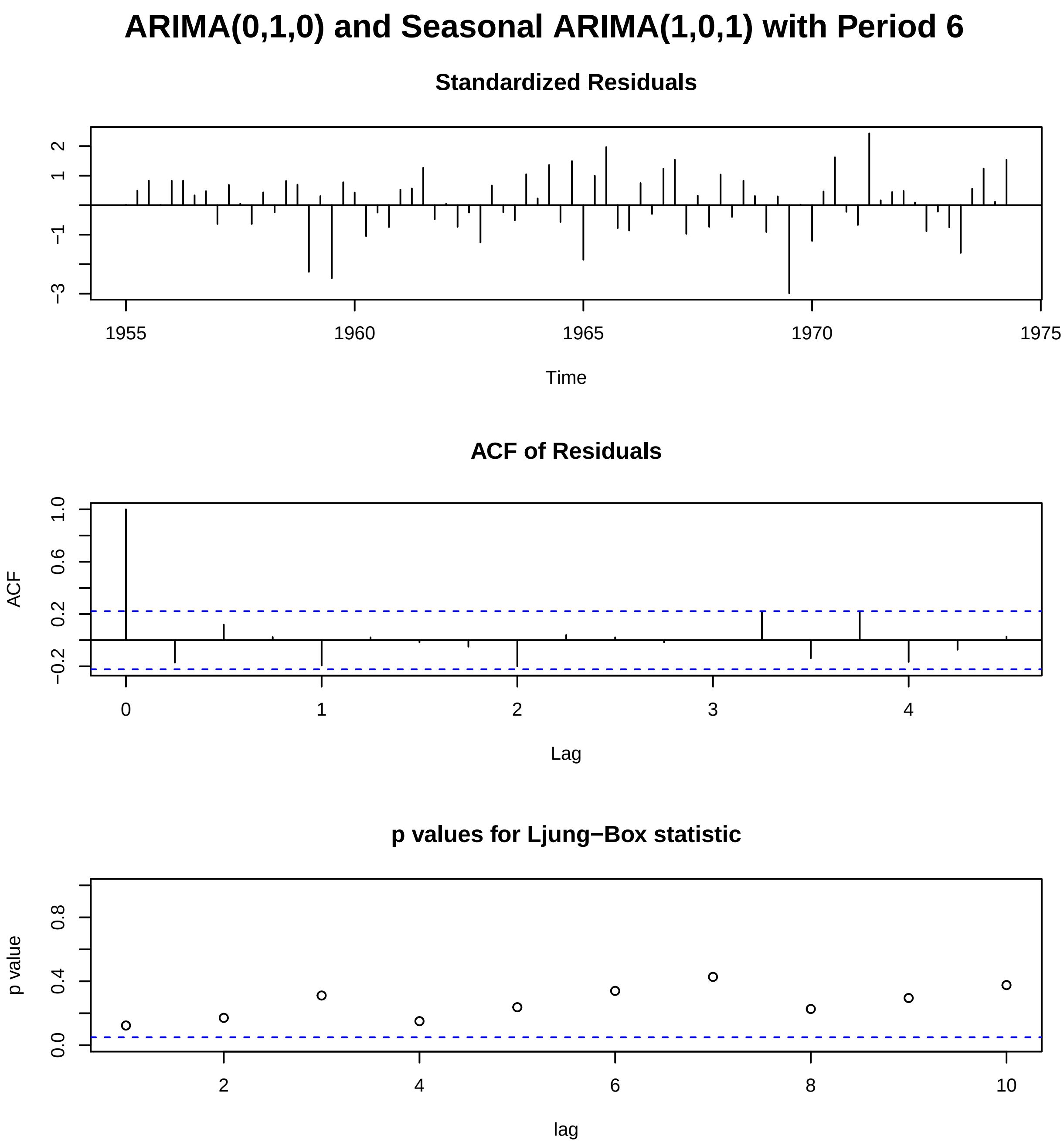


The residuals for the AR(1) model resemble a white-noise process, the ACF shows no correlation patterns between residuals, and the Ljung-Box statistic is borderline significant for higher lags. This indicates possible dependence among residuals.

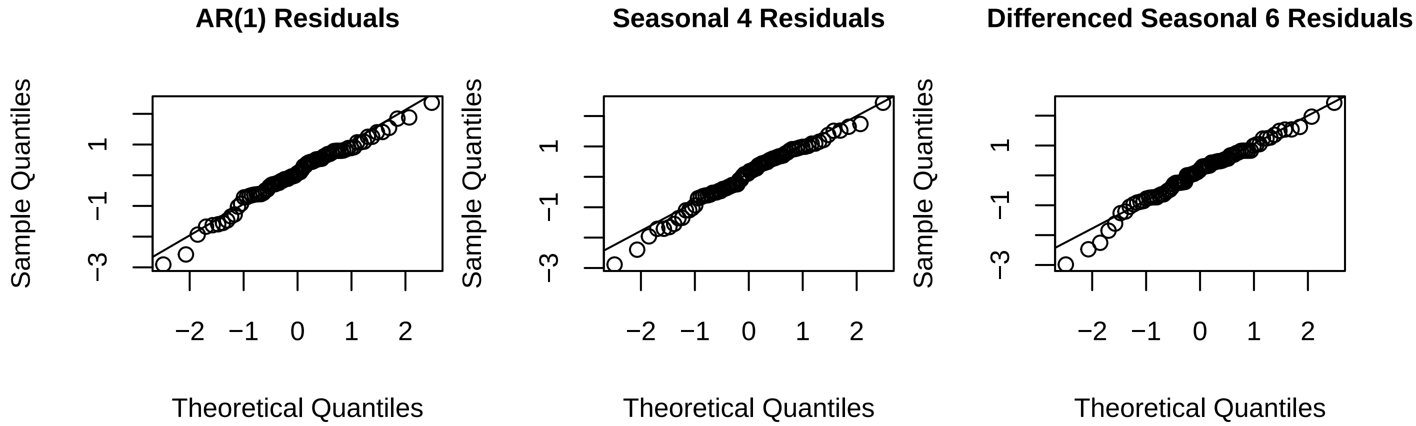
AR(1) and Seasonal AR(1) with Period 4



The residuals for the $AR(1)xAR(1)_4$ model resemble a white-noise process, the ACF shows no correlation patterns between residuals, and the Ljung-Box statistic is not significant for all tested lags.



The residuals for the $ARIMA(0,1,0) \times ARIMA(1,0,1)_6$ model resemble a white-noise process, the ACF shows no correlation patterns between residuals, and the Ljung-Box statistic is not significant for all tested lags.



Shapiro-Wilks Test

Model	AR(1)	Seasonal 4	Seasonal 6
W	0.981	0.983	0.979
p-value	0.295	0.369	0.237

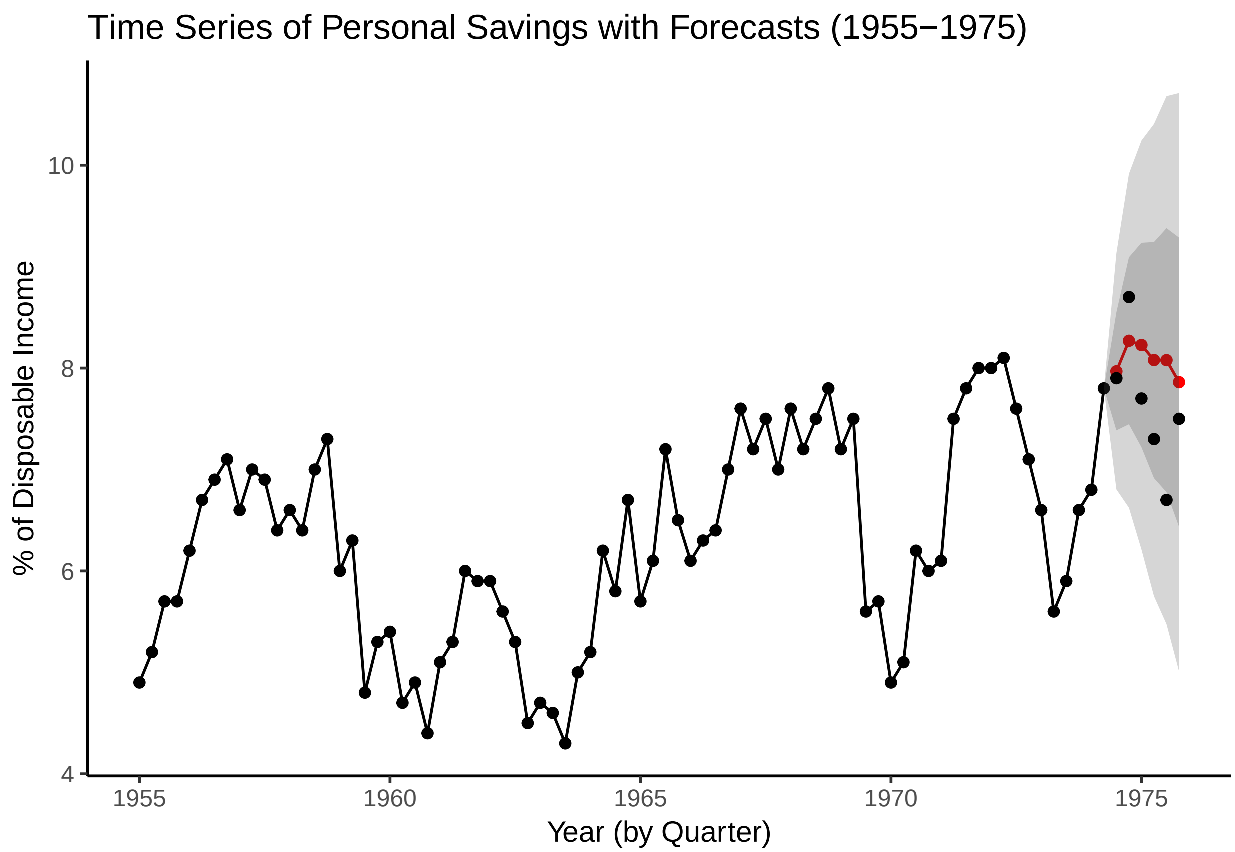
Based on their Q-Q plots and their Shapiro-Wilk tests, none of the models show evidence of residual nonnormality.

Runs Test

Model	AR(1)	Seasonal 4	Seasonal 6
p-value	0.73	0.37	0.18
Observed runs	42	44	45
Expected runs	30	40	39
n1	38	35	32
n2	40	43	46
k	0	0	0

Our runs tests support the Ljung-Box tests, which indicated that there is no sufficient evidence to reject residual independence.

Forecasting



Discussion

The forecasting for our model allows us to predict how personal savings rates will change in the near future.

Appendix: R Code

```
# load the time series
savings = read.csv("savings.csv",header=TRUE, nrows=104)
savings = savings %>% select(2)
savings.entire = savings[1:84,]
savings<-ts(savings, start=c(1955),frequency=4)

# plot the original time series
plot(savings, xlab="Year (by Quarter)",
     ylab= "% of Disposible Income",
     main= "Time Series of Personal Savings in the US (1955-1980)")
points(y=savings,x=as.vector(time(savings)),pch=as.vector(season(savings)), cex=.75)
```

```
# set aside points to validate our forecasts
savings.test = savings[79:84,]
```

```r
# remove the last 5 years because of the huge dip due to recession
savings = savings[1:78,]
savings<-ts(savings, start=c(1955),frequency=4)

# plot the shortened time series with labels for quarters
plot(savings, xlab="Year (by Quarter)",
     ylab= "% of Disposible Income",
     main= "Time Series of Personal Savings in the US (1955-1980)")
points(y=savings,x=as.vector(time(savings)),pch=as.vector(season(savings)), cex=.75)
abline(lm(savings~time(savings)), col='blue')
```

```r
# should we do a transformation?
boxcox = BoxCox.ar(savings)
boxcox
```

```r
# plot the acf and pacf of the original series
acf(savings, lag.max = 25)
pacf(savings, lag.max = 25)
```

```r
# calculate the differenced time series
diffs = (savings-zlag(savings))[2:78]

# plot the differenced time series
plot(diffs, xlab="Year (by Quarter)",
     ylab= "% of Disposible Income",
     main= "Differenced Time Series of Personal Savings in the US (1955-1980)", type="o")
abline(lm(diffs~time(diffs)), col="blue")
```

```r
# plot the acf and pacf of the differenced time series
acf(diffs, lag.max = 25)
pacf(diffs, lag.max = 25)
```

```r
# use the eacf and best subsets to find a candidate model
eacf(savings)
sub = armasubsets(y=savings,nar=7,nma=7, y.name='test', ar.method='ols')
plot(sub)
```

```r
# plot the eacf and best subsets for the differenced series
eacf(diffs)
sub = armasubsets(y=diffs,nar=7,nma=7, y.name='test', ar.method='ols')
plot(sub)
```

```r
# fit an AR(1) process
AR1model = arima(savings, order = c(1, 0, 0), seasonal = list(order = c(0, 0, 0)), method=c('ML'))
AR1model
```

```r
# run some general diagnostics on the models
tsdiag(AR1model)
tsdiag(SAR4model)
tsdiag(SAR6model)
```

```r
# test the residuals for normality
shapiro.test(ARresids)
shapiro.test(SAR4resids)
shapiro.test(SAR6resids)
```

```r
# test the residuals for independence
runs(ARresids)
runs(SAR4resids)
runs(SAR6resids)
```